

b UNIVERSITÄT BERN

Faculty of Business, Economics and Social Sciences

**Department of Economics** 

## **Fragility of Secured Credit Chains**

Piero Gottardi, Vincent Maurin, Cyril Monnet

23-04

February, 2023

# **DISCUSSION PAPERS**

# Fragility of Secured Credit Chains\*

Piero Gottardi<sup>†</sup> Vincent Maurin<sup>‡</sup> Cyril Monnet<sup>§</sup>

This Version: February, 2023

#### Abstract

We present a model of secured credit chains in which assets generated from intermediation activity and pledged as collateral create fragility. A dealer stands between a borrower and a financier. The dealer borrows from the financier to fund her project, subject to a moral hazard problem, In addition, the dealer can intermediate between the financier and the borrower, forming a credit chain. Intermediation profits can thus act as collateral for the loan to fund the dealer's own project. When these profits are risky, however, using them as collateral may undermine the dealer's incentives, generating fragility in the chain. The arrival of news about the value of the revenue of the intermediation activity further increases fragility. This fragility channel generates a premium for safe or opaque collateral. The environment considered in our model applies to various situations, such as trade credit chains, securitization and repo markets.

**Keywords**: Collateral, Secured Lending, Intermediation, Fragility

**JEL Codes**: G23, G30

<sup>\*</sup>We thank Giacinta Cestone, Jason Donaldson, Frederic Malherbe, and seminar participants at Cass Business School, the Federal Reserve Board, the University of Madison-Wisconsin, the summer meeting of the Finance Theory Group, UCL and the Virtual Finance Theory Seminar for useful comments.

<sup>&</sup>lt;sup>†</sup>University of Essex.

<sup>&</sup>lt;sup>‡</sup>Stockholm School of Economics.

<sup>§</sup>Corresponding author. University of Bern, SZ Gerzensee. Email: cyril.monnet@unibe.ch. Address: Schanzenheckstrasse 1, 3001 Bern, Switzerland.

### 1 Introduction

In many markets, creditors borrow from lenders via intermediaries. Along such credit chains, the lender's claim on the intermediary is often secured by the intermediary's claim on the borrower. For example, when they borrow, supplier firms can pledge trade credit extended to their customers as collateral, a practice known as factoring. Securitization allows banks to park loans in a Special Purpose Vehicle (SPV) to easily raise debt against them. In the repo market, dealer banks can re-pledge to money market funds the asset they receive as collateral from hedge funds. In all these examples, the loans extended by lenders are secured by implicit or explicit collateral, and lenders frequently have recourse to the balance sheet of the intermediary.

While these arrangements provide a high level of protection to lenders, concerns about contagion risk along secured credit chains are pervasive. In the repo market, financial regulators warn that secured lending based on collateral circulation can generate fragility. Securitization was said to spread mortgage risk in the financial system. In these narratives, intermediaries are often blamed for focusing on short-term profits and taking excessive risk. Once such risk is present, negative shocks would propagate along credit chains, exposing these chains to fragility. In this paper, we present a model of secured credit chains to analyze their potential fragility.

Our main result is that intermediaries may choose to form credit chains even if it exposes themselves and lenders to fragility. Intermediation profits provide additional collateral for intermediaries, which facilitates financing of other activities on their balance sheet. However, the risk of the intermediation activity can contaminate the rest of the intermediary's balance sheet. Despite this negative effect, intermediaries may find it optimal to become fragile in order to reap intermediation profits. We show that fragility is less severe when the intermediary's loan to the borrower is opaque. Finally, the intermediary has incentives to source safe collateral to mitigate fragility.

To explain the mechanisms underlying these results, we now describe our model in more detail. Our credit chain has three risk-neutral agents: a borrower, a lender and an intermediary, called dealer. Both the borrower and the dealer have profitable projects but no funds of their own. The lender has deep pockets but no project. Hence, there are gains from trade but credit is subject to two frictions. First, the dealer's project is

<sup>&</sup>lt;sup>1</sup>For example, the Financial Stability Board (FSB, 2017) remarks that "Collateral re-use can increase the interconnectedness among market participants and potentially contributes to the formation of contagion channels and risks." See ICMA (2019) for a rebuttal of the FSB arguments.

subject to moral hazard, as in Innes (1990). The probability of success of the dealer's project depends on her unobservable effort level, and effort is costly. Moral hazard affecting the dealer's project generates both an increasing cost for raising funds and an endogenous source of default risk. Second, the market is segmented as the borrower can only obtain funding, directly, from the dealer, not from the lender. The dealer can channel funds from the lender to the borrower through a credit chain, by taking on more debt to finance both her own project and her intermediation activity.

Each loan, from the lender to the dealer, and from the dealer to the borrower is secured by the debtor's cash flows. In particular, the dealer can pledge the asset acquired through her intermediation business, including the cash flow of the loan, or the assets pledged by the borrower.<sup>2</sup> We consider the case where the dealer cannot ring-fence assets on her balance sheet. Hence, a loan secured by the intermediation cash flows of the dealer also provides recourse to the rest of the dealer's balance sheet and vice versa. We argue in Section 7 that this recourse feature is relevant to various situations to which our model applies: factoring, repo, and securitization.

We take as a reference point the case where the dealer does not intermediate funds and only finances her own project by taking a small loan from the lender. Due to the moral-hazard problem, the dealer chooses a sub-optimal level of effort and may thus default on the loan. We take the probability of default as a measure of fragility, and study how fragility changes when the dealer intermediates, thus forming a credit chain. The borrower's project has positive NPV. By taking a larger loan to finance both her project and the intermediation activity, the dealer can thus retain a larger share of her project when it succeeds. In other words, intermediation profits act as collateral and increase the dealer's skin-in-the-game for her own project. When the revenue generated by the intermediation activity is safe enough, we show this skin-in-the-game effect is the only effect present. Hence, taking a larger loan to fund the intermediation activity is always profitable and improves the dealer's incentives, thus decreasing her probability of default. In this case, credit chains reduce fragility.

We show, however, that when the revenue from intermediation is sufficiently risky, using it as collateral may increase fragility, as FSB (2017) suggests. The reason is that this risky revenue provides the dealer some hedging against the failure of her own

<sup>&</sup>lt;sup>2</sup>In the repo market, a dealer bank would only enter a reverse repo with a borrower if she can re-use the collateral pledged by the borrower, when she enters a repo with a lender to match her repo book. Singh (2011) argues that dealer banks' ability to re-use collateral is essential to their role as repo intermediaries.

project. Fixing the expected value of the intermediation revenue, more risk implies a higher payoff in case of success. Hence, when the realized cash flow from intermediation is high, the dealer can still repay her entire debt even though her project fails. This feature weakens her incentives to exert effort, thus increasing the default probability of her project. This hedging effect is the second consequence of using intermediation revenue as collateral and goes in the opposite direction of the skin-in-the-game effect. The hedging effect is due to the fact that the dealer's larger debt must be repaid by her project when intermediation fails to generate some cash flow.

When the risk in intermediation revenue exceeds a given threshold, the hedging effect dominates the skin-in-the-game effect. Incentives are weaker, and as a result, the probability of default is higher when the intermediary chooses to intermediate funds and lend to the borrower. In this case, the formation of a credit chain generates fragility in the system. Provided the risk in intermediation revenue is not too high, however, the dealer still prefers a large loan because the profits from intermediation exceed the losses from the negative effect on her incentives. When instead collateral risk is very high, the same effect implies that intermediation is not worth it even if its NPV is positive. Due to the risk contamination, the dealer chooses to intermediate based on the collateral value of the intermediation profits rather than the NPV of the intermediation business.

Our model is general and can be applied to various environments with intermediation and secured lending. In Section 7 we describe three such applications: trade credit, securitization, repos, and we argue our mechanism for fragility is present there. In these markets some intermediary, either a bank or a firm, uses the loans it makes as collateral to raise financing. In all three markets considered, loans taken by the intermediary are typically recourse. This recourse feature exposes the intermediary's balance sheet to contamination from risky intermediation collateral, as in our model. With securitization, for instance, sponsors provide guarantees to the creditors of their Special Purpose Vehicle (SPV) that go beyond the value of the loans held by the SPV. Hence, while securitization allows banks to capture intermediation gains, their balance sheet becomes more fragile when the SPV loans are risky. In general, fragility is the price to pay for the development of secured credit chains with risky collateral.

The third insight from our work is that the fragility of credit chains can further increase through an additional *news channel*. To this end, we extend the model to allow the dealer to receive some news about the cash flow of her loan to the borrower before she chooses the level of effort for her own investment. The dealer can then optimally adjust her effort to this information. She exerts less effort when she learns

that her intermediation revenue is low because the lender will then claim most of the cash flow generated by her project. State-contingency in effort choice induces a positive correlation between the cash flow of the borrower's project and that of the dealer's project. This endogenous correlation generates contagion: a negative shock to the borrower's investment cash flow increases the default probability of the dealer's project.

There is more than just correlation as the intermediary's expected default risk also increases with news. When the intermediation and project cash flows are correlated via the effort decision of the dealer, the value of intermediation profits as collateral for the lender decreases. The collateral value is low exactly when the lender needs it, that is, when the dealer's project fails. He then charges a higher interest rate which in turn induces the dealer to choose a lower level of effort in expectation. Our analysis shows that the news channel exacerbates fragility of secured credit chains.

We also endogenize the arrival of news by examining the case when the dealer can pay a cost to acquire information about the value of the intermediation cash flow. The decision to acquire that information is not observable by the lender. In this situation, the higher the risk of the intermediation revenue is, the higher is the propensity of the dealer to covertly acquire information about it. Anticipating this behavior, the lender will charge a higher interest rate, as explained above. As a consequence, dealers are worse off - ex ante - when the cost of acquiring information is low, which means they prefer an opaque environment where information about collateral is hard to obtain.

Finally, we endogenize the risk of the revenue from intermediation. To this end, we assume borrowers also face a moral hazard problem. In this case, the riskiness of the borrowers' project and therefore of the cash flow from the loan granted by the dealer vary with the face value of this loan. The higher the face value is, the lower is the borrower's incentive to exert effort. We show that the use of the intermediation cash flow as collateral provides the dealer with incentives to sacrifice intermediation profits, by lowering the face value of the loan granted, in order to reduce the riskiness of that cash flow. Although fragility may still arise in equilibrium, dealers are willing to pay a premium for safer collateral.

#### Literature review

Our paper relates to a large literature on fragility and contagion in credit chains and networks. Kiyotaki and Moore (1997) study the propagation of default along credit chains, and Allen and Gale (2000) show that the structure of the network affects the propagation of risk. Subsequent works extended these results by considering either

simple interactions in complex networks or richer relationships in simplified networks. Eisenberg and Noe (2001), Acemoglu et al. (2015) and Cabrales et al. (2017) belong to the first category, and analyze the topology of resilient networks. Our work with endogenous lending contracts belongs to the second category, together with Farboodi (2017) and Di Maggio and Tahbaz-Salehi (2015). As in our model, the intermediary in Di Maggio and Tahbaz-Salehi (2015) is subject to a moral hazard problem. However, in their paper, intermediaries have no investment opportunities, while our focus is on the contagion between the dealer's project and her intermediation business. Similar to Farboodi (2017), in our model the dealer chooses to expose herself to fragility to reap intermediation profits. Unlike in her paper, we study contagion between different loan contracts. With our focus on the spillover between the various activities of dealers, our channel for secured funding fragility differs from the role of fire sales, discussed by Brunnermeier and Pedersen (2009), Kuong (2020) or Biais et al. (forthcoming).

As we explained, collateral re-use and securitization contribute to the formation of secured credit chains and, as such, they are two natural applications of our model.<sup>3</sup> The role of collateral re-use in shadow banking is discussed by Singh and Aitken (2010), Singh (2011) and Singh (2013). Some theoretical analyses of collateral re-use and its role in expanding borrowing are Bottazzi et al. (2012), Andolfatto et al. (2017), Infante (2019) and Gottardi et al. (2019). In this latter work we also showed that collateral re-use can explain the formation of intermediation chains. Our contribution is to show that the circulation of collateral along credit chains may generate fragility. Our mechanism is different from Park and Kahn (2019) who emphasize asset misallocation costs if the intermediary fails to return re-used collateral. In our model, all agents have a common valuation for assets and fragility is due to risk contamination between the re-used collateral and the intermediary's balance sheet.

Loan securitization also contributes to the formation of secured credit chains as intermediaries finance collateralized loans by issuing debt via Special Purpose Vehicles. Several works, including Keys et al. (2010), Purnanandam (2011) and Piskorski et al. (2015) show that securitization led issuers to apply lax standards for subprime loans.

<sup>&</sup>lt;sup>3</sup>Pyramiding shares many features with collateral re-use, with one difference that the asset repledged is the cash flow of the loan granted rather than the asset pledged by the borrower to secure this loan. Similarly to the results on collateral re-use, Gottardi and Kubler (2015) show that pyramiding relaxes collateral constraints by allowing an efficient use of existing collateral (see also Geanakoplos and Zame, 2010). See Maurin (2020) and Muley (2016) for a comparison between pyramiding and re-use.

<sup>&</sup>lt;sup>4</sup>See Bubb and Kaufman (2014) for a critic of these results. Plantin (2011) shows theoretically that a greater level of securitization, even though it leads to less screening by lenders, needs not generate an inefficient outcome.

Theoretically, Chemla and Hennessy (2014) and Vanasco (2017) show that investors purchasing securitized loans face asymmetry of information as intermediaries acquire private information when screening borrowers before selling these loans. All these works argue that securitization can reduce the quality of the loans extended by an intermediary. We show instead that securitization can also make credit chain fragile by increasing the default risk of other investments on intermediaries' balance sheet.

At a fundamental level, our analysis highlights a negative effect of higher pledge-ability of assets, a theme also present in Donaldson et al. (2020). These authors show that an increase in asset pledgeability leads firms to issue secured debt in order to dilute pre-existing unsecured debt. In our model, intermediation opportunities similarly generate additional collateral to secure financing. Our mechanism is different, however, as we highlight the contamination from the risk of the intermediation profits used as collateral to other balance sheet assets.<sup>5</sup> A contamination effect under joint financing is also present in Banal-Estañol et al. (2013) but the mechanism is different, as it relies on default costs, while ours is due to moral hazard.<sup>6</sup> Besides, our focus on intermediation chains leads to different predictions if collateral quality is endogenous.

Finally, we identify a news channel for fragility along credit chains whereby access to information about the value of assets used as collateral increases default risk. Our result, suggesting that opacity about collateral cash flows may be optimal is reminiscent of Dang et al. (2015), Gorton and Ordoñez (2014) and Monnet and Quintin (2017). Our news channel, however, is different from the Hirshleifer (1971) effect at play in these papers. In our model, fragility arises because the intermediary correlates her effort choice with the collateral value when she can acquire information about this value.

The rest of the paper is structured as follows. Section 2 presents the model. We study a benchmark economy without intermediation in Section 3. Our main results about intermediation with secured lending chains and fragility are gathered in Section 4. Section 5 shows that fragility worsens in the presence of news about assets pledged as collateral. In Section 6, we endogenize the quality of the collateral, that is the risk of the revenue of the intermediation activity. Finally, Section 7 discusses applications of our model and Section 8 concludes. All proofs are in the Appendix.

<sup>&</sup>lt;sup>5</sup>Bernhardt et al. (2020) clarify that increased cash flow pledgeability never harms the firm issuing the debt in the environment of Donaldson et al. (2020). Similarly, in our paper, the intermediary is always better-off when she can intermediate because she can forgo the investment opportunity if the contamination effect is too strong.

<sup>&</sup>lt;sup>6</sup>See also Bahaj and Malherbe (2020).

## 2 Model

### 2.1 Technology and Preferences

The economy has two dates t = 0, 1. There is one good, called cash. There are three risk neutral agents: B, whom we call the borrower, D, the dealer intermediary, and L, the lender (we can equivalently think there is a plurality of agents acting as lenders). The latter has a large initial endowment of cash. B and D have instead no cash, but they are both endowed with a risky project that requires an investment of size 1 in the initial period. The lender may thus be asked to provide 2 units of cash overall.

The project of the borrower matures at date 1 and pays off  $X_B$  with probability  $p_B$  and 0 otherwise. The dealer's project also matures at date 1 and pays off  $X_D$  in case of success and 0 otherwise.

The probability  $p_D$  of success of this project is endogenously determined by D's effort choice. More precisely, D chooses  $p_D$  at the end of date 0 at a utility cost  $\frac{1}{2}X_Dp_D^2$ . We refer to  $p_D$  both as the effort choice and the probability of success of D's project.<sup>8</sup> We assume that the payoff  $X_D$  is larger than  $X_B$ , which will simplify the analysis of borrowing contracts, as we explain below.

Assumption 1.  $X_B < X_D$ 

### 2.2 Frictions and Contracting

There are frictions and some restrictions on admissible trades that can limit the gains arising from L lending to agents B and D.

Moral Hazard

The dealer's project is subject to moral hazard as in Innes (1990) or Holmström and Tirole (1998): she cannot commit ex-ante to an effort choice. The socially optimal level of effort maximizes the expected payoff of D's project net of the effort cost, and is given by  $p_D^* = 1$ . It is thus optimal that the dealer's project always succeeds, but when D finances her project with a loan, she will choose a level of effort  $p_D < 1$ , due to the moral hazard problem. D's project can be interpreted as a portfolio of loans with cash

<sup>&</sup>lt;sup>7</sup>Although we consider for simplicity the case where the dealer has no funds of her own, the results extend to the case where the dealer has some limited funds, so she must still borrow to finance her activities.

<sup>&</sup>lt;sup>8</sup>The quadratic cost function is used for tractability. All that matters is that the cost is increasing and convex in the probability of success.

flows contingent on D's monitoring effort. For most of the analysis, the probability of success of the borrower's project is taken as exogenous. In Section 6, we relax this assumption to consider the case where B's project is also subject to moral hazard.

#### Segmented Market

The borrower and the lender cannot trade directly, while the dealer can trade with both of them. D can thus borrow from L to finance both her project and her loan to B, playing the role of an intermediary. The segmentation of the market reflects the institutional settings we present as applications of our model in Section 7.

#### Loan Contracts

Loan contracts between B and D, and between D and L, specify the amount borrowed and the repayment. We require the contract repayment to be monotonically increasing with the borrower's total cash flow, as in Innes (1990). It then follows that a debt contract with a fixed repayment is optimal for all loan contracts. For the one-unit loan from D to B, this is immediate: given B's project binary payoff structure, all contracts are debt contracts. When D intermediates, however, the loan from L to D, would be secured both by D's own project and D's loan to B. In this case, optimality of debt follows from the monotonicity constraint on D's total cash flows and our assumption  $X_B < X_D$ , which implies the MLRP holds (see Innes (1990)).  $^{10}$ 

Notice that the monotonicity constraint on the borrower's total cash flow ensures that the dealer has no gains from dividing a two-unit loan into two one-unit loan. More precisely, it has similar implications as the assumption that the dealer cannot ring-fence her assets to prevent that creditors of one-unit loan have recourse to her other asset. We will argue that all applications considered in Section 7 feature debt contracts with recourse, and thus that our contracting restriction applies.

#### Bargaining Power

Given the assumed segmentation of the market, B is only able to fund his project if D chooses to intermediate funds with L. We focus on the case in which the dealer has all the bargaining power both with the borrower or the lender. We show in Appendix A that our main results are robust to different specifications of bargaining power.

<sup>&</sup>lt;sup>9</sup>A common motivation for this assumption is that lenders would otherwise have incentives to sabotage the borrower's project.

<sup>&</sup>lt;sup>10</sup>Suppose the dealer extends a loan with repayment  $R_B \leq X_B$  to the borrower. The dealer's total cash flow is the sum of the cash flow of the loan to B, 0 or  $R_B$  and the cash flow of her own project, 0 or  $X_D$ . MRLP holds if for any  $(Y_1, Y_2) \in \{0, R_B, X_D, X_D + R_B\}^2$  with  $Y_1 < Y_2$ , the probability of total cash flow  $Y_2$  relative to that of  $Y_1$  increases with the dealer's effort choice  $p_D$ . Ordering total cash flows from smallest to largest, the probability distribution is  $\{(1-p_B)(1-p_D), (1-p_B)p_D, p_D(1-p_B), p_Dp_B\}$ . One can then easily verify that MRLP holds.

Finally, we assume the net present value of the borrower's project is positive but not too large:

## Assumption 2. $p_B X_B \in (1, \frac{3}{2})$ .

We similarly assume that D's project has positive net present value, taking into account the fact that the probability of success of this project is determined endogenously by D's incentives to exert effort. More specifically, the following condition states that the payoff  $X_D$  of D's project in case of success is high enough to ensure that lenders break even when they grant a one unit loan to D secured only by D's project:

### Assumption 3. $X_D \ge 4$ .

The upper bound on  $p_B X_B$  and lower bound on  $X_D$  imposed in the two assumptions above ensure that D always prefers to finance her project over intermediating when she only borrows 1 unit of funds. As the upper bound on  $p_B X_B$  is lower than 2, it also implies D must pledge her project to secure a large 2-unit loan.

### 3 No Intermediation

As a benchmark, we first analyze the situation without intermediation, that is, when D cannot lend to B. Then, the only loan relationship is between D and L.<sup>11</sup>

At t = 0, D borrows one unit from L to finance her own project. The lender's participation constraint is

$$p_D R_D \ge 1. \tag{1}$$

where  $R_D$  is the face value of the debt. The dealer cannot commit to the socially optimal choice of effort  $(p_D^* = 1)$  and chooses  $p_D$  to maximize her own payoff,

$$p_D \max\{X_D - R_D, 0\} - \frac{1}{2}X_D p_D^2,$$
 (2)

given the face value of the debt  $R_D$ . If the dealer's project succeeds, she repays her debt and retains the residual cash-flow  $X_D - R_D$  if positive. If instead her project fails,

 $<sup>^{11}</sup>$ Our no-intermediation benchmark can also be interpreted as a situation where D can lend to B, but where she cannot (re)pledge the cash flow of this loan to L. We show in Appendix B that in this case, D chooses not to intermediate unless intermediation is very profitable. Perhaps surprisingly, we will show later in Section 4 that D might still choose not to intermediate even when the cash flow of the loan to B is pledgeable.

she makes no payment to her creditors (she has no other assets in this case). Solving for the profit-maximizing value of  $p_D$  in (2), we obtain

$$p_D = \frac{X_D - R_D}{X_D},\tag{3}$$

which is decreasing in the face value of the debt  $R_D$  because a higher repayment obligation weakens incentives.

The dealer has all the bargaining power. She will thus set  $R_D$  so as to maximize her expected utility (2) subject to L's participation constraint (1), and anticipating her optimal ex-post effort choice given by (3). Plugging (3) into (2), one can see that the expected utility of the dealer is decreasing in the face value of her loan  $R_D$ . Hence D will choose the lowest value of  $R_D$  that satisfies the participation constraint of the lender which, after substituting (3), can be rewritten as follows:

$$(X_D - R_D) R_D \ge X_D \tag{4}$$

From this expression we obtain the following result.

**Proposition 1.** Without intermediation, the face value of the lender's loan to the dealer is given by

$$R_D = \frac{2}{1 + \sqrt{1 - \frac{4}{X_D}}}\tag{5}$$

The dealer's effort choice and her utility are respectively:

$$p_D = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{X_D}} \tag{6}$$

$$U_D = \frac{1}{2}p_D^2 X_D \tag{7}$$

The default probability  $(1 - p_D)$  is strictly positive and so is the net interest rate  $R_D - 1$  to compensate lenders for the risk of default. The default risk reflects the moral hazard problem of D. If she could commit to the optimal effort level  $p_D^* = 1$ , then  $R_D - 1$  would be zero, equal to the opportunity cost of funds for the lender. Without commitment, D's effort level is lower because some of the cash flow from her project accrues to the creditor. The larger the project's cash flow  $X_D$  (fixing the unit financing cost), however, the larger the share of the cash flow of the project D can retain after repaying L. Hence, when  $X_D$  is larger, D has more skin in the game, which mitigates

the moral hazard friction and reduces the interest rate, as can be seen from (5).

### 4 Credit Chain

In this section we consider the case where the dealer is able to channel funds from the lender to the borrower. As before, D may choose to borrow only 1 unit of funds from L to finance her project. However, D can now borrow 2 units of funds from L, lend 1 unit to B and invest the second unit in her own project. To borrow these 2 units, D can either get a single 2-unit loan backed by the two assets she acquires with the loan, her project and the cash flow of the loan extended to B, or two distinct 1-unit loans backed by each asset. With no-ring-fencing, these two credit arrangements are equivalent  $^{12}$ . To simplify the exposition, we analyze the version where D takes a single 2-unit loan from L.

With intermediation, D, who has all the bargaining power, sets the face value  $R_B$  of the loan she extends to B and the face value  $R_D^i$  of the loan she gets from L, where the superscript i identifies the case with intermediation. If she lends to B, the dealer sets  $R_B = X_B$  to capture the entire cash flow of B's project when successful.<sup>13</sup>

We now turn to the lending relationship between L and D. When she intermediates, D raises a 2-unit loan backed by two assets: her project, as before, and the cash flow of the loan to B. We denote  $R_D^i$  (resp.  $p_D^i$ ) the face value set by D for her loan from D (her effort choice). We guess and later verify that  $R_D^i \leq X_D$ , so the dealer can fully repay her loan to L when her project pays off. Because she intermediates, she can now also pay back at least a portion of her debt when her own project fails but her loan to B pays off. The payoff to D in this case is min  $\{R_D^i, X_B\}$ . For any given face value  $R_D^i$  of the debt, the dealer chooses the level of effort that maximizes her expected utility,

$$\max_{p_D^i} \left\{ p_D^i (X_D + p_B X_B - R_D^i) + (1 - p_D^i) p_B \max \left\{ X_B - R_D^i, 0 \right\} - \frac{1}{2} X_D \left( p_D^i \right)^2 \right\}.$$
 (8)

<sup>&</sup>lt;sup>12</sup>We show this equivalence formally in Appendix D. In all the applications considered in Section 7, loans have this recourse feature.

 $<sup>^{13}</sup>$ The environment is thus equivalent to one where D owns both projects. This equivalence breaks down if instead B has all the bargaining power with D, a case we analyze in Appendix A to show our results are robust. It breaks down too when B's project is also subject to moral hazard, a situation considered in Section 6. In both cases, the distinction between a credit chain and a single borrower financing multiple investments becomes essential.

Solving (8) for the optimal effort choice, yields

$$p_D^i = \frac{X_D - R_D^i + p_B X_B - p_B \max\{X_B - R_D^i, 0\}}{X_D}.$$
 (9)

As in the case without intermediation, the dealer sets the face value of the debt  $R_D^i$  taking as given her ex-post choice  $p_D^i$  in (9). She chooses  $R_D^i$  to maximize her expected utility, in (8), subject to the lender's participation constraint:

$$p_D^i R_D^i + (1 - p_D^i) p_B \min\{R_D^i, X_B\} \ge 2.$$
 (10)

Before stating the result, it useful to study how intermediation affects the dealer's incentives. Comparing (9) with (3), we see that two new terms appear in the numerator of the right-hand-side of the expression in (9) describing D's effort choice. These terms correspond to two opposite effects of intermediation on D's incentives.

The first term,  $p_B X_B - (R_D^i - R_D)$ , captures a "skin-in-the-game" effect. It is equal to the additional fraction of her project D can retain when successful with a 2-unit loan and intermediation compared to a 1-unit loan without intermediation. The larger this term, the stronger D's incentives are. To see clearly how this effect operates, consider the case  $R_D^i \geq X_B$ , where the participation constraint of L in (10) can be rewritten as

$$p_D^i(R_D^i - p_B X_B) \ge 2 - p_B X_B \tag{11}$$

Contrasting equations (8) and (11) when  $R_D^i \geq X_B$  with the corresponding equations (2) and (1) in the no-intermediation case, we see that, when she intermediates, D gets the same payoff as if she would borrow only  $2 - p_B X_B$  units with a net repayment  $R_D^i - p_B X_B$  to finance her project. Hence, D uses the expected intermediation profit,  $p_B X_B - 1$ , to reduce the net borrowing need for her own project below 1 unit. The corresponding reduction in her net debt level  $(R_D^i - p_B X_B < R_D)$  means that D has more skin-in-the game in her project, which strengthens her incentives to exert effort.

When  $R_D^i \geq X_B$  the skin-in-the-game is the only effect present and intermediation is unambiguously profitable for D. When  $X_B > R_D^i$ , however, this is no longer true as the second term in (9), equal to  $-p_B \max\{X_B - R_D^i, 0\}$ , has a strictly negative value. It captures an additional, negative effect of intermediation on incentives. We call it a "hedging effect", because, as we can see from the expression of the dealer's expected utility in (8), the loan to B provides a partial hedge to D against the failure of her own

project. When D's project fails, she still obtains a positive payoff whenever B's project succeeds. Naturally this hedge weakens D's incentives.

Furthermore, the hedging effect is more likely to be present, and is stronger, the riskier the revenue of the intermediation activity is. To see this, consider varying  $p_B$  and adjusting  $X_B$  so that the expected payment of the loan to B,  $p_B X_B$ , remains constant. As  $p_B$  decreases, both  $X_B$  and the variance of the payoff increases, and the hedging term  $p_B \max \{X_B - R_D^i, 0\}$  in (9) becomes larger. We will show that, when the intermediation revenue is sufficiently risky, this negative hedging effect on incentives becomes so strong as to trump the positive skin-in-the-game effect we described above.

In the proposition below, we characterize the optimal choice of the dealer, in terms of quantity borrowed, face value of the debt and effort level, for all levels of risk of the loan to B. To capture loan risk, we vary  $p_B$  while keeping the expected value of the loan  $p_B X_B$  constant. Hence, a lower value of  $p_B$  corresponds to a higher level of risk. To determine the optimal loan size we compare D's utility with the optimal 2-unit loan under intermediation, obtained solving problem (8) subject to (10), with the optimal 1-unit loan without intermediation, characterized in Proposition 1.

**Proposition 2.** Consider all positive values of  $p_B$  and  $X_B$  such that  $p_B X_B$  is a given constant satisfying Assumption 2. When D can intermediate, there exist thresholds  $\underline{p}_B$  and  $\bar{p}_B$  with  $0 < \underline{p}_B < \bar{p}_B \le 1$  such that:

- 1. when intermediation revenue is very risky  $(p_B \leq \underline{p}_B)$ , D prefers to borrow 1 unit to finance only her project,
- 2. when intermediation revenue is somewhat risky  $(p_B \in (\underline{p}_B, \bar{p}_B))$ , D borrows 2 units from L. As intermediation becomes less risky, that is, as  $p_B$  increases, the effort choice  $p_D^i$  increases and the face value  $R_D^i$  decreases and  $R_D^i < X_B$ .
- 3. when intermediation revenue is safer  $(p_B \geq \bar{p}_B)$ , D borrows 2 units from L. The effort choice  $p_D^i$  and the face value  $R_D^i$  are constant with respect to  $p_B$ , and  $R_D^i \geq X_B$ . In this case,  $p_D^i > p_D$ , that is, the dealer exerts more effort with intermediation than without it.

<sup>&</sup>lt;sup>14</sup>This argument ignores the effect of a decrease in  $p_B$  on the face value  $R_D^i$ . As the negative hedging effect becomes stronger when  $p_B$  decreases, the lender will react by increasing the face value  $R_D^i$ . As we show below, the net effect of  $p_B$  on the term  $p_B R_D^i$  is still positive when accounting for the endogenous adjustment of  $R_D^i$ . Our next result confirms that a decrease in  $p_B$  while keeping  $p_B X_B$  constant weakens the dealer's incentives.

Proposition 2 delivers a surprising result: when the loan to B is very risky, D chooses not to intermediate. Remember that under Assumption 2, B's investment has positive NPV, and D, having all the bargaining power, is able to appropriate the full return from this investment. When the loan to B is risky, however, intermediation profits have a strong negative (hedging) effect on D's incentives to exert effort on her project. In other words, the risk from the intermediation activity contaminates the rest of D's balance sheet. When intermediation profits are too risky ( $p_B \leq \underline{p}_B$ ), this effect is so strong as to induce D to turn down a profitable investment opportunity.

The second part of Proposition 2 describes the cases in which D chooses to intermediate (Cases 2 and 3).<sup>15</sup> The negative hedging effect is still present when the risk of the intermediation revenue is in a middle range: in this case the probability of success of D's project decreases with intermediation risk. The hedging effect disappears when intermediation risk is low  $(p_B \ge \bar{p}_B)$  in which case only the skin-in-the-game effect is active, and D's incentives are stronger with than without intermediation.

With intermediation, D borrows 2 units from L secured both by her project cash flow and the revenue from her loan to B. With respect to the situation without intermediation, we can view the intermediation profits  $p_BX_B - 1$ , given by the difference between the expected cash flow of the loan to B and the unit cost of funds, as additional collateral which helps D finance her project. The usefulness of this collateral, however, depends on the riskiness of the intermediation loan. As we showed, the risk in the collateral can contaminate D's project, and when the risk is too high, D forgoes the intermediation profits. To summarize, because D is financially constrained, her decision to intermediate is not driven solely by the net present value of such opportunity, but also by the value of intermediation profits as collateral which depends on their riskiness.

We now analyze whether the credit chain can be more fragile than the situation without intermediation. Proposition 2 shows that D may prefer to intermediate when the negative hedging effect is present and counteracts the positive skin-in-the-game effect, provided the first one is not too large (Case 2). One may think this happens because the skin-in-the-game effect prevails, which implies incentives are still stronger with intermediation than without. However, this needs not be the case. There is a range of values of risk of the cash flow of the loan from D to B for which D prefers a 2-unit loan even though this leads to a higher level of default on her own investment.

 $<sup>^{15}</sup>X_B$  decreases with  $p_B$  since  $p_BX_B$  is constant, and since in region 2  $R_D^i$  decreases with  $p_B$ , the transition from region 2 to region 3 is only due to the fact that  $X_B$  falls faster than  $R_D^i$ .

**Proposition 3.** For the same set of values of  $p_B$ ,  $X_B$  considered in Proposition 2, there exists a range of levels of risk of the intermediation revenue  $(\underline{p}_B, p_B^*)$ , with  $p_B^* \in (\underline{p}_B, \overline{p}_B)$ , such that for all  $p_B$  in that range, the dealer strictly prefers to borrow 2 units with intermediation and exerts less effort than without intermediation, that is,  $p_D^i < p_D$ .

For the levels of intermediation risk identified in the proposition, intermediation makes D's balance sheet more fragile, as the probability of default on her own investment increases. To understand why the dealer may choose to expose herself - and the lender - to a higher level of default, it is useful to examine the expression of D's utility at the optimal loan contract. Using equations (8) and (9) we obtain

$$U_D^i = \frac{1}{2} (p_D^i)^2 X_D + p_B \max \{X_B - R_D^i, 0\}.$$
 (12)

The second term in (12) is D's expected payoff conditional on her own investment failing, which reflects the strength of the hedging effect. When  $p_B \leq \bar{p}_B$ , this second term is positive as shown in Proposition 2. Comparing (12) with the corresponding value (7) with no intermediation, evaluated at the optimal effort level for each case, it is easy to see that  $U_D^i$  can be larger than  $U_D$  even when there is a higher default risk with intermediation  $(p_D^i < p_D)$ . The reason is that the intermediation profits of D, when the hedging effect is present, generate a second, strictly positive term in (12) that may dominate the cost of a higher failure rate. So D may prefer to engage in risky intermediation activity even if using investment profits as collateral increases the fragility of her other investments on her balance sheet.

These results are illustrated in Figure 1 for the following parameter values:  $X_D = 5.3$ ,  $p_B X_B = 1.1$ . The red, solid curve in the top (bottom) panel of Figure 1 is the level of effort (utility) of the dealer at the optimal 2-unit loan with intermediation as a function of the riskiness of the intermediation loan, described by  $p_B$ . Below the threshold  $\bar{p}_B = 0.48$ , both the level of effort and the utility increase with  $p_B$ , that is, they decrease when intermediation risk increases. The blue solid lines present the corresponding variables for the optimal 1-unit loan without intermediation; they are both independent of intermediation risk, as in this case D does not lend to B. In the top and the bottom panels, the intersections between the blue line and the red line define the thresholds, respectively,  $p_B^*$  and  $\underline{p}_B$ . In Figure 1 we see that fragility arises in the region [0.34, 0.43] where D prefers intermediation despite the increased fragility of her balance sheet.

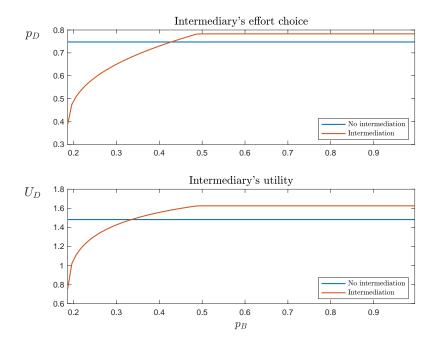


Figure 1: Fragility with intermediation. Threshold values are:  $\underline{p}_B = 0.34$ ,  $p_B^* = 0.43$ ,  $\bar{p}_B = 0.48$ .

## 5 Information and Fragility

In this section we show that intermediation can generate additional fragility through a news channel. We extend the analysis to allow the dealer to acquire, at a cost  $\gamma$ , a signal about the realization of the cash flow of the borrower's project. Without loss of generality, we assume the signal is fully informative and reveals whether B's project succeeded or not. D acquires the signal at the end of period 0, before choosing her level of effort. In addition to the moral hazard problem concerning her project, D is unable to commit to an information acquisition choice.

In Section 5.1 we study how the arrival of news affects D's incentives and hence the face value of the loan she can enter. We show that the fragility induced by intermediation is amplified when D receives news about the cash flow of the loan she extends. Despite increased fragility, we show in Section 5.2 that D chooses to acquire information at the interim stage, provided the cost  $\gamma$  is not too high.

### 5.1 Intermediation with news

Information about the cash flow of the borrower's project has no effect on the analysis of the case without intermediation. When the dealer borrows one unit, B's project is

not funded and there is no information to acquire.

With intermediation, the arrival of news regarding B's cash flow matters because these cash flows (together with D's project) secure D's 2-unit loan. Hence, such information affects D's incentives to exert effort on her project: her effort decision at the end of date 0 is now contingent on the signal received.

To analyze this effect more formally, say the news can be either good (g), when B's project is successful, or bad (b) when B's project fails. For each news realization  $s \in \{b, g\}$ , D now chooses her effort  $p_{Ds}^{i,n}$  to maximize her expected payoff conditional on the news received. With a slight abuse of notation, we let  $X_{Bs}$  denote the expected cash flow of B's project when news s arrives, with  $X_{Bb} = 0$  and  $X_{Bg} = X_B$ . Let also  $R_D^{i,n}$  denote the repayment due by the dealer to the lender for a 2-unit loan when it is known she will receive information about the value of B's cash flow. D's effort choice problem for news realization  $s \in \{b, g\}$  is then:

$$\max_{p_{D,s}^{i,n}} \left\{ p_{Ds}^{i,n} \left( X_D + X_{Bs} - R_D^{i,n} \right) + (1 - p_{Ds}^{i,n}) \max \left\{ X_{Bs} - R_D^{i,n}, 0 \right\} - \frac{1}{2} X_D \left( p_{Ds}^{i,n} \right)^2 \right\}$$
(13)

As in the previous section, we guess and then verify that, for the contract chosen by the dealer, the face value of the debt is such that  $R_D^{i,n} \leq X_D$ , that is, D can always fully repay the lender when her project succeeds. Given this property, D's effort choice when news s arrives is given by

$$p_{Ds}^{i,n} = \frac{X_D + X_{Bs} - R_D^{i,n} - \max\{X_{Bs} - R_D^{i,n}, 0\}}{X_D}$$
(14)

The key difference with respect to the expression obtained in (9) for the optimal level of effort without news is that the effort choice of the dealer is positively correlated with the realized value  $X_{Bs}$  of B's cash flow. In state b,  $X_{Bb} = 0 < R_D^{i,n}$ , so intermediation profits have no collateral value, which means the dealer's debt is backed only by her project. In this event D captures a lower share of her project returns and thus chooses to exert less effort. Hence, bad news about intermediation profits induces D to lower her effort level which increases her default probability. News generates contagion.

News allows the dealer to tailor her effort choice to the value of her intermediation profits. She thus enjoys an ex-post – that is, once her debt's face value is set – information rent. To find the value of this rent, we can again rewrite the dealer's utility using

the expression for the optimal effort choice derived in equation (14). We obtain

$$U_D^{i,n} = \frac{1}{2} \mathbb{E} \left[ \left( p_{D,s}^{i,n} \right)^2 \right] X_D + p_B \max \left\{ 0, X_B - R_D^{i,n} \right\}$$

$$= \frac{1}{2} \left( \mathbb{E} \left[ p_{D,s}^{i,n} \right] \right)^2 X_D + \frac{1}{2} \mathbb{V}ar \left[ p_{D,s}^{i,n} \right] X_D + p_B \max \left\{ 0, X_B - R_D^{i,n} \right\}$$
(15)

If D did not receive any news, her effort would be constant and her utility, for the same face value  $R_D^{i,n}$  of the debt, would be given by the sum of the first and third terms of equation (15).<sup>16</sup> Hence, D's benefit of receiving information – her ex-post information rent – is captured by the second term,  $\frac{1}{2} \mathbb{V}ar\left[p_{D,s}^{i,n}\right] X_D$ . The information rent is proportional to the variance of the effort choice because D uses this information to correlate her effort with the value of the intermediation revenue.

Although the dealer enjoys an ex-post information rent from the arrival of news, she may not benefit ex-ante from the arrival of news. The ex-post rent of the dealer constitutes an ex-post loss for the lender due to the correlation between D's effort choice and the value of her intermediation profits, induced by the arrival of news. To see why this correlation is costly to the lender, observe that D seizes the cash flows of the loan to B when the dealer's project fails. But this project is more likely to fail when the intermediation revenue is low, because bad news reduces D's incentives to exert effort. Hence, the cash flow of D from her intermediation business offers a worse collateral protection to lenders precisely when they need it the most. Anticipating this effect, the lender will charge a higher interest rate than in the case without news to ensure

$$p = \frac{X_D + p_B X_B - R_D^{i,n} - p_B \max\left\{X_B - R_D^{i,n}, 0\right\}}{X_D}.$$

This expression is equal to  $\mathbb{E}\left[p_{D,s}^{i,n}\right]$  where  $p_{D,s}^{i,n}$  is given by (14). Substituting then this value in the expression (12) of D's utility obtained in the previous section yields the result.

<sup>17</sup>To see that the lender's ex-post payoff is lower with news, suppose the face value of D's loan were the same with or without news, that is,  $R_D^{r,n} = R_D^r$ . As discussed above, D's expected level of effort would be the same with or without news, that is,  $\mathbb{E}[p_{Ds}^{r,n}] = p_D^r$ . Using equation (10) in the case with news, the expected payoff of the lender with news is then:

$$U_L = \mathbb{E}[p_{Ds}^{r,n}]R_D^r + (1 - \mathbb{E}[p_{Ds}^{r,n}])p_B \min\{X_B, R_D^r\} - \mathbb{C}ov\left[p_{Ds}^{r,n}, \min\{X_{Bs}, R_D^r\}\right]$$
(16)

The first two terms of this equation give the lender's utility without news. Hence, the positive correlation between D's effort choice,  $p_{Ds}^{r,n}$  and the value of the collateral  $X_{Bs}$  show that the lender's ex-post utility is lower with news.

<sup>&</sup>lt;sup>16</sup>This can be seen using (9) to derive D's optimal effort choice without news when debt face value is  $R_D^{i,n}$  and to show it is equal to

his participation constraint is still satisfied. This in turn has a negative effect on D's incentives, lowering her *expected* effort and utility.

Our next result shows that the negative effect of a higher debt burden with news trumps the ex-post benefit from information for the dealer.

**Proposition 4.** With intermediation, the expected probability of default of the dealer is always higher and her expected utility lower in the presence of news than without news.

With news, any level of risk in the cash flow of the loans extended by D always lowers the profits generated by intermediation. This result contrasts with Proposition 2 in which we showed that, below a certain level of risk, a risky intermediation profit was as good collateral as a safe one to support D's financing of her project.

We show next that, although the credit chain is even more fragile with news, fragility may still be the optimal choice for the dealer: she may borrow 2 units so as to reap the intermediation profits, even when she is more likely to default than when she borrows only one unit.

**Proposition 5.** In the presence of news there exist thresholds  $\underline{p}_B^n > \underline{p}_B$  and  $p_B^{*,n} > p_B^*$  such that the dealer prefers a 2-unit loan with intermediation when  $p_B \geq \underline{p}_B^n$  and there is fragility if  $p_B \in [\underline{p}_B^n, p_B^{*,n}]$ . Hence, with news, fragility arises for lower levels of risk of the loan to borrowers.

The above result shows that the weakening of incentives induced by the arrival of news reduces the region of risk levels of B's project for which intermediation occurs. At the same time, with news the region where fragility occurs shifts and includes higher values of  $p_B$ , for which the cash flow of the borrower's project is safer. As we saw, even without news, the dealer chooses sometimes to expose herself to fragility in order to reap intermediation profits. With news, the ex-post information rent captured by D described by the second term of equation (15) is an additional force increasing her propensity to make choices leading to fragility.

Figure 2 illustrates these results for the same parameter values used in Figure 1. The yellow curves show the probability of success (in the top panel) and the expected utility (in the bottom panel) of D when she intermediates funds between B and L in the presence of news. We see that for every value of  $p_B$  these curves lie strictly below the red curves, showing the corresponding values without news, reported in Figure 1 and again for convenience in Figure 2. This shows the negative effect of news on incentives and welfare. As a consequence, for values of  $p_B$  lying between 0.34 and 0.61

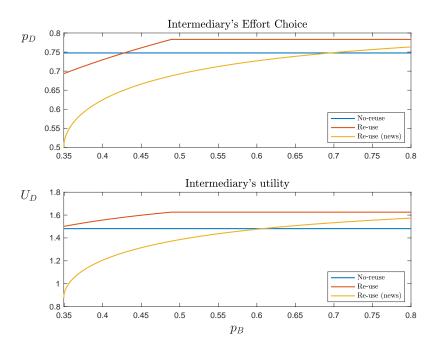


Figure 2: News-driven fragility. Threshold values are:  $\underline{p}_{B}^{n}=0.61, p_{B}^{n,*}=0.70.$ 

the dealer chooses to intermediate funds without news, but refrains from doing so when she receives news. We also see, in line with Corollary 5 that with news the fragility region shifts to the right, where the risk of the intermediation revenue is lower, as effort is lower. Fragility obtains with news when  $p_B$  lies in the interval [0.61, 0.70], while without news fragility happens in the region [0.34, 0.43] (see Figure 1).

## 5.2 Information Acquisition

So far, we took the arrival of information about the borrower's cash flow, and thus intermediation profits as exogenous. We now consider the case where the dealer can choose to acquire information at the interim stage. We showed in Proposition 4 that D is unambiguously worse-off in the presence of news. The decision to acquire information, however, takes place after the loan contract with L is signed, and D cannot commit not to acquire it. The next result then establishes that, even when information is costly, the dealer chooses to acquire it, provided the cost is sufficiently small. In stating the result, we use  $Var[X_B]$  to denote the variance of the borrower's project funded when intermediation occurs and focus on the case where this project is sufficiently safe  $(p_B \geq \bar{p}_B)$ , for simplicity.

**Proposition 6.** When  $p_B \geq \bar{p}_B$ , the dealer chooses to acquire information at the interim stage provided the information cost  $\gamma$  is sufficiently smaller than the variance of the revenue of intermediation:

 $\gamma \le \frac{\mathbb{V}ar[X_B]}{2X_D}.\tag{17}$ 

The result follows directly from our discussion of equation (15) above. As we explained, the rents earned by D when she receives news are captured by the second term of (15), proportional to the variance of effort. From equation (14) we see that the variance of effort depends only on the variance of the intermediation revenue. Intuitively, the dealer's willingness to pay for information increases when the revenue of her loan extended to B is more volatile as the benefits from tailoring her effort level to the realized value of this revenue are larger.

Ex-ante, D would like to commit not to acquire information because she anticipates that a rational lender would charge a higher interest rate when she is informed. However such commitment is not credible unless the cost of information is high. Once the face value  $R_D^{i,n}$  of the loan to D has been set, D is always willing to pay the cost if it satisfies condition (17) to enjoy the ex-post information rent. The right-hand-side of (17) is proportional to the variance of D's intermediation revenue. Hence, riskier intermediation profits are bad collateral for D also because she is more likely to acquire information about them. As D's willingness to intermediate depends on such collateral value – rather than the NPV of intermediation – endogenous information acquisition further reduces the benefits of risky intermediation.

We thus showed that the production of information regarding the project of the final borrower along the lending chain is harmful when intermediation profits are used as collateral by the dealer. However, it will happen in equilibrium when information costs are low. This result, saying that opacity is bliss, is reminiscent of the findings in Gorton and Ordoñez (2014) or Dang et al. (2015) who show that information about collateral returns may be detrimental for lending and welfare. However, the mechanism is different. In those papers, when lenders acquire information ex-ante, they choose not to lend to positive NPV borrowers with bad collateral, while, under opacity, all borrowers would receive financing. The mechanism is then a variant of Hirshleifer (1971)'s effect. Instead, in our model, information about collateral is detrimental because dealers use it to correlate ex-post the effort on their project with the collateral cash flow. Lenders anticipate this behavior, and charge a higher interest rate.

## 6 Endogenous Intermediation Risk

In this section we endogenize the risk of the cash flow of the loan to B, the final borrower along the chain, by allowing the dealer to affect the riskiness of this loan. Because intermediation profits constitute additional collateral, we analyze in fact D's optimal choice of collateral. To this end, we assume the probability of success of B's project is now also the result of some costly, unobservable effort of the borrower. The interest rate set by D in the loan to B then affects B's effort choice. Hence, the dealer indirectly determines the probability of success of B's project and so the risk of her loan to B. We assume D must choose the terms of her loan to B before the terms of her loan with E. In other words, E0 can commit to a level of riskiness for her loan to E1.

To model the borrower's effort choice, we assume B chooses the probability of success of his investment  $p_B$  at a cost  $\frac{1}{2}c_Bp_B^2X_B$  with  $c_B>1$  and  $X_B\in[4c_B,8c_B]$ . The latter condition is the counterpart of Assumption 2 when the success probability of B's project is endogenously chosen: the bounds on  $X_B$  ensure that lending to B is profitable but not so profitable that D could borrow 2 units from L secured only by the revenue of her intermediation activity. The condition  $c_B>1$  then implies that the first-best level of effort is lower than one. <sup>19</sup> The set-up is otherwise identical to Section 4 and we focus on the version of the model without news for simplicity.

We first determine B's effort choice. Let  $R_B$  denote the face value of the 1-unit loan granted to B. Proceeding as in Section 3, we find B's choice of effort is

$$p_B = \frac{X_B - R_B}{c_B X_B}. (18)$$

If the dealer were to set the face value  $R_B$  so as to maximize the value of the expected payment  $p_B R_B$  received from her loan to B, she would choose  $R_B^{max} = X_B/2$ , inducing an effort level  $p_B^{max} = 1/(2c_B)$ . The intermediary's expected revenue from her loan to B would be  $X_B/(4c_B)$ . We already showed however that D does not consider the NPV of the loan to B but its collateral value because she uses the cash flows of this loan as collateral with D. Precisely, we showed in Section 4 that, for a given value of the expected cash flow of the loan of the dealer to B, cash flow riskiness entails a cost for

 $<sup>^{18}\</sup>mbox{Without}$  such commitment, as is well known, D would have incentives to engage in risk-shifting. Still, as before, D cannot commit to the effort choice for her own investment.

<sup>&</sup>lt;sup>19</sup>As we have seen in Proposition 3, fragility arises when  $p_B$  is not too high. The condition  $c_B > 1$  ensures that the (now endogenous) value of  $p_B$  may lie in this fragility region.

the dealer. As such riskiness now endogenously depends on B's incentives, the dealer may find optimal to reduce the volatility of the cash flow of the loan to B by lowering the face value  $R_B$  of this loan below  $R_B^{max}$ . Hence D may prefer to sacrifice profits on her loan to B in order to obtain a safer cash flow from that loan.

**Proposition 7.** With endogenous loan risk there exists a threshold value  $\underline{X}_B < 8c_B$  of the cash flow of B's project when successful such that, for all  $X_B \geq \underline{X}_B$ , the dealer finds it optimal to sacrifice intermediation profits in exchange for a safer cash flow of her loan to B.

Proposition 7 shows that the dealer is willing to set a face value for her loan to B below the value  $R_B^{max}$  to induce B to exert more effort. Then the cash flow from her loan to B becomes safer. This sacrifice of NPV is optimal for D as it allows to mitigate the harmful consequences of the hedging effect. This result illustrates our previous observation that D's benefits from intermediation are not driven by intermediation revenues but by the collateral value of intermediation profits. Therefore, when intermediaries have insufficient funds of their own and must rely on external finance to fund their investment opportunities, the quest for safe collateral provides them with an incentive to channel intermediation towards safer projects, despite their lower returns. This result implies, for example, that there is an endogenous premium for safe collateral when it circulates along collateral chains. This finding resonates well with the evidence that re-used collateral in the swaps and derivatives markets is mostly in the form of highly liquid and safe Treasuries (see e.g. ISDA (2019)).

Interestingly, our results also imply that the final borrower along the credit chain earns more profits when he borrows from a dealer than if he could borrow directly from the financiers who have the funds. Suppose B could borrow directly from L, leaving all the bargaining power to the lender for symmetry. In such a situation the lender would simply maximize the expected revenue from this loan and thus choose face value  $R_B^{max}$ . In contrast D prefers to set a lower face value thus increasing B's surplus from the transaction, as shown in the previous proposition.

Despite the preference of the dealer for lowering the risk of the assets she acquired with her intermediation activity, fragility may still arise when the risk of the loans extended by the dealer is endogenous. Figure 3 provides a numerical illustration of this claim. The value of  $X_D = 5.3$  is the same as in Figure 1 and 2, and we set  $c_B = 1.3$ . The left panel reports the borrower's probability of success (yellow solid curve) at the loan contract optimally set by D, as a function of  $X_B$ , the cash flow of B's project

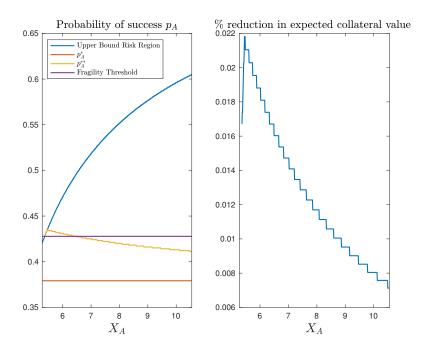


Figure 3: Optimal Collateral Risk.

when successful. The purple line represents the fragility threshold characterized in Proposition 3 for these parameter values.<sup>20</sup> When  $X_B \geq 7.53$ , the yellow curve lies below the purple line, that is, fragility arises in equilibrium.<sup>21</sup> Fragility arises despite D's incentives to reduce the risk of the loan she makes. The left panel of Figure 3 shows that these incentives are active for all values of  $X_B$  as B's chosen level of effort always lies strictly above the benchmark value  $p_B^{max}$  (the orange line). The right panel in the figure quantifies how much of the expected revenue of the intermediation activity the dealer D chooses to sacrifice to reduce the risk of this revenue.

To conclude this section, we now discuss the difference between the situation we considered with a credit chain and one where D would directly own B's investment. As we observed before, when D has all the bargaining power, the outcomes in these two situations are equivalent if the riskiness of B's project is exogenous. As a consequence, the results obtained in the previous sections have also implications for the joint financing of projects. In particular, they show that D's project (subject to moral hazard) can become riskier when financed jointly with another positive NPV investment

<sup>&</sup>lt;sup>20</sup>This value is the same as in Figure 1 because the fragility threshold  $p_B^*$  of Section 4 is independent of  $X_B$  and only depends on  $X_D$  which has the same value here (see the proof of Proposition 3).

<sup>&</sup>lt;sup>21</sup>For  $X_B \in [5.20, 5.26]$ , D prefers not to re-use collateral because of the negative hedging effect. For  $X_B \in [5.26, 7.53]$ , D re-uses collateral and the optimal choice of collateral riskiness is such that  $p_B$  exceeds the fragility threshold.

(B's project) rather than on a standalone basis. While a large literature has identified benefits from joint financing, our results suggest that this mode of funding is prone to fragility and contagion when investments are risky. This finding is reminiscent of Banal-Estañol et al. (2013), although, in that paper, contamination is caused by default costs rather than moral hazard of the borrower, as in our setup.

Importantly, when instead the success probability of B's project is also endogenous and subject to moral hazard, as in this section, the intermediation chain we considered is no longer equivalent to the situation in which D owns both investments. It is easy to verify that the chain exhibits more fragility, because providing incentives along the credit chain is costly. Even though she has all the bargaining power, D must leave some rents to the borrower to sustain his incentives. In contrast, when D owns both projects and directly chooses the effort for both, she earns the entire cash flow of the two projects net of the financing cost and hence chooses to exert more effort.

## 7 Applications

Our model is stylized and is not meant to be a perfect fit for a specific economic application. However, we believe that our analysis highlights some important forces at play in several markets. In this section, we discuss three such applications in detail: securitization, trade credit, and repos.<sup>22</sup> We rely on the specification of the model where the dealer raises two distinct 1-unit loans from L, secured respectively by D's own investment and her intermediation revenue. As explained at the beginning of Section 4, this specification is equivalent to the one with a single 2-unit loan under the assumption that D cannot ring-fence assets on her balance sheet, which means the creditor of each loan has recourse to all assets on her balance sheet. In all three applications, we thus argue that the creditor has recourse to the balance sheet of D. Hence, the fragility channel we identified is likely to be active.<sup>23</sup>

<sup>&</sup>lt;sup>22</sup>We thank Francisco Urzua for pointing out a fourth potential application: In business groups, holdings often borrow and pledge collateral obtained from subsidiaries where assets are typically located. See for example Ghatak and Kali (2001).

 $<sup>^{23}</sup>$ In our model all loans taken by the intermediaries are recourse. As we show in Appendix D, however, the fact that the loan secured by D's project is recourse is not essential for the fragility result. What matters is that the loan taken by D, and secured with the loan to B provides recourse. In fact, we show that if only this latter loan is recourse, fragility is even stronger than in our benchmark analysis.

#### Trade Credit

Our first application regards chains of trade credit. Trade credit is one of the major sources of funds for firms. Instead of borrowing money from a bank, a firm can obtain the inputs it needs by using trade credit. With this instrument, the firm (the borrower) obtains inputs from a supplier by promising to pay for those inputs at a later date. The supplier (the intermediary, here) records these loans as "account receivables" on its balance sheet. This has some analogy with the relationship between B and D in the environment we considered. In turn, the supplier may obtain funding from a financial lender (L in our model), by pledging or selling the trade receivable. This practice is sometimes known as factoring when the supplier uses invoices in order to borrow. Factoring can be recourse or non-recourse. With non-recourse factoring, the factoring firm is left empty-handed if the borrower (B) fails to pay. With recourse factoring instead, the supplier is on the hook to repay the factoring firm when the borrower fails, as in our model. In Europe, while non-recourse factoring is increasing, recourse factoring has been prevalent.<sup>24</sup>

The findings by Petersen and Rajan (2015) also suggest that such trade credit chains are a common arrangement. They show that firms with better access to credit from financial institutions offer more trade credit, that is, they may play a role as intermediaries. In our model only the intermediary has access to credit from lenders who can fund the trade credit position extended by D to B. In addition, Berger and Udell (1990) and Omiccioli (2005) show that firms use account receivables to secure borrowing from banks. Once again, this is akin to D securing her own loan from L by using the cash flow of the loan she extended to B as collateral. Interestingly, Omiccioli (2005) shows that this behavior is concentrated among small and risky firms. Our analysis suggests that the use of account receivables as collateral contributes to making firms riskier, thereby providing an explanation for this finding.

### Securitization

With securitization an intermediary (the originator) can park loans off-balance sheet in a Special Purpose Vehicle (SPV) to free some balance sheet space. The SPV funds

<sup>&</sup>lt;sup>24</sup>Data from Factor Chain International, the industry representative body, show that more than 60% of factoring happens in Europe (see https://fci.nl/en/industry-statistics).

these loans by selling bonds.<sup>25</sup> The firm who sets up the SPV is called the sponsor and can be the same agent as the loan originator. In this interpretation of our model, D is the intermediary/sponsor of the SPV, the loan to B is held by the SPV while D only keeps her project on her balance sheet. As in our model, the arrangement with a SPV still generates fragility as long as the creditors of the SPV have recourse to the balance sheet of the sponsor. In practice, sponsors offer recourse via implicit or explicit guarantees in order to improve the rating of the SPV's debt (see Acharya et al. (2013)). The simplest form of credit enhancement is an explicit recourse arrangement, whereby the creditors of the SPV would receive a payment directly from the enhancer should the SPV fail to pay. A more common form of credit enhancement is an irrevocable letter of credit. With such credit enhancement, the creditors of the SPV effectively have recourse to the balance sheet of the sponsor. Hence, as in our model, the intermediary is on the hook to repay the SPV creditors.

Viewing credit enhancement as a recourse arrangement, our model sheds light on how securitization can benefit the originator bank, by allowing it to expand its lending activity, while making its balance sheet more risky. We show that cross-subsidization between securitization and other activities, induced by SPV credit enhancements, can generate contagion and fragility, thus formalizing the argument in Acharya et al. (2013). Our mechanism is different from the narrative in Keys et al. (2010) and others who argue that securitization leads to fragility because banks have no incentive to exert due diligence for loans they plan to sell. In our model, the enhancement guarantee puts the balance sheet of the intermediary at stake affecting her incentives to exert due diligence for the assets remaining on her balance sheet, ultimately increasing the probability of default of these assets. Because cross-subsidization sometimes both increases lending and reduces fragility in our model, pure ring-fencing between banks' own trading activities and their intermediation business may not always be efficient though.

## Repurchase Agreements

The third application of our model is given by the bilateral repurchase agreement (repo) market. In this market, financial institutions borrow funds, usually short term, by selling assets with the agreement to buy them back at a later date at an agreed price.

<sup>&</sup>lt;sup>25</sup>This process often involves the pooling of different loans and their tranching into different debt claims to cater to an heterogeneous investor clientele. These features of securitization are important but they are not relevant to our argument.

Essentially, the sale of these assets amounts to borrowing funds collateralized by the assets sold. Risky assets such as MBS or equity can be used as collateral in repos, and dealer banks often act as intermediaries in repo markets.<sup>26</sup> For example Aldasoro and Ehlers (2018) provide evidence that French banks (among others) are intermediating the US dollar funding needs of Japanese banks with onshore US money market funds. Key to this intermediation process is the ability of financial institutions to re-use the asset they obtained in a previous repo. In our model, the loan to B finances the acquisition of an asset that is then pledged as collateral to D. Because D then borrows against this asset, she effectively re-uses the collateral pledged by B. Infante et al. (2018) document high re-use rate of collateral even for non-Treasuries among US dealers and FSB (2017) has identified re-use as a key source of risk. Finally, repos are recourse loans, as we point out in Gottardi et al. (2019). So a lender in the repo market has an (unsecured) claim to the borrower entire balance sheet in case the collateral value is not high enough to cover the borrower's debt. Hence, our model can explain why credit chains in repo markets can be cause for concerns when loans are secured by risky assets.

### 8 Conclusion

Our paper shows that the ability of intermediaries to borrow against the revenue from their intermediation business can induce a trade-off between a higher level of total borrowing along secured credit chains and greater fragility. We first show that the larger level of credit when intermediation occurs has a stabilizing effect when the yield of the loans extended by intermediaries is relatively safe: in that case, intermediation and the formation of credit chains make the system less fragile. When instead the yield of such loans is risky, a trade-off arises: intermediaries still choose to reap intermediation profits but by so doing they may expose themselves and the whole system to fragility. We show that such fragility is exacerbated in the presence of news about the value of the loan yields. We also show that intermediaries are willing to pay a premium to lower the risk of the loans they extend in their intermediation business. Our findings apply to the repo market, but also to securitization and other forms of asset-based financing.

While our analysis focuses for simplicity on a short intermediation chain, it would be interesting to see how our results change when considering a longer chain of trades or a richer network of credit relationships. As we have seen, the intermediation of

<sup>&</sup>lt;sup>26</sup>See Julliard et al. (2019) for the UK and Baklanova et al. (2015), for the US.

funds can generate fragility but also induces intermediaries to source safer loans, in order to mitigate this fragility. Accounting jointly for these effects, it is thus unclear whether more complex networks or longer intermediation chains lead to more fragility. We also believe our model could provide a basis to compare different market structures. In the present paper we maintained the assumption that borrowers and lenders could only trade through the intermediaries, as in OTC markets. Recent regulatory efforts to reduce the fragility of these markets led to a push toward centralization of trades, via, for instance Central Counterparties (CCP). Market centralization could indeed shorten credit chains. However, there are also widespread concerns that risk would be concentrated on a single agent rather being spread over a collection of intermediaries.

# Appendix

## A Bargaining Power to Borrowers

In this section, we consider the case in which borrowers have the bargaining power, that is, B sets the terms when borrowing from D and, as in the main text, D sets the terms when borrowing from L. We denote  $R_B$  the face value of the loan from D to B with  $R_B \leq X_B$ . Unlike in the main text, D's payoff from the loan is not equal to the payoff from B's investment. The dealer pledges the loan as collateral when borrowing from L. Effectively, the loan is an asset with payoff  $R_B$  (resp. 0) with probability  $p_B$  (resp.  $1-p_B$ ). We focus on the case without news for simplicity. Without intermediation, the analysis is identical to Section 3

We now turn to the case where D can lend to B by raising additional funds from L. Let  $\underline{R}_B := 1/p_B$  be the break-even rate, which is the minimum face value the dealer should accept to lend 1 unit to B, given the probability of success  $p_B$ . We show below that our main results from Section 4 survive and, in particular, that intermediation can generate fragility.

**Proposition A.1.** There exists  $\bar{p}_{BB} > \underline{p}_{B}$  with  $\underline{p}_{B}$  defined in Proposition 2, such that

- 1. Agent D takes a 2-unit loan if and only if  $p_B \ge \underline{p}_B$
- 2. Agent D exerts less effort when intermediating if  $p_B \in [\underline{p}_B, \bar{p}_{BB}]$ .

The face value of the loan to B satisfies  $R_B > \underline{R}_B$  in the fragility region  $[\underline{p}_B, \overline{p}_{BB}]$ , and  $R_B = \underline{R}_B$  for  $p_B \geq \overline{p}_{BB}$ .

*Proof.* When D intermediates funds between L and B, the objective of B when offering a loan is to minimize the face value of the loan extended by D subject to D's participation constraint. Formally, B's problem writes:

$$\min R_B$$
 subject to  $U_D^i(R_B) \ge U_D$  (A.1)

with  $U_D^i(R_B)$  the utility of D when she intermediates, and she obtains cash flow  $R_B$  from the loan to B in case of success. We first determine  $U_D^i(R_B)$  and then solve for  $R_B$ .

For the first step, observe that the only difference with our analysis in Section 4 is the payoff of the loan to B in case of success, which is  $R_B$  rather than  $X_B$ . We can thus use the results in Proposition 2 to characterize  $U_D^i(R_B)$ . In particular, for any  $R_B \geq \underline{R}_B$ , extending the notation of Proposition 2, there exists thresholds  $\underline{p}_B(R_B)$  and  $\bar{p}_B(R_B)$  such that the statements of Proposition 2 hold, substituting  $X_B$  with  $R_B$ .

In the second step of the analysis, we determine  $R_B$  using (A.1). Conjecture first that  $R_B$  is such that  $p_B \geq \bar{p}_B(R_B)$  which, by definition, implies that  $R_B \leq R_D^i$ , that is the face value of the loan to B is lower than the face value of the loan the dealer gets from L. The utility of D is given by  $U_D^i(R_B) = \frac{1}{2} \left[ p_D^i(R_B) \right]^2 X_D$ . Using equation (7), D's participation constraint binds in program (A.1) if and only if

$$p_D = p_D^i(R_B) = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2 - p_B R_B}{X_D}}$$

where  $p_D^i(R_B)$  is given by equation (E.7) substituting  $X_B$  with  $R_B$ . It follows immediately that  $R_B = \underline{R}_B$  is the solution to problem (A.1). Let thus  $\bar{p}_{BB} := \bar{p}_B(\underline{R}_B)$  be the threshold below which  $R_D^i \geq \underline{R}_B$  does not hold.

We then turn to the case  $p_B \leq \bar{p}_{BB}$  to characterize the threshold below which D prefers a 1-unit loan. Observe that the maximum face value B can set is  $X_B$ . This implies that the threshold of interest is given by  $\underline{p}_B := \underline{p}_B(X_B)$ , which is the same as in Proposition 2.

We can now show that  $p_D(R_B) < p_D$  for all  $p_B \in [\underline{p}_B, \bar{p}_{BB}]$ . By definition of  $\bar{p}_{BB}$ , the face value of the loan satisfies  $R_B \geq R_D^i$ . Using the results from Proposition 2 and the characterization of  $U_D^i(R_B)$  in the proof of that result, we obtain

$$U_D^i(R_B) = \frac{1}{2} \left[ p_D^i(R_B) \right]^2 X_D + p_D^i(R_B) (R_B - R_D^i(R_B)) = \frac{X_D}{2} + R_B - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1 - p_D^i(R_B))^2}{X_D}}} (A.2)$$

where  $p_D(R_B)$  and  $R_D(R_B)$  are given by equations (E.11) and (E.10) respectively, substituting  $X_B$  with  $R_B$ . By the participation constraint of agent D, we have  $U_D(R_B) = U_D$ . Equation (A.2) thus implies that  $p_D^i < p_D$  when  $p_B \in [p_B, \bar{p}_{BB})$ .

Finally, we show that  $R_B > \underline{R}_B$  when  $p_B \leq \bar{p}_{BB}$ . To see this, suppose by contradiction that  $R_B = \underline{R}_B$ . From equation (A.2), we can see that  $U_D^i$  would be strictly increasing for  $p_B \in [\underline{p}_B, \bar{p}_{BB}]$  because  $p_D(R_B)$  is strictly increasing with  $p_B$  for a given value of  $R_B$ . This result contradicts the condition that agent D's participation constraint binds for all  $p_B$ .

## B Equilibrium without intermediation

In this section, we rationalize the no-intermediation benchmark as an endogenous choice of D when she can lend to B but cannot (re)pledge the cash flows of this loan to L. By this we mean that D cannot seize any cash flow D from B's loan repayment. We show that D may choose not to intermediate for a larger set of parameters than in Proposition (2). The reason is intuitive: in this case, the 2 unit loan from the lender is backed only by the dealer's project, which severely undermines her incentives compared to a 1 unit loan.

At t = 0, D chooses whether to borrow 1 unit from L to fund only her project, or 2 units to fund also the loan to B. The 1-unit-loan case is described in Proposition B.1. For the 2-unit-loan case, we let  $R_{D,2}$  be the face value of D's debt and  $p_{D,2}$  her effort choice. Because he cannot seize thee cash flows of D's loan to B, the lender only receives a positive payment when this project is successful, that is, with probability  $p_{D,2}$ . The lender's participation constraint is then

$$p_{D,2}R_{D,2} \ge 2.$$
 (B.1)

As in the 1-unit loan case, the dealer chooses effort level  $p_{D,2}$  given  $R_{D,2}$  to maximize her payoff

$$p_B X_B + p_{D,2} \max \{X_D - R_{D,2}, 0\} - \frac{1}{2} X_D p_{D,2}^2$$
 (B.2)

As the cash flows of the loan to B are not pledgeable, D earns the expected revenue  $p_BX_B$  whether her own project succeeds or defaults. The profit-maximizing value of  $p_{D,2}$  given  $R_{D,2}$  is given by (3) replacing  $R_D$  with  $R_{D,2}$  The intermediary thus chooses  $R_{D,2}$  to maximize her expected utility (B.2) given her ex-post effort choice and D's participation constraint (B.1). We obtain the following result.

**Proposition B.1.** When the dealer can intermediate, but cannot pledge the cash flow of the loan to B, she is able to borrow 2 units from L only when  $X_D \geq 8$ . The optimal face value of the 2-unit loan is then

$$R_{D,2} = \frac{4}{1 + \sqrt{1 - \frac{8}{X_D}}} \tag{B.3}$$

The dealer's effort choice and her utility are respectively:

$$p_{D,2} = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2}{X_D}} \tag{B.4}$$

$$U_{D,2} = p_B X_B + \frac{1}{2} p_{D,2}^2 X_D \tag{B.5}$$

Comparing the effort choice with a 1-unit loan given by (6) to its counterpart with a 2-unit loan given by (B.4), we see the dealer exerts less effort when she intermediates. This is intuitive because her project now backs a larger loan. While in both cases, the dealer would like to commit to full effort, the ex-post optimal effort choice decreases with the face value of the debt, as shown by (3). When she intermediates, however, the dealer enjoys the revenue  $p_B X_B$  as shown by equation (B.5). Hence, as we show below, intermediation is profitable if the intermediation profits compensate for the reduction in effort on her project.

Corollary 1. When the dealer cannot use the cash flow from her loan to B as collateral, the dealer chooses not to intermediate if  $X_D \leq 8$  or if  $X_D \geq 8$  and

$$p_B X_B - 1 \le \frac{1}{2} X_D \left[ \sqrt{\frac{1}{4} - \frac{1}{X_D}} - \sqrt{\frac{1}{4} - \frac{2}{X_D}} \right] - \frac{1}{2}$$
 (B.6)

As shown in Proposition B.1, when the yield of D's project is too low  $(X_D \leq 8)$ , the dealer cannot get a 2 unit loan. When instead  $X_D \geq 8$ , both 1-unit and 2-unit loans are feasible but D still prefers a smaller loan when her intermediation profit  $(p_B X_B - 1)$  is smaller than the negative effect on D's incentives of a larger loan, captured by the term on the right-hand side of (B.6).

It is useful to compare the conditions under which D does not intermediate in Corollary 1 to that obtained in Proposition 2 when D can pledge the cash flow of the loan to B. Without this ability, the benefits from intermediation are much lower because there is no skin-in-the-game effect. Even if collateral is safe, D intermediates only if profits are large enough to compensate for weaker incentives with a large loan. Unlike in Proposition 2, however, condition B.6 shows that intermediation risk plays no role when D cannot pledge the cash flow of the loan to B. Intuitively, there can no contamination between the risk of the loan to B and D's own project if the loan to B does not back the loan from L to D.

## D Recourse Loans and Fragility

In this section, we analyze the version of the model in which D obtains two distinct loans of one unit each. These two loans can equivalently be financed by the same investor or by two different investors. For ease of exposition, we call  $L_D$  the creditor secured by D's own investment and  $L_B$  the lender secured by D's loan to B. With two distinct loans, an important feature of the lending relationships between the intermediary and lenders is whether loans provides recourse. A lender has recourse if he has an (unsecured) claim to D's other assets when the payoff of the asset securing the loan falls short of the promised repayment of the loan.

We first show in Section D.1 that the model with two loans is equivalent to our benchmark model if both creditors have recourse. Motivated by the empirical applications discussed in Section 7, we then show in Section D.2 that fragility is even stronger than in our benchmark model if recourse is only given to creditor  $L_B$  whose claim is secured by intermediation cash flows (the cash flow of D's loan to B).

### D.1 Symmetric Recourse

In this case, for  $i \in \{B, D\}$ , lender  $L_i$  has an unsecured claim to the asset pledged by D to creditor  $L_j$  with  $j \neq i$ . We let  $R_{DB}$  and  $R_{DD}$  denote the face value of the loan secured respectively by D's loan to B and D's project. We guess and verify that the face value of the loans are such that  $X_D > R_{DB} + R_{DD}$ , that is, D can repay both loans in full using only the cash flow of his own investment when it succeeds. We are left to determine agents' payoff when D's project fails but the loan to B succeeds. Lender  $L_B$  receives  $R_{DB} < X_B$  while Lender  $L_D$  gets min  $\{X_B - R_{DB}, R_{DD}\}$  and D gets payoff max  $\{0, X_B - R_{DD} - R_{DB}\}$ . Lenders  $L_B$  and  $L_D$ 's participation constraint are respectively

$$p_D R_{DB} + (1 - p_D) p_B R_{DB} \ge 1,$$
  
$$p_D R_{DD} + (1 - p_D) p_B \min \{ X_B - R_{DB}, R_{DD} \} \ge 1.$$

Agent D's effort decision is the solution to the following problem:

$$\max_{p_D} p_D(X_D - R_{DD} - R_{DB} + p_B X_B) + (1 - p_D) p_B \max \{0, X_B - R_{DD} - R_{DB}\} - \frac{1}{2} p_D^2 X_D \quad (D.1)$$

taking as given  $R_{DD}$  and  $R_{DB}$ .

We now show that D's problem is identical to the problem with a single lender who lends

2 units with face value  $R_D := R_{DB} + R_{DD}$ . To see this, let us first derive D's effort choice,

$$p_D = \frac{X_D + p_B X_B - R_D - p_B \max\{0, X_B - R_D\}}{X_D},$$
 (D.2)

The mapping between  $R_D$  and  $p_D$  in (D.2) is the same as in the single-lender case, where  $p_D$  is given by (9). Next, saturating the lenders' participation constraint and summing over the left-hand-side of the constraints, we obtain

$$p_D R_D + (1 - p_D) p_B \min \{X_B, R_D\} = 2$$

Again, this equation is identical to the participation constraint of a single lender lending 2 units with face value  $R_D = R_{DB} + R_{DD}$ , given by (10). It follows from these two observations that the equilibrium face value of the total debt incurred by D and the equilibrium effort choice are again given by Proposition 2. This observation also implies that our conjecture  $X_D > R_D$  is satisfied under the assumptions of the model.

#### D.2 Asymmetric Recourse

We now consider the situation in which only the "intermediation" loan extended by  $L_B$  is recourse. We guess and verify again that  $X_D > R_{DB} + R_{DD}$ . In this case, only the participation constraint of lender  $L_D$  is different with respect to Section D.1. Because lender  $L_D$  receives a payoff of zero when D's investment fails, his participation constraint is now given by:

$$p_D R_{DD} > 1$$
.

The effort decision of agent D is the solution to the following problem

$$\max_{p_D} p_D(X_D - R_{DD} - R_{DB} + p_B X_B) + (1 - p_D)p_B(X_B - R_{DB}) - \frac{1}{2}p_D^2 X_D$$
 (D.3)

The second term of (D.3) is different from the second term of (D.1) because when D's own investment fails, lender  $L_D$  does not have recourse to the payoff of the loan to B. We can then prove the following result.

**Proposition D.1.** With asymmetric recourse, the intermediary's default probability is higher than with symmetric recourse and than without intermediation for all values of  $p_B$ . Despite the additional fragility due to asymmetric recourse, D prefers to intermediate when intermediation profits are sufficiently safe, that is, when  $p_B$  is high enough.

*Proof.* To prove the first result, we derive the effort choice of D. Solving for  $p_D$  in (D.3) gives

$$p_D = \frac{X_D - R_{DD} - (1 - p_B)R_{DB}}{X_D} \tag{D.4}$$

Comparing equations (D.4) and (D.2), it follows immediately, that for given values  $R_{DB}$  and  $R_{DD}$ , the effort choice is strictly lower with asymmetric recourse. Comparing now the participation constraints of creditor  $L_D$ , for a given effort choice  $p_D$ , the face value  $R_{DD}$  must be strictly higher with asymmetric recourse. From these two observations, we can conclude that the effort choice of agent D is weakly lower with recourse. A similar argument shows that the effort choice is also lower than without intermediation.

To prove the second result, consider the limit case when  $p_B \to 1$ . Then, comparing equations (D.4) and (6), the effort choice is the same with intermediation asymmetric recourse as without intermediation. However, the utility derived by agent D in the former case is given by

$$U_D = \frac{1}{2}p_D^2 X_D + X_B - 1,$$

which is strictly higher than her utility level without intermediation given by (7). Hence, by continuity, for  $p_B$  close enough to 1, agent D prefers to intermediate in the asymmetric recourse model.

## E Proofs

#### E.1 Proof of Proposition 1

Letting L's participation constraint, equation (4), bind, we obtain

$$R_{D,l}^2 - X_D R_{D,l} + l X_D = 0$$

The value of  $R_{D,l}$  is the lowest root of this second-order equation with discriminant  $\Delta_l = X_D^2 - 4lX_D$ . We have

$$R_{D,l} = \frac{X_D - \sqrt{\Delta_l}}{2}$$

Replacing  $\Delta_l$  by its value, we obtain equation (5). Expression (6) is obtained by plugging equation (5) in equation (3). Finally, observe that

$$U_{D,l} = p_{D,l}(X_D - R_{D,l}) - \frac{1}{2}X_D p_{D,l}^2 = X_D p_{D,l}^2 - \frac{1}{2}X_D p_{D,l}^2$$

where we used (6) to substitute for  $R_{D,l}$ . Equation (7) immediately follows.

### E.2 Proof of Proposition 2

In Step 1, we characterize the optimal contract for a 2-unit loan when D intermediates. In Step 2, we compare this outcome to the 1-unit loan outcome without intermediation characterized in Proposition 1.

#### Step 1. Equilibrium with intermediation.

Case i) Conjecture  $R_D^i \in [X_B, X_D]$ .

We first solve for the face value  $R_D^i$  under this conjecture and then verify it. The effort choice in equation (9) becomes

$$p_D^i = \frac{X_D - R_D^i + p_B X_B}{X_D} \tag{E.5}$$

We can thus rewrite L's participation constraint as a function of  $R_D^i$  only. From equation (10) we get

$$p_D^i(R_D^i - p_B X_B) \ge 2 - p_B X_B$$
$$(X_D - R_D^i + p_B X_B)(R_D^i - p_B X_B) \ge X_D(2 - p_B X_B)$$

The variable  $\tilde{R}_D^i = R_D^i - p_B X_B$  is thus a solution to the following equation

$$\left(\tilde{R}_D^i\right)^2 - X_D \tilde{R}_D^i + X_D (2 - p_B X_B) = 0$$

Solving for the smallest root of the equation above, we obtain

$$R_D^i = R_B + \frac{1}{2} \left( X_D - \sqrt{X_D^2 - 4X_D(2 - p_B X_B)} \right)$$
 (E.6)

The effort choice is obtained by plugging equation (E.6) in (E.5) to obtain

$$p_D^i = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2 - p_B X_B}{X_D}} \tag{E.7}$$

Comparing equations (6) and (E.7) shows that  $p_D^i > p_D$ , because  $p_B X_B > 1$  under Assumption 2. It is also immediate that  $p_D^i$  and  $R_D^i$  are independent of  $p_B$  as  $p_B X_B$  is fixed.

We are left to verify the conjecture  $R_D^i \in [X_B, X_D]$  and to characterize the threshold  $\bar{p}_B$  mentioned in the statement of the proposition. The condition  $R_D^i \leq X_D$  is equivalent to

$$2R_B \le X_D + \sqrt{X_D^2 - 4X_D(2 - p_B X_B)}$$

which is implied by Assumptions 2 and 3. The condition  $R_D^i \geq X_B$  writes

$$X_{D} - \sqrt{X_{D}^{2} - 4X_{D}(2 - p_{B}X_{B})} \ge 2(1 - p_{B})X_{B}$$

$$p_{B} \ge \bar{p}_{B} := \frac{2p_{B}X_{B}}{X_{D} + 2p_{B}X_{B} - \sqrt{X_{D}^{2} - 4X_{D}(2 - p_{B}X_{B})}}$$
(E.8)

where  $\bar{p}_B \leq 1$ . This concludes Step 1 of the proof for the case  $p_B \geq \bar{p}_B$ .

Case ii)  $R_D^i \le \min\{X_B, X_D\}.$ 

From equation (8), the optimal choice of effort by agent D is

$$p_D^i = \frac{X_D - (1 - p_B)R_D^i}{X_D} \tag{E.9}$$

We can thus rewrite L's participation constraint (10) as follows

$$X_D p_B R_D^i + (X_D - (1 - p_B) R_D^i)(1 - p_B) R_D^i \ge 2X_D$$
$$-(1 - p_B)^2 (R_D^i)^2 + X_D R_D^i - 2X_D \ge 0$$

A solution to this equation exists if and only if  $X_D \ge 8(1 - p_B)^2$ , that is, if

$$p_B \ge \hat{p}_B := 1 - \sqrt{\frac{X_D}{8}}.$$

In this case,  $R_D^i$  is given by the smallest root of the second order polynomial above, that is

$$R_D^i = \frac{X_D - \sqrt{X_D^2 - 8X_D(1 - p_B)^2}}{2(1 - p_B)^2} = \frac{4}{1 + \sqrt{1 - \frac{2(1 - p_B)^2}{X_D}}}$$
(E.10)

We will then check conditions such that conjecture  $R_D^i R_D^i \leq \min\{X_B, X_D\}$  is satisfied. The second expression in (E.10) shows that  $R_D^i$  is decreasing with  $p_B$ . To obtain the equilibrium effort choice, plug in (E.10) in equation (E.9):

$$p_D^i = \frac{1}{2} - \frac{p_B}{2(1 - p_B)} + \sqrt{\frac{1}{4(1 - p_B)^2} - \frac{2}{X_D}}$$
 (E.11)

Let us now study the monotonicity of  $p_D^i$  as a function of  $p_B$ . Differentiating the right-hand-side of (E.11) with respect to  $p_B$  we obtain,

$$\frac{\partial p_D^i}{\partial p_B} = -\frac{1}{2(1-p_B)^2} + \frac{1}{4(1-p_B)^3} \frac{1}{\sqrt{\frac{1}{4(1-p_B)^2} - \frac{2}{X_D}}}$$

Hence,  $p_D^i$  is increasing with  $p_B$  because

$$0 \le 1 - 2(1 - p_B)\sqrt{\frac{1}{4(1 - p_B)^2} - \frac{2}{X_D}} = 1 - \sqrt{1 - \frac{8(1 - p_B)^2}{X_D}}$$

We must first verify the conjecture  $R_D^i \leq \min\{X_B, X_D\}$ . The condition  $R_D^i \leq X_B$  is equivalent to  $p_B \leq \bar{p}_B$ . Expression (E.10) shows  $R_D^i \leq 4$  which implies  $R_D^i \leq X_D$  under Assumption 3.

Finally, we check that the interval  $[\hat{p}_B, \bar{p}_B]$  is not empty to ensure Case ii) arises in equilibrium for some values of  $p_B$ . If  $X_D \geq 8$ , we have  $\hat{p}_B \leq 0$ , which proves the result because  $\bar{p}_B > 0$ . Consider thus the case  $X_D \in [4, 8]$ . The threshold  $\bar{p}_B$  is strictly increasing with  $p_B X_B$  while  $\hat{p}_B$  does not depend on  $p_B X_B$ . It is thus enough to verify that  $\bar{p}_B \geq \hat{p}_B$  for  $p_B X_B = 1$ . In particular, we have

$$\bar{p}_B(p_B X_B = 1, X_D = 4) = \frac{1}{3} > 1 - \frac{1}{\sqrt{2}} = \hat{p}_B(X_D = 4).$$

As  $\bar{p}_B$  is strictly increasing with  $X_D$  while  $\hat{p}_B$  is strictly decreasing with  $X_D$ , we can conclude that the inequality above holds, in fact, for all values of  $X_D$ . Hence, the interval  $[\hat{p}_B, \bar{p}_B]$  is non-empty. This concludes Step 1 of the proof for the case  $p_B \leq \bar{p}_B$ .

#### Step 2. Optimality of intermediation

We now prove the existence of a threshold  $\underline{p}_B$  with satisfies  $\underline{p}_B \leq \bar{p}_B$ , and such that D prefers a 2-unit loan with intermediation to a 1-unit loan if and only if  $p_B \geq \underline{p}_B$ . Our analysis in Step 1 implies that  $\underline{p}_B \geq \hat{p}_B$  because a 2-unit loan is not feasible for  $p_B \leq \hat{p}_B$ . To further characterize  $\underline{p}_B$ , it is useful to derive D's utility with intermediation. Using equations (9) and (8), we obtain equation (12).

We show first  $U_D^i > U_D$  when  $p_B \ge \underline{p}_B$ . From the analysis of Step 1, the condition  $p_B \ge \underline{p}_B$  implies  $R_D^i \ge X_B$ . Comparing equations (7) and (12) with  $R_D^i \ge X_B$ , the result follows from the finding  $p_D^i \ge p_D$  derived in Step 1. Hence, it must be that  $\underline{p}_B < \bar{p}_B$ .

Consider now the case  $p_B \leq \bar{p}_B$ . We first show that  $U_D^i$  decreases with  $p_B$ . Rewriting (12),

$$U_{D}^{i} = \frac{1}{2} (p_{D}^{i})^{2} X_{D} + R_{B} - p_{B} R_{D}^{i}$$

$$= \frac{1}{2} \left( 1 - \frac{(1 - p_{B})}{X_{D}} R_{D}^{i} \right)^{2} X_{D} + R_{B} - p_{B} R_{D}^{i}$$

$$= \frac{X_{D}}{2} + p_{B} X_{B} - R_{D}^{i} \left( 1 - \frac{(1 - p_{B})^{2}}{2X_{D}} R_{D}^{i} \right)$$

$$= \frac{X_{D}}{2} + p_{B} X_{B} - \frac{R_{D}^{i}}{4} \left( 4 - 1 + \sqrt{1 - \frac{8(1 - p_{B})^{2}}{X_{D}}} \right)$$

$$= \frac{X_{D}}{2} + p_{B} X_{B} - \frac{R_{D}^{i}}{2} - \frac{X_{D}}{8(1 - p_{B})^{2}} \left( 1 - \sqrt{1 - \frac{8(1 - p_{B})^{2}}{X_{D}}} \right) \left( 1 + \sqrt{1 - \frac{8(1 - p_{B})^{2}}{X_{D}}} \right)$$

$$= \frac{X_{D}}{2} + p_{B} X_{B} - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1 - p_{B})^{2}}{X_{D}}}}$$
(E.12)

where to derive the second, third, and final line, we used equation (E.9), (E.10) and (E.11) respectively. It follows from (E.12) that  $U_D^i$  is strictly increasing with  $p_B$  when  $p_B \leq \bar{p}_B$ .

Two cases are then possible. If  $U_D^i(\hat{p}_B) \geq U_D$ , then  $\underline{p}_B := \hat{p}_B$  by definition because intermediation is preferred for all  $p_B \geq \hat{p}_B$  If instead  $U_D^i(\hat{p}_B) < U_D$ , because  $U_D^i$  is strictly increasing with  $p_B$  for  $p_B \in [\hat{p}_B, \bar{p}_B]$ , then,  $\underline{p}_B$  is the unique value of  $p_B \in [\hat{p}_B, \bar{p}_B]$  implicitly defined by  $U_D^i(\underline{p}_B) = U_D$ . This concludes the proof.

## E.3 Proof of Proposition 3

We first show that, if it exists, the threshold  $p_B^*$  belongs to the interval  $(\underline{p}_B, \bar{p}_B)$ . We then prove existence.

For the first step, observe from Proposition 1 and 2 that, for  $p_B \geq \bar{p}_B$ , D enjoys a higher utility and exerts more effort with intermediation than without it. Second, we showed that D prefers a 1-unit loan if  $p_B \leq \underline{p}_B$ . Hence, the threshold  $p_B^*$ , if it exists, must belong to the interval  $(\underline{p}_B, \bar{p}_B)$ .

For the second step, we showed in Proposition 2 that the effort choice with re-use,  $p_D^i$ , is increasing with  $p_B$ . To show that  $p_B^*$  exists, we are thus left to show that  $p_D^i(\underline{p}_B) < p_D$  because  $p_D^i(\bar{p}_B) < p_D$ . Consider the two cases analyzed in the proof of Proposition 2. Suppose first that  $\underline{p}_B = \hat{p}_B$ , which is the case when  $U_D^i(\hat{p}_B) > U_D$ . Then, we have

$$p_D^i(\hat{p}_B) = \frac{1}{2} - \frac{\hat{p}_B}{1 - \hat{p}_B} = 1 - \sqrt{\frac{2}{X_D}}$$

Using equation (3), the inequality  $p_D^i(\hat{p}_B) < p_D$  holds if and only if

$$\begin{split} \frac{1}{2} - \sqrt{\frac{2}{X_D}} &\leq \sqrt{\frac{1}{4} - \frac{1}{X_D}} \\ \Leftrightarrow & \frac{1}{4} - \sqrt{\frac{2}{X_D}} + \frac{2}{X_D} \leq \frac{1}{4} - \frac{1}{X_D} \end{split}$$

The last equation holds because  $X_D \ge 4$  by Assumption 3. Consider now the case  $\underline{p}_B > \hat{p}_B$ , such that  $U_D^i(\underline{p}_B) = U_D$ . Then,

$$U_D = \frac{1}{2} p_D^2 X_D = U_D^i(\underline{p}_B) = \frac{1}{2} \left( p_D^i \left( \underline{p}_B \right) \right)^2 X_D + \underline{p}_B \left[ X_B - R_D^i \left( \underline{p}_B \right) \right]$$

where the expression for  $U_D^i$  is given by equation (12). As  $X_B > R_D^i$  when  $p_B < \bar{p}_B$ , this implies that  $p_D^i(\underline{p}_B) < p_D$ . Hence, in both cases, there exists  $p_B^* \in (\underline{p}_B, \bar{p}_B)$  such that  $p_D^i < p_D$  if and only if  $p_B \in (\underline{p}_B, p_B^*)$ .

We can further derive an analytical expression for  $p_B^*$  by solving for the equation  $p_D$ 

 $p_D^i(p_B)$ . We obtain

$$\sqrt{\frac{1}{4} - \frac{1}{X_D}} = -\frac{p_B^*}{2(1 - p_B^*)} + \sqrt{\frac{1}{4(1 - p_B^*)^2} - \frac{2}{X_D}}$$

$$\frac{1}{4} - \frac{1}{X_D} + \frac{(p_B^*)^2}{4(1 - p_B^*)^2} + \frac{p_B^*}{1 - p_B^*} \sqrt{\frac{1}{4} - \frac{1}{X_D}} = \frac{1}{4(1 - p_B^*)^2} - \frac{2}{X_D}$$

$$\frac{1}{4} + \frac{1}{X_D} + \frac{p_B^*}{1 - p_B^*} \sqrt{\frac{1}{4} - \frac{1}{X_D}} = \frac{1 + p_B^*}{4(1 - p_B^*)}$$

$$\Rightarrow \qquad p_B^* = \frac{2}{X_D + 2 - \sqrt{X_D^2 - 4X_D}}$$

#### E.4 Proof of Proposition 4

The proof is in several steps. We first derive the values of  $R_D^{i,n}$  and  $p_{Ds}^{i,n}$  under the two different cases  $R_D^{i,n} \geq X_B$  (Step 1) and  $R_D^{i,n} \leq X_B$  (Step 2). We then compare the expected level of effort and the utility of agent D in the two regions  $p_B \geq \bar{p}_B$  (Step 3) and  $p_B < \bar{p}_B$  (Step 4), characterized in Proposition 2. Let  $\bar{X}_B = p_B X_B$  be the expected revenue of the loan to B.

Step 1. Values of 
$$R_D^{i,n}$$
 and  $p_{Ds}^{i,n}$  when  $R_D^{i,n} \ge X_B$ 

The participation constraint of the lender writes

$$p_B(p_{Dg}^{i,n}R_D^{i,n} + (1 - p_{Dg}^{i,n})X_B) + (1 - p_B)p_{Db}^{i,n}R_D^{i,n} \ge 2$$

$$\mathbb{E}[p_D^{i,n}]R_D^{i,n} + (1 - p_{Dg}^{i,n})p_BX_B \ge 2X_D$$

Using equation (14) to substitute for  $p_{Dg}^{i,n}$  and  $p_{Db}^{i,n}$ , we obtain

$$(X_D - R_D^{i,n} + \bar{X}_B)R_D^{i,n} + (R_D^{i,n} - X_B)\bar{X}_B \ge 2X_D$$

Denoting  $\tilde{R}_D^{i,n} = R_D^{i,n} - \bar{X}_B$ , we have

$$\left(\tilde{R}_{D}^{i,n}\right)^{2} - X_{D}\tilde{R}_{D}^{i,n} + X_{D}(2 - \bar{X}_{B}) + p_{B}(1 - p_{B})X_{B}^{2} = 0$$

This second order equation has real solutions if and only if

$$0 \le X_D^2 - 4X_D(2 - \bar{X}_B) - 4R_B(X_B - \bar{X}_B)$$

which is equivalent to

$$p_B \ge \hat{p}_B := \frac{4\bar{X}_B^2}{4\bar{X}_B^2 + X_D^2 - 4X_D(2 - \bar{X}_B)} \tag{E.13}$$

The loan face value is then the lowest root of the second order polynomial, given by

$$R_D^{i,n} = \bar{X}_B + \frac{1}{2} \left( X_D - \sqrt{X_D^2 - 4X_D(2 - \bar{X}_B) - 4R_B(X_B - \bar{X}_B)} \right)$$

$$= \bar{X}_B + 2 \frac{2 - \bar{X}_B + \frac{R_B(X_B - \bar{X}_B)}{X_D}}{1 + \sqrt{1 - 4\frac{2 - \bar{X}_B}{X_D} - \frac{4R_B(X_B - \bar{X}_B)}{X_D^2}}}$$
(E.14)

The expression for  $\mathbb{E}[p_{Ds}^{i,n}]$  is obtained by plugging the expression for  $R_D^{i,n}$  obtained above in equation (14) and taking the average over the states  $s \in \{b, g\}$ . We obtain

$$\mathbb{E}[p_{Ds}^{in}] = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{(2 - \bar{X}_B)}{X_D} - \frac{R_B(X_B - \bar{X}_B)}{X_D^2}}$$
 (E.15)

As  $\mathbb{E}[p_{Ds}^{in}]$  is decreasing with  $X_B$  and  $X_B = \bar{X}_B/p_B$  where  $\bar{X}_B$  is fixed, it follows that  $\mathbb{E}[p_{Ds}^{in}]$  is decreasing with  $p_B$ .

Let us now derive D's utility. From equation (13), we have

$$U_D^{i,n} = \frac{1}{2} \mathbb{E} \left[ \left( p_{Ds}^{i,n} \right)^2 \right] X_D$$

$$= \frac{1}{2X_D} \mathbb{E} \left[ \left( X_D - R_D^{i,n} + X_{Bs} \right)^2 \right]$$

$$= \frac{1}{2X_D} \left[ \left( X_D - R_D^{i,n} + \bar{X}_B \right)^2 + \mathbb{V}ar[X_{Bs}] \right]$$

$$= \frac{1}{8X_D} \left[ \left( X_D + \sqrt{X_D^2 - 4X_D(2 - \bar{X}_B) - 4R_B(X_B - \bar{X}_B)} \right)^2 + 4\bar{X}_B(X_B - \bar{X}_B) \right]$$

$$= \frac{1}{8X_D} \left[ 2X_D^2 - 4X_D(2 - 2p_B X_B) + 2X_D \sqrt{X_D^2 - 4X_D(2 - \bar{X}_B) - 4R_B(X_B - \bar{X}_B)} \right]$$

$$= \frac{1}{4} \left[ X_D + \sqrt{X_D^2 - 4X_D(2 - \bar{X}_B) - 4\bar{X}_B(X_B - \bar{X}_B)} - 2(2 - \bar{X}_B) \right]$$
(E.16)

As  $U_D^{i,n}$  is decreasing with  $X_B$  and  $X_B = \bar{X}_B/p_B$  with  $\bar{X}_B$  fixed, it follows that  $U_D^{i,n}$  is increasing with  $p_B$ .

Step 2. Values of  $R_D^{i,n}$  and  $p_{Ds}^{i,n}$  when  $R_D^{i,n} < X_B$ 

The conjecture  $R_D^{i,n} < X_B$  together with equation (14) imply that

$$p_{Dg}^{i,n} = 1, \quad p_{Db}^{i,n} = \frac{X_D - R_D}{X_D}$$

Using again equations (14) and (16) with the effort choices derived above, the participation

constraint of the lender now writes

$$p_B R_D^{i,n} + (1 - p_B) p_{Db}^{i,n} R_D^{i,n} \ge 2$$
$$-(1 - p_B) \left( R_D^{i,n} \right)^2 + R_D^{i,n} X_D - 2X_D \ge 0$$

This second-order equation has a solution if  $X_D \ge 8(1-p_B)$ . The solution is the smallest root of the second-order polynomial above, given by

$$R_D^{i,n} = \frac{X_D - \sqrt{X_D^2 - 8X_D(1 - p_B)}}{2(1 - p_B)} = \frac{4}{1 + \sqrt{1 - \frac{8(1 - p_B)}{X_D}}}$$
(E.17)

The expected level of effort in this case is obtained thanks to equation (14):

$$\mathbb{E}[p_{Ds}^{in}] = p_B + (1 - p_B) \left[ 1 - \frac{R_D^{i,n}}{X_D} \right]$$

$$= 1 - \frac{X_D - \sqrt{X_D^2 - 8X_D(1 - p_B)}}{2X_D}$$

$$= \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{8(1 - p_B)}{X_D}}$$
(E.18)

The expression above shows that  $\mathbb{E}[p_{Ds}^{in}]$  is also increasing with  $p_B$  in this case. Finally, the ex-ante utility of agent D is given by

$$U_D^{i,n} = p_B \left(\frac{X_D}{2} - R_D^{i,n} + X_B\right) + \frac{1}{2}(1 - p_B) \left(p_{D,b}^{i,n}\right)^2 X_D$$

$$= p_B \frac{X_D}{2} - p_B R_D^{i,n} + \bar{X}_B + \frac{1}{2}(1 - p_B) \left[X_D - 2R_D^{i,n} + \frac{\left(R_D^{i,n}\right)^2}{X_D}\right]$$

$$= \frac{X_D}{2} + \bar{X}_B - R_D^{i,n} \left(1 - \frac{(1 - p_B)R_D^{i,n}}{2X_D}\right)$$

$$= \frac{X_D}{2} + \bar{X}_B - \frac{R_D^{i,n}}{4} \left(4 - 1 + \sqrt{1 - \frac{8(1 - p_B)}{X_D}}\right)$$

$$= \frac{X_D}{2} + \bar{X}_B - \frac{R_D^{i,n}}{2} - 1 \tag{E.19}$$

The utility  $U_D^{i,n}$  of agent D is increasing with  $p_B$  because  $R_D^{i,n}$  is decreasing with  $p_B$ , as can be seen from equation (E.17).

Step 3. Proof that  $U_D^{i,n} \leq U_D^i$  and  $\mathbb{E}[p_{D,s}^{i,n}] < p_D^i$  for  $p_B \geq \bar{p}_B$ .

By definition of  $\bar{p}_B$  in Proposition 2, the face value of the loan in the absence of news satisfies  $R_D^i \geq X_B$ . Comparing equation (E.6) for  $R_D^i$  and equation (E.14) for  $R_D^{i,n}$  shows  $R_D^{i,n} \geq R_D^i$ . Hence,  $R_D^{i,n} \geq X_B$  also holds for  $p_B \geq \bar{p}_B$ . The result that the expected level of effort is lower with news follows directly from the comparison between equations (E.7) and (E.15). The dealer's utility is given by equation (12) with  $R_D^i \geq X_B$  without news and equation (E.16) with news. We showed  $U_D^{i,n}$  is increasing with  $p_B$  for  $p_B \in [\bar{p}_B, 1]$ , while  $U_D^i$  is constant over this interval. Because,  $U_D^i = U_D^{i,n}$  for  $p_B = 1$ , we have  $U_D^i > U_D^{i,n}$  for all  $p_B \in [\bar{p}_B, 1)$ . This concludes the proof for the case  $p_B \geq \bar{p}_B$ .

Step 4. Proof that 
$$U_D^{i,n} \leq U_D^i$$
 and  $\mathbb{E}[p_{D,s}^{i,n}] < p_D^i$  for  $p_B < \bar{p}_B$ 

When  $p_B < \bar{p}_B$ , the face value of the loan in the absence of news satisfies  $R_D^i \le X_B$  by definition of  $\bar{p}_B$  in Proposition 2. In the model with news, two cases are possible, with, either  $R_D^{i,n} \le X_B$ , or  $R_D^{i,n} > X_B$ . Consider first the case  $R_D^{i,n} \le X_B$ . Then the comparison between equations (E.11) and (E.18) shows that  $\mathbb{E}[p_{D,s}^{i,n}] < p_D^i$  because  $(1 - p_B)^2 < (1 - p_B)$ . The comparison between equations (E.12) and (E.19) shows that  $U_D^{i,n} < U_D^i$  for the same reason.

Suppose now the equilibrium with news is such that  $R_D^{i,n} > X_B$ . We first show that the expected level of effort is lower than in the model without news. Using equation (14), the expected level of effort with news is

$$\mathbb{E}[p_{D,s}^{i,n}] = \frac{X_D + p_B X_B - R_D^{i,n}}{X_D} \le \frac{X_D - (1 - p_B) R_D^{i,n}}{X_D} \le \frac{X_D - (1 - p_B) R_D^i}{X_D} = p_D^{i,n}$$

where the first inequality follows from  $X_B \leq R_D^{i,n}$  and the second from  $R_D^{i,n} \geq X_B \geq R_D^i$ .

We are then left to show that  $U_D^{i,n} \leq U_D^i$  in this case. For this, suppose agent D could commit to the maximum effort level  $\tilde{p}_{Dg} = 1$  in state g. Given the face value  $\tilde{R}_D^{i,n}$  that the lender would require, D's effort choice in state b would be given by  $\tilde{p}_{Db}^{in} = \frac{X_D - \tilde{R}_{D,n}}{X_D}$  using equation (14). Fixing the face value of the loan  $\tilde{R}_D^{i,n}$ , these effort levels are the same than in the case analyzed in Step 2. Hence, the fictitious face value  $\tilde{R}_D^{i,n}$  and agent B's utility  $\tilde{U}_B^{i,n}$  would be given by equation (E.17) and (E.19), respectively. We have shown above that  $\tilde{U}_D^{i,n} \leq U_D^i$  for all values of  $p_B$ . Because the ability to commit in state g is valuable, we have  $U_D^{i,n} \leq \tilde{U}_D^{i,n}$  which implies  $U_D^{i,n} \leq U_D^i$  also in the case when  $R_D^{i,n} > X_B$ . This concludes the proof for the case  $p_D < \bar{p}_D$ .

## E.5 Proof of Proposition 5

The proof of Proposition 4 shows that the utility of agent D with news  $U_D^{i,n}$  is increasing with  $p_B$ . In addition,  $U_D^{i,n} \leq U_D^i$  for all values of  $p_B$  with a strict inequality except for  $p_B = 1$ . By Proposition 2, we also know there exists a threshold  $\underline{p}_B$  such that  $U_D^i \geq U_D$  if and only if

 $p_B \ge \underline{p}_B$ . These two results combined there exists a threshold  $\underline{p}_B^n \ge \underline{p}_B$  such that  $U_D^{i,n} \ge U_D$  if and only if  $p_B \ge p_B^n$ .

Because  $\mathbb{E}[p_{D,s}^{i,n}]$  is increasing with  $p_B$  and always lower than  $p_D^i$ , a similar argument establishes that there exists a threshold  $p_B^{*,n} \geq p_B^*$  such that  $\mathbb{E}[p_{D,s}^{i,n}] \geq p_D$  if and only if  $p_B \geq p_B^{*,n}$ . We are thus left to show that the fragility region with news, that is, the region  $[\underline{p}_B^n, p_B^{*,n}]$  is non-empty. As we proved that  $U_D^{i,n}$  is increasing with  $p_B$ , it is enough to show that D's utility with re-use (and news) is higher than without re-use for  $p_B = p_B^{*,n}$ .

Suppose first that  $R_D^{i,n} \ge X_B$  for  $p_B = p_B^{*,n}$ . Then, D's utility is given by equation (E.16). By definition of  $p_B^{*,n}$ ,  $\mathbb{E}[p_D^{i,n}] = p_D$  when  $p_B = p_B^{*,n}$ , and, thus

$$U_{D}^{i,n} = \frac{1}{2} \mathbb{E} \left[ \left( p_{Ds}^{i,n} \right)^{2} \right] X_{D} > \frac{1}{2} \left( \mathbb{E} \left[ p_{Ds}^{i,n} \right] \right)^{2} X_{D} = \frac{1}{2} p_{D}^{2} X_{D} = U_{D}$$

by Jensen's inequality. This proves D strictly prefers re-using collateral for  $p_B = p_B^{*,n}$ . By continuity, the fragility region  $[\underline{p}_B^n, p_B^{*,n}]$  is non-empty.

Suppose now that  $R_D^{i,n} < X_B$  for  $p_B = p_B^{*,n}$ . Then, D's utility is given by equation (E.19), which we can rewrite as

$$U_D^{i,n} = \frac{1}{2} p_B X_D + \frac{1}{2} (1 - p_B) \left( p_{D,b}^{i,n} \right)^2 X_D + p_B (X_B - R_D^{i,n})$$
$$= \frac{1}{2} \mathbb{E} \left[ \left( p_{D,s}^{i,n} \right)^2 \right] X_D + p_B (X_B - R_D^{i,n})$$

As the second term of  $U_D^{i,n}$  is positive when  $R_D^{i,n} < X_B$  and because  $\mathbb{E}[p_{D,s}^{i,n}] = p_D$  for  $p_B = p_B^{*,n}$ , it follows again by Jensen's inequality that  $U_D^{i,n} > U_D$  for  $p_B = p_B^{*,n}$ . This concludes the proof for this case.

## E.6 Proof of Proposition 6

Suppose D takes a 2-unit loan to then lend to B. Let  $R_D^i$  be the face value of the loan assuming D does not acquire information. Our discussion in the main text shows that the value of acquiring information is given by the second term of (15). Using equation (13), we have

$$\frac{1}{2} \mathbb{V}ar[p_{Ds}^{i,n}] X_D = \frac{1}{2} \frac{\mathbb{V}ar[X_B]}{X_D}$$

The comparison of the benefit of information above with the cost  $\gamma$  leads to Condition (17). Hence, if  $\gamma$  satisfies condition (17), D will acquire information in equilibrium.

#### E.7 Proof of Proposition 7

To show that the threshold  $\underline{X}_B$  exists, we must find conditions such that D prefers to intermediate funds and optimally sets  $R_B < R_B^{max}$ .

We first derive the condition such that D chooses  $R_B < R_B^{max}$  provided D intermediates. Denote  $\bar{p}_B(p_BR_B)$  the threshold  $\bar{p}_B$  introduced in Proposition 2 defined now as a function of the endogenous expected value of the collateral payoff  $p_BR_B$ . Suppose first  $p_B > \bar{p}_B(p_BR_B)$  holds in equilibrium, with  $p_BR_B = p_BX_B(1-c_Bp_B)$  where  $R_B$  is given by equation (18). When  $p_B > \bar{p}_B$ , Case 2 of Proposition 2 applies. Agent D's utility is increasing with the expected value of her intermediation cash flow and this utility does not depend on other moments of the distribution of this cash flow. This observation implies that the profit-maximizing face value is  $R_B = R_B^{max} = \frac{X_B}{2}$  and, we obtain,  $p_B = p_B^{max}$ . We are left to verify the initial conjecture  $p_B > \bar{p}_B(p_BR_B)$  holds. Using equation (E.8), which defines  $\bar{p}_B$ , this condition writes

$$1 > \frac{X_B}{X_D + \frac{X_B}{2c_B} - \sqrt{X_D^2 - 4X_D \left(2 - \frac{X_B}{4c_B}\right)}}$$
 (E.20)

The right hand side of this inequality is increasing with  $X_B$  and it is equal to  $2c_B \ge 2$  for  $X_B = 8c_B$ . Hence, there exists  $\underline{X}_{B,1} < 8c_B$  such that  $p_B > \bar{p}_B(p_B R_B)$  holds if and only if  $X_B < \underline{X}_{B,1}$ .

If instead  $X_B \geq \underline{X}_{B,1}$ , it must be that  $p_B \leq \bar{p}_B(p_B R_B)$ . This implies that Case 1 of Proposition 2 applies. We showed in the proof of this Proposition that agent D's utility is given by equation (E.12). Hence, D's optimization problem is given by

$$\max_{p_B} U_D^i(p_B) = \frac{X_D}{2} + p_B(1 - c_B p_B) X_B - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1 - p_B)^2}{X_B}}}$$
(E.21)

subject to 
$$p_B \leq \bar{p}_B (p_B (1 - c_B p_B) X_B)$$

The constraint ensures that the optimal choice of  $p_B$  lies below the threshold  $\bar{p}_B$  so that agent D's utility is indeed given by  $U_D^i(p_B)$  for any feasible choice  $p_B$ . As will be clear shortly, this constraint is redundant. The second term of the objective function is increasing with  $p_B$ . This observation, together with the fact that the second-term is maximized for  $p_B = \frac{1}{2c_B}$  implies D chooses  $p_B > \frac{1}{2c_B}$ . There is no benefit in increasing  $p_B$  beyond  $\bar{p}_B$ . Indeed, the expected value of the intermediation cash flow would further decrease without any risk reduction benefit because risk is irrelevant for  $p_B \geq \bar{p}_B$ . This observation confirms that the constraint is redundant.

Finally, we are left to verify that agent D chooses to lend to B in equilibrium. Intermediation is preferred if  $U_D^i(p_B) > U_D$  with  $U_D^i(p_B)$  defined in equation (E.21) and  $p_B$  the profit

maximizing choice. Because  $p_B$  is preferred to  $p_B^{max} = \frac{1}{2c_B}$ , a sufficient condition for the result is that  $U_D^i(p_B^{max}) \geq U_D$ . When  $X_B \geq \underline{X}_{B,1}$ , using equation (E.21), this condition writes

$$U_D \le \frac{X_D}{2} + \frac{X_B}{4c_B} - 1 - \frac{2}{1 + \sqrt{1 - \frac{8(1 - \frac{1}{2c_B})^2}{X_D}}}$$

As the left-hand side of the inequality is independent of  $X_B$  and the right-hand side is increasing with  $X_B$ , this condition defines a lower bound  $\underline{X}_{B,2}$  on  $X_B$ . It is easy to verify that the condition holds strictly for  $X_B = 8c_B$  and thus that  $\underline{X}_{B,2} < 8c_B$ . Hence, for all  $X_B \ge \max\left\{\underline{X}_{B,1}, \underline{X}_{B,2}\right\}$ , D prefers to intermediate and the optimal level of effort by B satisfies  $p_B > p_B^{max}$ , which means D sacrifices intermediation rents.

# References

- Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, 2015, Systemic risk and stability in financial networks, *American Economic Review* 105, 564–608.
- Acharya, Viral V., Philipp Schnabl, and Gustavo Suarez, 2013, Securitization without risk transfer, *Journal of Financial Economics* 107, 515 536.
- Aldasoro, Inaki, and Thomas Ehlers, 2018, The geography of dollar funding of non-us banks, Technical report, BIS.
- Allen, Franklin, and Douglas Gale, 2000, Financial contagion, *Journal of Political Economy* 108, 1–33.
- Andolfatto, David, Fernando M. Martin, and Shengxing Zhang, 2017, Rehypothecation and liquidity, *European Economic Review* 100, 488–505.
- Bahaj, Saleem, and Frederic Malherbe, 2020, The forced safety effect: How higher capital requirements can increase bank lending, *The Journal of Finance* 75, 3013–3053.
- Baklanova, Viktoria, Adam Copeland, and Rebecca McCaughrin, 2015, Reference guide to U.S. repo and securities lending markets, Staff Reports 740, Federal Reserve Bank of New York.
- Banal-Estañol, Albert, Marco Ottaviani, and Andrew Winton, 2013, The Flip Side of Financial Synergies: Coinsurance Versus Risk Contamination, *The Review of Financial Studies* 26, 3142–3181.
- Berger, Allen, and Gregory Udell, 1990, Collateral, loan quality and bank risk, *Journal* of Monetary Economics 25, 21–42.
- Bernhardt, Dan, Kostas Koufopoulos, and Giulio Trigilia, 2020, Is there a paradox of pledgeability?, *Journal of Financial Economics* 137, 606–611.
- Biais, Bruno, Florian Heider, and Marie Hoerova, forthcoming, Variation margins, fire sales, and information-constrained optimality, *Review of Economic Studies* to appear.
- Bottazzi, Jean-Marc, Jaime Luque, and Mario R. Páscoa, 2012, Securities market theory: Possession, repo and rehypothecation, *Journal of Economic Theory* 147, 477–500.

- Brunnermeier, Markus K., and Lasse Heje Pedersen, 2009, Market liquidity and funding liquidity, *The Review of Financial Studies* 22, 2201.
- Bubb, Ryan, and Alex Kaufman, 2014, Securitization and moral hazard: Evidence from credit score cutoff rules, *Journal of Monetary Economics* 63, 1 18.
- Cabrales, Antonio, Piero Gottardi, and Fernando Vega-Redondo, 2017, Risk Sharing and Contagion in Networks, *The Review of Financial Studies* 30, 3086–3127.
- Chemla, Gilles, and Christopher A. Hennessy, 2014, Skin in the game and moral hazard, The Journal of Finance 69, 1597–1641.
- Dang, Tri Vi, Gary Gorton, and Holmström Bengt, 2015, Ignorance, debt and financial crises, Technical report.
- Di Maggio, Marco, and Alireza Tahbaz-Salehi, 2015, Collateral shortages and intermediation networks, Research paper, Columbia Business School.
- Donaldson, Jason Roderick, Denis Gromb, and Giorgia Piacentino, 2020, The paradox of pledgeability, *Journal of Financial Economics* 137, 591 605.
- Eisenberg, Larry, and Thomas H. Noe, 2001, Systemic risk in financial systems, *Management Science* 47, 236–249.
- Farboodi, Maryam, 2017, Intermediation and voluntary exposure to counterparty risk, Technical report.
- FSB, 2017, Re-hypothecation and collateral re-use: Potential financial stability issues, market evolution and regulatory approaches, Technical report.
- Geanakoplos, John, and William R. Zame, 2010, Collateralized Security Markets, Levine's Working Paper Archive 661465000000000040, David K. Levine.
- Ghatak, Maitreesh, and Raja Kali, 2001, Financially interlinked business groups\*, Journal of Economics & Management Strategy 10, 591–619.
- Gorton, Gary, and Guillermo Ordoñez, 2014, Collateral crises, American Economic Review 104, 343–78.
- Gottardi, Piero, and Felix Kubler, 2015, Dynamic competitive economies with complete markets and collateral constraints, *The Review of Economic Studies* 82, 1119–1153.

- Gottardi, Piero, Vincent Maurin, and Cyril Monnet, 2019, A theory of repurchase agreements, collateral re-use, and repo intermediation, *Review of Economic Dynamics*
- Hirshleifer, Jack, 1971, The private and social value of information and the reward to inventive activity, *The American Economic Review* 61, 561–574.
- Holmström, Bengt, and Jean Tirole, 1998, Private and public supply of liquidity, *Journal of Political Economy* 106, 1–40.
- ICMA, 2019, Frequently Asked Questions on repo, Technical report.
- Infante, Sebastian, 2019, Liquidity windfalls: The consequences of repo rehypothecation, *Journal of Financial Economics* 133, 42 63.
- Infante, Sebastian, Charles Press, and Jacob Strauss, 2018, The ins and outs of collateral re-use, Feds notes, Federal Reserve Board.
- Innes, Robert D, 1990, Limited liability and incentive contracting with ex-ante action choices, *Journal of Economic Theory* 52, 45–67.
- ISDA, 2019, Margin survey year-end 2019, Technical report.
- Julliard, Christian, Zijun Liu, Seyed E. Seyedan, KAramfil Todorov, and Kathy Yuan, 2019, What Drives Repo Haircuts? Evidence from the UK Market, Technical report.
- Keys, Benjamin J., Tanmoy Mukherjee, Amit Seru, and Vikrant Vig, 2010, Did Securitization Lead to Lax Screening? Evidence from Subprime Loans, *The Quarterly Journal of Economics* 125, 307–362.
- Kiyotaki, Nobuhiro, and John Moore, 1997, Credit chains, Technical report.
- Kuong, John Chi-Fong, 2020, Self-Fulfilling Fire Sales: Fragility of Collateralized Short-Term Debt Markets, *The Review of Financial Studies* hhaa115.
- Maurin, Vincent, 2020, Asset scarcity and collateral rehypothecation, Technical report, Stockholm School of Economics.
- Monnet, Cyril, and Erwan Quintin, 2017, Rational Opacity, *The Review of Financial Studies* 30, 4317–4348.

- Muley, Ameya, 2016, Reuse of collateral: Rehypothecation and pyramiding, Technical report, MIT.
- Omiccioli, Massimo, 2005, Trade Credit as Collateral, Temi di discussione (Economic working papers) 553, Bank of Italy, Economic Research and International Relations Area.
- Park, Hyejin, and Charles M. Kahn, 2019, Collateral, rehypothecation, and efficiency, Journal of Financial Intermediation 39, 34–46, Post-Crisis Evolution of Banking and Financial Markets.
- Petersen, Mitchell A., and Raghuram G. Rajan, 2015, Trade Credit: Theories and Evidence, *The Review of Financial Studies* 10, 661–691.
- Piskorski, Tomasz, Amit Seru, and James Witkin, 2015, Asset Quality Misrepresentation by Financial Intermediaries: Evidence from the RMBS Market, *Journal of Finance* 70, 2635–2678.
- Plantin, Guillaume, 2011, Good Securitization, Bad Securitization, Imes discussion paper, Bank of Japan.
- Purnanandam, Amiyatosh, 2011, Originate-to-distribute model and the subprime mort-gage crisis, *Review of Financial Studies* 24, 1881–1915.
- Singh, Manmohan, 2011, Velocity of pledged collateral: Analysis and implications, IMF Working Papers 11/256, International Monetary Fund.
- Singh, Manmohan, 2013, The changing collateral space, IMF Working Papers 13/25, International Monetary Fund.
- Singh, Manmohan, and James Aitken, 2010, The (sizable) role of rehypothecation in the shadow banking system, IMF Working Papers 10/172, International Monetary Fund.
- Vanasco, Victoria, 2017, The downside of asset screening for market liquidity, *The Journal of Finance* 72, 1937–1982.

# F Online Appendix: Loan contract under monotonicity

Suppose B and D projects are such that  $X_B < X_D$ . Rewrite D's total cash flows with intermediation can be ordered as follows:

$$X_1 = X_B \le X_2 = X_D \le X_3 = X_D + X_B$$

Denote  $R_1 \leq R_2 \leq R_3$  the contract repayment assuming monotonicity. Repayments should also satisfy  $R_i \in [0, X_i]$ . The problem of the intermediary is to maximize her utility

$$U = p_B(1 - p_D)(X_B - R_1) + p_D(1 - p_B)(X_D - R_2) + p_B p_D(X_B + X_D - R_3) - \frac{1}{2}p_D^2 X_D$$

subject to the constraint that effort  $p_D$  is optimally chosen ex-post

$$p_D^* = \frac{X_D + p_B X_B - [(1 - p_B)R_2 + p_B R_3] - p_B (X_B - R_1)}{X_D}$$
$$= 1 - \frac{(1 - p_B)R_2 + p_B R_3 - p_B R_1}{X_D},$$

and the participation constraint of lenders who lend 2 units

$$(1 - p_D)p_B R_1 + p_D [(1 - p_B)R_2 + p_B R_3] \ge 2$$
$$p_B R_1 + p_D [(1 - p_B)R_2 + p_B R_3 - p_B R_1] \ge 2$$

Given an ex-post optimal effort choice, the intermediary's utility can be written

$$U = p_B X_B - p_B R_1 + \frac{1}{2} (p_D^*)^2 X_D$$

We see that the effort choice and the dealer's utility, and hence the lenders' participation constraints, only depend on  $(1 - p_B)R_2 + p_BR_3$ . Hence, we set  $\bar{R} = R_2 = R_3$  without loss, and only need to verify later that the optimal value of  $\bar{R}$  is lower than  $X_D$ . We now denote  $\underline{R} = R_1$ . Plugging the optimal effort choice  $p_D^*$  into the participation constraint

and the dealer's utility, we can write the problem as

$$\max_{\underline{R},\bar{R}} U(\underline{R},\bar{R}) = p_B X_B - p_B \underline{R} + \frac{1}{2} \left[ 1 - \frac{\bar{R} - p_B \underline{R}}{X_D} \right]^2 X_D$$
 subject to 
$$2X_D \leq \bar{R} X_D - (\bar{R} - p_B \underline{R})^2$$
 
$$\underline{R} \leq X_B$$
 
$$\underline{R} \leq \bar{R}$$

Developing  $U(\underline{R}, \bar{R})$ , we get

$$U(\underline{R}, \bar{R}) = p_B X_B + \frac{X_D}{2} - \bar{R} + \frac{(\bar{R} - p_B \underline{R})^2}{2X_D}$$

It is obious that the lender's participation constraint should bind. Otherwise, the following perturbation would strictly increase profit

$$\Delta \bar{R} = -\epsilon, \qquad \Delta \underline{R} = -\frac{\epsilon}{p_H}$$

It is feasible under the constraints on repayments if  $\epsilon > 0$  is small enough unless  $\underline{R} = 0$ . But in this case, it would still be optimal to decrease only  $\bar{R}$  because we are guessing  $\bar{R} \leq X_D$ . Plugging the participation constraint into the dealer's utility function we get

$$U(\underline{R}, \bar{R}) = p_B X_B + \frac{X_D}{2} - 2 - \frac{(\bar{R} - p_B \underline{R})^2}{2X_D}$$

The first three terms give the dealer's profit if she could commit to full effort. The last is the incentive cost. Observe that U is strictly decreasing in  $(\bar{R} - p_B R)^2$  along the participation constraint" of lenders. This means either that  $\underline{R} \leq \bar{R}$  should bind or that  $\underline{R} \leq X_B$  should bind. In both cases, this establishes the optimality of a standard debt contract. In the second case, the contract has face value  $\bar{R} \geq X_B$  which implements payment  $\underline{R} = X_B$  because the dealer defaults in this state. Note that the first case should correspond to the case where the hedging effect is active. The second case would corresponds to the case when only the skin-in-the game effect is active because then all the cash flows of the loan to B are pledged to the lenders.