UNIVERSITÄT BERN

Faculty of Business, Economics and Social Sciences

Department of Economics

Interest Rate Smoothing in the Face of Energy Shocks

Stefano Maria Corbellini

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Schanzeneckstrasse 1
CH-3012 Bern, Switzerland
http://www.vwi.unibe.ch

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Abstract

This paper analyzes the monetary policy trade-off between defending purchasing power of consumers and keeping moderate debt cost for borrowers, in the framework of a heterogeneous agent New Keynesian open economy hit by a foreign energy price shock. Raising the interest rate indeed combats the loss in purchasing power due to the energy shock through a real exchange rate appreciation: however, this comes at the expense of higher interest payments for debtors. The trade-off can be resolved by adopting a milder interest rate policy during the crisis in exchange for a prolonged contraction beyond the energy shock time span. This interest rate smoothing approach allows to still experience a real appreciation today, while spreading the impact on debt costs more evenly over time. This policy counterfactual is analyzed in a quantitative model of the UK economy under the 2022-2023 energy price hike, where the loss of consumers' purchasing power and the vulnerability of mortgage costs to higher policy rates have been elements of paramount empirical relevance.

*University of Bern

1 Introduction

The years 2022 and 2023 witnessed a substantial rise in energy prices, exacerbating the inflationary pressures that had been steadily building since 2021; in response to the inflation surge, central banks in advanced economies have raised the policy rates, aiming to curb inflationary pressures and to safeguard the real income of consumers. Generally these interventions increased interest rates on variable-rate mortgages and fixed-rate mortgages due for renewal during the period of rate hikes. The case of the UK economy is particularly illustrative of this phenomenon: housing mortgages' cost are typically renegotiated every 5 years or less, making their interest rate particularly sensible to the movements in the policy rate set by the Bank of England (BoE).

The Central Banks' trade-off between shielding real income of consumers and maintaining moderate mortgage interest rates poses challenges for the formulation of a monetary policy reaction to an energy price shock. A contractionary interest rate policy effectively safeguards households' wages purchasing power by fostering a real exchange rate appreciation (by uncovered interest rate parity); on the other side, it increases the cost of mortgages.

The main theoretical result of the paper is that the trade-off between the protection of households' real income and preventing high interest rates for borrowers can be resolved once we account for monetary policy manipulating *the whole path of future interest rates*. If the central bank indeed commits to monetary tightening in the future, this implies a current real appreciation of domestic goods - through uncovered interest rate parity holding across the whole yield curve - that protects real wages' purchasing power ; therefore there is room to adopt a milder monetary policy at the onset of the shock, in order not to increase too much the financial burden on borrowers. The result of the paper echoes Silvana Tenreyro's argument in her final speech as Monetary Policy Committee member at the Bank of England, which stated that the monetary authority should commit in advance to a determined path of future interest rates, in order to partially offset the need to raise current rates in reaction to the surge in energy prices.

This paper analyses this trade-off in a small open economy new keynesian setting where agents are heterogeneous because of uninsurable idiosyncratic income risk. Agents trade in a liquid assets and are endowed with perpetual liabilities (mortgages) whose interest rate is in part fixed and in part variable, i.e. directly connected with the monetary policy rate. The presence of mortgages creates a quantitatively relevant adverse effect of contractionary monetary policy on the budget constraints of households. Agents' heterogeneity is a key assumption to make both the components of the trade-off (increases in temporary mortgage costs and falls in the real wage) quantitatively relevant from a welfare perspective:

households indeed are unable to fully absorb income and mortgage cost shocks due to a precautionary saving motive, which especially holds true for the ones closer to the borrowing limit. Moreover, a full heterogeneous agents environment allows to have both a real wage fall and mortgage cost increases to be quantitatively relevant in affecting consumption over the whole crossection of agents (differently from a two-agents models, where these effects would only be numerically important for the borrowing constrained agents).

Once obtained the theoretical results in terms of benefit of interest rate smoothing, I proceed to a quantitative assessment of the implications of the model in the UK economy. The model is fed with the actual current and expected interest rate hike implemented by the BoE, as well as by the actual energy price data. The model is constructed and calibrated to match data both in an "aggregate" dimension (CPI inflation, real exchange rate, real wage, aggregate mortgage cost) and to align with the incidence of mortgages on the cross-sectional households' consumption patterns. The reference panel data for this analysis, "Understanding Society", reports nearly exclusively food expenditure among various expenditure items: therefore, I focus on comparing the model's outcomes to the data in terms of the effects of mortgage cost increases on food consumption.

The quantitative results of the paper point out that a *smoothed* interest rate policy - characterized by the interest rate peaking at 1 percentage point less than in BoE implemented policy, and requiring an additional three years to land on the new long-term level - is able to attain the same real exchange appreciation over the energy crisis, while reducing the food consumption difference between mortgagors and non-mortgagors by 4% over 2022, thanks to the reduced interest rate surge.

Contribution to the literature The model builds on the framework by Auclert, Rognlie, Souchier, and Straub (2023b), which study fiscal and monetary response to energy shocks in a HANK-type small open economy. Other recent literature studying the behavior of heterogeneous agents open economy in face of foreign shocks are Auclert, Rognlie, Souchier, and Straub (2023a) and Fukui, Nakamura, and Steinsson (2023) - for the case of depreciation shocks, and de Ferra, Mitman, and Romei (2020) - for sudden stops in capital inflows. This paper complements this strand of literature by analysing the trade-off - faced by a monetary policy reacting to the energy price shock - between fighting real wages deterioration and keeping moderate welfare costs for borrowers.

Pieroni (2023) studies the inflation - output gap trade-off faced by monetary policy during an energy supply shock in a closed economy HANK environment. Also in his framework the government's choice is characterized by a tension between raising interest rates to fight inflation, and the aim of not penalizing too much borrowers though the cost of debt channel.

However, it restricts monetary policy to a Taylor-rule without room for monetary smoothing. The 2022-2023 energy crisis gives rise to other sources of welfare loss, which have been analyzed by recent literature: Olivi, Sterk, and Xhani (2023) study optimal monetary policy when consumption baskets vary across households: their model does not display neither an open economy dimension (so an appreciation channel of monetary policy) nor a debt cost channel of interest rate policy, which are the key factors of the trade-off examined in my work.

My paper, while assessing the trade-off between purchasing power defense and mortgage cost moderation, explicitly takes into account distributional effects of interest rate hikes, effects which are investigated empirically and theoretically in Del Negro, Dogra, Gundam, Lee, and Pacula (2024). Factoring inequality outcomes in the assessment of monetary policy performance is a robust implication of optimal policy analysis in heterogeneous agents' models such as in Bhandari, Evans, Golosov, and Sargent (2021), Wolf (2023), Ragot (2017), Acharya, Challe, and Dogra (2021), Dávila and Schaab (2023) and Smirnov (2023). My paper naturally relates to this branch of literature by accounting for the asymmetric effect on monetary policy across the households' crossection in formulating an alternative monetary policy with respect to the benchmark one followed by the BoE over the energy crisis. In accordance with the findings from optimal policy literature, the proposed alternative suggests a "milder" contraction during the most severe stages of the economic cycle, to avoid excessively burdening borrowers. Chan, Diz, and Kanngiesser (2023) reach a similar conclusion, showing that their two-agent models—featuring a hand-to-mouth household—experience adverse effects from a contractionary interest rate hike in the context of an energy shock, even when the household is not directly engaged in borrowing.

The modelization of the heterogeneous agents' setting follow closely Nuño and Thomas (2022) and Achdou, Han, Lasry, Lions, and Moll (2021).

The paper is organized as follows: section 2 presents the model; section 3 analyzes the real appreciation - mortgage cost trade-off of the central bank, and provides the analytical result behind the interest rate smoothing policy prescription. Section 4 lays the ground for the quantitative application: it first presents the macro trends of the UK economy over the energy crisis and computes the empirical effect of mortgages on food consumption of households over the crossection; then proceeds to calibration and validation of the model. Section 5 explores the quantitative results of the model by comparing the benchmark BoE policy with a smoothed policy alternative. Section 6 concludes.

2 Model

The following general open economy framework builds on Auclert et al. (2023a) and Auclert et al. (2023b), while introducing two novel elements: long term bonds and mortgages (the latter modeled as perpetual debt, as in Burya and Davitaya (2022)), and food and non-food consumption (in order to construct a model-counterpart of food consumption variations analyzed in section 4).

2.1 Domestic households

A small open economy (the "domestic" economy) is populated by a unit mass of households, heterogeneous with respect to their wealth and their labor productivity. The discounted utility of a generic household i in economy j reads:

$$E_0 \int_0^\infty e^{\rho t} \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\phi}}{1+\phi} \right] dt \tag{2.1.1}$$

where ρ is a subjective discount rate, σ is the coefficient of risk aversion, c_t is a Dixit-Stigliz consumption aggregator of a food c_t^f and non-food good c_t^{nf}, with elasticity ν and time-varying relative weight φ_t:

$$c_t = [\varphi_t^{\frac{1}{\nu}} c_{ft}^{\frac{\nu-1}{\nu}} + (1-\varphi_t)^{\frac{1}{\nu}} c_{nt}^{\frac{\nu-1}{\nu}}]^{\frac{\nu}{1-\nu}} \tag{2.1.2}$$

The Dixit-Stigliz formulation gives rise to the standard characterization of the price level as a harmonic average of the food and non-food goods:

$$p_t = [\varphi_t p_{ft}^{1-\nu} + (1-\varphi_t) p_{nt}^{1-\nu}]^{\frac{1}{1-\nu}} \tag{2.1.3}$$

Labor supply n_t is a bundle of a unit mass of labor varieties k supplied by the household:

$$n_t = \int_0^1 n_{kt} dk \tag{2.1.4}$$

where each variety's supply n_{kt} - equal across all household - is determined by a union, whose optimization problem will be discussed later.

I follow Nuño and Thomas (2022), in assuming that households trade in a nominal risk-free long-term bond a_t, among themselves and with foreign investors. A bond issued a time t promises a stream of nominal payments $\{\delta e^{-\delta(s-t)}\}_{s\in(t,\infty)}$ summing up to one unit of domestic currency over the infinite lifetime of the bond. A fraction ω of households is also endowed with *mortgage stock*, equal across all of them, that enter the budget constraint under the form of an nominal perpetual debt paid at interest rate i_t^d, and whose proceeds are rebated equally to each domestic household. Therefore the remaining fraction $1-\omega$ of households which are non-mortgagors (or "outright owners") still enjoy the stream of proceeds of mortgage revenues. The real levels of mortgage stocks $D_t^r \equiv D/p_t$ follows a law of motion which takes into account the effect of inflation $\pi_t \equiv \dot{p}_t/p_t$ on its denominator:

$$\dot{D}_t^r = -D_t^r \pi_t \tag{2.1.5}$$

The drift in the asset's dynamics is determined by the saving of the household, converted in asset units by division by the price X_t of the currently traded bond, net of the real reduction of asset amount by the amortization rate δ and inflation π_t:

$$\dot{a}_t = \frac{\delta a_t + z_t w_t n_t + d_t - c_t - D_t^r i_t^d + \Pi_t}{X_t} - (\delta + \pi_t) a_t \tag{2.1.6}$$

where $w_t \equiv W_t/p_t$ is the real wage, z_t is an idiosyncratic productivity shock that follows a diffusion process with parameters $\mu(z), \varsigma^2$; i_t^D is a household-specific interest rate on mortgages d_t and Π_t are dividends rebated to the household, generated respectively by the profits o firms and by the pooled economy-wide revenues from mortgages.

Each household's mortgage debt stock D is made up by a variable rate amount D^v and a fixed rate amount D^f, such that $D = D^v + D^f$. Both D^v and D^f have real value determined with the same process of (2.1.5): so the ratios D^v/D and D^f/D are constant over time. The variable rate mortgage yields interest rate i_t, anchored to the one provided by a security issued by the central bank (see section 2.5). The fixed rate mortgage consists instead in the sum of a continuum of mortgages of the same size D^f/S, indexed with subscript s and ranging from 0 to S:

$$D^f = \int_0^S D^f(s) ds \tag{2.1.7}$$

Each $D^f(s)$ entails a household-specific interest rate $i_t^f(s)$: this implies $i_t^f = \frac{1}{S}\int_0^S i_t^f(s) ds$.

At each period t, only the mortgage $s(t)$ gets its interest rate updated, where $s(t)$ is the remainder of the division of t/S: this introduces a S-interval periodicity in the update of each mortgage s. When a mortgage $s(t)$ is renewed, it is paired with an interest rate $i_t^f(s) = i_{\tau\in[t,t+S)}^f(s)$, constant until next time of renewal $t+S$. I assume that this interest rate is set to the level that would guarantee to the foreign household the same total payment amount of domestic currency over the next S time interval that would be accrued if $D^f(s)$

were behaving as a variable rate mortgage (given the information set of the economy at time t). In other terms, the fixed interest rate is equal to the average of the variable rates over the time until the next mortgage rate renewal:

$$i_t^f(s) = i^f_{\tau \in [t, t+S)}(s) = \frac{1}{S} \int\limits_{[t,t+S)} i_\tau d\tau \tag{2.1.8}$$

It is here worth to highlight that the updating mechanism for i_t^d (2.1.8) is arbitrarily assumed in a stylized way to capture the forward-looking nature of the fixed rate of mortgage, and it will prove to be suitable to let the aggregate mortgage rate i_t^d track its empirical counterpart in section 4.3. Given the exogenous and non-tradable nature of the mortgage perpetuity D, the interest rate update rule for both fixed and variable mortgages is indeed detached from any market force in the model[1]. Let us define the aggregate interest rate on mortgages i_t^d as the weighted average of the fixed and variable rate:

$$i_t^d = \frac{D^f}{D} i_t^f + \frac{D^v}{D} i_t \tag{2.1.9}$$

Households aim at maximizing lifetime utility (2.1.1) by choosing consumption, asset holding under constraints (2.1.6) and the borrowing limit. The intertemporal problem of the household can be formulated recursively under the form of a Hamiltonian-Bellman-Jacobi equation for household with productivity realization z, asset holding a:

$$\rho V_t(a,z) = \max_{a_t, c_t} \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\phi}}{1+\phi} + s_t(a,z) \frac{\partial V_t}{\partial a} \right] + \mu(z) \frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2} \frac{\partial^2 V_t}{\partial z^2} + \frac{\partial V_t(a,z)}{\partial t} \tag{2.1.10}$$

where

$$s_t(a,z) = \begin{cases} \frac{\delta a_t + z_t w_t n_t + d_t - c_t - D_t^r i_t^d + \Pi_t}{X_t} - (\delta + \pi_t) a_t & \text{if mortgagor} \\ \frac{\delta a_t + z_t w_t n_t + d_t - c_t + \Pi_t}{X_t} - (\delta + \pi_t) a_t & \text{if non-mortgagor} \end{cases} \tag{2.1.11}$$

We can define the joint density of wealth and productivity $f_t(a,z)$. Its dynamics over time

[1] The non-tradability of the perpetuity could be relaxed by assuming that the latter was sold only once in the life of the economy, by a private perfectly competitive intermediary with property rights equally split across all households, to a subset of agents (since then called "mortgagors") hit by a preference shock to current consumption such to drive them to wish to relax their current borrowing limit at the expense of future perpetual payments), while no unexpected shock had yet hit the economy. At the trade time, the perpetuity D would be expected to yield the same interest rate $\bar{i}$ as the long-term debt a_t, for the whole infinite horizon on the economy. After that moment, mortgagors would be locked-in with their mortgage position D and converge to a steady state distributions of assets and states - that one that will be treated in section 2.8.

are governed by a Kolmogorov-forward equation:

$$\frac{\partial f_t(a,z)}{\partial t} = -\frac{\partial}{\partial a}[s_t(a,z)f_t(a,z)] - \mu(z)\frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2}\frac{\partial^2 V_t}{\partial z^2} \tag{2.1.12}$$

I will assume that the process for z is normalized such that the idiosyncratic productivity realizations aggregate to one:

$$\int_0^1 z f_t(a,z)dz = 1 \tag{2.1.13}$$

Lastly, let us define C_t as aggregate consumption in the domestic economy - the integral of $c_t(a,z)$ over all states a, z.

2.2 Final good producers

A mass of perfectly competitive firms produce either the food or non-food good, according to a CES production function in energy input y_{Et} (supplied by the foreign economy) and non-energy domestic input y_{Dt} (supplied by domestic producers):

$$y_{jt} = [(1-\alpha_E)^{\frac{1}{\epsilon}} y_{Dt}^{\frac{\epsilon-1}{\epsilon}} + \alpha_E^{\frac{1}{\epsilon}} y_{Et}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{1-\epsilon}} \qquad j = f, n \tag{2.2.1}$$

where ϵ is the elasticity of substitution between energy and non-energy goods. Notice that, being the production function for the food and non-food good exactly equal, the marginal cost mc_t^f, mc_t^n for both goods is the same, and it immediately follows that $mc_t^f = mc_t^n = p_{ft} = p_{nt} = p_t$ by perfect competition. The CES production function gives rise to the following formulation for the latter nominal marginal cost (equal to the final consumer's price p_t):

$$mc_t^f = mc_t^{nf} = p_{ft} = p_{nt} = p_t = [(1-\alpha_E)p_{Dt}^{1-\epsilon} + \alpha_E p_{Et}^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{2.2.2}$$

where p_{Dt} and p_{Et} are respectively the prices of the non-energy and energy inputs.
The non-energy input y_{Dt} is in turn itself a CES aggregator of a home-produced good y_{Ht} and foreign-produced good y_{Ft}:

$$y_{Dt} = [(1-\alpha)^{\frac{1}{\eta}} y_{Ht}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} y_{Ft}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{1-\eta}} \tag{2.2.3}$$

Where η is the elasticity of substitution between the domestic and foreign good. The price of the non-energy good can be derived as:

$$p_{Dt} = [(1-\alpha)p_{Ht}^{1-\eta} + \alpha p_{Ft}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{2.2.4}$$

Final producers and producers of the non-energy good solve an optimal variety expenditure problem, which delivers a standard Dixit-Stigliz demand formulation for energy, domestic and foreign goods:

$$y_{Et} = \alpha_E \left(\frac{p_{Et}}{p_t}\right)^{-\epsilon} y_{jt} \tag{2.2.5}$$

$$y_{Ht} = (1-\alpha_E) \left(\frac{p_{Dt}}{p_t}\right)^{-\epsilon} (1-\alpha) \left(\frac{p_{Ht}}{p_{Dt}}\right)^{-\eta} y_{jt} \tag{2.2.6}$$

$$y_{Ft} = (1-\alpha_E) \left(\frac{p_{Dt}}{p_t}\right)^{-\epsilon} \alpha \left(\frac{p_{Ft}}{p_{Dt}}\right)^{-\eta} y_{jt} \tag{2.2.7}$$

2.3 Intermediate good producers

The intermediate domestic good y_{Ht} is produced by a competitive mass of firms[2] which operate under a technology linear in aggregate labor N_t and aggregate productivity A:

$$Y_{Ht} = AN_t \tag{2.3.1}$$

This implies that dividends are zero ($d_t = 0$). Aggregate labor N_t is a Dixit-Stigliz aggregator of labor varieties:

$$N_t = \left(\int N_{kt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.3.2}$$

where N_{kt} is the aggregate labor demand for variety k. The zero profit condition equates the real wage per unit of output to the price of the domestic good:

$$w_t \frac{1}{A} = \frac{p_{Ht}}{p_t} \tag{2.3.3}$$

Firms also face an optimal choice of the labor variety mix, leading to the standard optimal labor variety demand:

$$N_{kt} = \left(\frac{W_{kt}}{W_t}\right)^{-\varepsilon} N_t \tag{2.3.4}$$

where W_{kt} is the nominal wage in labor market k.

[2]Auclert et al. (2023a) instead assumes a monopolistically competitive sector with flexible prices

2.4 Unions

Each union k determines the labor supply of variety k, i.e. n_{kt} - equal across all households - standing ready to satisfy labor demand:

$$n_{kt} = N_{kt} \tag{2.4.1}$$

Following Wolf (2021), the union chooses the nominal wage W_{kt} at which it supplies labor in order to maximize the utility of the *average* agent; this utility is considered net of a nominal adjustment cost and a real wage stabilization motive (the latter being introduced as in Auclert et al. (2023b):

$$\max \int_{\tau \geq 0} \exp \left[-\rho\tau \left(\{ u\left(C_{t+\tau}\right) - v\left(N_{t+\tau}\right) \} - \frac{\psi}{2} \pi_t^{W2} N_{t+\tau} - \frac{\zeta}{2} \frac{(\varepsilon-1)\tilde{N}u'\tilde{C}}{\tilde{w}} \left(w_{k,t+\tau} - \tilde{w}\right)^2 \right) \right] \tag{2.4.2}$$

where $\tilde{N}$, $\tilde{C}$ and $\tilde{w}$ are respectively the final steady state levels of labor, aggregate consumption and the real wage and ζ is a parameter measuring the extent of the real wage stabilization motive. The latter is an important element to produce a positive pattern of inflation even in the tail of the energy shock, when energy price inflation would turn negative. As shown in the appendix, I solve the maximization problem subject to constraint (2.3.4) and the real labor earnings specification derived from the household block, obtaining the New Keynesian Phillips curve for inflation in the labor market:

$$\pi_t^W = \frac{1}{\rho - \dot{N}_t / N_t} \left[\kappa \left(\chi N_t^{\phi} - \frac{\varepsilon - 1}{\varepsilon} w_t C_t^{-\sigma} - \zeta \frac{\varepsilon - 1}{\varepsilon} \frac{\tilde{N}}{N_t} \tilde{C}^{-\sigma} (w_t - \tilde{w}) \frac{w_t}{\tilde{w}} \right) + \dot{\pi}_t^W \right] \tag{2.4.3}$$

where the slope κ is given by $\frac{\varepsilon}{\psi}$.

2.5 Central bank

The central bank trades a short term (instantaneous) risk-free asset with foreign households - as in Nuño and Thomas (2022). and sets its nominal return i_t. I assume the central bank not to follow any rule, but instead to set the prospective i_t for $[t, \infty)$ according to a fully arbitrary path contingent to the information set of the policy-maker at time t. Given the perfect foresight nature of the model, the planned path for i_t updates only if an unexpected ("MIT") shock hits the model a time t. This modelization choice allows to replicate a close fit of actual interest rate policy data, as showed in section 4.2. Equilibrium implication of this unconventional assumptions for monetary policy will be discussed in section 2.7.

2.6 Foreign economy

The rest of the world displays a representative household with constant consumption C^* of a non-energy good ($C^* = y^*$). The good is produced by a foreign representative firm, with technology symmetric to the final producers in the domestic economy ((2.2.3)):

$$y^* = [\alpha^{\frac{1}{\eta}} {y_{Ht}}^{*\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} {y_{Ft}}^{*\frac{\eta-1}{\eta}}]^{\frac{\eta}{1-\eta}} \tag{2.6.1}$$

where y^*_{Ht} and y^*_{Ft} are respectively the quantities of domestic and foreign input used by the foreign representative firm; note that the coefficient $(1-\alpha)$ is paired with y^*_{Ft}, due to home bias, mirroring expression (2.2.3).

Exported domestic goods are priced in foreign currency. Therefore, the foreign firms features the following Dixit-Stigliz demand for the domestic good:

$$y^*_{Ht} = \alpha \left(\frac{p^*_{Ht}}{p^*_t} \right)^{-\eta} y^*_{HF} \tag{2.6.2}$$

Where p^*_{Ht} and p^*_t are the home good price and the foreign price level in foreign currency, respectively. The foreign price index p^*_t is given by the standard CES formulation, symmetric to (2.2.2):

$$p^*_t = [(1-\alpha) p^{*1-\eta}_{Ft} + \alpha p^{*1-\eta}_{Ht}]^{\frac{1}{1-\eta}} \tag{2.6.3}$$

with p^*_{Ft} being the price of the foreign good in foreign currency; I assume p^*_{Ft} to be itself a Dixit-Stigliz aggregator of a mass of varieties N^*, i.e. $p^*_{Ft} = \left(\int_0^{N^*} \tilde{p}^{*1-\eta}_{Ft}(n) dn \right)^{\frac{1}{1-\eta}}$. For $N^* \to \infty$, imposing symmetry across the foreign varieties' prices $\tilde{p}^*_{Ft}(n)$ implies $p^*_{Ft} \to p^*_t$ - namely, the foreign economy is "big" with respect to the domestic one, so its price index is not affected by domestic economy's price fluctuations.

Monetary policy in the foreign economy ensures full price stability:

$$p^*_t = p^*_{Ft} = 1 \tag{2.6.4}$$

where I normalize p^* to 1. I assume the law of one price to hold, hence I obtain:

$$p^*_{Ht} = p_{Ht} \mathcal{S}_t \tag{2.6.5}$$

$$p_{Ft} = p^*_{Ft} / \mathcal{S}_t = 1/\mathcal{S}_t \tag{2.6.6}$$

where $\mathcal{S}_t$ is the nominal exchange rate. Defining the real exchange rate as $Q_t = \mathcal{S}_t \frac{p_t}{p^*_t} = p_t \mathcal{S}_t$, and substituting y^* by C^* by foreign economy's good market clearing, we can rewrite foreign demand (2.6.2) as:

$$y^*_{Ht} = \alpha \left(\frac{p_{Ht}}{p_t} Q_t\right)^{-\eta} C^* \tag{2.6.7}$$

From the equation above, it can be noticed how a real appreciation (i.e. an increase in Q_t), leads foreign consumers to express a lower demand for the domestic good, which becomes relatively less convenient.

In the light of the foreign price stability and law of one price assumptions, and using the definition $Q_t = p_t \mathcal{S}_t$ we can also rearrange the domestic price index (2.2.2) formulation to obtain the real price of energy and the domestic and foreign goods as a functions of real exchange rate Q_t and energy price in foreign currency p^*_{Et}, that I assume to be exogenous :

$$\frac{p_{Et}}{p_t} = p^*_{Et}/Q_t \equiv p_E(Q_t, p^*_{Et}) \tag{2.6.8}$$

$$\frac{p_{Dt}}{p_t} = \left(\frac{1 - \alpha_E p_E(Q_t, p^*_{Et})^{1-\epsilon}}{1 - \alpha_E}\right)^{\frac{1}{1-\epsilon}} \equiv p_D(Q_t, p^*_{Et}) \tag{2.6.9}$$

$$\frac{p_{Ht}}{p_t} = \left[\frac{1}{1-\alpha}\left(\frac{1 - \alpha_E p_E(Q_t, p^*_{Et})^{1-\epsilon}}{1 - \alpha_E}\right)^{\frac{1-\eta}{1-\epsilon}} - \frac{\alpha}{1-\alpha} p_F(Q_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv p_H(Q_t, p^*_{Et}) \tag{2.6.10}$$

$$\frac{p_{Ft}}{p_t} = 1/Q_t \equiv p_F(Q_t) \tag{2.6.11}$$

The real price of energy p_{Et}/p_t depends positively on the foreign nominal price of energy p^*_{Et}, and negatively on the real exchange rate Q_t: domestic goods' appreciation indeed makes imported energy relatively cheaper. Conversely, the price of the non-energy good p_{Dt} is negatively related to the price of energy, so it is decreasing in p^*_{Et} and increasing in Q_t. p_{Ft}/p_t depends negatively on the real exchange rate: real appreciations indeed reduce the price of the foreign good relatively to the domestic one. The real price of the domestic good, p_{Ht}/p_t, depends negatively on both the real price of energy and the real price of foreign goods: therefore, a real appreciation (i.e. and increase in Q_t) boosts the real price of domestic goods by making energy and foreign goods relatively cheaper. An increase in energy price p^*_{Et} instead lowers p_{Ht}/p_t by reducing the relative price of domestic goods with respect to energy.

I assume that the foreign household can invest both in an short term foreign asset yielding nominal return $i^* + \xi_t$ (with ξ_t being a time varying component), and in the domestic central bank's security mentioned in section 2.5: to rule out arbitrage opportunities, the return from the two assets needs therefore to be equal (uncovered interest parity, "UIP"):

$$i_t = i^* - \frac{d\mathcal{S}_t}{\mathcal{S}_t dt} + \xi_t \tag{2.6.12}$$

The condition can also be expressed in real terms:

$$i_t - \pi_t = i^* - \pi^* - \frac{dQ_t}{Q_t dt} + \xi_t \tag{2.6.13}$$

where $\pi^* = 0$ due price stability in the foreign economy. The foreign households, being able to invest also in domestic long term bonds, discounts the coupon payments of the latter at the central bank's security short term interest rate, allowing us to pin down the price of bonds at time t:

$$X_t = \int_t^{\infty} \delta e^{-\left[\int_t^s (i_\tau + \delta(\tau - t))d\tau\right]} ds \tag{2.6.14}$$

2.7 Equilibrium

Given a path for the interest rates i_t and energy prices p^*_{Et}, an initial distribution of wealth and productivity $f_0(a,z)$, and foreign consumption C^*, a competitive equilibrium is defined as a path for households' choices (a_t,c_{ft},c_{nt},c_t), firms' choices (N_t, y_{ft}, y_{nt},y_{Ht},y_{Et}), unions' choices (n_t,π^W_t), prices ($\mathrm{p}_H(Q_t, p^*_{Et})$,$p_E(Q_t, p^*_{Et})$,$p_F(Q_t)$,$w_t$,$Q_t$,$X_t$), aggregate quantities (Y_{ft},Y_{nt},Y_{Ht},C_t) and distributions ($f_t(a,z)$, consistent with the Kolmogorov forward dynamics (2.1.12)) such that households and firms optimize, and the following market clearing conditions in the goods and labor market are satisfied, as well as the uniform rebating rule for mortgage payment revenues:

$$\begin{aligned}
Y_{Ht} =& (1-\alpha_E)\left(\frac{p_{Dt}}{p_t}\right)^{-\epsilon}(1-\alpha)\left(\frac{p_{Ht}}{p_{Dt}}\right)^{-\eta}(Y_{ft}+Y_{nt}) + \alpha\left(\frac{p^*_{Ht}}{p^*_t}\right)^{-\eta} C^* = \\
=& (1-\alpha_E)\left(\frac{1-\alpha_E p_E(Q_t,p^*_{Et})^{1-\epsilon}}{1-\alpha_E}\right)^{-\frac{\epsilon}{1-\epsilon}}(1-\alpha)\left(\frac{1-\alpha(p_F(Q_t)/p_D(Q_t,p^*_{Et}))^{1-\eta}}{1-\alpha}\right)^{-\frac{\eta}{1-\eta}}(Y_{ft}+Y_{nt}) \\
&+ \alpha\left(p_H(Q_t,p^*_{Et})Q_t\right)^{-\eta} C^* && (2.7.1) \\
C_t =& Y_{ft} + Y_{nt} && (2.7.2) \\
Y_{Ht} =& A N_t && (2.7.3) \\
N_t =& n_t && (2.7.4) \\
\Pi_t =& \omega D^r_t i^d_t && (2.7.5)
\end{aligned}$$

where (2.7.1) is market clearing in the domestic good's market[3], (2.7.2) is market clearing in the final goods' market[4], (2.7.3) is market clearing the labor market, and (2.7.4) stands

[3]Condition (2.7.1) is retrieved by substituting for p_{Dt}/p_t and p_{Ht}/p_{Dt} by using the price indexes (2.2.2) and (2.2.4) and results (2.6.8)-(2.6.11).

[4]Since $p_{ft} = p_{nt} = p_t$, the aggregate demands for the food and non-food goods write $C_{ft} = \varphi_t C_t$ and $C_{nt} = (1-\varphi_t)C_t$. By market clearing in the two markets, we have $Y_{ft} = C_{ft}$ and $Y_{nt} = C_{nt}$, hence, since

for the assumptions of households complying with the unions' choices in setting their labor supply (by symmetry among unions, $\int_0^1 n_{kt} dk \equiv n_t \; \forall k$). The goods market clearing condition (2.7.1) in particular is given by the sum of domestic demand (the first term on the right hand side) and foreign demand (the second term on the right hand side).
It is here worth stressing that the variables that we need to take as exogenous in order to compute the equilibrium are not only the energy shocks P^*_{Et}, but also the interest rates i_t, unlike standard modelization choices which would introduce policy rules to endogenize monetary policy (as a Taylor rule). Assuming an arbitrary path for interest rate opens up the possibility for inflation and output indeterminacy due to mean-zero sunspot shocks (see Cochrane (2011); I acknowledge here the limitation of this approach and decide to focus only on purely deterministic equilibria.

2.8 Steady state

In order to obtain a stationary value for D^r_t, I need calibrate the model to obtain price stability ($\pi = 0$): this is achieved by imposing the stationary interest rate $\bar{i}$ equal to $i^* + \xi$ in the steady state version of UIP ((2.6.13)), with ξ being a stationary value for ξ_t. The model exhibits an infinite number of steady states, each one indexed by a value for the stationary real stock of mortgage $\bar{D}^r$. This is due to the fact that any nonzero inflation path $\pi_t \in [0, \infty)$, for given initial stock D, determines a different limit value of D^r_t (for $t \to \infty$) - determined by the extent to which the inflation path reduces the real mortgage stock over time. The following discussion will characterize a steady state for *given* $\bar{D}^r$.
The real domestic price $p_H(Q)$ is determined uniquely by the steady state $\bar{Q}$, and so is $\bar{w}$, by (2.3.3). Therefore, by (2.1.6), each household's consumption in home and foreign good $c(a, z)$ is determined uniquely by $\bar{Q}$, the steady state interest rate $\bar{i}$ (which also pins down the mortgage rate $i^d = i$), labor $\bar{N}$ and the states a, z (provided that I already substitute for the mortgage proceeds' rebating rule (2.7.5) and the labor supply compliance (2.7.4)). This implies that the drift function $s(a, z)$ depends only on $\bar{i}$, $\bar{Q}$ and $\bar{N}$ and the states a, z. Then, by setting to 0 the left hand side of (2.1.12), we can obtain the whole steady state distribution $f(a, z)$ as a function of $\bar{i}$, $\bar{Q}$ and $\bar{N}$.
Aggregate consumption $\bar{C}$ is defined as the integral over steady state consumption for each combination of states, given the stationary distribution $\bar{f}(a, z)$: $\bar{C} = \int \bar{c}(a, z) \bar{f}(a, z) da dz$; since both the idiosyncratic consumption levels $\bar{c}(a, z)$ and the distribution $\bar{f}(a, z)$ are determined by $\bar{i}$, $\bar{Q}$ and $\bar{N}$, we can then retrieve the following parsimonious functional formulation

$\varphi_t \in (0, 1)$, we obtain $C_t = Y_{ft} + Y_{nt}$

for $\bar{C}$:

$$\bar{C} = C(\bar{i}, \bar{Q}, \bar{N}) \tag{2.8.1}$$

Given $\bar{D}^r$, $\bar{i} = i^*$ and the stationary price of energy $\bar{p}^*_E$, equations (2.4.3),(2.7.1),(2.7.2),(2.7.3),(2.8.1),(2.6.13) define a system of six equations in six variables: $\bar{\pi}, \bar{Y}_H, \bar{N}, \bar{C}, (\bar{Y}_{ft} + \bar{Y}_{nt}), \bar{Q}$. If mortgages are 0 ($\bar{D}^r = 0$), the model shocks are small enough in size to guaranteee that the dynamics revert to the *initial* steady state: heterogeneous agents small open economy models can indeed feature stable steady states thanks to the convergence property of the asset distribution (beyond Auclert et al. (2023a), see also Nuño and Thomas (2022) de Ferra et al. (2020)[5]).

However, allowing for $\bar{D}^r > 0$ leads to convergence to a different final steady state from the initial one, due to the different final mortgage stock D^r, if inflation π_t is different from 0 at any point in time.

Notice that the discussion so far relies on the assumption of no structural parametric changes over the dynamics of the model, which would mechanically lead to a different final steady state. This however will be the case for the quantitative analysis of section 4 and 5, which will postulate a different final stationary interest rate both in the domestic and foreign economy, $\tilde{i} = i^* + \tilde{\xi} > \bar{i} = \bar{i}^* + \bar{\xi}$ (with $\bar{\xi}$ and $\tilde{\xi}$ being respectively the initial and final stationary value for ξ_t) providing an additional reason behind the attainment of a different final steady state, in addition to the inflation-driven adjustment of the mortgage stock.

3 Trading off appreciations with mortgage costs

In this section I analyse the impact of an energy price shock on crossectional household income, and later will introduce the trade-off faced by monetary policy in its reaction. Starting from a steady state configuration for the domestic economy, I will take into account an unexpected and temporary rise in the price of energy p^*_{Et} ($P^*_{Et} > \bar{P}^*_E$ and $P^*_{Es} = \bar{P}^*_E$ for $s \in (t, \infty)$). Given the results obtained in model outline, we can express the real income of a mortgage-holding household with states a, z (i.e. $\delta a_t + z_t w_t n_t - D^r_t i^d_t + \Pi_t$), net of the coupon payment δa_t, as follows:

$$z p_H(Q_t, p^*_{Et}) Y_{Ht} - (1 - \omega) D^r_t i^d_t (i_{ss} \in [t, t + S))) \tag{3.0.1}$$

Where labor income is a function of domestic output Y_{Ht} and and the real price of domestic good $p_H(Q_t, p^*_{Et})$, while the mortgage rate i^d_t is expressed as a function of all the future short-term interest rates until $t + S$ (i.e. $i^d_t(i_{ss} \in [t, t + S)))$). The equilibrium expression

[5] Then it is not needed to resort to debt-elastic interest rates as commonly done in representative agent models without international risk sharing.

$(1-\omega)D_t^r i_t^d$ stands for mortgage payment net of revenues Π_t. Notice that I decide not to include coupon payments δa_t in the income specification (3.0.1), as they do not depend directly on the energy price variation, nor on the interest rate policy (they depend instead indirectly on these shocks through the endogenous response of the household in adjusting its asset stock a_t).

A jump in p^*_{Et} makes domestic goods relatively more attractive than energy, increasing overall world demand for domestic goods relatively to demand for the foreign ones. This effect is captured in equation (2.7.1) , and has a positive impact on domestic output Y_{Ht} (expenditure switching channel, ES). On the other side, an increase in p^*_{Et} lowers the firms' revenue per unit of output, and then wages, i.e. the term $p_H(Q_t)$ in equation (3.0.1) (terms of trade channel, TT). This last effect is produced by the higher price of energy relative to domestic goods, which passes through on domestic real wages.

If ES is stronger, households will enjoy a higher current wage income, while if TT dominates they will suffer from a current wage income loss. By looking at equation (2.7.1), with elasticities ϵ and η low enough the effect of energy price on demand for domestic good is muted: therefore the expenditure switching channel is dominated by the terms of trade channel. This is the case I will focus from now onwards, as it allows the energy price shock to induce a real income loss (as in Auclert et al. (2023b)).

Let us now assume rigid prices and zero wedges in the UIP[6]: $\pi_s = \xi_s = 0 \ \forall s > t$. Let us define the final steady state real exchange level $\bar{Q}$ [7]. If the central bank reacts to the shock by producing an increase in the interest rate i_t by a contractionary monetary policy, that implies $dQ_t < 0$ by the UIP condition (2.6.13). In order to have this movement being consistent with a reversion to the initial steady state, Q_t needs to jump at the onset of the shock: the economy experiences a real appreciation. Intuitively, the domestic currency temporarily soars before depreciating over time back to its steady state level: this reduces the incentive to invest in domestic assets and restores indifference between the two countries' investment opportunities. This will be hence labelled as the *UIP channel* of an interest rate hike. The real exchange rate appreciation in turn passes through the real domestic wages by the firms' pricing condition (2.3.3), restoring some purchasing power for the household: analytically, in equation (3.0.1),the real wage term $p_H(Q_t, p^*_{Et})$ is negatively affected by the shock to p^*_{Et} but positively affected by the increase in Q_t. The interest rate hike fights the fall in real income by a domestic real appreciation.

However, the rise in the interest rate i_t affects real income (3.0.1) also through a higher

[6] This is obtained by assuming fully rigid nominal wages, so non-energy good prices P_{Dt}, together with the fact that $P^*_{Es} = \bar{P}^*_E$ for $s > t$.

[7] Following the discussion of Section 2.8, the initial shock to prices lowers the final real mortgage stock and implies a different final steady state real exchange rate from the initial one.

outflows in terms of mortgage payment, as the aggregate mortgage rate i_t^d rises due to the increase in the short-term interest rate (by the mechanisms unraveled in equations (2.1.8) and (2.1.9) (*debt-cost* channel of an interest rate increase). The effects of an interest rate hike on consumption of mortgagors poses a trade-off to central bank's policy: on one side, the whole households' crossection suffers a weaker real income loss, on the other, mortgagors incur into a higher cost of debt.

The key aspect, however, is that whether the interest rate hike is frontloaded or smoothed can make a lot of difference to mitigate this trade off. Indeed you can achieve an appreciation of the current exchange rate even if the interest rate hike is smoothed over time. Let us consider the policy maker willing to attain the level $Q_t = Q^* > \bar{Q}$. The forward iteration of the UIP condition (2.6.13) up to infinity yields:

$$\ln Q^* - \ln \bar{Q} = \int_t^{\infty} (i_\tau - i^*) d\tau \tag{3.0.2}$$

So the current real exchange rate depends on the whole sum of future interest rates.

The question to be posed here is whether the trade-off between current appreciation and mortgage cost increase can be relaxed by distributing the latter over a protracted time span, leveraging the forward looking nature of Q_t. This can be engineered by an increase in the *future* interest rates short term rates $\int_t^{\infty} (i_\tau - i^*) d\tau$ (from now onwards, I will refer to this policy as *monetary smoothing*); notice that this would nevertheless come at the expense of Q_t and i_t being persistently above steady state beyond t, when it would be not anymore needed.

What does this interest rate smoothing strategy implies for the current variation in the mortgage cost, $i_t^d - i^*$? I will answer to this question by considering first two simple extreme cases ($S \to 0$ and $S = \infty$), and then I will analyze the general case for any fixed term horizon.

1. Case $S \to 0$ (short maturity mortgages). The fixed rate behaves as a variable rate ($i_t^f = i_t$) (we can see that by plugging the limit $S \to 0$ inside (2.1.8)). So, by equation (2.1.9), the variation in mortgage cost ($i_t^d - i^*$) boils down to $i_t - i^*$. A smoothed pattern for the policy rate i_t over time maps exactly into the same pattern for i_t^d, so interest rate smoothing is extremely effective in shifting the mortgage cost burden of an appreciation forward in time.

2. Case $S \to \infty$ (long maturity mortgages). The size of the sub-mortgages getting their interest rate updated in the interval dt, i.e. (dt/S), goes to 0. The aggregate fixed mortgage interest rate at t is the average of the previously renewed mortgage rates

down to time $t - S$ (set at i^* since the economy was in steady state before t) and the current renewed rate at the forward looking value $\frac{1}{S} \int\limits_{[t,t+S)} i_\tau d\tau$ (see equation (2.1.8)). Therefore, the variation in i_t^f (i.e. $(\dot{i})_t^f$ is given by:

$$(\dot{i})_t^f = \frac{1}{S}\left(\frac{1}{S}\int\limits_{[t,t+S)} i_\tau d\tau - i^*\right) \tag{3.0.3}$$

where for $S \to \infty$, the expression above equals zero. Hence, by (2.1.9), the overall variation in mortgage cost is $(\dot{i})_t^d = \frac{D^v}{D}(\dot{i})_t$. The total mortgage rate deviation is pinned down only by variable rate mortgages variations. Therefore, the rationale to implement interest rate smoothing is more limited and given exclusively by the aim to smooth out variable rate mortgage cost increases over time.

The two simple cases above represent two extreme cases with respect to the extent to which interest rate smoothing shifts ahead the mortgage cost burden: significantly in the case $S \to 0$ and minimally in the case $S \to \infty$. Hence it is reasonable to expect that this policy would be more desirable the lower is the mortgage horizon S, as showed below. Consider the variation at t of mortgage cost (according to equation (2.1.9)):

$$(\dot{i})_t^d = \left(1 - \frac{D_v}{D}\right)(\dot{i})_t^f + \frac{D^v}{D}(\dot{i})_t \tag{3.0.4}$$

We can then substitute for (3.0.3):

$$(\dot{i})_t^d = \left(1 - \frac{D_v}{D}\right)\frac{1}{S^2}\int\limits_{[t,t+S)} (i_\tau - i^*)d\tau + \frac{D^v}{D}(\dot{i})_t \tag{3.0.5}$$

Substituting for (3.0.2) we obtain:

$$(\dot{i})_t^d = \left(1 - \frac{D_v}{D}\right)\frac{1}{S^2}\left(\ln Q^* - \ln \bar{Q} - \int\limits_{[t+S,\infty)} (i_\tau - i^*)d\tau\right) + \frac{D^v}{D}\left(\ln Q^* - \ln \bar{Q} - \int\limits_{(t,\infty)} (i_\tau - i^*)d\tau\right) \tag{3.0.6}$$

And finally rearranging, we obtain:

$$\dot{(i)}_t^d = \left[\left(1-\frac{D_v}{D}\right)\frac{1}{S^2}+\frac{D^v}{D}\right](\ln Q^* - \ln \bar{Q}) - \underbrace{\left[\left(1-\frac{D_v}{D}\right)\frac{1}{S^2}\int\limits_{[t+S,\infty)}(i_\tau - i^*)d\tau + \frac{D^v}{D}\int\limits_{(t,\infty)}(i_\tau - i^*)d\tau\right]}_{\text{smoothing effect}} \tag{3.0.7}$$

Equation (5.5) provide the key analytical result to understand why monetary policy smoothing can relax the trade-off between appreciation of Q_t and increase in i_t^d. Adopting a smoothed policy allows to achieve the target Q^* at the expense of a lower mortgage rate i_t^d - effect captured in the term $\int\limits_{t+S}^{\infty}(i_\tau - i^*)d\tau$ (the raise in interest rates beyond the fixed mortgage term $t+S$ entails indeed no effect on the currently updating fixed rate i_t^f) and in the term $\int\limits_{(t,\infty)}(i_\tau - i^*)d\tau$ (the raise in interest rates beyond t has not effect on the current variable rate i_t).

For a higher mortgage term S (higher maturity), interest rate smoothing is less effective in mitigating the increase in mortgage costs (the impact becoming minimal for $S \to \infty$, as discussed previously). This is due to:

1. the impact of future monetary contraction on today's rate i_t^d is active for a longer time span $[t, t+S]$ (analytically, the"innocuous" forward guidance term $\int\limits_{t+S}^{\infty}(i_\tau - i^*)d\tau$ shrinks).

2. a smaller fraction of mortgages are updated at t, so shifting the debt cost burden ahead in time is quantitatively less important in the determination of i_t^d (analytically, this is given by the smaller term $\frac{1}{S}$).

The results indicate that smoothing the interest rate path during an energy shock is beneficial from a welfare perspective, as it allows for real exchange rate appreciation while reducing the immediate pressure on mortgage costs. By avoiding sharp rate hikes, policymakers can mitigate the financial burden on households during the shock period.

However, this approach has a long-term cost. Prolonged monetary accommodation leads to higher future mortgage rates due to delayed monetary tightening, extending beyond the energy shock. This creates a forward guidance challenge, where the policymaker must assess whether short-term relief outweighs the future burden. A detailed quantitative analysis, as outlined in the next sections, is necessary to determine the overall welfare impact of this trade-off.

4 A quantitative application to the UK economy

4.1 The UK case in data

The surge in energy prices starting from 2021 had significant consequences for the UK economy. As depicted in Figure 4.1, the industrial energy price index for electricity, gas, and other fuels surged by approximately 150% from 2021 to 2023. This surge in energy prices translated into a surge in CPI inflation, which peaked at 11% in 2023. Real wages, as illustrated in Figure 4.1, experienced a fall from the second half of 2021 onwards, resulting in a decline in the purchasing power of workers and households. In response to the inflation

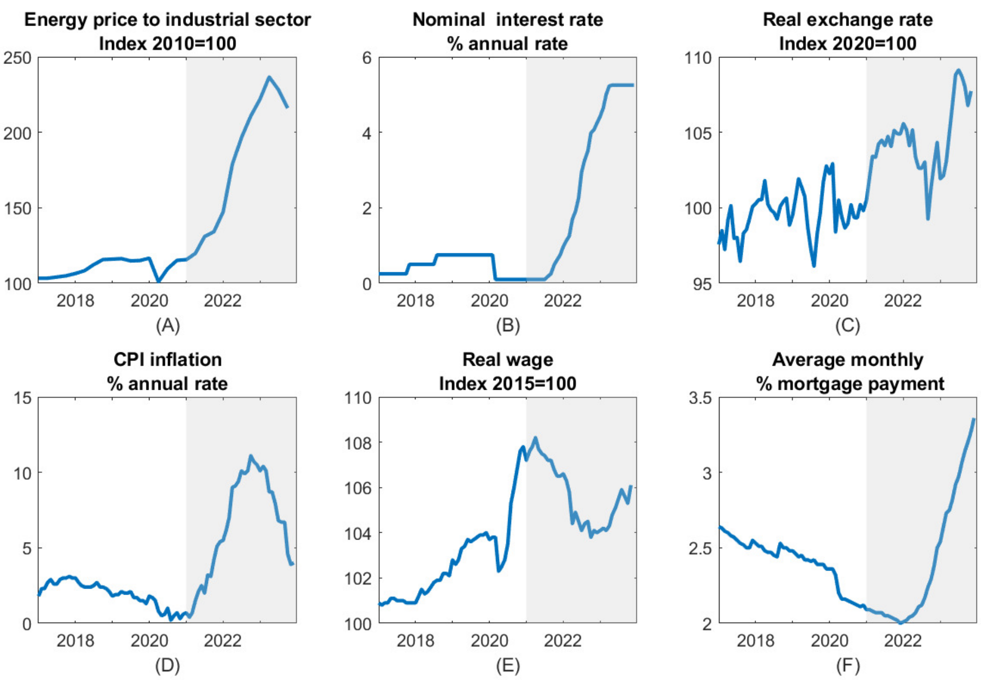


Figure 4.1: Energy prices to industry (quarterly data), policy rate, real exchange rate, CPI inflation rate, real wage and aggregate mortgage rate. Source: Office for National Statistics, BoE and FRED database)

surge brought on by the increase in energy prices, the Bank of England responded decisively. Between 2021 and 2024, the bank significantly raised nominal interest rates, climbing from 0.25% to approximately 5%. This shift in nominal interest rates held implications for mortgages' cost dynamics. Notably, approximately a quarter of the total outstanding mortgage stock were poised to conclude their fixed-rate terms between the final quarter of 2022 and the culmination of 2023, getting their interest rate revised upwards and impacting negatively on households' finances; moreover, a 12% of the total outstanding mortgage stock is made up by variable rate mortgages[8]. These features of the mortgage market determined a discernible increase in the aggregate economy-wide mortgage rate, which climbed from 2% to almost 3.5% between 2021 and 2024. The facts presented above demonstrate the challenging trade-off faced by the Bank of England. Striking a balance between restoring real wage values and keeping borrowing rates moderate for mortgages was a complex task: while the former objective required a tight monetary policy to contain inflation, the latter was calling for a loose interest rate setting.

In what follows, I will further dig into the relevance of the increase in mortgage rates in affecting crossectional consumption. Leveraging data from the "Understanding Society" survey, a longitudinal panel that tracks information across various households in the UK over time, I explore the dynamics within two interview waves: 2020-2021 and 2021-2022.

In particular, I restrict the the analysis to households interviewed both in 2021 and in 2022, in order to track their consumption behavior over time. I include in the sample only households categorized as either housing mortgagors or outright homeowners. Households with tenure status changing between the two interview waves are also excluded, leading to a final sample of 2,477 households. The survey associates to each household its food consumption consumed at home, in addition to income, demographic and geographical characteristics.

Table 1: Descriptive Statistics (monthly), households in 2021 interview wave

Per capita food consumption (£)			Income (£)		% mortgagors
Decile	Mean	Std	Mean	Std	
Bottom 20%	98.6	23.2	4'099	2'242	63%
Bottom 40%	124.7	31.9	4'084	2'348	61%
Bottom 60%	148.1	43.1	4'143	2'649	59%
Bottom 80%	174.2	59.4	4'137	2.742	56%
100%	222.7	133.9	4.128	2.779	54%

Due to the importance of distributional outcomes of a mortgage cost surge in the current framework, it is convenient to express descriptive statistics of the sample with respect to different subsamples of the distribution of food consumption in the pre-energy shock interview wave (i.e. 2021). The total sample of households is indeed split into 5 subsamples according to the position held by each household in the 2021 consumption distribution, namely the bottom 20%,40%,60%,80%,100% of the distribution. I restrict my analysis to the variation in annual food consumption, due to the limited range of expenditure items captured in the survey. For each household, I compute the percentage variation in per-member household food

[8]Source: Office of National Statistics

consumption (given by the ratio between household food consumption and household size $C_f(i,t) = food\_consumption(i,t)/size(i,t)$) between 2022 and the initial wave response:

$$\Delta_{c,f}(i,2022) = \left[\frac{C_f(i,2022)}{CPI\_food(2022)} - \frac{C_f(i,2021)}{CPI\_food(2022)}\right] \Big/ \frac{C_f(i,2021)}{CPI\_food(2022)} \tag{4.1.1}$$

where $C_f(i,2022)$ is the food consumption value for household i reported in 2022, and $C_f(i,2021)$ is the value stated by the same respondent in the previous 2021 interview. The variations is adjusted for changes in the food price index, in order to track only movements in real expenditure for food.

In order to capture distributional effects of mortgage cost increases along the households' crossection, I regress the consumption variation $\Delta^j_{c,f}(i,2022)$ on a dummy $I_M(i)$, which assumes value 1 if the household owns its house through a mortgage and 0 if it is an owner outright; in the regression I control for the total net household real income variation between the two interview waves, $\Delta income(i,2022) = \frac{income(i,2022)}{CPI(2022)} / \frac{income(i,2021)}{CPI(2021)}$. An additional vector X of regressors include government office regions as a geographical controls, and both number of children and number of people in working age as demographical controls. The empirical specification, for each quintile $\mathcal{Q}^j$ of the consumption distribution for $C(i,2021)$, writes:

$$\Delta^j_{c,f}(i,2022) = \beta^j_0 + \beta^j_1 * I_M(i) + \beta^j_2 * \Delta income(i,2022) + \beta^j_3 X_t(i) + \varepsilon(i) \tag{4.1.2}$$
$$\forall\ i \text{ s.t. } C_f(i,2021) \leq \mathcal{Q}^j(C_f(i,2021))$$

The results up are summed up in Table 2.

Consistently with the prediction of heterogeneous agents literature, households which are able to afford lower consumption levels have also a low capacity to financially absorb income shocks (like a mortgage cost increase). We can indeed notice how the coefficients of the "Mortgagor" dummy increase in size and significance as we consider subsamples closer to the bottom of the consumption distribution. In particular, controlling for locations and demographic characteristics, the the bottom 20% and 40% of the distribution displayed respectively a 30% and 16% consumption loss of mortgagors with respect to outright owners - with a statistical significance of 1%; the other samples (bottom 60%, 80% and 100%) feature instead lower and not significant consumption effect from mortgage holding. Overall, controlling for geographical locations does not change significantly the estimated impact of mortgage holding, while controlling for demographics drops this impact from 42% to 30%, suggesting that household's composition is a determinant of the mortgagor/owner outright

Table 2: Regression results for consumption variation $\Delta_c^j(i, 2022)$

Variable	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Mortgagor	-0.3006***	-0.1570***	-0.0830	-0.0653	-0.0692
	(0.1306)	(0.0590)	(0.0988)	(0.0745)	(0.0597)
Demographic controls	Yes	Yes	Yes	Yes	Yes
Regional controls	Yes	Yes	Yes	Yes	Yes
Mortgagor	-0.2916***	-0.1525***	-0.0852	-0.0634	-0.0717
	(0.1034)	(0.0590)	(0.0983)	(0.0742)	(0.0595)
Demographic controls	Yes	Yes	Yes	Yes	Yes
Regional controls	No	No	No	No	No
Mortgagor	-0.4210***	-0.2750***	-0.1088	-0.0575	-0.0228
	(0.0894)	(0.0510)	(0.0846)	(0.0636)	(0.0513)
Demographic controls	No	No	No	No	No
Regional controls	Yes	Yes	Yes	Yes	Yes
Mortgagor	-0.4156***	-0.2732***	-0.1095	-0.0570	-0.0249
	(0.0892)	(0.0510)	(0.0841)	(0.0633)	(0.0511)
Demographic controls	No	No	No	No	No
Regional controls	No	No	No	No	No
Δ% income control	Yes	Yes	Yes	Yes	Yes
Bottom % of $C(i, 2021)$	20%	40%	60%	80%	100%
Observations	495	991	1486	1982	2477

Note: Standard errors in parentheses. *Significant at the 10% level. **Significant at the 5% level. ***Significant at the 1% level

status of the household, as well as of its consumption variation over the 2021-2022 time span. In what follows, I will tailor the calibration of the model to calibrate the empirical estimates in the case with all controls (first line of Table 2), reported graphically in Figure 4.2.

4.2 Calibration

The main channels of effect of real exchange rate policy in the model are the "open economy" dimension, that generates the adverse effects of the price of energy on domestic real wages through a terms-of-trade effect, and the "mortgage" dimension, which mediates the transmission of contractionary interest rate policy on crossectional consumption through the mortgage cost variation faced by households. Therefore my calibration strategy aims at matching salient features of the UK economy along both these dimensions.

Parameters. Following the calibration of Chan et al. (2023), tailored to the UK economy, I set the energy share in production α_e to 0.05 and the elasticity between labor and energy ϵ to 0.15, the price elasticity of world demand for domestic exports η to 0.35, and the export

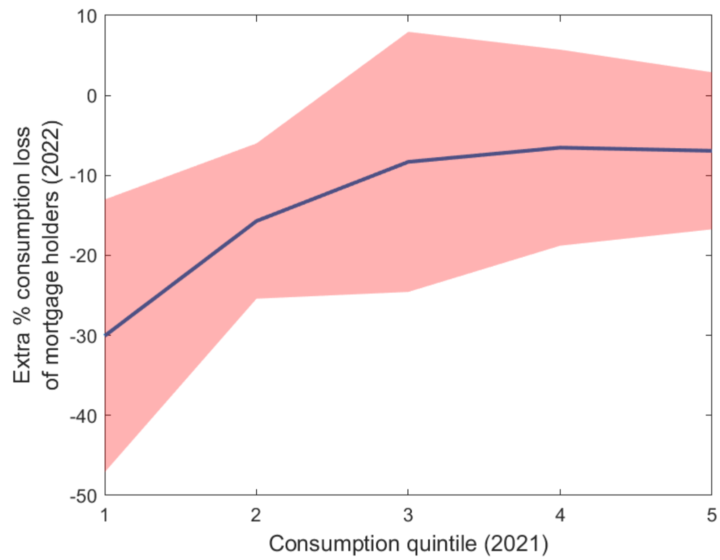


Figure 4.2: Coefficient β^j of mortgagor dummy in the regression for consumption variation $\Delta_c^j(i, 2022)$, with all controls, for households lying below each 2021 wealth quintile D_j. Shaded area: 90% confidence bandwidth

share α to 0.25. The time step Δ is 1/3 (monthly unit periods). The slope of the Phillips curve is set to 0.0049 as in Auclert et al. (2023a). The real wage stabilization motive $\zeta = 25$ guarantees that the pressure on nominal wages in the labor market is such to push inflation to a 8% peak above the steady state. By a proper choice of value for foreign consumption C^*, I obtain an initial steady state real exchange rate $\bar{Q}$ equal to 1, which serves to mediate the effect of the energy shock on real wages down to a -3% at the beginning of 2023, in line with data (see Figure 4.4)[9].

With regards to the household crossectional dimension, I follow Chan et al. (2023) quantitative model for energy shock effects on the UK in setting $\sigma = 1$, while I set $\phi = 2$ and $\rho = 0.05$ as in the open economy HANK calibration of Auclert et al. (2023a); the borrowing limit is close to 0 ($\bar{a} = -0.2$) according to literature's standard practice. The long-term bonds amortization rate δ is set to 0.021, consistent with a bond duration of 4.5 years (see Nuño and Thomas (2022)). The fraction of mortgagors replicates the data for the full "Understanding Society" survey sample (54%, see Table 1). The average mortgage duration S is set to 5.5 years to match the aggregate mortgage rate path as in Figure 4.4, and the fraction of variable rate mortgages is set to 0.12 according to the most recent Office of National Statistics

[9]Different values for $\bar{Q}$ have indeed the consequence of scaling up and down the whole path for Q_t (reported in Figure 4.3 in percentage deviations from steqady state), with mitigating or amplificating effect on the domestic price of energy P^*_{Et}/Q_t.

Parameter	Definition	Value	Source/Target
		Households	
ρ	Household discount factor	0.05	Auclert et al. (2023b) - HANK with energy shocks
σ	Household risk aversion	1	Chan et al. (2023) - Quantitative model for UK
ϕ	Inverse Frisch elasticity	2	Auclert et al. (2023b) - HANK with energy shocks
$\mu(z)$	Mean of the diffusion process	$0.3(1-z)$	Literature
ς^2	Variance of the diffusion process	4	Shape of crossectional Δ consumption (section 4.4)
δ	Amortization rate, LT bonds	0.021	Nuño and Thomas (2022)
$\bar{a}$	Borrowing limit	-0.02	Literature
		Mortgages	
D	Mortgage stock	-50	Magnitude of crossectional Δ consumption (section 4.4)
S	Mortgage duration	66	Aggregate mortgage rate path
D_v/D	% variable rate mortgages	12%	ONS UK
ω	% mortgagors	54%	Understanding society survey (2021)
		Labour Unions	
ε	Labor demand elasticity	10	Literature
κ	Slope of Phillips curve	0.0049	Auclert et al. (2023a) - HANK open economy
ζ	Real wage stabilization	25	Inflation peak $\approx$ 8% above pre-crisis mean
		Firms and international trade	
α_e	Energy share in production	0.05	5% energy share in production
ϵ	CES degree energy-labour in production	0.15	UK estimates
η	Elasticity of world demand for domestic goods	0.35	Chan et al. (2023) - Quantitative model for UK
α	Foreign preference for domestic exports	0.25	Export share $\approx$ 0.25
C^*	Foreign consumption	1.29	$\bar{Q}=1.4$ such that real wage fall $\approx$ 3%
		Monetary Policy	
$\bar{i}+\bar{\xi}$	Initial steady state interest rate (with $\bar{\xi}=0$)	0.5% yearly	Pre-energy crisis path
$\tilde{i}+\tilde{\xi}$	Final steady state interest rate	3% yearly	Post-energy crisis path (BoE projections)

Table 3: Calibrated parameters

(ONS) data. The mortgage stock is calibrated at $D=-50$, to match the magnitude of the consumption effect of mortgages as in Figure 4.2 (as carried out in section 4.4). In order to replicate not only the magnitude of the curve of effects in Figure 4.2, but also its shape, while assuming a standard calibration value $\mu(z)=0.3(z^{mean}-z)$ as in Achdou et al. (2021), I set the variance of the diffusion $\varsigma^2=4$. The high idiosyncratic risk indeed creates a strong precautionary motive in a household the closer it is to the borrowing limit, letting its consumption absorb the mortgage cost shock in order not to affect the precautionary asset buffer.

Shocks. The model is fed with an energy price shock and an interest rate policy following a lognormal time profile starting from $t=01/2022$ and and tracking the data pattern, as showed in Figure 4.3. The right tail of the lognormal model input for i_t (i,.e. at the right of the *argmax* of the curve) is truncated when the implied value for i_t would fall below the final steady state $\tilde{i}$, i.e. at year September 2027: from then onwards, i_t is set at the new level $\tilde{i}$. The lognormal time profile is described in the following equations:

$$P^*_{Et} = \bar{P}^*_E + K_e Lognormal_{\mu_e,\sigma_e^2}(t)\ \forall t > 04/2022 \tag{4.2.1}$$

$$i_t = \begin{cases} \bar{i} + K_i Lognormal_{\mu_i,\sigma_i^2}(t)\ \forall t \in [01/2022, \text{argmax}(i_t)] \\ \max\{\bar{i} + K_i Lognormal_{\mu_i,\sigma_i^2}(t)\ ,\ \tilde{i}\}\ \forall t > \text{argmax}(i_t) \end{cases} \tag{4.2.2}$$

where $K_e = 29.2, K_i = 1.07, \mu_e = 3.25, \mu_i = 4.1, \sigma_e = 0.7, \sigma_i = 1$ are parameters set to match closely the data counterpart. Policy rate data are retrieved from realized and expected future interest rates (from BoE monetary policy committee's projections): the latter point out to a gradual interest rate cut - already initiated in April 2024 - to be implemented at a progressively slower pace. Consistently with this assumption, I assume a final "landing" stationary value for the BoE rate i_t of 3% (annualized). As mentioned previously, in order to obtain $\tilde{\pi} = 0$ in the final steady state, I assume the final stationary value for the foreign interest rate $i^* + \tilde{\xi}$ to be equal to an annualized 3% as well (see equation (2.6.13)). The initial steady state interest rate $\bar{i}$ is instead set at 0.5% annualized, consistently with the pre-energy crisis existing policy rate data (see Figure 4.3, centre plot).

As far as energy price is concerned, the observed counterpart is given by the quarterly overall fuel cost to industry index (detrended with respect to the same time span of the exchange rate series ,i.e. 2017-2021) - recalling that the energy enters the model as a domestic firms' input.

Q_t is set to steady state until December 2021 - the onset of the energy crisis shock. Afterwards it is computed as the filtered version of the real exchange data series[10], whose computation details are left in the appendix. Using UIP condition (2.6.13), I can backward-engineer the pattern of UIP shocks ξ_t such that the imposed time profile of Q_t is consistent with the data input. In this way, even though Q_t is endogenous in the model, I can successfully reproduce its path in simulation. The pattern of Q_t reported in Figure 4.3 is characterized by a different final steady state: as discussed in section 2.8, this is due to the fact that the final steady state displays a different interest rate in both economies :$\tilde{i} = i^* + \tilde{\xi} > \bar{i} = i^* + \bar{\xi}$, where $\tilde{\xi}$ is the final stationary value for ξ_t.

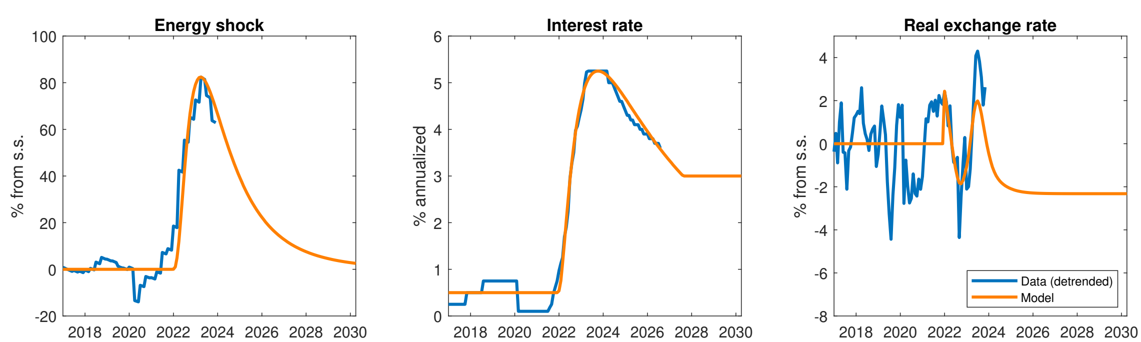


Figure 4.3: Input of the model: Energy shock, interest rate, real exchange rate, vs. data (Re-elaboration from series by Office for National Statistics, BoE and FRED). Energy shock and real exchange rate removed trends are computed on the 2017-2021 time sample, while the interest rate is presented in raw data.

[10]Real exchange data for UK are recovered by the FRED database.

4.3 Model validation at the aggregate level

The model is solved under perfect foresight, by looping over the final steady state real mortgage stock $\bar{D}^r$, and aggregate consumption, building on the solution method by Achdou et al. (2021) - details are reported in the appendix. The impulse responses for inflation, real wage and aggregate mortgage rate i_t^d are reported in Figure 4.4 and compared with the data counterparts, which are build from the dataset of BoE and ONS; UK inflation from 01/2022 (CPI) is presented in % points and absolute difference from the 2% BoE target; the aggregate mortgage rate is presented in % points and absolute difference from the plateau reached in 2021 after a steady decrease ongoing since year 2016 (see Figure 4.1) . Real wages are presented in percentage deviation from a trend computed on a shorter time span (2017-2019) due to the impact of the pandemic period on the variable's path. The magnitude and hump-shape (resp. u-shape) of CPI inflation (resp. real wage) is successfully replicated by the model output, with inflation peaking at 9% above the steady state level, and real wages falling beyond 3%. The aggregate mortgage rate follows a similar upward trend as the data, while deviating by up to 0.5 percentage point.

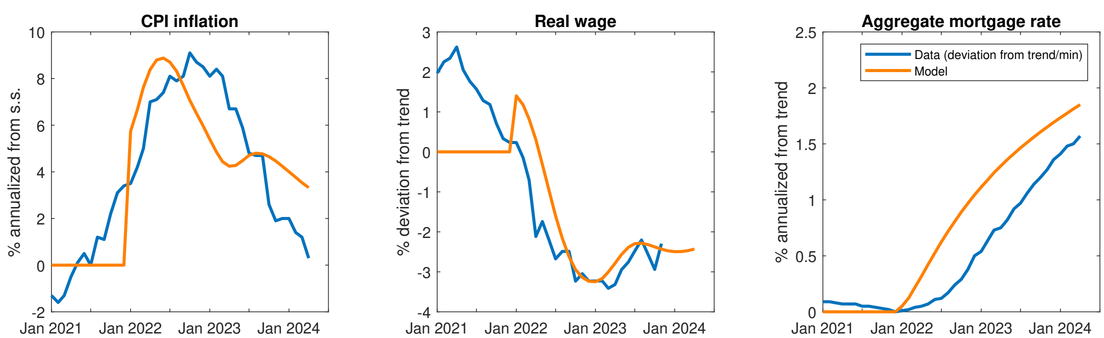


Figure 4.4: Output of the model: Inflation, real wage and aggregate mortgage rate i_t^d, vs. data (Re-elaboration from series by Office for National Statistics and BoE). CPI inflation is take in difference from the pre-crisis 2%. The real wage is presented with the linear trend removed, calculated based on the 2017-2019 pre-COVID time sample. Aggregate mortgage rates is showed in absolute differences from the 2% 2021 plateau.

4.4 Model validation in the crossectional consumption response

So far calibration choices were not discussed in detail with respect to the per-household mortgage stock D and the variance of the diffusion process ς^2. The goal of this section is to present a calibration choice of these parameters, suitable to let the models replicate the difference in the 2021-2022 percentage consumption variation between the mortgagors and non-mortgagors (Figure 4.2). The features of the diffusion process of idiosyncratic shocks are

indeed a paramount element of the model to determine the differences in precautionary saving across households according to their position held in the initial consumption distribution - and hence, the difference in consumption responses to the increase in mortgage costs.
The main challenge that needs to be addressed by the validation method consists in producing a discrete sample of mortgage and non-mortgage households with food consumption and income variations between 2021 and 2022, in order to implement a regression of the type (4.1.2) on the simulation output.

4.4.1 A model-generated crossectional effect of mortgages

In order to compare the effect of the mortgage cost increase on crossectional household consumption with the data output in Table 2, I need to perform a regression of the same type on the data delivered by the model: that requires, for each household starting at node a, z in period 12/2021, to identify the model implied variation of food consumption between 2021 and 2022 $\Delta^j_{c,f}(2022)(a, z)$ and variation of income $\Delta income(2022)(a, z)$, which I formulate as the ratio of the average expected consumption and income in 2022, on their initial steady state value (as of January 2021), given the initial state node a, z:

$$\Delta^j_{c,f}(2022)(a, z) = \frac{\frac{1}{12}\sum_{t\in 2022} E\left[c_{ft}|c_{f,ss} = c_f(a, z)\right]}{c_{f,ss}} - 1 \tag{4.4.1}$$

$$\Delta^j_{y}(2022)(a, z) = \frac{\frac{1}{12}\sum_{t\in 2022} E\left[y_t|y_{ss} = y(a, z)\right]}{y_{ss}} - 1 \tag{4.4.2}$$

where income y_t is defined as the resources flow accrued to the household:

$$y_t = \delta a_t + z_t w_t n_t + \Pi_t - D^r_t i^d_t \tag{4.4.3}$$

The asset a and shock z are discretized along grids with dimension I and J respectively, which deliver discretized vectors $\{g_t\}_t, \{C_{ft}\}_t, \{y_t\}_t$ with size $I * J \times 1$, which comprise respectively the density, food consumption and income for each state node a, z. Following Achdou et al. (2021), we can also derive for each period a transition matrix $\mathcal{G}^{t+1}_t$ such that $g_{t+1} = \mathcal{G}^{t+1}_t g_t$; therefore, by multiplying the transition matrices from $t = 12/2021$ to any $t \in 2022$, we obtain the transition matrix that map g_{ss} to g_t:

$$g_t = \mathcal{G}^t_{ss} g_{ss} \tag{4.4.4}$$

Each column of the matrix $\mathcal{G}^t_{ss}$ (hence, each row of the transpose $(\mathcal{G}^t_{ss})^T$) represents the distribution of outcomes in t conditional on state a, z in staedy state. Then I can recover the expected consumption (resp., income) in 2022, conditional on the household being characterized by states a, z in steady state (i.e. in 12/2021), and hence the variations introduced in equations (4.4.1)-(4.4.2):

$$\Delta^j_{c,f}(2022)(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}^t_{ss})^T * C_{ft}}{c_{f,ss}} - 1 \approx \ln\left[\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}^t_{ss})^T * C_{ft}\right] - \ln c_{f,ss} \tag{4.4.5}$$

$$\Delta^j_{y}(2022)(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}^t_{ss})^T * y_t}{y_{ss}} - 1 \approx \ln\left[\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}^t_{ss})^T * y_t\right] - \ln y_{ss} \tag{4.4.6}$$

At this stage, discretizing g_{ss} into a frequency allows to obtain a countable number of households - indexed by i - each one with consumption $c_{ss}(i)$. I can then rank the resulting sample of model household according to $c_{ss}(i)$, to obtain the initial discretized distribution of consumption. Notice that, alongside the derivation carried out in this section, I can also formulate an expression for the variation in total consumption basket c_t, analogous to (4.4.5)

$$\Delta^j_{c}(2022)(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}^t_{ss})^T * C_{t}}{c_{ss}} - 1 \approx \ln\left[\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}^t_{ss})^T * C_{t}\right] - \ln c_{ss} \tag{4.4.7}$$

4.4.2 From total nondurable to food consumption

The total nondurable consumption values for each model household ($c_t(i)$) are necessary inputs to generate the model-implied idiosyncratic food consumption levels and hence to draw a comparison with the empirical section results', as previously discussed. Given the consumption aggregator (2.1.2) and the result $p_{ft} = p_t$, food consumption is given by:

$$c_{ft}(i) = \varphi_t c_t(i) \tag{4.4.8}$$

Similarly to Aguiar and Bils (2015)[11], for any t I can carry out a first order approximation of $c_t(i)$ around $c_\tau(i)$, where τ is any benchmark date:

$$\ln c_{ft}(i) - \ln c_{f,\tau}(i) = (\ln\varphi_t - \ln\varphi_\tau) + (\ln c_t(i) - \ln c_\tau(i)) \tag{4.4.9}$$

Therefore the equation boils down to:

$$\ln c_{ft}(i) = \Phi_t + \ln c_t(i) \tag{4.4.10}$$

[11]The authors perform instead a linear approximation around the crossectional average of $c_t(i)$.

where Φ_t is a time varying coefficient. Let us assume that φ_t (and then Φ_t) varies only on a yearly basis; taking time difference between the expected value of 2022 consumption and 12/2021 (steady state), we get:

$$\Delta_{c,f}(i, 2022) = \Delta\Phi_{2022} + \Delta_c(i, 2022) \tag{4.4.11}$$

where the quantities $\Delta_{c,f}(i, 2022)$ and $\Delta_c(i, 2022)$ are defined respectively by (4.4.5) and (4.4.7), and $\Delta\Phi_{2022}$ is given by $\Delta\Phi_{2022} = \Phi_{2022} - \Phi_{2021}$.

Let us now run a regression on the model output, mirroring the empirical counterpart (4.1.2), with the exception of being performed on total nondurable consumption instead of exclusively food:

$$\begin{aligned} &\Delta_c^j(i, 2022) = \gamma_0^j + \gamma_1^j * I_M(i) + \gamma_2^j * \Delta y(i, 2022) + \varepsilon(i) \\ &\forall\ i \text{ s.t. } c_{12/2021}(i) \leq \mathcal{Q}^j(c_{12/2021}) \end{aligned} \tag{4.4.12}$$

where $\Delta y(i, 2022)$ is the percentage income variation of household i between 12/2021 and year 2022 (given by expression (4.4.6), $I_M(i)$ is the previously defined indicator function for mortage holders, and $\mathcal{Q}^j(c_{12/2021})$ is the $j-th$ quintile of the steady state model consumption distribution. Results are summarized in Table 4.

Table 4: Regression results for consumption variation $\Delta_c^j(i, 2022)$ (model output)

Variable	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Mortgagor	-0.2406	-0.1172	-0.0730	-0.0747	-0.0649
Bottom % of $C(i, 2021)$	20%	40%	60%	80%	100%
Δ% income control	Yes	Yes	Yes	Yes	Yes

Note: All values are significant as the regression is performed on the whole model population

Once having estimated the coefficients γ_0^j, γ_1^j, γ_2^j, we substitute for the linear prediction (4.4.12) inside (4.4.11):

$$\Delta_{c,f}(i, 2022) = \Delta\Phi_{2022} + \gamma_0^j + \gamma_1^j * I_M(i) + \gamma_2^j * \Delta y(i, 2022) + \varepsilon(i) \tag{4.4.13}$$

The coefficient γ_1^j provides the impact of mortgage holding on 2021 -2022 on food consumption variation, a model counterpart of the empirical estimate of β_1^j retrieved in section 4.1 and plotted in Figure 4.2 for each sample of the model consumption distribution in 12/2021. Note that, while the model accounts percentage variations in food consumption deviating from the ones in total consumption by the factor $\Delta\Phi_{2022}$, the average *difference* between percentage consumption variations of mortgagors and non-mortgagors is the same both with respect to food and total consumption, and measured by the factor γ_1^j.

Table 5: Consumption variation $\Delta^j_{c,f}(i, 2022)$. Model vs. Data.

Variable		**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Mortgagor	Data	-0.3006***	-0.1570***	-0.0830	-0.0653	-0.0692
		(0.1306)	(0.0590)	(0.0988)	(0.0745)	(0.0597)
	Model (γ_1^j)	-0.2406	-0.1172	-0.0730	-0.0747	-0.06494
Bottom % of $C(i, 2021)$		20%	40%	60%	80%	100%

Note: Standard errors in parentheses. *Significant at the 10% level. **Significant at the 5% level. ***Significant at the 1% level

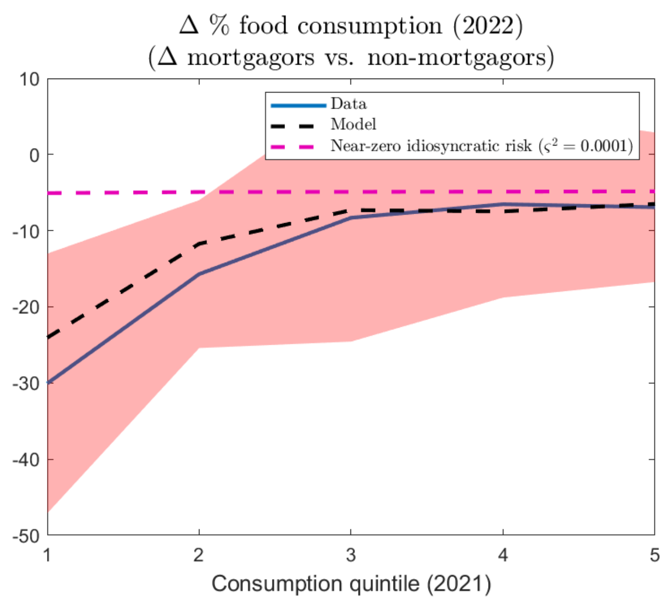


Figure 4.5: Coefficient β^j of mortgagor dummy in the regression for food consumption variation $\Delta^j_{c,f}(i, 2022)$, for households lying below steady state consumption deciles $\mathcal{Q}^j$. Shaded area: 90% confidence bandwidth of the empirical results. Model vs Data.

Figure 4.5 compares the crossectional effects of the mortgage cost increase as from the simulation's outcome, to the empirical counterpart illustrated in Figure 4.2, and to the outcome which would arise in a setting with near-zero idiosyncratic shock ($\varsigma^2 = 0.0001$). The model replicates closely the negative relationship between the position held in the consumption distribution at the end of 2021 (i.e. in steady state) and the extra-consumption loss with respect to owners outright over the crossection of mortgagors, with households at the bottom of the distribution suffering most in food consumption terms. No confidence bandwidths arise in the model-based regression, as the latter is performed on the whole model population. In the near-zero idiosyncratic shock case, the heterogeneity dimension of the model is shut down, as all agents have nearly the same propensity to consume: consequently the impact on consumption of the mortgage cost increase is equal across all quintiles of the steady state distribution (around 7% loss with respect to non-mortgagors). Therefore the heterogeneity dimension of the model is a key element to match the stronger impact of the shock at the bottom of the steady state consumption distribution; however, for higher quintiles the effects become increasingly similar in magnitude, due to the consumption smoothing behavior of households in HANK being more aligned to the ones in the complete markets environment, thanks to the higher wealth stock working as a buffer against idiosyncratic shocks.

5 Smoothing interest rate policy

5.1 The equilibrium effect of the benchmark BoE policy

The impulse response of the variables under the interest rate set by the BoE (from onwards labelled as i_t^{bmk}, where "bmk" being short for "benchmark"), which were showcased in the previous section, underlie a real appreciation effect that fights the real income loss due to the energy price shock, along the lines discussed in section 3. Through the UIP condition, a persistent increase in the interest rate produces an upward shift of the whole real exchange rate path. In order to show that, Figure 5.1 compares the equilibrium pattern for CPI inflation, nominal and real ($i_t - \pi_t$) interest rate, real exchange rate, real wage and aggregate mortgage rate to the one that would materialize with a milder interest rate policy ("moderate hike", in short mh) implemented. Such alternative policy is constructed as imposing the parameters $\sigma_i^{mh} = 0.75$ and $K_i^{mh} = 0.7$ (lower than the $\sigma_i = 1$ and $K_i = 1.07$ of the benchmark). While lowering σ_i reduces the mass in the tails of the interest rate path, the decrease in K_i shrinks the whole path downwards. The parametrization allows to implement the landing on the new steady state interest rate in the same year of the benchmark (2027), while mitigating the hike especially in the first stages of the crisis.

The lower nominal interest rate hike translates into a stronger drop in the real interest rate, as the former makes up less for inflation. Through UIP, this not only implies a lower appreciation in the real exchange rate, but even a depreciation: Q_t falls by more than 6% from its steady state level. The combined effect of the energy shock and the real depreciation determines a stronger fall in the real wage (down to an extra 2% over 2022) in the moderate hike case with respect to the benchmark scenario. Nevertheless, the milder rise in the nominal interest rate allows to produce a lower path for the aggregate mortgage rate (by around 0.4%

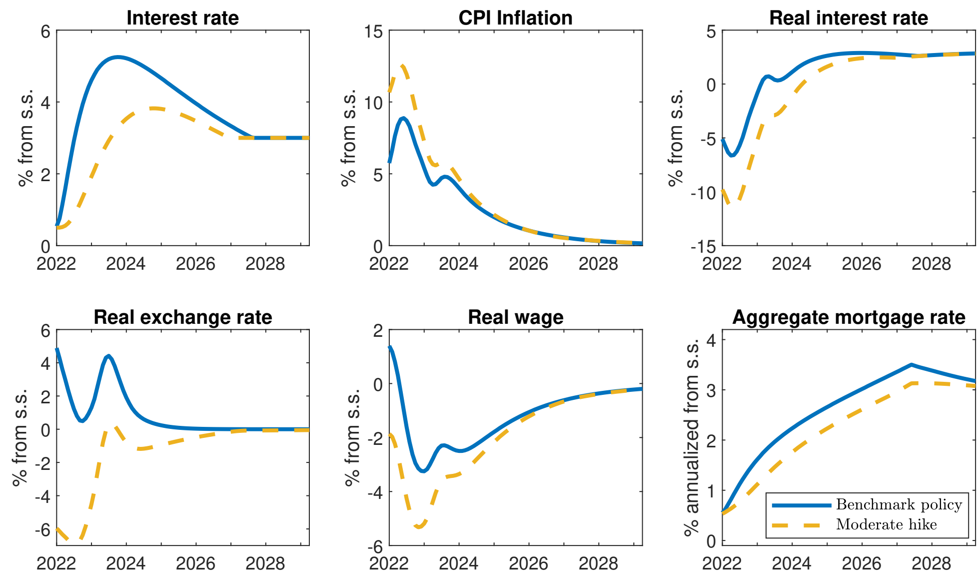


Figure 5.1: Impulse response functions to the energy shock. Benchmark policy vs. Moderate hike.

for five years from the onset of the shock).

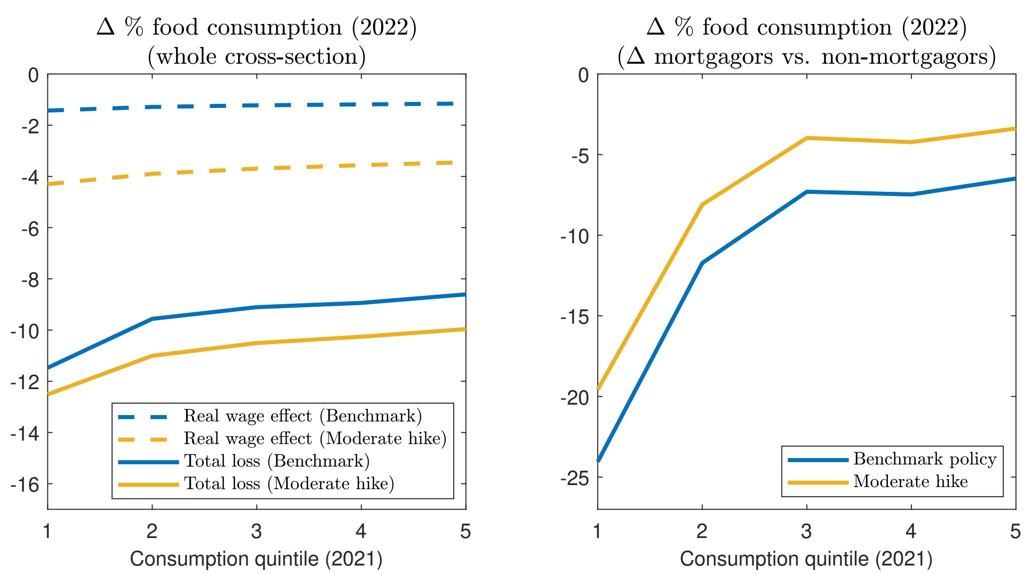


Figure 5.2: Left: food consumption % fall over 2022 for each consumption quintile of the 12/2021 consumption distribution (total $\Delta_{c,f}^{total}$ and decomposed by real wage effect $\Delta_{c,f}^{w}$). Right: 2022 Δ% consumption difference between mortgage and non-mortgagors ($\Delta_{c,f}$ from equation (4.4.11)). Benchmark policy vs. Moderate hike.

Figure 5.2 showcases the average % variation in food consumption between steady state (12/2021) and year 2022, isolating the effect of real wage fall alone, for the households lying below each j quintile of the steady state consumption distribution. The overall variation in consumption ($\Delta_{c,f}^{total}$) is defined as the average of the total expected extra variation in consumption over the crossection with respect to a scenario without any aggregate shock. The effect of wages is isolated by subtracting from this variation the one that would be obtained by exogenously fixing the real wages to steady state in the partial equilibrium outcome of the households' block. The total $\Delta_{c,f}^{total}$ and real wage-driven $\Delta_{c,f}^{w}$ variations can then be defined as follows:

$$\tilde{\Delta}_{c,f}^{j,total}(2022) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t})^{T} * C_{ft}}{c_{f,ss}} - \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t,no\ shock})^{T} * C_{ft}^{no\ shock}}{c_{f,ss}} \tag{5.1.1}$$

$$\tilde{\Delta}_{c,f}^{j,w=\bar{w}}(2022) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t,\ w=\bar{w}})^{T} * C_{ft}^{w=\bar{w}}}{c_{f,ss}} - \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t,no\ shock})^{T} * C_{ft}^{no\ shock}}{c_{f,ss}} \tag{5.1.2}$$

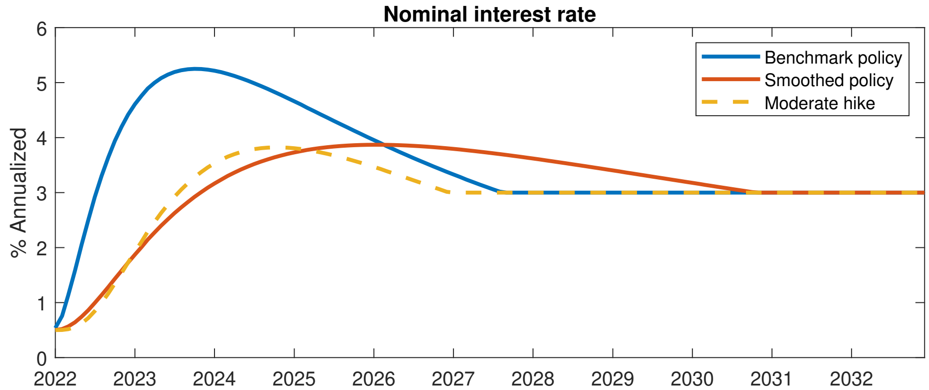
$$\tilde{\Delta}_{c,f}^{j,w}(2022) = \tilde{\Delta}_{c,f}^{j,total}(2022) - \tilde{\Delta}_{c,f}^{j,w=\bar{w}}(2022) \tag{5.1.3}$$

where $\mathcal{G}_{ss}^{t}$ and C_{ft} are respectively the transition matrix from 12/2021 to time t, and the food consumption level.

Figure 5.2 well captures the trade-off implied by the "moderate hike" policy between real exchange rate appreciation and mortgage costs. The alternative policy produces a worse impact of energy shock on consumption through a real wage fall - as real exchange rate appreciation is milder - by approximately 4% with respect to the 2% of the benchmark, across all consumption quintiles (see left plots). However, the lower interest rate involves a better performance in terms of consumption of mortgagors, who can enjoy a weaker increase in the aggregate mortgage rate and see their consumption inequality gap with non-mortgagors being reduced by approximately 4 percentage points across all consumption quintiles.

5.2 A smoothed interest rate policy alternative

In what follows, I will introduce a new candidate policy ("smoothed policy", in short *sm*), which assumes a lognormal profile specified in the same way as in the benchmark policy (equation (4.2.2)),except for the "location" parameter μ_i^{sm} and the scaling coefficient K_i^{sm} which are such that $\mu_i^{sm} \neq \mu_i$ and $K_i^{sm} \neq K_i$, while keeping σ_i^{sm} equal to the benchmark σ_i^{bmk}. In order to make this alternative policy *smoother* than the benchmark one, I impose the assumption $\mu_i^{sm} > \mu_i$: an increase in the location parameter indeed reduces the height of the peak and shifts the whole distribution to the right, as it is illustrated in Figure 5.3, where the smoothed policy path is compared to the benchmark and the moderate hike previously considered in Figure 5.1. Furthermore, the scaling size parameter is set higher than the benchmark ($K_i^{sm} > K_i$) such to generate a prospective cumulate sum of interest



Benchmark policy	σ_i^{bmk}	1	μ_i^{bmk}	4.1	K^{bmk}	1.07
Smoothed policy	σ_i^{sm}	1	μ_i^{sm}	4.9	K^{sm}	1.69
Moderate hike	σ_i^{mh}	0.75	μ_i^{mh}	4.1	K^{mh}	0.7

Figure 5.3: Benchmark policy vs. smoothed policy and moderate hike. Graaphical path and functional form

rates $\int_{01/2022}^{\infty} i_t dt$ sizable enough to determine the same effect on the real exchange appreciation through UIP as the one produced by the benchmark policy. This can be seen in the bottom-left plot of Figure 5.4, where until 2024 the real exchange rate path under the smoothed policy is quantitatively similar to the benchmark, implying a similar patterns of real wages as well, which experience a 3% fall with respect to steady state over 2023. This in turn implies that the effect of the energy shock on consumption through the real wage is equal between the benchmark and smoothed policy (in both cases comprising a 2% food consumption loss), as shown in the left graph of Figure 5.5. On the other side, the mortgage rate i_t^d in the smoothed policy takes lower values until 2026, with a peak reduction of up to 0.7 percentage points. As highlighted in the bottom-right plot, this translates into a significantly lower consumption drop for mortgagors with respect to non-mortgagors (by 4% in the 2021-2022 time window): smoothed policies are successful in partially closing the inequality gap between the two types of agents, without affecting the performance in terms of real exchange rate appreciation during the energy crisis.

By comparing the smoothed policy with the simple moderate hike, we can observe that the equally mild initial rise in the interest rate implemented by both policies delivers an equal relief on mortgagors' consumption (right graph of Figure 5.5); however, only the smoothed policy is able to achieve that without affecting negatively the real exchange rate, since it sustains it at the same level implied by the benchmark policy through the 3-years further protracted interest rate hike. Therefore, the smoothed policy overperforms the moderate hike and matches the benchmark policy in reducing the consumption loss due to real wage fall.

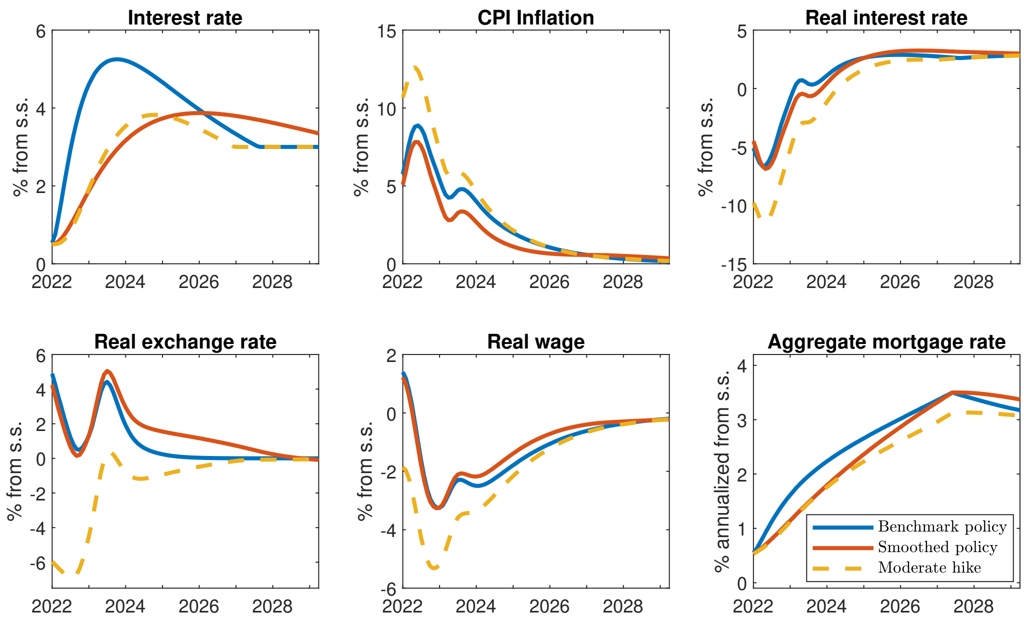


Figure 5.4: Impulse response functions to the energy shock. Benchmark policy vs. Smoothed policy and Moderate hike.

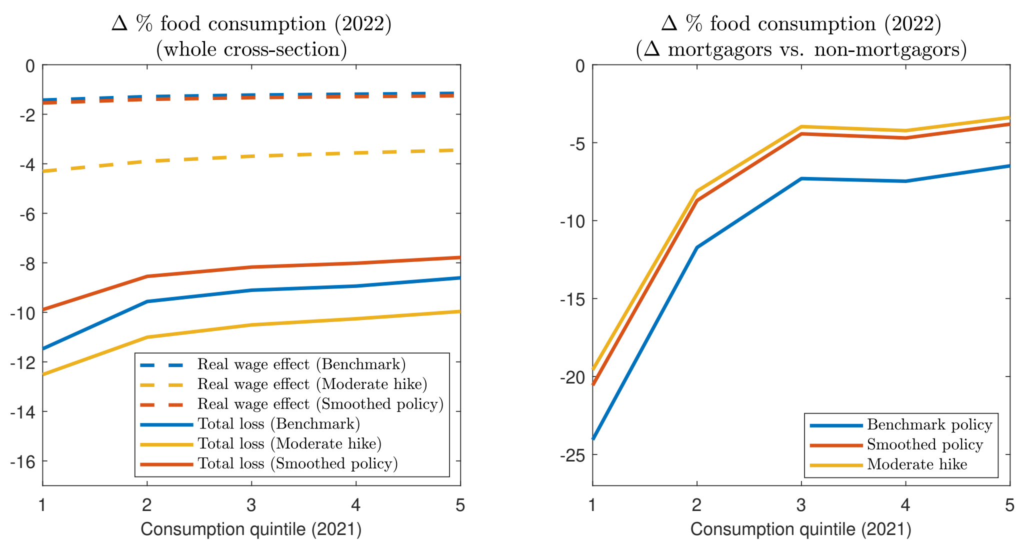


Figure 5.5: Left: food consumption % fall over 2022 for each consumption quintile of the 12/2021 consumption distribution (total $\Delta_{c,f}^{total}$ and decomposed by real wage effect $\Delta_{c,f}^{w}$). Right: 2022 % consumption fall difference between mortgage and non-mortgagors ($\Delta_{c,f}$ from equation (4.4.11)). Benchmark policy vs. Smoothed policy and Moderate hike.

From the discussion above, we can see how the quantitative results confirm the theoretical prescriptions coming from the stylized model of section 3: a smoothing motive of the interest rate policy relaxes the trade-off between real appreciation and mortgage cost increase, which instead was still relevant in the simple moderate hike case: the consumption loss due to real wage fall is indeed the approximately the same between the benchmark and the smoothed policy, while the latter can achieve more moderate mortgage rates and therefore a lower impact on consumption of mortgagors.

5.3 Welfare implications

As a further step with respect to the policy experiment carried out so far, I proceed to investigate the welfare implications of adopting the smoothed interest rate policy. Given the perfect foresight nature of the model, the discounted welfare of any household is embedded in its value function $V_{t_0}^{m}(a,z)$ or $V_{t_0}^{nm}(a,z)$, where t_0 is the time index for the first period of the simulation, and m and nm are respectively indexes for mortgagor and an non-mortgagor household. The analysis of the previous section pointed out that interest rate smoothing, during initial stages of the energy crisis, relieves the mortgage cost burden without giving up real wage defence; however, the interest rate remains higher for a longer time, making real exchange and wages' appreciation more persistent until 2028 - and less needed, as the energy price would have already decreased substantially (see Figure 4.3); moreover the interest rate smoothing involves an undesirable longer protraction of high mortgage rates, as can be observed in the bottom-right plot of Figure 5.4, where i_t^d under the smoothed policy overtakes the one produced by the benchmark policy from year 2028 onwards. The adverse effect of this kind of "forward guidance" intervention needs to be taken into account in order to quantitatively evaluate the welfare implications of the smoothing policy: such implications are nonetheless encoded in the initial level of the value functions $V_{t_0}^{m}(a,z)$ or $V_{t_0}^{nm}(a,z)$, which can be averaged across the initial idiosyncratic shocks to obtain average value functions per asset level $V_{t_0}^{m}(a)$ and $V_{t_0}^{nm}(a)$. Figure 5.6 reports on the left an "inequality" measure given by the difference between $V_{t_0}^{m}(a)$ and $V_{t_0}^{nm}(a)$: mortgagors are worse off than non-mortgagors in both policy scenarios, due to mortgage costs burdening both over the dynamics and in the final steady state; however, implementing the smoothed policy allows to reduce inequality between the two household class, thanks to its mitigation effect on mortgage rates. In the current scenario a policymaker caring about inequality would then consider the smoothed policy as a "less costly" measure, from a welfare perspective, to tackle the impact of the energy shocks on the economy. Total utilitarian welfare, defined as the average discretized value function at time 0, i.e. $\sum_{t=12/2021}^{T} \beta^t \sum_{a,z} g_t(a,z)v(a,z)$ (with $\beta = \frac{1}{1+\rho\Delta}$ and T being the last simulation period) increases from -5.7639 to -5.7501, pointing out that the reduction in inequality is not achieved at the expense of a lower economywide utility. In order to substantiate the welfare increase in terms of consumption unit, I compute in the benchmark scenario the consumption subsidy that would need to be accrued to every household over 2022, taking the equilibrium consumption and labor choices as given, in order to yield the same total welfare of the smoothed policy outcome. In other terms, I seek to compute the subsidy k^* such that:

$$\sum_{t=12/2021}^{\infty} \beta^t \sum_{a,z} g_t(a,z) u(c_t^{bmk}(1+k_t), n_t^{bmk}) = \sum_{t=12/2021}^{\infty} \beta^t \sum_{a,z} g_t(a,z) u(c_t^{sm}, n_t^{sm}) \tag{5.3.1}$$

$$\text{with } k_t = \begin{cases} k^* \text{ if } t \leq 12/2022 \\ 0 \text{ if } t > 12/2022 \end{cases}$$

The resulting 2022 subsidy k^* is equal to 1.1%, implying that the consumption path of all the households (and consequently aggregate consumption) would need to be shifted upward over 2022 by this percentage amount in order to guarantee the achievement of the same total welfare as in the smoothed policy case (see right plot of Figure 5.6).

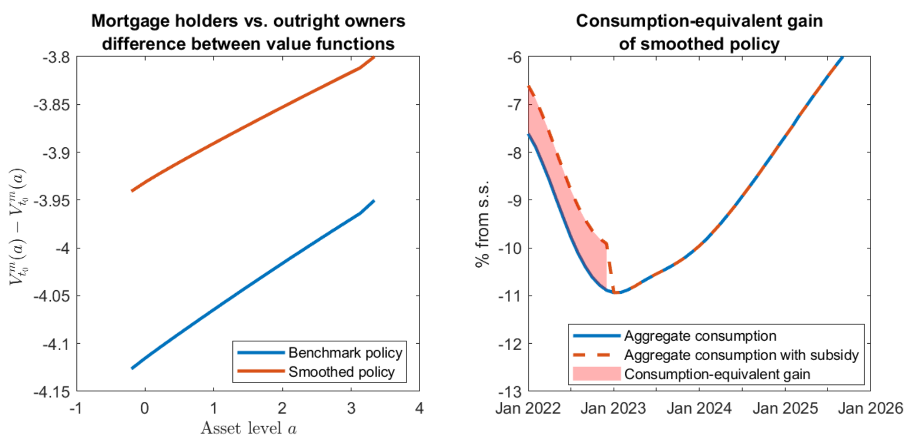


Figure 5.6: Value functions at the first simulation period, for each household class and asset level, Benchmark policy vs. Smoothed policy. (left plot). Consumption-equivalent gain of adopting the Smoothed policy (right plot)

5.4 Testing the model implications: increasing fixed mortgage horizon

A corollary policy prescriptions coming from the discussion of section 3 is that a shorter time horizon S for fixed rates' renewal leads to a stronger effect of interest rate smoothing in relaxing the trade-off between exchange rate appreciation and mortgage costs, due to the higher sensibility of the current mortgage rate to future short term interest rate variations.

I can test this implication in the current quantitative setting, by assessing the impulse responses under the same shock and three candidate policies of last section, with the exception of S being now set to three years instead of the 5.5 years calibrated so far. Note that the pressure of contractionary policy on real exchange rate determination (through UIP (2.6.13)) is the same as in the previous section, as the policy paths for i_t are the same as the ones considered before. On the other side, given the lower stickiness of fixed rate mortgages, the

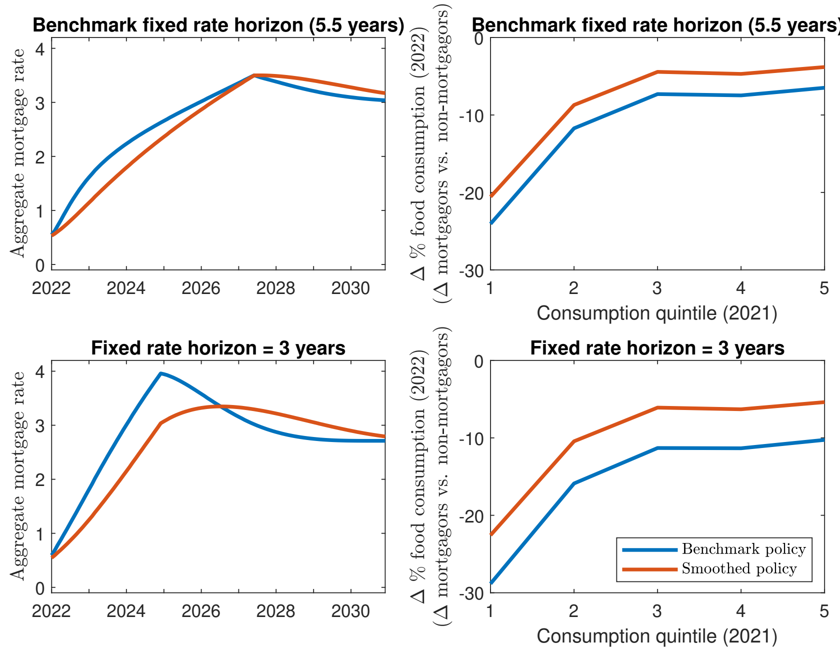


Figure 5.7: Policy's impact of mortgages for different fixed rate time horizons. Benchmark vs. Smoothed policy.

overall mortgage rate i_t^d displays for all the three policies a stronger reaction in magnitude with respect to the baseline, with the benchmark policy's mortgage rate peak amounting to 4% (as opposed to the 3.5% peak of the baseline), as showed in Figure 5.7.

What is now the impact of the different policies on mortgage rates? As a result of the increased influence of the fixed-rate mortgage channel on the overall mortgage channel i_t^d, the impact of rising mortgage costs on mortgagors' consumption is amplified in both the policy options (see the bottom-right plot), with the consumption effects of mortgage cost increases now being different by 7 percentage points between the two policies (with respect to the 4% difference of the benchmark case). This confirms the analytical prediction outlined in section 3.

6 Conclusion

The trade-off between shielding the real wage of households and maintaining moderate costs for mortgagors in response to an energy price shock through an interest rate hike presents a complex challenge. While an increase in the interest rate can protect the purchasing power of households via real exchange rate appreciation, it also leads to higher mortgage rates. The benchmark contractionary policy implemented by the Bank of England (BoE) during 2022-2023 resulted in significant consumption losses for mortgagors, particularly those at the lower end of the consumption distribution. To address these challenges, this paper has explored an alternative strategy that employs milder and prolonged interest rate hikes. This approach achieves the same real exchange rate appreciation but allows for the spread of mortgage cost increases over an extended period, thereby mitigating the immediate burden on mortgagors. The effect is decreasing in the length of fixed rate mortgage contracts.
This strategy presents a balanced approach to monetary policy, that would lead to more equitable welfare outcomes in the face of energy price shocks. A natural extension for this paper would therefore consist in a fully microfounded normative analysis, in the spirit of the literature about optimal policy in HANK.

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Appendix

A Derivation of the New Keynesian Phillips curve

Following Wolf (2023), I assume that unions seek to maximize the *utility the average household* [12], i.e, a fictitious agent consuming the average amount over the household's crossection, and subject to the same supply schedule across labor varieties - set by the unions. The utility is evaluated net of an inflation and real wage stabilization cost Ψ_t. The maximization problem writes:

$$\max \int_{\tau \geq 0} \exp\left[-\rho\tau\left(\left\{u\left(C_{t+\tau}\right) - v\left(N_{t+\tau}\right)\right\} - \Psi_t\right)\right] = \tag{A.0.1}$$

$$\max \int_{\tau \geq 0} \exp\left[-\rho\tau\left(\left\{u\left(C_{t+\tau}\right) - v\left(N_{t+\tau}\right)\right\} - \frac{\psi}{2}\pi_t^{W2} N_{t+\tau} - \frac{\zeta}{2}\frac{(\varepsilon-1)\tilde{N}u'(\tilde{C})}{\tilde{w}}\left(w_{k,t+\tau} - \tilde{w}\right)^2\right)\right] \tag{A.0.2}$$

subject to 1) the average real labor earning at time $t+\tau$ being given by:

$$Z_{t+\tau} = \frac{1}{P_{t+\tau}}\int_0^1 W_{kt+\tau}\left(\frac{W_{kt+\tau}}{W_{t+\tau}}\right)^{-\varepsilon} N_{t+\tau}dk \tag{A.0.3}$$

2) the envelope condition:

$$\frac{\partial C_{t+\tau}}{\partial W_{kt+\tau}} = \frac{\partial Z_{t+\tau}}{\partial W_{kt+\tau}} = \frac{1}{P_{t+\tau}}\int_0^1 W_{kt+\tau}\left(\frac{W_{kt+\tau}}{W_{t+\tau}}\right)^{-\varepsilon} N_{t+\tau}dk \tag{A.0.4}$$

and 3) the effect of the kth-variety nominal wage W_{kt} on labor supply, that , due to the N_{kt} determination $N_{kt} \equiv \int_0^1 \left(\frac{W_{kt}}{W_t}\right)^{-\varepsilon} N_t dk$, and the symmetry $N_{kt} = N_t$, writes:

$$\frac{\partial N_t}{\partial W_{kt}} = \frac{\partial N_{kt}}{\partial W_{kt}} = -\varepsilon\frac{N_{kt}}{W_{kt}} = -\varepsilon\frac{N_t}{W_{kt}} \tag{A.0.5}$$

The problem can be formulated as a Hamilton-Bellman-Jacobi equation:

$$\rho J(W,t) = \max_{\pi^w}\left[\left\{u\left(C_t\right) - v\left(N_t\right)\right\} - \frac{\psi}{2}\pi_t^2 N_t - \frac{\zeta}{2}\frac{(\varepsilon-1)\bar{N}u'(\bar{C})}{\bar{w}}\left(w_{k,t+\tau} - \bar{w}\right)^2\right] + J_W(W,t)W\pi^W + J_t(W,t) \tag{A.0.6}$$

where $J(W,t)$ is the real value of a union with wage W. Taking the envelope and first order

[12] This is a convenient assumption to model the way union aggregates preferences, because it allows to abstract inflation dynamics from distributional outcomes; an alternative is to assume maximization of the average utility of households for some arbitrary weights

conditions and imposing symmetry across all k, we get:

$$J_W(W,t)W = \psi \pi^W N \tag{A.0.7}$$

$$\left(\rho - \pi^W\right) J_W(W,t) = \frac{\varepsilon}{W}\left[N v'(N) - \frac{\varepsilon-1}{\varepsilon} N w u'(C) - \zeta \frac{\bar{N}}{\frac{\varepsilon}{\varepsilon-1}\bar{w}} u'(C)(w-w)w\right] + \tag{A.0.8}$$

$$+ J_{WW}(W,t)W\pi^W + J_{Wt}(W,t) \tag{A.0.9}$$

Differentiating (A.0.7) with respect to time gives

$$J_{WW}(W,t)\dot{W} + J_{Wt}(W,t) = \frac{\psi \bar{N} \dot{\pi}^W}{W} + \frac{\psi \dot{N} \pi^W}{W} - \frac{\psi \pi^W \bar{N}}{W}\frac{\dot{W}}{W} \tag{A.0.10}$$

Substituting the above expression and (A.0.7) inside (A.0.8) we obtain the Phillips curve as presented in section 2.4 (equation (2.4.3)), with $\kappa^W = \frac{\varepsilon}{\psi}$.

B Equilibrium conditions

The model equilibrium is described by the following set of conditions:

$\rho V_t(a,z) = \max\limits_{a_t, c_t}\left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\phi}}{1+\phi} + s_t(a,z)\frac{\partial V_t}{\partial a}\right] + \mu(z)\frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2}\frac{\partial^2 V_t}{\partial z^2} + \frac{\partial V_t(a,z)}{\partial t}$

$c_t(a,z)^{-\sigma} = \frac{\partial V_t(a,z)}{\partial a}$

$s_t(a,z) = \begin{cases} \frac{\delta a_t + z_t w_t n_t + d_t - c_t - D_t^r i_t^d + \Pi_t}{X_t} - (\delta + \pi_t)a_t & \text{if mortgagor} \\ \frac{\delta a_t + z_t w_t n_t + d_t - c_t + \Pi_t}{X_t} - (\delta + \pi_t)a_t & \text{if non-mortgagor} \end{cases}$

$\frac{\partial f_t(a,z)}{\partial t} = -\frac{\partial}{\partial a}[s_t(a,z)f_t(a,z)] - \mu(z)\frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2}\frac{\partial^2 V_t}{\partial z^2}$

$C_t = \int\limits_0^1 c_t(a,z)f_t(a,z)dadz$

$w_t \frac{1}{A} = p_H(Q_t, P_{Et}^*)$

$\dot{D}_t^r = -D_t^r \pi$

$i_t^d = \frac{D^f}{D} i_t^f + \frac{D^v}{D} i_t$

$i_t^f = \frac{1}{S}\int\limits_0^S i_t^f(s)ds$

$i_t^f(s) = i^f_{\tau \in [t,t+S)}(s) = \frac{1}{S}\int_{[t,t+S)} i_\tau d\tau$

$\pi_t^W = \frac{1}{\rho - \dot{N}_t/N_t}\left[\kappa\left(\chi N_t^\phi - \frac{\varepsilon-1}{\varepsilon} w_t C_t^{-\sigma} - \zeta \frac{\varepsilon-1}{\varepsilon}\frac{\tilde{N}}{N_t}\tilde{C}^{-\sigma}(w_t - \tilde{w})\frac{w_t}{\tilde{w}}\right) + \dot{\pi}_t^W\right]$

$X_t = \int_t^\infty \delta e^{-\left[\int_t^s i_s + \delta(s-t)\right]} ds$

$i_t - \pi_t = i^* - \pi^* - \frac{dQ_t}{Q_t} + \xi_t$

$Y_{Ht} = (1-\alpha_E)\left(\frac{1-\alpha_E p_E(Q_t, p_{Et}^*)^{1-\epsilon}}{1-\alpha_E}\right)^{-\frac{\epsilon}{1-\epsilon}}(1-\alpha)\left(\frac{1-\alpha(p_F(Q_t)/p_D(Q_t, p_{Et}^*))^{1-\eta}}{1-\alpha}\right)^{-\frac{\eta}{1-\eta}}(Y_{ft} + Y_{nt}) +$
$\alpha\left(p_H(Q_t, p_{Et}^*)Q_t\right)^{-\eta} C^*$

$Y_{ft} + Y_{nt} = C_t$

$Y_{Ht} = AN_t$

$N_t = n_t$
$\Pi_t = \omega D_t^r i_t^d$
$d_t = Y_{Ht} - AN_t$
$w_t = W_t/p_t$
$\pi_t = \dot{p}_t/p_t$
$\pi_t^W = \dot{W}_t/W_t$

Which in order, are: the Hamiltonian-Bellman-Jacobi equation, the optimality condition of the household, the drift function, the Kolmogorov-Forward equation, the definition of aggregate consumption, the domestic good producers' pricing, the evolution of the real mortgage stock, the definition of the mortgage rate, of the fixed mortgage rate, and of the fixed rate of submortgage s. Then we have the Phillips curve, the pricing of long term bonds, the UIP condition, the market clearing conditions and the mortgage revenues rebating rule. Finally we have the definition of dividends, real wage, price inflation and wage inflation.

C Real exchange rate path in the benchmark scenario

Q_t is set to the initial steady state $\bar{Q}$ until April 2022 - the onset of the energy crisis shock. Afterwards it is denoted by Q_t' and it is computed as the filtered version of the real exchange data series (Figure 4.3). Q_t' is made up by the following two subsets: 1) $\bar{Q}$+the detrended real exchange rate index for UK for $01/2022 < t < 07/2023$ (denoted by $\hat{Q}_t^{data}$). The linear trend is computed according to the pre-energy crisis period 01/2017-04/2022. I choose 2017 as starting year for the trend computation sample, when the time series for the real exchange rate presents a structural break due to Brexit. 2) a diffusion process Q_t for $t \geq 08/2023$ with no innovation, persistence $\rho = 0.85$, and with starting point $Q'_{08/2023} = \hat{Q}^{data}_{08/2023}$. This represents the normalization "tail" of monetary contraction following the decline of energy price pressures.

$$Q_t' = \tilde{Q} + \hat{Q}_t^{data} \quad 01/2022 < t < 07/2023 \tag{C.0.1}$$

$$dQ_t' = (\rho - 1)(Q_t - \tilde{Q}) \quad \forall t \geq 08/2023 \tag{C.0.2}$$

Where Q' is the final steady state real exchange rate. The input Q_t is given then by:

$$Q_t = \bar{Q} \quad \forall t < 01/2022 \tag{C.0.3}$$

$$Q_t = filter(Q_t') \quad \forall t \geq 01/2022 \tag{C.0.4}$$

D Solution algorithm

D.1 Steady state

Under the benchmark policy, the model is solved numerically with the method presented in Achdou et al. (2021), by iteration over the aggregate consumption value. Prior to considering the solution over the dynamics it is necessary to solve for the final steady state of the model (for given D^r) through the following steps:

1. Use the calibrated value for $\tilde{Q}$ to obtain the wage

$$w = p_H(\tilde{Q}, p_E^*)/A \tag{D.1.1}$$

2. As discussed in section, 2.8, steady state requires $\pi = 0$. Therefore, since the real wage is constant, also nominal wage inflation is 0, $\pi^W = 0$.

3. Imposing stationarity in the Phillips curve (2.4.3), we get

$$N = \left(\frac{\varepsilon - 1}{\varepsilon} w \tilde{C}^{-\sigma} \frac{1}{\chi} \right)^{\frac{1}{\phi}} \tag{D.1.2}$$

4. Solve the household problem by iteration on the HJB equation (see Nuño and Thomas (2022) for the case with long-term bonds). Notice $i^d = i$ in the initial steady state and $i^d = i'$ in the final steady state.

5. Compute aggregate consumption $C = \int\limits_a \int\limits_z c(a, z) da dz$

6. From the equilibrium condition (2.7.1)-(2.7.2), we obtain:

$$\begin{aligned} A\tilde{N} =& (1 - \alpha_E) \left(\frac{1 - \alpha_E p_E(\tilde{Q}, p_E^*)^{1-\epsilon}}{1 - \alpha_E} \right)^{-\frac{\epsilon}{1-\epsilon}} (1 - \alpha) \left(\frac{1 - \alpha(p_F(\tilde{Q})/p_D(\tilde{Q}, p_E^*))^{1-\eta}}{1 - \alpha} \right)^{-\frac{\eta}{1-\eta}} \tilde{C} \\ &+ \alpha \left(p_H(Q, p_E^*) \bar{Q} \right)^{-\eta} C^* \end{aligned} \tag{D.1.3}$$

From which we can retrieve the value for foreign consumption C^* consistent with the stationary equilibrium

Once computed the final steady state, I already exploited the degree of freedom provided by C^*, so , in order to compute the initial steady state, as well as a different final steady state characterized by a different D, I need to solve the system of equations (D.1.1),(D.1.2),(D.1.3), together with aggregate demand (equation (2.8.1))

$$\bar{C} = C(\bar{i}, \bar{Q}) \tag{D.1.4}$$

that is a system of four variables ($\bar{w}$,$\bar{Q}$,$\bar{N}$,$\bar{C}$) in four equations. Since (D.1.4) has to be solved numerically as in points 4-5, I proceed as follows:

1. Guess $\bar{C}$
2. Use (D.1.1),(D.1.2),(D.1.3) to get $\bar{Q}$, $\bar{w}$,$\bar{N}$
3. Use $\bar{w}$ and to solve for the households' optimization and aggregate into an updated guess $\bar{C}'$ ((as in point 4 and 5 of final steady state computation))
4. Update the guess:
$$\bar{C} = \bar{C} + \vartheta^S(\bar{C}' - \bar{C}) \tag{D.1.5}$$
until convergence of the quantity $|\bar{C} - \bar{C}'|$ to a threshold small enough. The sign and magnitude of the coefficient ϑ depends on the parameters of the model. For my parametrization and initial guess for $\bar{C}$, imposing a positive ϑ^S leads to an explosive feedback-loop between $\bar{C}$ and $\bar{w}$, while a negative ϑ^S (=0.1) allows to reach convergence.

D.2 Dynamics

Let us now turn the attention to the solution over the dynamics following an unexpected shock to p^*_{Et}, under perfect foresight. The algorithm unfolds as follows:

- Assume a long time horizon T for the discretized variables' path
- Start with the inputs for i_t, Q_t, p^*_{Et}
- Compute $\{w_t\} = \{p_H(Q_t, p^*_{Et})/A\}$
- Use the sequence $\{i_t\}$ to compute the path for long-term bond prices $\{X_t\}$

Then go through the following loop

1. Guess a value for the final steady state real mortgage stock $D^{r\prime}$ and compute the final steady state through the same steps showcased in the second part of section D.1
2. Guess a value for $\{C_t\}$

3. Compute $\{N_t\}$ as a function of $\{Q_t\}, \{p^*_{Et}\}, \{C_t\}$ (see equilibrium conditions (2.7.1)-(2.7.2))

4. Use $\{C_t\}, \{N_t\}, \{w_t\}$ to compute π^W_t backward, starting from $\pi^W_T = 0$

5. Compute $\pi_t = \frac{w_{t-1}}{w_t}\frac{1}{\pi^W_t}$ $\forall t \leq T$ (notice $w_{-1} = \bar{w}$)

6. Use the UIP condition (2.6.13) to back out the path of wedges $\{\xi_t\}$ such that the assumed values for Q_t are consistent with the resulting inflation path $\{\pi_t\}$

7. Starting from $D^r_{-1} = D$, use $\{\pi_t\}$ to compute the path for the real mortgage stock $\{D^r_t\}$ up to time T (leading to a final value D^r_T not necessarily equal to the guess $D^{r\prime}$

8. At each t, compute $i^d_t = \frac{D^f}{D} i^f_t + \frac{D^v}{D} i_t$. Following the assumptions of section 2.1, we can express $i^f_t = \frac{1}{S}((S-1)i^f_t + \frac{1}{S}\sum_{\tau=0}^{\infty} i_{t+\tau})$ (S needs to be $\in \mathbb{N}$).

9. Solve the household problem with long term bonds holding (see Nuño and Thomas (2022) backward, starting from the value functions of the final steady state computed in point 1.

10. Compute the new path for aggregate consumption $\{C'_t\} = \{\sum a, zc_t(a,z)dadz\}$

11. Update C_t as $C_t = (1-\vartheta)C_t + \vartheta C'_t$ for an arbitrary coefficient $\vartheta \in (0,1)$

12. Iterate until convergence of $\max |\{C_t\} - \{C'_t\}|$ to some low threshold value.

13. Update D'_r as $D'_r = (1-\vartheta^D)D'_r + \vartheta^D D^r_T$ for an arbitrary coefficient $\vartheta^D \in (0,1)$

14. Iterate until convergence of $\max |D^{r\prime} - D^r_T{}'|$ to some low threshold value.

D.3 Alternative policies

In order to solve the model for the alternative policies, we do not take anymore $\{Q_t\}$ as an input and back out the UIP wedges $\{\xi_t\}$ consistent with equilibrium, but we instead take $\{\xi_t\}$ as exogenous and solve for $\{Q_t\}$. In order to accomplish this task, I augment the model with an inner loop over the *real interest rate*, in order to determine the inflation path given the policy on i_t. The modified algorithm writes:

- Assume a long time horizon T for the discretized variables' path
- Start with the inputs for i_t and p^*_{Et}
- Use the sequence $\{i_t\}$ to compute the path for long-term bond prices $\{X_t\}$

1. Guess a value for the final steady state real mortgage stock $D^{r\prime}$ and compute the final steady state through the same steps showcased at the beginning of the current section.
2. Guess a value for $\{C_t\}$
3. Go through the following loop
   (a) Guess a path for the real interest rate $r_t \equiv i_t - \pi_t$
   (b) Obtain the implied path for inflation $\pi_t = i_t - r_t$
   (c) Substitute for $\{r_t\} \equiv \{i_t - \pi_t\}$ and the wedges $\{\xi_t\}$ inside the UIP condition (2.6.13) for every t. Iterate the condition backward, starting from $Q_T = \bar{Q}$, to recover the path for Q_t.
   (d) Compute $\{w_t\} = \{p_H(Q_t, p^*_{Et})/A\}$
   (e) Compute $\{N_t\}$ as a function of $\{Q_t\}, \{p^*_{Et}\}, \{C_t\}$ (see equilibrium conditions (2.7.1)-(2.7.2))
   (f) Use $\{C_t\}, \{N_t\}, \{w_t\}$ to compute π^W_t backward, starting from $\pi^W_T = 0$
   (g) Compute the implied inflation from energy prices and labor market forces: $\pi'_t = \frac{w_{t-1}}{w_t}\frac{1}{\pi^W_t}$ $\forall t \leq T$ (notice $w_{-1} = \bar{w}$)
   (h) Update the real rate at each t according to the variation between inflation determined by the guess and the resulting inflation from the last point:
   $r_t = r_t - \vartheta(\pi'_t - \pi_t)$ for an arbitrary coefficient $\vartheta \in (0, 1)$
   (i) iterate until convergence of $\max |\{r_t\} - \{r'_t\}|$ to some low threshold value.
4. Go through the point 7-14 as for the benchmark policy algorithm, and iterate until convergence of $\max |D^{r\prime} - D^{r}_{T}{}'|$ to some low threshold value.

E Extension: price stickiness

In this section I study a version of the model where price stickiness characterize firms instead of unions. The extension is relevant to explore the robustness of the results of the model in presence of slow pass-through of energy prices onto final product prices. I will hereafter assume the wage adjustment cost ψ equal to 0, while introducing an alternative layer of price stickiness at the final producers' level.

For simplicity I assume away food and non food duality in the final good supply, which instead is now given by a range of varieties from 0 to 1 - each variety being produced by a different firm in a monopolistically competitive market. In presence of Rotemberg's price adjustment costs, the recursive problem of a final producer writes:

$$(\rho^F - \pi)J(p,t) = \max_{\pi} \left(\frac{p}{P_t} - m_t \frac{1}{\tau^F}\right)\left(\frac{p}{p_t^j}\right)^{-\upsilon} C_t - \frac{\tilde{\psi}}{2}\pi^2 C_t + J_p(p,t)p\pi + J_t(p,t) \tag{E.0.1}$$

where ρ_t^F is the discount factor of any firm, $J(p,t)$ is the real value of a firm with price p, P_t is the price level, C_t is aggregate consumption, m_t is the real marginal cost, τ^F is a government's subsidy, and $\tilde{\psi}$ is a coefficient measuring the extent of price adjustment costs. The first order and envelope conditions for the firm are

$$J_p(p,t) = \frac{\tilde{\psi}\pi C}{p}$$
$$\left(\rho^F - \pi\right) J_p(p,t) = -\left(\frac{p}{P} - m\frac{1}{\tau^F}\right)\upsilon\left(\frac{p}{P^j}\right)^{-\upsilon-1}\frac{C}{P} + \left(\frac{p}{P}\right)^{-\upsilon}\frac{C}{P} + J_{pp}(p,t)p\pi + J_{tp}(p,t)$$

By perfect competition within food and non-food industry, and symmetry among firms, we will have $p = P$, and hence

$$J_p(p,t) = \frac{\tilde{\psi}\pi C}{p} \tag{E.0.2}$$
$$\left(\rho^F - \pi\right) J_p(p,t) = -(1 - m\frac{1}{\tau^F})\upsilon\frac{C}{p} + \frac{C}{p} + J_{pp}(p,t)p\pi + J_{tp}(p,t) \tag{E.0.3}$$

Differentiating (E.0.2) with respect to time gives

$$J_{pp}(p,t)\dot{p} + J_{pt}(p,t) = \frac{\tilde{\psi}C\dot{\pi}}{p} + \frac{\tilde{\psi}\dot{C}\pi}{p} - \frac{\theta C}{p}\frac{\dot{p}}{p}\pi$$

Substituting into condition (E.0.3) and dividing by $\theta C/p$ gives

$$\left(\rho^F - \frac{\dot{C}}{C}\right)\pi = \frac{1}{\theta}(-(1 - m\frac{1}{\tau^F})\upsilon + 1) + \dot{\pi}$$

Since profits of firms are accrued to households, we assume discounting of firms is weighted by the marginal utility of the latter, i.e. $\rho^F = \rho + (\sigma - 1)\frac{\dot{C}}{C}$, that implies:

$$\left(\rho+(\sigma-1)\frac{\dot{C}}{C}\right)\pi=\frac{1}{\theta}(-(1-m\frac{1}{\tau^F})v+1)+\dot{\pi} \tag{E.0.4}$$

Notice that the marginal cost is given by the following Dixit-Stigliz price aggregator

$$m_t=\tau^F\left((1-\alpha_e)\left(\left((1-\alpha)\left(\frac{p_{h,t}}{p_t}\right)^{1-\eta}+\alpha\left(\frac{p_{ft}}{p_t}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{1-\varepsilon}+\alpha_e\left(\frac{p_{e,t}}{p_t}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{E.0.5}$$

Importantly, now that perfect competition is ruled out, a profit term Π_t^F coming from the dividends of the final producers is rebated to households; following Wolf (2021), I assume that this dividend term is weighted by household productivity (i.e. it enters the budget constraint as $\Pi_t^F z$, summing to Π_t^F over the crossection, thanks to assumption (2.1.13)); this allows not to have cyclical inequality implied by the dividends' rebating scheme. For simplicity, I assume that profit are zero in steady state thanks to a proper level of τ^F - financed by firm's profit itself in la lump-sum fashion, as in Corbellini (2024).

The algorithm solution needs to be updated from its version of section D.2: point 3 is replaced by a joint computation of the equilibrium w_t, m_t, N_t , by a system of market clearing conditions (2.7.1)-(2.7.3), marginal cost expression (E.0.5) and the union's first order condition (2.4.3) with $\psi=0$. The results in terms of m_t is in turn propedeutical to compute inflation π_t at each point in time using (E.0.4), which replaces point 4.

Figure E.1 reports the impulse response functions of the economy in presence of sticky prices and flexible wage setting.

By looking at the top-centre and bottom-left subplot, we can note how also in this case the implementation of the smoothed policy yields quantitative results which confirm the theoretical implications of the model, even though significantly smaller: the real exchange rate is better stabilized with respect to the moderate hike case, with the aggregate mortgage rate closely tracking the moderate hike counterpart until 2024.

F Sensitivity Analysis

The following robustness checks aims at generating the impulse responses to the same energy shock analyzed in the body of the text, under different parametrizations of the key quantities determining the extent of the appreciation-mortgage cost trade-off, namely the mortgage stock amount D and the steady state real exchange rate $\bar{Q}$, that is determined

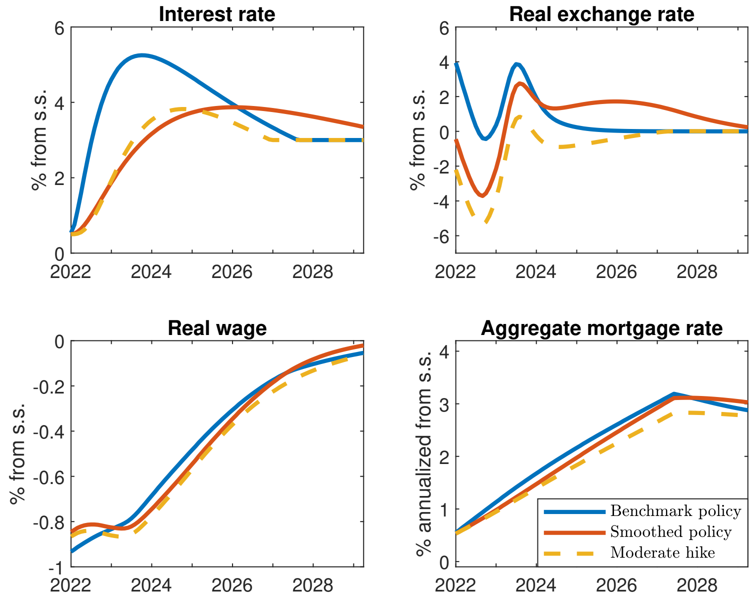

Figure E.1: Impulse response functions to the benchmark energy shock. Benchmark vs. Smoothed policy and Moderate hike. Sticky prices with flexible wage setting.

through leaving a degree of freedom on the foreign consumption parameter C^*, as discussed in section 4.2.

Real exchange movements are a key force behind the monetary policy trade-off explored by the model. Varying values of $\bar{Q}$ effectively result in scaling the entire path of Q_t (shown in Figure 4.3 as percentage deviations), which either dampens or enhances the domestic energy price P^*_{Et}/Q_t and then the impact on real wages. Figure F.1 reports the impulse response functions of the economy under a lower value for $\bar{Q}$, i.e. $\bar{Q} = 1.1$ - recovered by the steady state computation by choosing a proper value of foreign consumption C^*.

As the real exchange rate is characterized by different path in absolute levels but not in percentage deviations with respect to the steady state, the impulse response of Q_t is unchanged with respect to the baseline scenario of Figure 5.4. Real wages instead react more strongly than in the baseline analysis, with the benchmark policy implying a peak fall in w_t of 4% - stronger than the 3% trough illustrated in Figure 5.4. Again, the intuition in terms of trade-off relaxation by the smoothing of the interest rates follows exactly as in the analysis in the body of the paper.

Next, I consider an alternative calibration of the mortgage stock D: I compare the effects of mortgage cost increase on consumption under the benchmark scenario ($D = -50$), with the case of a calibration $D = -30$ (Figure F.2). The mechanisms and trade-off of the baseline scenario are yet unaltered.

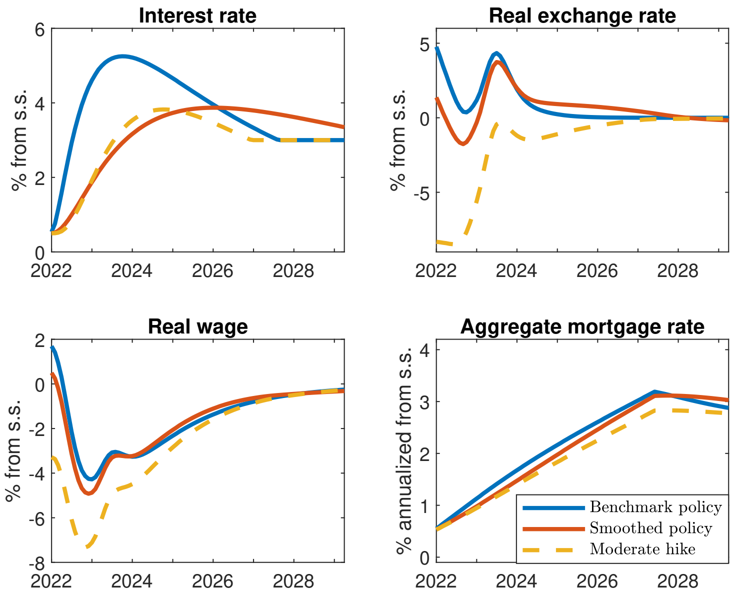

Figure F.1: Impulse response functions to the benchmark energy shock. Benchmark vs. Smoothed policy and Moderate hike. Case $\bar{Q} = 1.1$

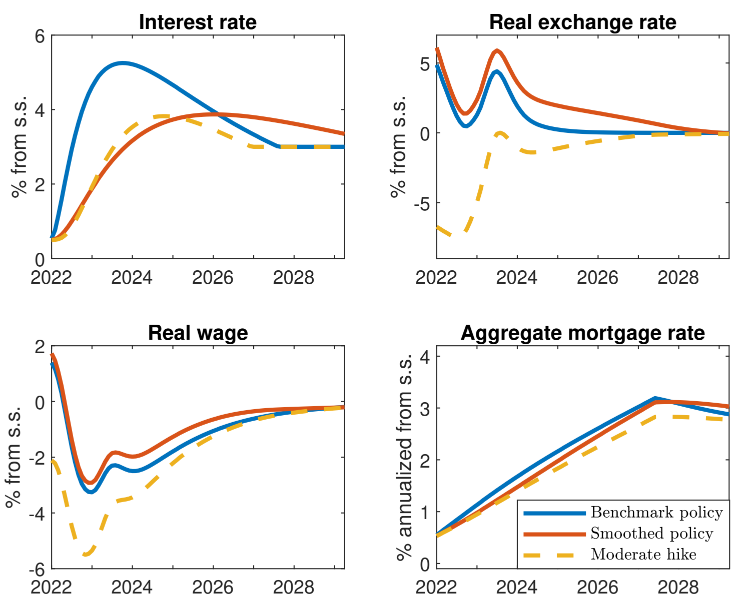

Figure F.2: Impulse response functions to the benchmark energy shock. Benchmark vs. Smoothed policy and Moderate hike. Case $D = -30$