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Natural Rate of Interest**

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Gibson's Paradox and the Natural Rate of Interest*

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Abstract

We argue that Gibson's paradox has nothing to do with the Gold Standard per se, and it rather originates from low-frequency variation in the natural rate of interest under certain types of monetary regimes that make inflation I(0) and (approximately) zero-mean. Although the Gold Standard is the only historical example of such a regime, Gibson's paradox is a feature of a potentially wide array of monetary arrangements. In fact, once removing the deterministic component of the drift in the price level, the paradox can be recovered from the data generated under inflation-targeting regimes. By the same token, the paradox could arise under a regime targeting the level of the money stock, whereas it would not appear under arrangements targeting the levels of either prices or nominal GDP. We show that the mechanism underlying Gibson's paradox hinges on the interaction between the Fisher equation and an asset pricing condition determining the current value of money. Our interpretation points towards inefficiencies in the actual implementation of monetary policies.

Keywords: Gibson's Paradox; monetary regimes; natural rate of interest; Fisher equation; Gold Standard; inflation targeting; optimal monetary policy.
JEL Classification: E2, E3.

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1 Introduction

Gibson's paradox, the strong positive correlation between the price level and long-term nominal interest rates that had prevailed under the Gold Standard (see Figures 1*a*-1*b*), is one of the most robust stylized facts in empirical macroeconomics. It is also one of the most mysterious since, in spite of more than a century of theoretical and empirical investigations, there is no widespread consensus on what, exactly, had produced it. Writing in the mid-1970s, Friedman and Schwartz (1976, p. 288) pointed out that '*[t]he Gibson paradox remains an empirical phenomenon without a theoretical explanation.*' Following Friedman and Schwartz's (1982) observation about the temporal coincidence of Gibson's paradox with the Gold Standard, the only broad agreement in the literature appears to be that, since the paradox had only appeared under the Gold Standard, and it has instead been absent from post-WWII data, it had likely originated from the peculiar workings of monetary regimes based on commodity money.

In this paper we reconsider Gibson's paradox based on New Keynesian models for either the Gold Standard or inflation targeting regimes. Our evidence suggests that the paradox has nothing to do with the Gold Standard per se, and it rather originates from low-frequency variation in the natural rate of interest under certain types of monetary regimes that make inflation mean-reverting and (approximately) zero-mean. Although the Gold Standard is the only historical example of such a regime, Gibson's paradox is, in principle, a feature of a potentially wide array of monetary arrangements.

In fact, estimated New Keynesian models for inflation-targeting regimes suggest that—exactly as for the Gold Standard—very highly persistent shocks to the natural rate of interest generate a strong, positive long-horizon correlation between the price level and long-term nominal interest rates. Under existing inflation targeting frameworks this correlation is not apparent in the raw data mainly due to the presence of the drift in the price level induced by a positive inflation target. We show however that once removing the deterministic component of the drift in the price level a positive long horizon correlation between nominal long-term interest rates and the price level can be recovered from the data generated under inflation-targeting regimes. By the same token, we show that the paradox could arise under a regime targeting the level of the money stock, whereas it would not appear under arrangements targeting the levels of either prices or nominal GDP.

Although our evidence is mainly based on estimated New Keynesian models, a crucial *logical* point to stress from the outset is the following. The fact that, under the Gold standard, Gibson's paradox pertains to the very low frequencies (i.e., the long horizons) implies that what is required in order to generate it is a shock (or a set of shocks) with a very highly persistent (in the limit, permanent) impact on *both* prices *and* the long rate. As for prices there are several candidates, such as shocks to the overall stock of gold (due to the discovery of new gold fields, or technological breakthroughs such as the invention of the cyanide process), or permanent GDP shocks (due e.g. to improvements in technology). On the other hand, and crucially, under monetary regimes such as the Gold Standard or inflation-targeting frameworks, in which inflation had been, and is, strongly mean-reverting, long-horizon fluctuations in nominal interest rates can *only* be driven by low-frequency fluctuations in the natural rate of interest.

1.1 The mechanism underlying Gibson's paradox

Intuitively, the mechanism underlying the emergence of Gibson's paradox under specific types of monetary regimes hinges on the interaction between the Fisher equation and an asset pricing condition determining the current value of money. The Fisher equation implies that long-horizon fluctuations in the natural rate of interest automatically map, one-for-one, into corresponding fluctuations in nominal interest rates at all maturities. The asset pricing condition, on the other hand, implies that increases in the natural rate—which is the discount factor for the determination of the current value of money—map into decreases in the expected value of future money, and therefore in the current value of money, which are obtained via corresponding increases in the price level. Long-horizon increases (decreases) in the natural rate of interest therefore map into corresponding low-frequency increases (decreases) in the price level via the asset pricing condition, and in nominal interest rates via the Fisher equation. Although this mechanism works for nominal interest rates at all maturities, in practice it is especially apparent for long rates, which behave as the long-horizon components of short rates.

Although the workings of the asset pricing condition is especially apparent under a Gold Standard regime, in which the value of money is automatically linked to the value of gold, in fact an asset pricing condition determining the current value of money underlies any monetary regime. Since the Fisher equation also features in any meaningful economic model, this implies that Gibson's paradox has nothing to do with the Gold Standard per se, and it is in principle a feature of a potentially wide array of monetary arrangements.

A necessary condition for Gibson's paradox to be visible in the raw data, as it had been under the Gold Standard, is that the workings of neither the Fisher equation, nor the asset pricing condition, should be obscured by features introducing a disconnect between prices and nominal interest rates at long horizons. One such feature is the presence of a positive inflation target, which introduces an upward drift in the price level, thus obscuring any otherwise existing relationship between nominal interest rates and the price level. In particular, the presence of non-zero drift in prices causes Gibson's correlation to become hidden in the raw data, but other than that it does not affect it in any way, so that it is still possible to recover it by simply removing the drift. By the same token, (near) unit-root behavior in inflation introduces, via the Fisher equation, a second long-horizon component in nominal interest rates. To the extent that long-horizon fluctuations in inflation and the natural rate are weakly correlated, this introduces a wedge between low-frequency movements in prices and in nominal interest rates, thus distorting the relationship between the two variables originating from fluctuations in the natural rate. An even starker case is represented by a price-level targeting regime. Since such a regime ultimately fully neutralizes shocks to the price level, by returning it to target, it inherently causes prices and nominal interest rates to be disconnected at long-horizons.

1.2 Gibson's paradox and optimal monetary policy

An implication of the previous discussion is that the appearance of Gibson's paradox is clear indication of the sub-optimality of monetary policy. If the central bank were able to track fluctuations in the natural rate of interest, and to neutralize the impact on the economy of shocks to the natural rate, Gibson's paradox would *never* appear in the data, as in the Neo-Wicksellian approach to policy proposed by Woodford (2003).

In the light of the long-standing criticism of the inflexibility of the Gold Standard, the fact that Gibson's paradox had so starkly appeared under this regime should therefore come as no surprise. More intriguing is the fact that it can be recovered from the data generated under inflation targeting regimes.

A natural metric for assessing the sub-optimality of monetary policy is the fraction of the variance of macroeconomic time series that is explained by shocks to the natural rate. Based on standard SVAR methods, this metric can be straightforwardly computed in the frequency domain, thus allowing to assess the extent of sub-optimality both overall, and at different frequencies. Our main finding is that for all countries, and under either monetary regime, shocks to the natural rate had, and have explained non-negligible fractions of the variance of inflation and real GDP growth, thus pointing towards the sub-optimality of the way monetary policy has been implemented under either monetary framework.

The paper is organized as follows. The next section documents the evolution of the relationship between long-term nominal interest rates and the price level since the early XVIII century, whereas Section 3 briefly reviews the previous literature on Gibson's paradox. Section 4 outlines a theory of Gibson's paradox based on standard DSGE models. Section 5 estimates New Keynesian models for both the Gold Standard and inflation-targeting regimes, and analyzes their impulse-response functions in order to determine which shocks could plausibly have generated Gibson's paradox. In Section 6 we explore the long-horizon relationship between prices and nominal long-term interest rates induced by alternative monetary policy rules. Section 7 shows that, once controlling for the deterministic component of the drift in the price level induced by the presence of a positive inflation target, a positive long-horizon relationship between the price level and long-term nominal interest rates can

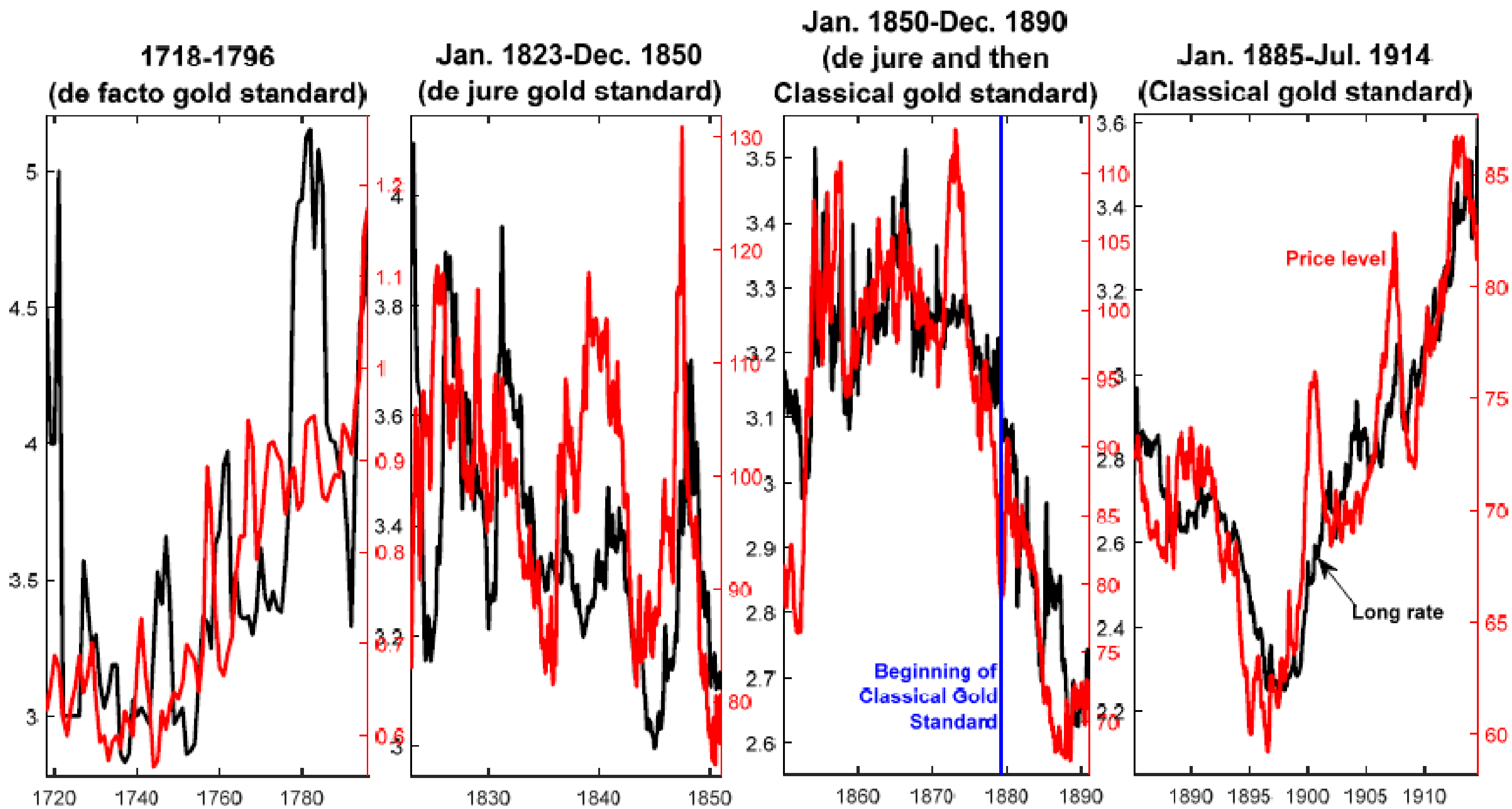

Figure 1a Gibson's paradox in the United Kingdom under the Gold Standard

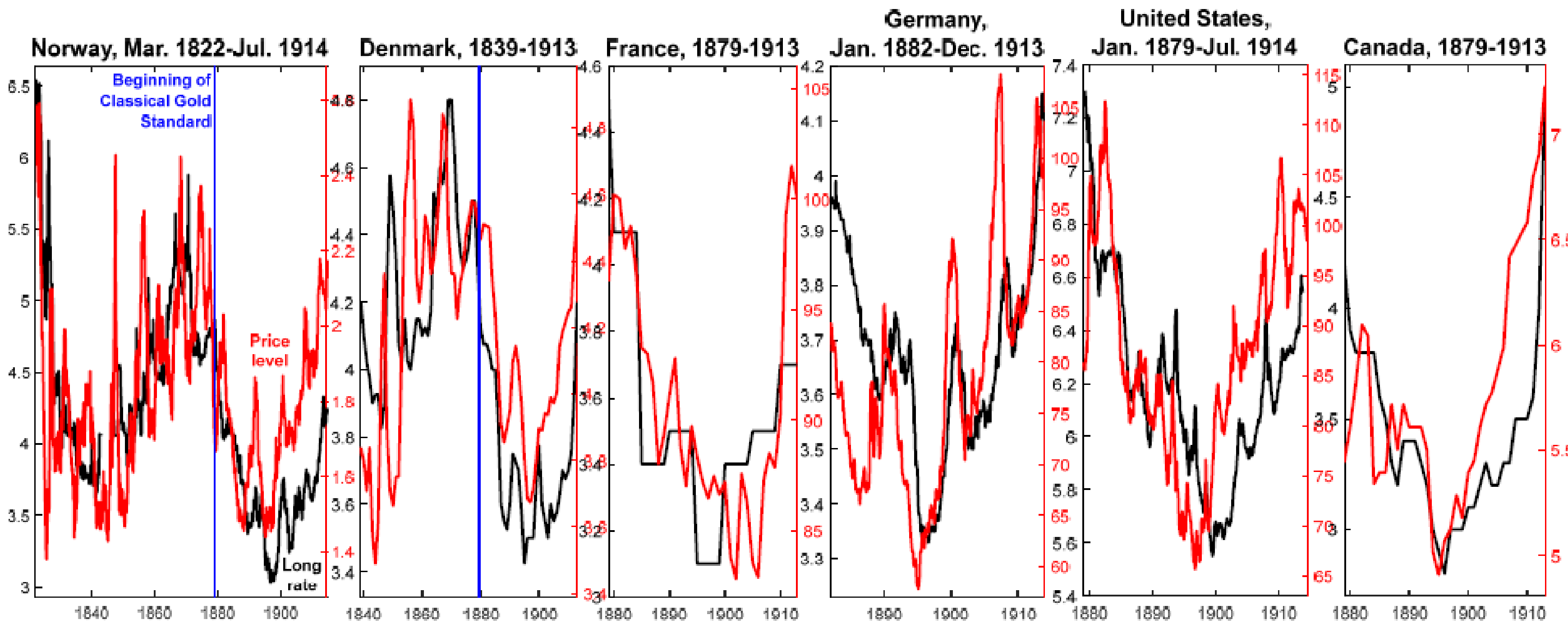

Figure 1b Gibson's paradox in other countries under the Gold Standard

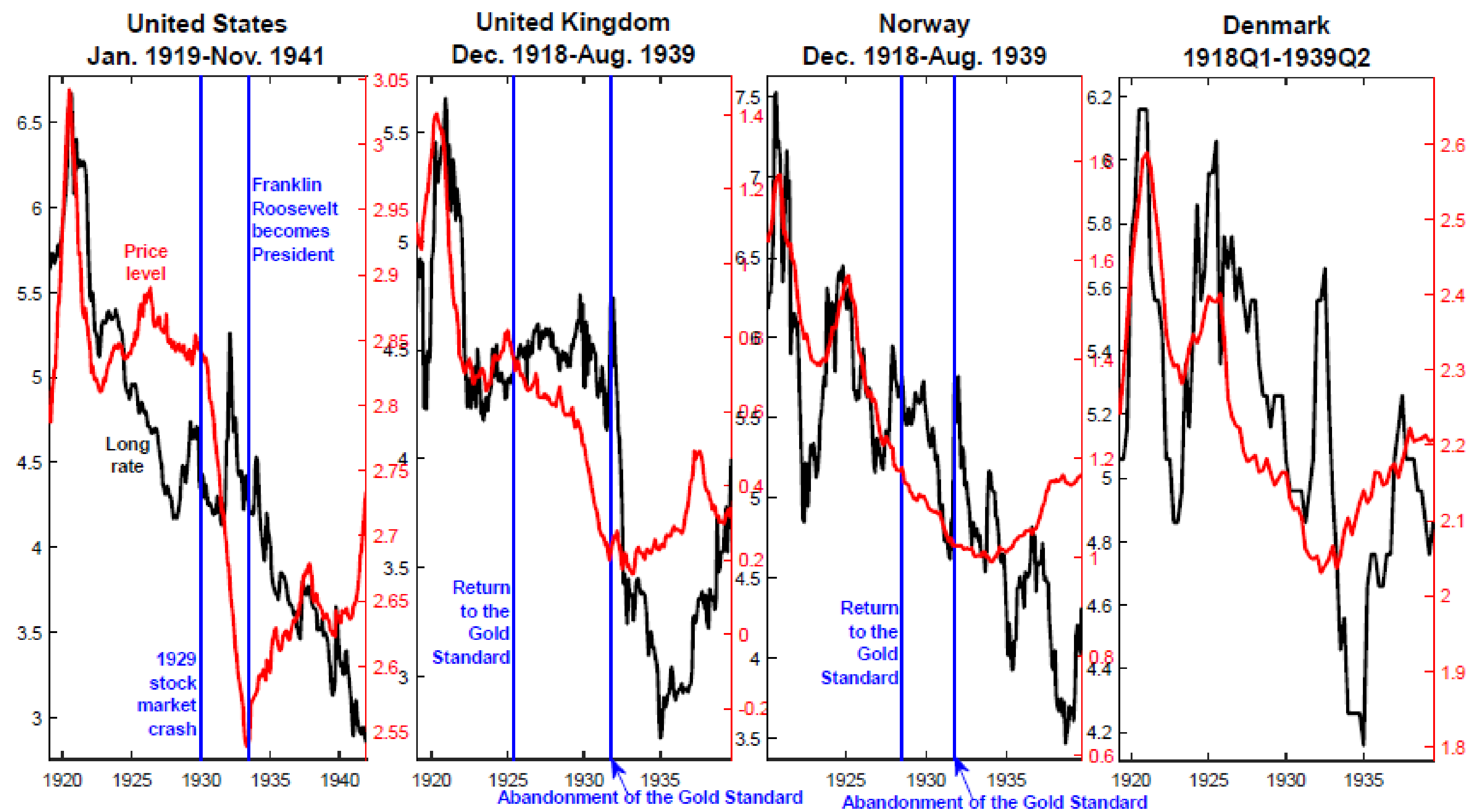

Figure 1c Gibson's paradox during the interwar period

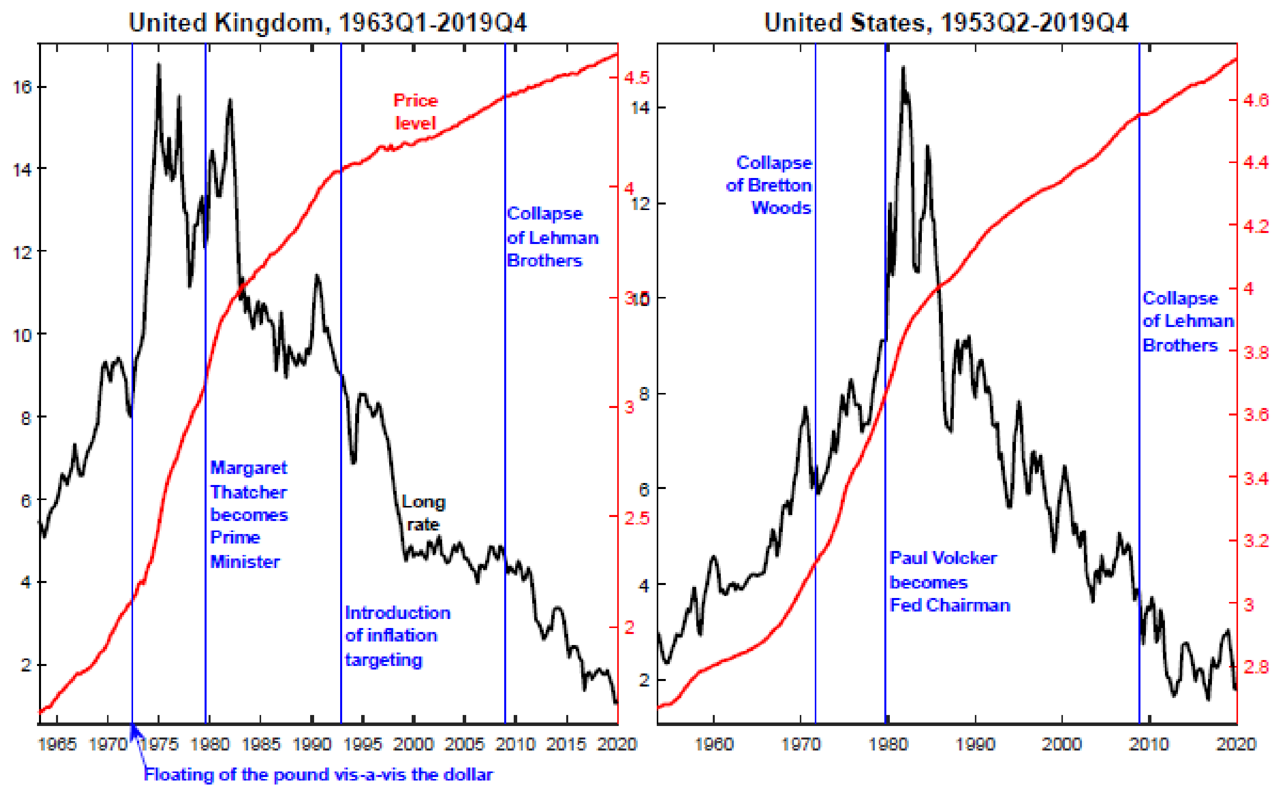

Figure 1d The price level and long-term nominal interest rates over the post-WWII period in the United Kingdom and the United States

be recovered from the data generated by inflation-targeting regimes. Section 8 illustrates our frequency-domain-based proposed metric for assessing the sub-optimality of monetary policy, and applies it to both the United Kingdom and the United States under the Gold Standard, and to inflation targeting regimes. Section 9 concludes.

2 Stylized Facts

Figures 1*a*-1*d* illustrate the evolution of the relationship between long-term nominal interest rates and the price level since the early XVIII century. Gibson's paradox had been near-uniformly apparent in the data up until the outbreak of World War I, sometimes strikingly so.[1] This is the case in particular for the United Kingdom since 1850, Norway since 1822, Denmark since 1839, and the United States during the Classical Gold Standard period (January 1879-July 1914). Interestingly, for the United Kingdom a positive low-frequency co-movement between the two series is clearly apparent also in the data from the XVIII century,[2] whereas the correlation had been weaker during the period between the re-establishment of the prewar gold parity[3] following the end of the Napoleonic Wars and the mid-XIX century.

In fact, the evidence for the Gold Standard period is so strong that in most cases statistical tests detect cointegration between the price level and long-term nominal interest rates. Specifically, Johansen's tests of the null of no cointegration, which is predicated on the assumption that the series feature exact unit roots,[4] uniformly detects cointegration based on monthly data,[5] with bootstrapped p-values[6] for the maximum eigenvalue tests ranging between 0.0000 and 0.0259. On the other hand,

[1] At first sight, a possible concern about the evidence reported in Figures 1*a*-1*c* is that, with near certainty, old price series are plagued by a non-negligible extent of measurement error (see e.g. Cogley and Sargent, 2015, and Cogley et al., 2015). Further, it can plausibly be assumed that the older the price indices, the greater the extent of measurement error they suffer from. (On the other hand, since interest rates had been quoted on financial markets, and their quotes had typically been recorded in official publications, the problem is likely virtually non-existent for long-term interest rates.) In fact, for the purpose of documenting the evolution of the long-horizon relationship between long-term interest rates and prices, since measurement error pertains to the price *level* its presence should not introduce any material distortion.

[2] Evidence for the period 1703-1717, during which the United Kingdom had been on a de facto *silver* standard, is qualitatively the same as for the period 1718-1796 (we do not report this evidence because the period is extraordinarily short, but it is available upon request). This suggests that what mattered was that the monetary standard was based on a commodity, rather than gold per se.

[3] The gold parity was officially re-established in May 1821.

[4] For all countries and sample periods evidence from Elliot, Rothenberg and Stock (1996) tests suggests that the null hypothesis of a unit root cannot be rejected. We interpret these results as pointing towards either an exact or a near unit root.

[5] I.e., for the United Kingdom based on the series plotted in the second, third, and fourth panel of Figure 1*a*, and for the United States, Norway, and Germany based on the series plotted in Figure 1*b*.

[6] We bootstrap the p-values as in Cavaliere, Rahbek, and Taylor (2012).

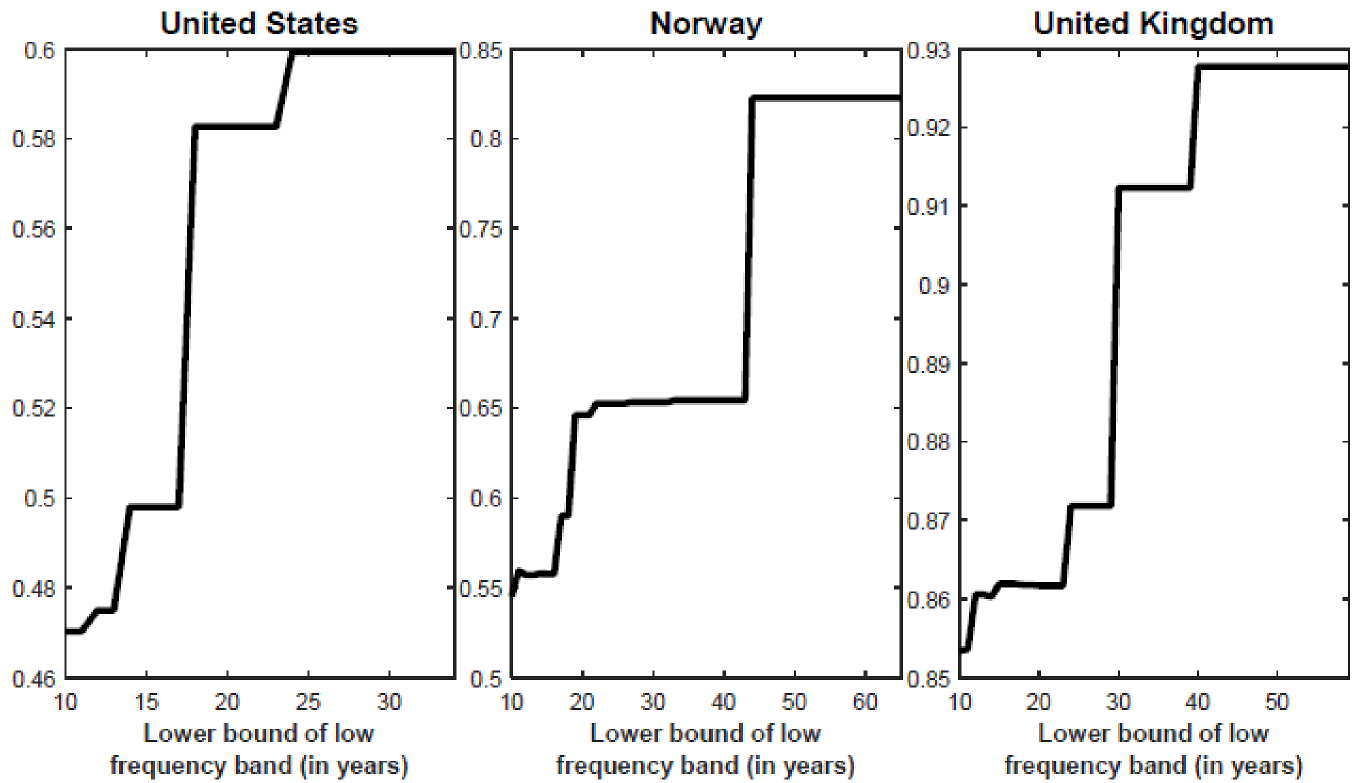

Figure 2 Evidence from Müller and Watson's low-frequency regressions: R^2 in the regression of log prices on the long-term nominal interest rate by frequency band

based on annual data the null hypothesis is never rejected.[7] By the same token, Wright's (2000) test,[8] which is valid for both exact and near unit roots, detects cointegration based on monthly data for all countries except Germany, whereas based on annual data it only detects it for France.[9]

Both the visual evidence in Figures 1*a*-1*b* and the previous discussion suggest that Gibson's paradox is a very low-frequency phenomenon. Figure 2 reports some simple evidence on this. The figure shows, for the United States, Norway, and the United Kingdom,[10] the R^2 by frequency band in the low-frequency regression of log prices on a long-term nominal interest rate based on the methodology proposed by Müller and Watson (2018, 2020). As the regression focuses on lower and lower frequencies the R^2 consistently increases. For example, for Norway it is equal to 'just' 0.55 for the frequency band associated with cycles slower then ten years, and it increases to 0.825 for cycles slower than 43 years or more. Evidence for the other two countries is qualitatively the same.

Evidence for the interwar period is uniformly weaker, but overall it appears to still point towards a positive long-horizon co-movement between long-term rates and the price level. Any evidence of a long-horizon co-movement between the two series however vanished altogether following the end of World War II, with the price level acquiring a consistently positive drift, and long-term nominal interest rates exhibiting instead a hump-shaped pattern mostly attributable to the rise, and then fall of inflation associated with the Great Inflation episode, and in more recent years to the progressive decline of the natural rate of interest. In fact, as we will show in Section 7, the positive long-horizon correlation between the price level and long-term nominal interest rates that is the hallmark of Gibson's paradox can be recovered from the data generated by inflation-targeting regimes.

3 Previous Literature on Gibson's Paradox

Although Keynes (1930) labelled the paradox after Gibson (1923), in fact a long-horizon positive correlation between prices and long-term nominal interest rates had already been discussed by Tooke (1844) and Wicksell (1898, 1907).

Wicksell and Keynes proposed an explanation centered on the workings of the commercial banking system. In reaction to an increase in the productivity of capi-

[7] I.e., for the United Kingdom based on the series plotted in the first panel of Figure 1*a*, and for Denmark, France, and Canada based on the series plotted in Figure 1*b*.

[8] We bootstrap Wright's (2000) test via the procedure proposed by Benati, Lucas, Nicolini, and Weber (2021).

[9] Overall, our results therefore question the evidence of Corbae and Ouliaris (1989), who based on Phillips and Ouliaris's (1990) tests and asymptotic critical values do not detect cointegration for the United States and the United Kingdom.

[10] We focus on these three countries because they feature long samples of monthly data. Evidence for the other countries in Figure 1*b* is in line with that in Figure A.1, with the partial exception of Denmark and Canada.

tal, and therefore in the demand for credit, commercial banks increase lending, and therefore the money supply, but they only do so with a lag. As a result nominal interest rates rise, but they consistently lag behind the natural rate of interest, and the resulting economic expansion leads to an increase in the price level. Cagan (1965) and Shiller and Siegel (1977) refuted Wicksell and Keynes' explanation on empirical grounds, by pointing out that an increase in the money supply due to an increase in bank lending increases the money multiplier, thus counterfactually inducing a positive correlation between the multiplier and the price level.[11]

Fisher (1930) argued that since expected inflation is positively correlated with prices, fluctuations in inflation expectations cause corresponding fluctuations in the same direction in both nominal interest rates and the price level. Several authors[12] expressed skepticism of this explanation, because Fisher's hypothesis that agents form inflation expectations based on inflation's past behavior implies that, in order to produce Gibson's paradox, they should be implausibly slow in adjusting their expectations in response to changes in actual inflation.[13] Further, from a rational expectations perspective a crucial problem with this explanation is that under the Gold Standard, during which Gibson's paradox had appeared, inflation had been near-uniformly indistinguishable from white noise,[14] thus implying that expected inflation had been essentially constant.[15]

Sargent (1973) builds an IS-LM-type model featuring inertia in the adjustment of wages and prices to their long-run equilibrium values in response to shocks. Following a permanent, one-off increase in the money supply at t=t_0 prices slowly increase towards their new long-run equilibrium, whereas the nominal interest rate jumps downwards at t_0 and then slowly reverts to its original value (see Figure 13, p. 443). During the transition between steady-states the model therefore generates Gibson's paradox. As stressed by Sargent (1973), *'[t]he key reason that the Gibson paradox may infest the data generated by the model is the failure of wages and prices to adjust sufficiently quickly to keep output always at its full-employment level.'*

Shiller and Siegel (1977) proposed an explanation based on the impact of unanticipated changes in the price level on the distribution of wealth between creditors and debtors. An unanticipated increase in the price level causes a decrease in the real value of nominal bonds, thus causing an increase in the real wealth of debtors, and

[11] Cagan (1965) further argued that, at least in the United States, changes in the money supply had originated to a dominant extent from changes in high-powered money, rather than changes in lending on the part of commercial banks. Jonung (1976) produced qualitatively the same evidence for Sweden.

[12] See Macauley (1938), Cagan (1965), and Sargent (1973).

[13] Fisher's (1930) own estimates implied that the average lag of expected inflation on actual inflation should have ranged between 7.3 and 10.7 years.

[14] See Barsky (1987) and Benati (2008).

[15] As pointed out by Sargent (1973), 'it is difficult *both* to accept Fisher's explanation of the Gibson paradox, *and* to maintain that the extraordinarily long lags in expectations are 'rational". In fact, Sargent's own estimates suggested that for the United States during the period 1880-1914 the optimal prediction for inflation had been essentially zero.

a corresponding decrease in the real wealth of creditors. Assuming that both agents want to maintain a certain fraction of their wealth in either long or short positions in bonds, debtors, having become wealthier, want to increase their supply of bonds by more than the decrease in their real value. Symmetrically creditors, having become poorer, want to increase their holdings of bonds by less than the change in their real value. At the initial equilibrium interest rate there is therefore an excess supply of bonds, and interest rates have to increase in order to restore equilibrium on the market. At first sight this explanation would appear of limited applicability, since it crucially hinges on *unanticipated* changes in the price level. In fact this is not the case, since under the Gold Standard the price level was statistically indistinguishable from a random walk.

Friedman and Schwartz (1982, pp. 527-587) featured a detailed discussion of the literature up until the early 1980s. Following their observation about the temporal coincidence of Gibson's paradox with the Gold Standard, most subsequent authors have proposed explanations based on the peculiar workings of monetary regimes based on commodity money.

Lee and Petruzzi (1986) proposed an explanation based on the reallocation of wealth between gold and financial assets induced by fluctuations in real interest rates. In response to an increase in real, and therefore nominal interest rates, investors shift their wealth from gold to financial assets. In turn, the decrease in the demand for gold causes a fall in its real price in terms of goods, that is, an increase in the overall price level.

Building upon Barro's (1979) benchmark model of the Gold Standard, Barsky and Summers' (1988)[16] analysis is centered on the nature of gold as a very long-lived asset. An increase in the real interest rate, and therefore in nominal rates, causes a decrease in the demand for monetary gold via a standard money demand function. At the same time, by increasing the carrying cost of gold it decreases its demand for non-monetary purposes (jewelry, art, ...), which in the United Kingdom during the period between the end of the Napoleonic Wars and World War I was about two-thirds of the overall gold stock. The resulting decrease in the overall demand for gold causes, as in Lee and Petruzzi (1986), a decrease in the real price of gold, which is obtained via an increase in the price level.

Finally, some authors questioned the very existence of Gibson's paradox. Benjamin and Kochin (1984) for example argued that *'[i]n significant part the movements of both the interest rate and the price level have been produced by war. Once the influence of war is taken into account, there is virtually no evidence of any linkage between the price level and the long-term interest rate.'* Benjamin and Kochin's evidence was refuted by Barsky and Summers (1988). In fact, the evidence in Figures 1*a*-1*c* clearly speaks against Benjamin and Kochin's position.

Less drastically, some authors questioned the solidity of Gibson's paradox. Macaulay

[16] Online Appendix D discusses the relationship between Barsky and Summers's (1988) analysis of the Gold Standard and ours.

(1938), for example, stated that *'the exceptions to this appearance of relationship are so numerous and so glaring that they cannot be overlooked'* By the same token, Dwyer (1984) argued that for the period before World War I *'the only statistically significant correlations are the correlations of prices and short-term interest rates in the United States and France.'* Again, our evidence for the Gold Standard period, as well as (e.g.) Barsky and Summers' (1988), clearly refutes this position, and it rather supports Keynes' (1930) assertion that Gibson's paradox is *'one of the most completely established empirical facts in the whole field of quantitative economics'.*

We now turn to a theory of Gibson's paradox.

4 A Theory of Gibson's Paradox

4.1 A model of the Gold Standard

Consider a closed-economy model with a representative agent maximizing the following utility function

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\theta_t^c U(C_t)+\theta_t^g V(g_t)+\theta_t^m L\left(\frac{M_t}{P_t}\right)\right]\right\},$$

in which β is the rate of time preference, with $0<\beta<1$; $U(\cdot)$ is a concave function, with C being the consumption of a perishable good; $V(\cdot)$ is a concave function of agent's gold holdings g, which are postulated to provide utility benefits. The representative agent also gets liquidity benefits from holding real money balances through the utility function $L(\cdot)$. The function is concave and displays a satiation point at $\bar{m}$, meaning that $L_m(\cdot)=0$ for $M_t/P_t\geq\bar{m}$ in which $L_m(\cdot)$ is the first derivative of the function $L(\cdot)$; M_t is the nominal stock of money held at time t and P_t is the price of the consumption good. Finally, $\theta_t^c,\theta_t^g,\theta_t^m$ are preference disturbances.

The agent is subject to the flow budget constraint

$$B_t+M_t+P_{g,t}g_t+P_tC_t=B_{t-1}(1+i_{t-1})+M_{t-1}+P_{g,t}g_{t-1}+P_tY_t-T_t+P_{g,t}(G_t-G_{t-1})$$

in which B_t are the holdings at time t of risk-free bonds denominated in units of currency with interest rate $1+i_t$; $P_{g,t}$ is the price of gold in units of currency; Y_t is the endowment of goods and G_t is the stock of gold in the economy, with $G_t\geq G_{t-1}$; T_t are lump-sum taxes levied by the Treasury. The representative agent's problem is subject to an appropriate borrowing limit condition.

The first-order condition with respect to B_t implies that

$$1=\beta E_t\left\{\frac{\theta_{t+1}^c U_c(C_{t+1})}{\theta_t^c U_c(C_t)}\frac{P_t}{P_{t+1}}\right\}(1+i_t);\qquad(1)$$

the one with respect to M_t is

$$\frac{\theta_t^c U_c(C_t)}{P_t}=\frac{\theta_t^m}{P_t}L_m\left(\frac{M_t}{P_t}\right)+\beta E_t\left\{\frac{\theta_{t+1}^c U_c(C_{t+1})}{P_{t+1}}\right\},\qquad(2)$$

while that with respect to gold holdings is

$$\frac{P_{g,t}}{P_t} = \frac{\theta_t^g}{\theta_t^c}\frac{V_g(g_t)}{U_c(C_t)} + \beta E_t\left\{\frac{\theta_{t+1}^c}{\theta_t^c}\frac{U_c(C_{t+1})}{U_c(C_t)}\frac{P_{g,t+1}}{P_{t+1}}\right\}. \tag{3}$$

The set of first-order conditions is completed by the exhaustion of the intertemporal budget constraint.

The other two agents in the economy are the Treasury and the central bank. The Treasury issues debt B_t^g at the nominal interest rate i_t levying lump-sum taxes according to the budget constraint

$$B_t^g = (1+i_t)B_{t-1}^g - T_t.$$

The central bank issues money with full convertibility into gold, fixing the convertibility rate between gold and dollars at a value that we normalize to $P_{g,t} = 1$. In order to do so, the central bank backs the money supply with gold

$$M_t = g_t^c$$

in which g_t^c are the central bank's holdings of gold.

In equilibrium goods, gold and asset markets clear, i.e.

$$C_t = Y_t$$

$$g_t + g_t^c = G_t$$

and

$$B_t = B_t^g.$$

Substituting these conditions into the first-order conditions we obtain

$$\frac{1}{1+i_t} = E_t\left\{\beta\frac{\theta_{t+1}^c}{\theta_t^c}\frac{U_c(Y_{t+1})}{U_c(Y_t)}\frac{P_t}{P_{t+1}},\right\} \tag{4}$$

$$\frac{1}{P_t} = \frac{1}{P_t}\frac{\theta_t^m}{\theta_t^c}\frac{L_m\left(\frac{g_t^c}{P_t}\right)}{U_c(Y_t)} + E_t\left\{\beta\frac{\theta_{t+1}^c}{\theta_t^c}\frac{U_c(Y_{t+1})}{U_c(Y_t)}\frac{1}{P_{t+1}}\right\}, \tag{5}$$

$$\frac{P_{g,t}}{P_t} = \frac{\theta_t^g}{\theta_t^c}\frac{V_g(G_t - g_t^c)}{U_c(Y_t)} + E_t\left\{\beta\frac{\theta_{t+1}^c}{\theta_t^c}\frac{U_c(Y_{t+1})}{U_c(Y_t)}\frac{P_{g,t+1}}{P_{t+1}}\right\}. \tag{6}$$

Since $P_{g,t} = 1$, the stochastic sequences $\{i_t, P_t, g_t^c\}_{t=t_0}^{\infty}$ solve the equilibrium conditions (4)-(6) for given exogenous sequences $\{G_t, Y_t, \theta_t^g, \theta_t^m, \theta_t^c\}_{t=t_0}^{\infty}$.

The first observation comes from equation (6). Iterating it forward we obtain that the relative price of gold is equal to the present-discounted value of its utility services:

$$\frac{1}{P_t} = E_t\left\{\sum_{T=t}^{\infty} R_{t,T}\frac{\theta_T^g}{\theta_T^c}\frac{V_g(G_T - g_T^c)}{U_c(Y_T)}\right\}, \tag{7}$$

where $R_{t,T} = \beta^{T-t}\theta_T^c U_c(Y_T)/(\theta_t^c U_c(Y_t))$. This expression pins down the value of money. Note, moreover, that combining equations (4) and (5), we obtain

$$\frac{\theta_t^m}{\theta_t^c}\frac{L_m\left(\frac{g_t^c}{P_t}\right)}{U_c(Y_t)} = \frac{i_t}{1+i_t}. \tag{8}$$

Since marginal utility with respect to real money balances is decreasing, the above equation implies a negative relationship between g_t^c/P_t and i_t and therefore a positive relationship between P_t/g_t^c and the nominal interest rate. Note moreover that equations (5) and (6) imply that

$$\frac{1}{P_t}\theta_t^m L_m\left(\frac{g_t^c}{P_t}\right) = \theta_t^g V_g(G_t - g_t^c), \tag{9}$$

i.e. the marginal benefits of gold as jewelry are equal to the marginal benefits of the liquidity services of money. Note that the above equation fully determines the price level as a function of the overall stock of gold G_t, and of central bank's gold holdings g_t^c. Define as $Z(\cdot,\cdot)$ the following function

$$Z(P_t, g_t^c) = \frac{1}{P_t}L_m\left(\frac{g_t^c}{P_t}\right).$$

Note that

$$Z_p(P_t, g_t^c) = -\frac{L_m(\cdot)}{P_t^2}\left[1 - \epsilon\left(\frac{g_t^c}{P_t}\right)\right],$$

in which $\epsilon(\cdot)$ is given $\epsilon(\cdot) = -(L_{mm}(\cdot)/L_m(\cdot))g_t^c/P_t$; Z is decreasing in P when the elasticity is less than the unitary value. Since in this case P_t in (9) is increasing in the overall stock of gold, this is the empirically relevant case. Combining (8) and (9), we get

$$P_t = \frac{i_t}{1+i_t}\left[\frac{\theta_t^c}{\theta_t^g}\frac{U_c(Y_t)}{V_g(G_t - g_t^c)}\right] \tag{10}$$

implying again, for given values of the objects within square brackets, a positive relationship between prices and the short-term nominal interest rate. Since, from an empirical standpoint, the long-term rate behaves as the low-frequency component of the short-term rate, this automatically implies the positive relationship between the long rate and the price level which is the hallmark of Gibson's paradox. On the contrary, the objects within brackets weaken the positive relationship between prices and the short-term nominal interest rate (and therefore the long rate) implied by the first portion of (10).

4.2 What had generated Gibson's paradox?

In order to understand which shocks are most likely to have generated the paradox, consider rewriting (4) as

$$\frac{1}{1+i_t} = E_t \left\{ \frac{1}{R^n_{t,t+1}} \frac{1}{\Pi_{t+1}}, \right\}, \tag{11}$$

for an appropriately defined random variable $R^n_{t,t+1} = [\theta^c_t U_c(Y_t)]/[\beta\theta^c_{t+1} U_c(Y_{t+1})]$ whose expectation represents the usually defined natural rate of interest at time t, while Π_{t+1} is the gross inflation rate between time t and t+1. Two are the drivers of the nominal interest rate, the natural rate of interest and the inflation rate, with increases in either of them leading to corresponding increases in the nominal interest rate. Low-frequency fluctuations in the natural rate of interest and/or the inflation rate are the sources of long-horizon variation in nominal interest rates.

Moreover, rewrite (7) as

$$\frac{1}{P_t} = E_t \left\{ \sum_{T=t}^{\infty} \frac{1}{R^n_{t,T}} \frac{\theta^g_T}{\theta^c_T} \frac{V_g(G_T - g^c_T)}{U_c(Y_T)} \right\} : \tag{12}$$

this expression shows that the forces driving the price level under a Gold Standard regime do so via their impact on the natural rate of interest, output, gold production and central bank's holding of gold (on top of the shocks θ^g_t and θ^c_t).

We can also write (12) as

$$\frac{1}{P_t} = \frac{\theta^g_t}{\theta^c_t} \frac{V_g(G_t - g^c_t)}{U_c(Y_t)} + E_t \left\{ \frac{1}{R^n_{t,t+1}} \frac{1}{P_{t+1}} \right\}. \tag{13}$$

Under a Gold Standard regime the value of money today depends on (i) the utility services provided by gold, which are the first addendum on the right-hand side of the expression, and (ii) the expected discounted value of money, which is the second addendum, where the discount factor is the natural rate of interest.

Key to explain Gibson's paradox is that the common element in equations (11) and (13) is indeed the natural rate of interest $R^n_{t,t+1}$, which everything else equal affects the price level and the nominal interest rate in the same way.

The intuition for why the natural rate impacts positively upon the nominal interest rate is straightforward, and it comes directly from the Fisher equation. Within the asset pricing equation (13), on the other hand, a higher natural rate of interest, by decreasing the expected value of future money, causes a corresponding decrease in the value of money today, thus increasing the price level.

From an empirical standpoint, however, the crucial aspect is that in order not to obscure the common factor between the nominal interest rate and the price level, the other elements in the previous two equations should not vary much compared to the natural rate. In particular, the inflation rate should be mute in (11), as well as both the utility services provided by gold and the inflation rate in equation (13).

A final important issue to be discussed pertains to the stationarity of inflation under the Gold Standard, which is consistent with the just-mentioned discussion of why the natural rate of interest is the only possible driver of Gibson's paradox. Based on our previous discussion of expression (11), and of long-horizon drivers of nominal interest rates, as matter of logic the fact that inflation had been I(0) leaves the natural rate of interest as the *only* possible driver of low-frequency variation in nominal interest rates.[17] Equation (13) is compatible with a stationary inflation rate, and with the price level being perturbed by fluctuations in the natural rate of interest. To see this, consider first-order approximations of equations (11) and (10), which imply expressions

$$\hat{\imath}_t = r_t^n + E_t \pi_{t+1}$$

and

$$p_t = (1-\beta)[(\hat{\theta}_t^c - \hat{\theta}_t^g) + \epsilon_g^{-1}(\hat{g}_t - s_g \hat{g}_t^c) - \sigma^{-1}\hat{Y}_t + \beta r_t^n + \beta E_t p_{t+1},$$

where a $\hat{}$ over a variable denotes a logarithmic deviation from the steady-state; p_t is the log of the price level and $\pi_t = p_t - p_{t-1}$; r_t^n is the natural rate of interest in the log-linear approximation; σ, ϵ_g and s_g are parameters detailed in the Online Appendix B. Since under the Gold Standard inflation had consistently been statistically indistinguishable from a zero-mean, white noise process,[18] in the previous two expressions $E_t \pi_{t+1} \simeq 0$, and $E_t p_{t+1} \simeq p_t$, which clearly highlights how the *only common driver* of prices and nominal interest rates had been the natural rate of interest, r_t^n.

Further, concerning inflation, a first-order approximation of (9) implies that

$$\pi_t = -\vartheta_g \Delta \hat{g}_t^c + \vartheta_G \Delta \hat{g}_t - \vartheta_\theta (\Delta \hat{\theta}_t^g - \Delta \hat{\theta}_t^m),$$

for positive parameters ϑ_g, ϑ_G and ϑ_θ, in which Δ is the first-difference operator, i.e. $\Delta \equiv 1 - L$, where L is the lag operator. The above equation shows that the inflation rate could be affected by fluctuations in the natural rate of interest only through the endogenous variation of the central bank's gold holdings, $\Delta \hat{g}_t^c$. Changes in $\hat{g}_t^c$ are however most likely stationary: in fact, for both the United States and the United Kingdom evidence from Elliot et al.'s (1996) tests points towards very strong rejection of the null of a unit root in $\Delta \hat{g}_t^c$.[19] In turn, this implies that the natural rate of interest is the only driver of long-horizon variation in nominal interest rates.

4.3 Alternative monetary regimes

The fact that, as we will show, the natural rate of interest had been the driver of Gibson's paradox under the Gold Standard raises the obvious possibility that, if

[17] Notice that equation (10) is compatible with a stationary inflation rate and with the price level being perturbed by movements in the natural rate of interest.

[18] See Barsky (1987) and Benati (2008).

[19] The bootstrapped p-values for the two countries range between 0.0000 and 0.0231.

fluctuations in the natural rate were sufficiently large compared to other sources of variation, the paradox might also appear under alternative monetary regimes.

Under this respect, the experience of inflation targeting regimes is intriguing. As it has been documented,[20] under these regimes inflation has consistently been strongly mean-reverting, and in many cases statistically indistinguishable from white noise. At the same time since the early 1990s, when these regimes started being introduced, the natural rate of interest has near-uniformly been drifting downwards by substantial amounts.[21] This suggests that inflation targeting regimes could well have generated Gibson's paradox, and that only the presence of (i) a positive drift in the price level associated with a positive inflation target, and (ii) a negative drift in long-term rates due to the progressive decrease in the natural rate, has prevented it from being manifestly apparent in the raw data as it had been under the Gold Standard. As we will see, our evidence in Sections 5.2 and 7 strongly suggests that this is indeed the case.

By the same token, in the light of the previous discussion the evidence in Figure 1*c* of a low-frequency relationship between the price level and long-term interest rates during the interwar period has a natural interpretation. As argued e.g. by Eggertsson (2008), the Great Depression had been characterized by a dramatic fall in the natural rate of interest. E.g., Benati (2023) estimates that in the United States it had fallen from nearly 5.5 per cent immediately before the 1929 stockmarket crash to between 1 and 2 per cent during the period between President Roosevelt's inauguration and the attack on Pearl Harbor. This suggests that variation in the natural rate might well have been dominant compared to other sources of variation, thus causing Gibson's paradox to appear in the data.

Formally, a common relationship between the Gold Standard and alternative monetary regimes is the Fisher equation (11). In a regime in which the central bank sets its policy in terms of money supply the relevant asset pricing condition for determining the value of money is equation (5), which can be written as

$$\frac{1}{P_t} = \frac{1}{P_t}\frac{\theta_t^m}{\theta_t^c}\frac{L_m\left(\frac{M_t}{P_t}\right)}{U_c(Y_t)} + E_t\left\{\frac{1}{R_{t,t+1}^n}\frac{1}{P_{t+1}}\right\}. \tag{14}$$

This expression exhibits some similarities with (13), with the value of money today depending on the discounted value of money in the future, where the discount factor is once again the natural real rate of interest. The main difference with the Gold Standard is that what now matters is the real liquidity value of money, as opposed to the utility benefits of gold. To the extent that variation in such liquidity value is

[20] See e.g. Benati (2008).

[21] E.g., Holston et al.'s (2017) estimates suggest that between the early 1990s and the mid-2010s the natural rate had decreased from 2.5% to slightly more than 1% in Canada. The corresponding figures for the Euro area are about 2.5 and -0.3%, whereas those for the United Kingdom are about 2.1 and 1.5%.

not sufficiently large as to interfere with the relationship between the natural rate of interest and prices, Gibson's paradox should emerge. In fact, our results in Section 6 suggests that this is indeed the case. In particular, we will show that the impulse-response functions (IRFs) to shocks to the natural rate are qualitatively the same as under the Gold Standard.

On the other hand, under a monetary framework in which the central bank controls the interest rate, there is apparently no similar asset pricing equation. However, consider for example an interest rate rule such as

$$1 + i_t = (1 + \bar{\imath}_t) \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi}$$

for some process $\bar{\imath}_t$, in which $\phi_\pi > 1$ and Π is a constant inflation target. This expression can still be combined with (11) in order to obtain the 'asset-pricing' condition

$$z_t \left(\frac{1}{P_t} \right)^{1+\phi_\pi} = E_t \left\{ \frac{1}{R^n_{t,t+1}} \frac{1}{P_{t+1}} \right\}, \tag{15}$$

with z_t given by

$$z_t = \frac{(P_{t-1}\Pi)^{\phi_\pi}}{(1 + \bar{\imath}_t)}$$

The condition (15) relates the current value of money to the expected value of money discounted by the natural rate of interest. Once again, the factor z_t in (15), as well as the inflation rate in (11), should be such as not to obscure the relationship between the natural rate and the price level.

Under a regime targeting the price level, on the other hand, the dynamic of inflation following the initial shock is such as to return the price level to target, thus blurring the relationship with the natural rate, and therefore with nominal interest rates. In particular, if, as in e.g. Laubach and Williams (2003) and Holston, Laubach and Williams (2017) the natural rate is a unit root process, a shock to the natural rate has a permanent impact on the natural rate itself, whereas it has no impact on the price level, thus completely destroying any long-horizon relationship between the long rate and the price level. In the more realistic case in which the natural rate is a near unit root process the same logic still holds.

We now turn to the New Keynesian model that we will take to the data.

4.4 The New Keynesian model

We start by discussing the structure of the model, and we then turn to the stochastic properties of the key driving processes.

4.4.1 The model

In order to estimate the model of Section 4.1 we make some additional assumptions that are motivated by the need to have a realistic empirical characterization of the data. Specifically, we add habit formation in consumption in order to capture the high inertia that characterizes empirical measures of the output gap, and price rigidities in order to provide a realistic description of the inflationary process.[22]

To model habit formation in consumption we consider a utility function of the form

$$U(C_t - hC_{t-1}) = \frac{(C_t - hC_{t-1})^{1-\sigma_x^{-1}}}{1-\sigma_x^{-1}} \tag{16}$$

Denoting by $x_t = C_t - hC_{t-1}$ we have that $\hat{x}_t = (1-h)^{-1}(\hat{C}_t - h\hat{C}_{t-1})$ and $\sigma_x^{-1} \equiv -(xU_{xx}/U_x)$. A first-order approximation of the Euler equation implies that

$$\hat{x}_t = E_t\hat{x}_{t+1} - \sigma_x(\hat{\imath}_t - E_t\pi_{t+1} - \epsilon_t), \tag{17}$$

where ϵ_t is a reparameterization of preference shocks and $\hat{\imath}_t$ is the short-term nominal interest rate in log-deviations with respect to the steady state. Assuming consumption is equal to output, and detrending output by the natural rate of output, y_t^n we obtain

$$\tilde{y}_t = \gamma E_t\tilde{y}_{t+1} + (1-\gamma)\tilde{y}_{t-1} - \sigma(\hat{\imath}_t - E_t\pi_{t+1} - r_t^n) + v_t, \text{ with } \epsilon_t^v \sim N(0, \sigma_v^2), \tag{18}$$

in which $\tilde{y}_t$ is the output gap; $\sigma = \sigma_x(1-h)/(1+h), \gamma = 1/(1+h)$; and the natural rate of interest is

$$r_t^n = \epsilon_t + \sigma^{-1}\gamma E_t\Delta y_{t+1}^n - \sigma^{-1}(1-\gamma)\Delta y_t^n. \tag{19}$$

As in Calvo's model, the Phillips curve is given by

$$\pi_t = k(\eta y_t + \sigma_x^{-1}\hat{x}_t) + \beta E_t\pi_{t+1} + u_t, \text{ with } u_t \sim N(0, \sigma_u^2)$$

with $\pi_t = p_t - p_{t-1}$, where p_t is the logarithm of the price level; k is a positive parameter; η is the inverse of the Frisch elasticity of labor supply; and β is the rate of time preference. Detrending the previous expression we obtain

$$\pi_t = \kappa\tilde{y}_t + \kappa_1\Delta\tilde{y}_t + \kappa_1\Delta y_t^n + \beta E_t\pi_{t+1} + u_t, \tag{20}$$

in which $\kappa = k(\eta + \sigma_x^{-1})$ and $\kappa_1 = k\sigma^{-1}(1-\gamma)$.

To complete the model under the Gold standard we take a first-order approximation of equation (9) obtaining a further restriction for the inflation process as

$$\pi_t = -\vartheta_g\Delta\hat{g}_t^c + \vartheta_G\Delta\hat{g}_t - \vartheta_\theta(\Delta\hat{\theta}_t^g - \Delta\hat{\theta}_t^m), \tag{21}$$

[22] Notice that empirical evidence (see in particular Kackmeister, 2007) suggests that under the Gold Standard prices were markedly *stickier* than after World War II. In fact this is what one should expect under a monetary regime in which average inflation had been essentially zero, as opposed to the positive values of the post-WWII period. On this see also Levy and Young (2004).

where ϑ_g, ϑ_G and ϑ_θ are all positive parameters detailed in Online Appendix B.

A first-order approximation of the equilibrium on the money market (see equation 8) implies an equilibrium relationship involving the levels of the central bank's gold stock, prices, output and interest rates:

$$\hat{g}_t^c - p_t = q_y \hat{x}_t - q_i \hat{\imath}_t - q_\theta(\hat{\theta}_t^c - \hat{\theta}_t^m), \tag{22}$$

for positive parameters q_y, q_i and q_θ detailed in Online Appendix B. By taking first differences of (22) we obtain

$$\begin{aligned} \Delta\hat{g}_t^c - \pi_t &= q_y(1+\rho)(\Delta y_t + \Delta y_t^n) - q_y\rho(\Delta y_{t-1} + \Delta y_{t-1}^n) + \\ &\quad -q_i(\Delta\tilde{\imath}_t + \Delta r_t^n) - q_\theta(\Delta\hat{\theta}_t^c - \Delta\hat{\theta}_t^m), \end{aligned}$$

where $\rho = (1-\gamma)\sigma_x/\sigma$.

To characterize the long-term interest rate, $\hat{\imath}_{L,t}$, we postulate a bond with a decaying coupon structure 1, δ, δ^2, δ^3..., where $\delta > 0$, and such that:

$$\delta = (1+i)\left(1 - \frac{1}{m}\right), \tag{23}$$

in which m is the maturity of the long-term bond expressed in quarters. Note that, in a first-order approximation, the price of the long-term bond Q_t is related to the short-term rate and to the expected future bond price as

$$\hat{Q}_t = -\hat{\imath}_t + \frac{\delta}{1+i} E_t \hat{Q}_{t+1},$$

where $1+i$ is the steady-state gross nominal rate. In Online Appendix C we show that this implies that the long-term interest rate is related to the short-term rate through

$$\hat{\imath}_{L,t} = \frac{1+i-\delta}{1+i}\hat{\imath}_t + \frac{\delta}{1+i} E_t \hat{\imath}_{L,t+1}. \tag{24}$$

Finally, note that that in a steady state $i_t^L = i_t = i$ for all t.

4.4.2 The driving processes

We assume that $\hat{\theta}_t^g$, $\hat{\theta}_t^c$, and $\hat{\theta}_t^m$ evolve according to the stationary AR(1) processes[23]

$$\hat{\theta}_t^x = \rho_x \hat{\theta}_{t-1}^x + \theta_t^x, \text{ with } \theta_t^x \sim N(0, \sigma_{\theta^x}^2) \tag{25}$$

for $x = g$, c, m, and $\mid \rho_x \mid <1$.

[23] In Section 5.2 we discuss how relaxing this assumption by making the three processes evolve as random walks produces, for our purposes, the same qualitative evidence, and in fact makes our results even stronger.

Turning to the two drivers of the natural rate of interest in expression (19), ϵ_t and Δy_t^n, although they are routinely modelled as random walks,[24] in what follows we postulate that they evolve according to the zero-mean[25] AR(1) processes

$$\epsilon_t = \rho_\epsilon \epsilon_{t-1} + \epsilon_t^\epsilon, \text{ with } \epsilon_t^\epsilon \sim N(0, \sigma_\epsilon^2) \tag{26}$$

$$\Delta y_t^n = \rho_n \Delta y_{t-1}^n + \epsilon_t^n, \text{ with } \epsilon_t^n \sim N(0, \sigma_n^2). \tag{27}$$

Although a random-walk specification for both ϵ_t and Δy_t^n is standard in the literature, strictly speaking it cannot be correct, because it would imply that both trend output growth and the natural rate of interest could take any value between plus and minus infinity. To anticipate, ϵ_t is uniformly estimated to be very highly persistent, and in fact quite close to a near-unit root process, whereas Δy_t^n is strongly mean-reverting. These assumptions imply that the expression for the natural rate of interest becomes

$$r_t^n = \epsilon_t + \sigma^{-1}[\gamma\rho_n - (1-\gamma)]\Delta y_t^n. \tag{28}$$

Notice that, for reasonable values of the habit persistence parameter h, $[\gamma\rho_n - (1 - \gamma)] > 0$, thus implying that, in line with the conventional wisdom encoded (e.g.) in Ramsey's optimal growth model, an increase in trend output growth maps into a corresponding increase in the natural rate of interest. In fact, although in estimation we impose no constraint on $[\gamma\rho_n - (1-\gamma)]$, this object is consistently estimated to be positive for all countries and sample periods.

Finally, we postulate that the logarithm of the overall stock of gold, g_t, evolves as a random-walk with drift, i.e.

$$g_t = g_{t-1} + \mu_g + \epsilon_t^g, \text{ with } \epsilon_t^g \sim N(0, \sigma_g^2). \tag{29}$$

This assumption requires some discussion. Taken literally, expression (29) implies indeed that the evolution of g_t had been exogenous, and entirely unrelated to macroeconomic developments.[26] This, however, cannot be literally true: e.g. gold scarcity, which characterized the period between the 1860s and the early 1890s,[27] and manifested itself in the guise of deflation[28]—and therefore an increase in the real price of gold in terms of goods—most likely stimulated the search for both new gold fields and more efficient extraction methods. At the margin this should have had an impact, so that both subsequent gold fields discoveries, and subsequent advances in the extraction process might have been spurred, at least in part, by previous gold scarcity. The

[24] See in particular Laubach and Williams (2003) and Holston, Laubach and Williams (2017).

[25] Since, as discussed below, in what follows we model deviations of both the short- and the long-term nominal interest rate from their sample averages, the fact that both (26) and (27) are zero-mean entails no loss of generality, and it is rather the appropriate way to specify the two processes.

[26] This was indeed the assumption made by Lee and Petruzzi (1986).

[27] See the discussion in Chapter 3 of Friedman and Schwartz (1963).

[28] See Figures 1*a*-1*b*. This is especially apparent in the last two panels of Figure 1*a*, and in the first two panels of Figure 1*b*.

implication is that expression (29) should not be taken literally. Rather, it should be regarded as an approximation to a more complex model of the evolution of g_t in which current gold scarcity or abundance, reflected in the real price of gold in terms of goods, has some marginal, non-zero impact on the future evolution of the overall stock of gold.

A more complex model in which gold mining is endogenous, and reacts to fluctuations in the real price of gold, could be developed along the lines of Barsky and Summers (1985). The key reason why we do not do so, and in what follows we work with specification (29), is that for strictly *logical* reasons this cannot make any material difference to our analysis, exactly as it did for Barsky and Summers (1988).[29]

Why endogeneity of gold makes no material difference The reason for this is straightforward. As discussed, the mechanism underlying the appearance of Gibson's paradox under the Gold Standard hinges on the interaction between the Fisher equation and the asset pricing condition determining the current value of money. As for the Fisher equation, endogeneity of gold makes no difference: an $x\%$ change in the natural rate of interest still causes an $x\%$ corresponding change in the long-term nominal rate. As for the asset pricing condition, a decrease (say) in the natural rate causes an increase in the current value of money, i.e. a fall in the price level, and therefore an increase in the real price of gold. This leads to more mining and therefore to an increase in the overall stock of gold. In turn, this causes an increase the price level, thus counteracting the initial decrease due to the fall in the natural rate. However—and this is the crucial point—this mechanism *cannot fully counteract* the forces originating from the initial fall in the natural rate, because in equilibrium the price level has to *decrease*. If endogeneity of gold fully counteracted these forces the price level would return to its initial value, thus violating the asset pricing condition. The implication is that, as a simple matter of logic, endogeneity of gold cannot make any material difference to our analysis.

4.5 The New Keynesian model under alternative monetary regimes

The corresponding New Keynesian model under alternative monetary policy regimes is described by

$$(\pi_t - \bar{\pi}) = \kappa \tilde{y}_t + \kappa_1 \Delta \tilde{y}_t + \kappa_1 \Delta y_t^n + \beta E_t(\pi_{t+1} - \bar{\pi}) + u_t, \tag{30}$$

where $\bar{\pi}$ is the central bank's inflation target, and by expressions (18) and (24). Equilibrium on the money market is characterized by a standard expression describing the demand for real money balances as a fraction of GDP as a function of the short-term

[29] See the discussion in Barsky and Summers (1988, footnote 8).

nominal interest rate. Following Benati et al. (2021) we take M1 as the relevant monetary aggregate (so that we focus on the demand for money for transaction purposes), and we assume that the demand for real money balances takes the 'Selden-Latané'[30] functional form, which is linear in money velocity and the short-term rate, so that the (inverse of the) demand for real money balances as a fraction of GDP is

$$V_t \equiv \frac{Y_t}{M_t} = \psi + \alpha i_t + \lambda_t \tag{31}$$

where V_t is money velocity, defined as the ratio between nominal GDP and the nominal money stock. As shown in Benati et al. (2021), for low-inflation, and therefore low-interest rate countries such as those analyzed herein the data clearly prefer the 'Selden-Latané' specification to either of the two money-demand specifications proposed by Cagan (1956) and Meltzer (1963), which have dominated post-WWII research on money demand. We assume that λ_t follows a stationary AR(1) process, $\lambda_t = \rho_\lambda \lambda_{t-1} + \epsilon_{\lambda,t}$, with $\epsilon_{\lambda,t} \sim N(0, \sigma_\lambda^2)$. The model can be closed, e.g., with a money supply rule or an interest rate rule. For estimation purposes, in the next section we will close it with a standard forward-looking Taylor rule with smoothing,

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1-\rho_i)[\phi_\pi E_t(\pi_{t+1} - \bar{\pi}) + \phi_y E_t \hat{y}_{t+1}] + \epsilon_{i,t}, \text{ with } \epsilon_{i,t} \sim N(0, \sigma_i^2). \tag{32}$$

Then, in order to explore how the long-horizon correlation between prices and long-term nominal interest rates crucially hinges on the specific features of the monetary policy rule, in Section 6 we will take the models estimated based on data from inflation-targeting regimes conditional on the monetary rule (32), and for each draw from the posterior we will replace (32) with an alternative calibrated monetary rule.

We now turn to the empirical evidence for the Gold Standard and inflation-targeting regimes.

5 Evidence from Estimated New Keynesian Models

In this section we discuss the evidence obtained by estimating the New Keynesian models for either the Gold Standard or inflation-targeting regimes. In order to explore the models' ability to replicate Gibson's paradox we focus on their IRFs to the structural shocks. We start by discussing details of the Bayesian estimation procedure, and we then turn to the evidence.

5.1 The data

Gold Standard For the United Kingdom, for which we only have a series for the stock of gold held at the Bank of England, whereas we have no data on the remaining

[30] From Selden (1956) and Latané (1960).

stock of gold in the economy, we estimate the model of Section 4.4 based on data for the Bank of England's discount rate, a consol yield, and the logarithms of real GDP, the wholesale price index, and the stock of gold held at the Bank of England. For the United States, for which we also have data on the stock of gold held outside the monetary authority,[31] we estimate the model based on a call money rate, a corporate bond yield, and the logarithms of real GNP, Warren and Pearson's (1933) wholesale price index, the overall stock of gold, and the stock of gold held by the monetary authority. For details on the data see Online Appendix A.1.

Inflation-targeting regimes For all ten countries[32] we estimate the model of Section 4.5 based on data for a short- and a long-term nominal interest rate, the velocity of M1 (computed as the ratio between nominal GDP and nominal M1), and the logarithms of real GDP and the GDP deflator. For details on the data see Online Appendix A.3. For all countries the benchmark results are based on the full sample periods for the inflation-targeting regimes.[33] For the countries with sufficiently long sample periods,[34] however, qualitatively the same results are obtained if we exclude the period following the collapse of Lehman Brothers (these results are available upon request).[35]

5.2 Bayesian estimation

5.2.1 Calibrated parameters

We calibrate $\beta = 0.9975$, and we set μ_y and the steady-state nominal interest rate, i, to the average values taken by Δy_t and the short-term nominal interest rate over the sample period. As for μ_g, for the United States, for which we have data on the stock of gold held both by the monetary authority and in the rest of the economy, we set it to the sample average of Δg_t, i.e. 0.0164. As for the United Kingdom, for which we only have data on the stock of gold held at the Bank of England, i.e. g_t^c, since Δg_t is an unobserved state variable the value of μ_g is not needed for estimation purposes. On the other hand, since g_t^c exhibits a clear upward trend over the sample period, in the extended state-space form of the model featuring also g_t^c and the log-levels of real GDP (y_t) and prices (p_t), we include in the equation for g_t^c an intercept μ_{g^c}, which

[31] For nearly the entire sample the Treasury, until the founding of the Federal Reserve in December 1913.

[32] Australia, Canada, Denmark, the Euro area, New Zealand, Norway, Sweden, Switzerland, the United Kingdom, and the United States.

[33] The sample periods are discussed in Online Appendix A.3.1. We end all samples in 2019Q4 in order to avoid our results being distorted by the impact of the COVID pandemic.

[34] Canada, Denmark, New Zealand, the United Kingdom, and the United States.

[35] The main rationales for doing so are (i) the extraordinary turbulence of most of that period; and (ii) the fact that for several countries it was characterized by the introduction of unconventional monetary policies. Although in estimation we use, whenever possible, a shadow rate instead of the standard monetary policy rate, it is an open question how much the combination of (i) and (ii) could have distorted results.

we calibrate to the sample average of Δg_t^c, i.e. 5.36×10^{-3} (for details, see Online Appendix E).

In the light of the evidence from estimating long-run money demand curves, which suggests that a unitary elasticity of money demand with respect to output is the empirically relevant case (see e.g. Benati et al. 2021), we set $\epsilon_m = \sigma$, which implies indeed that $q_y = 1$. Finally, we set $\epsilon_g = \epsilon_m$.

For the United States we calibrate the share of non-monetary gold, s_g, to its average value over the sample period (0.2403) based on the series for the stock of gold held by the monetary authority, and the remaining stock of gold in the economy, detailed in Online Appendix A.1. For the United Kingdom we set s_g to the same value as for for the United States. Although this choice is clearly arbitrary, we have explored the robustness of our results to alternative assumptions about s_g, estimating the model conditional on s_g equal to 0.1, 0.2, 0.3, 0.4 and 0.5, obtaining results that are qualitatively the same as those based on the calibration s_g=0.2403 (this evidence is available upon request).

5.2.2 Prior distributions

In performing Bayesian estimation we postulate prior distribution only for those parameters for which we have sufficiently reliable prior information. For all other parameters (specifically, all innovation variances, and all autoregressive coefficients) we do not postulate any prior, so that in fact they are estimated simply based on the likelihood of the data.

Table 1 reports the prior distributions for the New Keynesian models of Sections 4.4 and 4.5. In line with the literature, for both models we assume that the elasticity of intertemporal substitution, σ_x, is smaller than 1, and we postulate a Beta prior centered at 0.5, and with a standard deviation of 0.15. Although this prior has a non-negligible curvature, in fact it allows for essentially any value between 0 and 1. We also postulate the same prior for the extent of forward-lookingness in the IS curve, γ. We postulate a comparatively flat slope of the Phillips curve, κ, which we encode in a quite informative Gamma prior with a mode of 0.05 and a standard deviation of 0.01. In line with previous estimates in the literature the prior for the inverse of the Frisch elasticity of labor supply η is centered at 2.5, but the standard deviation of 0.25 allows for essentially any value between 1.5 and 3.5.

Turning to δ, for the United Kingdom under the Gold Standard, for which in estimation we use a consol (i.e., a perpetuity) as the long rate, we set $\delta = (1 + i)$, which obtains from expression (23) for $m \to \infty$. On the other hand, for the United States under the Gold Standard, and for Denmark under inflation-targeting, for which we do not have precise information about the maturity of the long-term rate we use in estimation, we postulate for $(1+i)$-δ a Gamma prior distribution with mode 2.6×10^{-3} and standard deviation 5.0×10^{-4}. In terms of the corresponding maturity of the long-term bond, this prior implies that m in expression (23) is greater than approximately ten years, but beyond that horizon it imposes little information. Finally, for all other countries under inflation-targeting, for which (as detailed in Online Appendix A.3) we take as the long-term rate the 10-year yield on government bonds, we calibrate m=40 and we compute the corresponding value for δ based on expression (23).

Table 1 Prior distributions of the structural parameters for the New Keynesian models

Parameter	Domain	Density	Mode	Standard deviation
κ	$\mathbb{R}^+$	Gamma	0.05	0.01
σ_x	[0, 1]	Beta	0.5	0.15
γ	[0, 1]	Beta	0.5	0.15
η	$\mathbb{R}^+$	Gamma	2.5	0.25
$(1+i)$-δ [a]	$\mathbb{R}^+$	Gamma	2.6×10^{-3}	5.0×10^{-4}
Gold Standard				
$\text{Corr}(i_{L,t}, p_t)_{25\text{Y}}$	[0, 1]	Beta	0.9	0.1
Inflation targeting regimes				
ϕ_π	$\mathbb{R}^+$	Gamma	1.5	0.15
ϕ_y	$\mathbb{R}^+$	Gamma	0.25	0.1
$-\psi$	$\mathbb{R}^+$	Gamma	1	0.1
α	$\mathbb{R}^+$	Gamma	10	1

[a] Only for the United States under the Gold Standard and Denmark under inflation targeting. For the United Kingdom under the Gold Standard we set δ=(1+i). For other countries under inflation targeting (see text) the maturity of the long-term rate is 10 years, and so we calibrate δ accordingly.

The priors for the parameters of the Taylor rule (ρ_r, ϕ_π, and ϕ_y) are standard. The priors for ψ for α are in line with the values in the literature on estimating long-run Selden-Latané money demand curves for M1 (see Benati et al., 2021).

Finally, for the Gold Standard (but not for inflation targeting regimes) we postulate a Beta prior with a mode of 0.9 for $\text{Corr}(i_{L,t}, p_t)_{25\text{Y}}$, the coefficient of correlation between forecast errors to the long rate and the price level at the 25 years ahead horizon. The rationale for doing so is in order to force the estimation algorithm towards regions of the parameter space that can effectively capture Gibson's paradox. The comparatively large standard deviation, which we set at 0.1, allows however for essentially any value between 0.3 and 1.

5.2.3 Drawing from the posterior distribution

We draw from the posterior distribution via Random Walk Metropolis (RWM) exactly as in An and Schorfheide (2006, Section 4.1). Online Appendix D describes the simulated annealing procedure we use in order to obtain the mode of the log posterior,

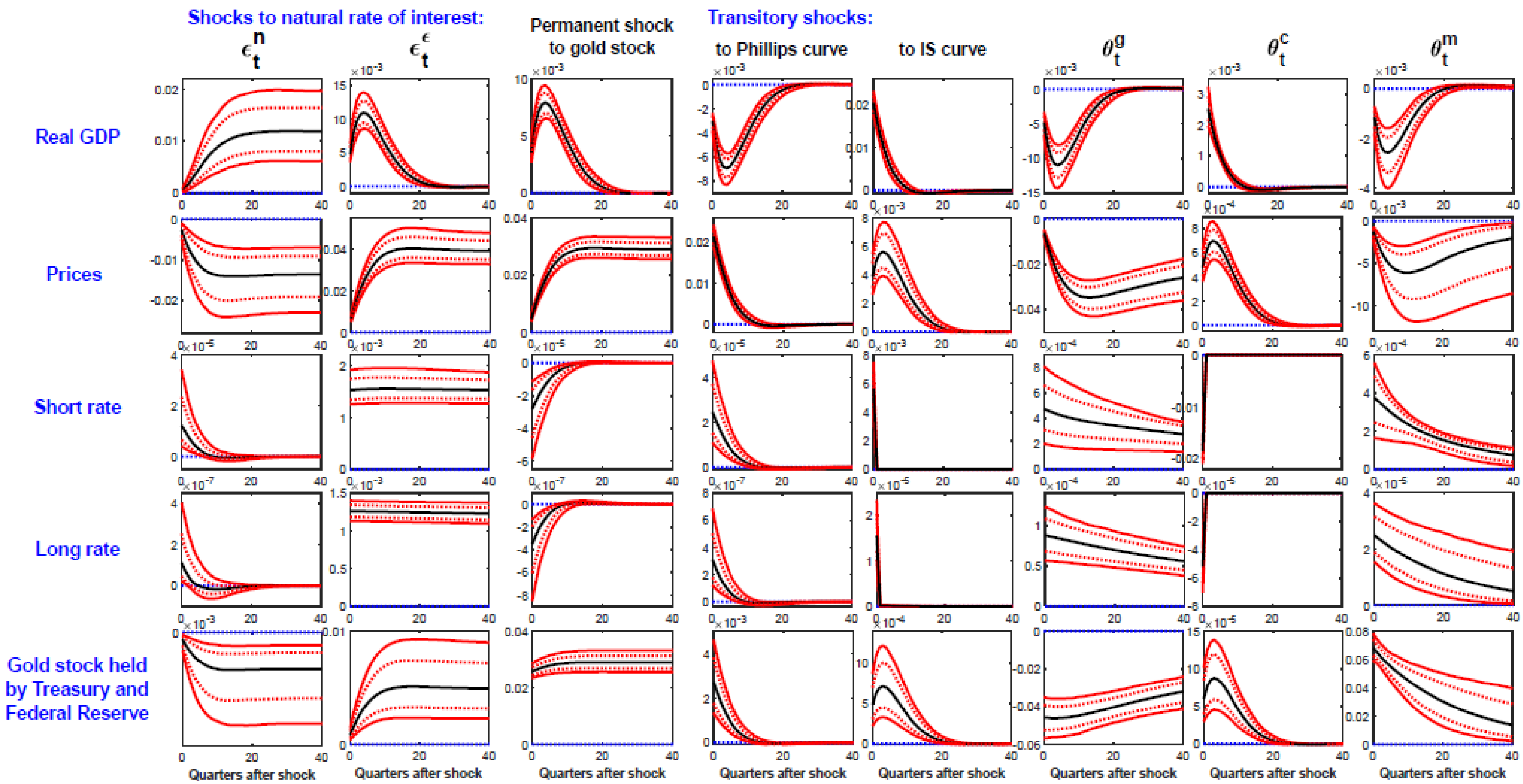

Figure 3 United States, 1879Q1-1914Q2: Impulse-response functions of the New Keynesian model to the structural shocks (medians of the posterior distributions, and 16-84 and 5-95 credible sets)

as well as the Markov Chain Monte Carlo (MCMC) algorithm we use in order to characterize the ergodic distribution.

We check convergence of the Markov chain to the ergodic distribution based on Geweke's (1992) inefficiency factors (IFs) of the draws for each individual parameter.[36] For all parameters the IFs are equal to at most 15, well below the values of 20-25 which are typically taken to indicate problems in the convergence of the Markov chain.

5.3 Evidence

5.3.1 The Gold Standard

Figure 3 shows, for the United States during the Classical Gold Standard period, the medians of the posterior distributions of the IRFs to the structural shocks, together with the 16-84 and 5-95 per cent credible sets, whereas Figure A.1 in the Online Appendix shows the corresponding evidence for the United Kingdom.[37]

In order to meaningfully interpret this evidence it is important to recollect that, as discussed in both the Introduction and Section 4.2, what is needed in order to replicate Gibson's paradox is a shock (or a set of shocks) that has a very highly persistent positive impact on both the price level and the long-term nominal interest rate. In Figures 3 and A.1 the only shock that accomplishes this is ϵ_t^{ϵ}—i.e., the shock to the component of the natural rate of interest that is unrelated to fluctuations in trend GDP growth—whereas all the other shocks are incapable of generating such long-horizon positive correlation.

In particular, the impact of ϵ_t^n on the long-term rate is strongly mean-reverting and mostly insignificant. This reflects the fact that ρ_n in (27) is estimated to be quite small, with a median of the posterior distribution equal to just 0.3662. By contrast, ϵ_t^{ϵ} is estimated to be a near-unit root process, with a posterior median of ρ_{ϵ} in (26) equal to 0.9993. It is important to recall that, as discussed in Section 5.2.2, we did not postulate any prior distribution for either ρ_n or ρ_{ϵ}. The implication is that the starkly different estimates we obtain for the two parameters uniquely reflect the information in the data. Finally, it should be noted that ϵ_t^n has a negative long-horizon impact on

[36] The inefficiency factors are defined as the inverse of the relative numerical efficiency measure of Geweke (1992),

$$RNE = (2\pi)^{-1} \frac{1}{S(0)} \int_{-\pi}^{\pi} S(\omega) d\omega,$$

where $S(\omega)$ is the spectral density of the sequence of draws from RWM for the parameter of interest at frequency ω. We estimate the spectral densities via the lag-window estimator as described in chapter 10 of Hamilton (1994). We also considered an estimator based on the fast-Fourier transform, and results were very similar.

[37] In order not to clutter the figures we do not report evidence for the overall stock of gold, g_t, since based on (29) this evidence is trivial.

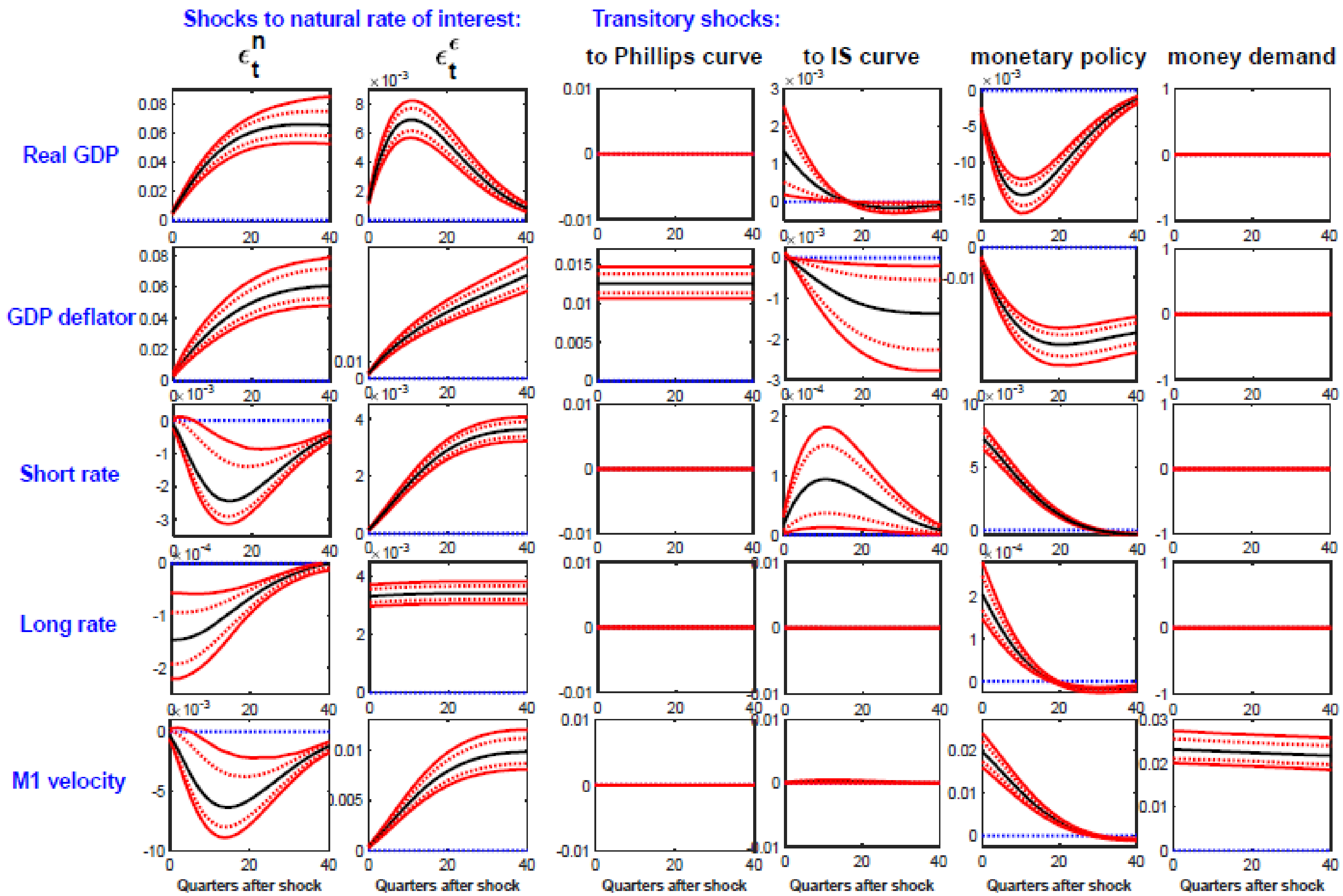

Figure 4 United Kingdom, 1993Q1-2019Q4: Impulse-response functions of the New Keynesian model to the structural shocks (medians of the posterior distributions, and 16-84 and 5-95 credible sets)

prices, so that even if it had a significantly more persistent impact on the long rate it would still produce a long-horizon correlation with the wrong sign.

Turning to θ_t^c and the shocks to the overall stock of gold, and to the IS and Phillips curves, all of them have a very short-lived impact on the long rate, which mean-reverts to zero either one period after impact (this is the case for θ_t^c and the IS curve shock, v_t), or within about five years.

On the other hand, θ_t^g and θ_t^m induce persistent *in*creases and *de*creases, respectively, in the long rate and the price level, so that the correlation they generate is the opposite of what is needed.

5.3.2 Inflation-targeting regimes

Turning to inflation-targeting regimes, the evidence for the United Kingdom in Figure 4, which is qualitatively the same as that for all other nine countries,[38] shows that exactly as for the Gold Standard only ϵ_t^ϵ can generate a positive long-horizon correlation between the price level and the long-term rate, whereas all other shocks have a mean-reverting impact, or even no impact at all by assumption, on the long rate.

The evidence for inflation targeting regimes suggests two considerations.

First, the evidence in Figure 4 begs the obvious question of why, exactly, Gibson's paradox is nowhere nearly apparent in the raw data generated by inflation-targeting regimes. In principle one possible explanation could be that ϵ_t^n, shocks to the IS and Phillips curve, and monetary policy shocks 'blur' the positive long-horizon correlation induced by ϵ_t^ϵ, thus making it *disappear* from the raw data. Notice indeed that both ϵ_t^n and the monetary policy shock induce a *negative* correlation between prices and the long rate, whereas shocks to the IS and Phillips curve have no impact on the long rate, and they have a negative and positive impact, respectively, on prices. In fact, as the evidence in Section 7 shows, this does not seem to be the case: once controlling for the deterministic component of the drift in the price level induced by the presence of a positive inflation target, a positive long-horizon correlation between the two series can indeed be recovered from the data generated by inflation-targeting regimes. This suggests that main reason why Gibson's paradox is not apparent in the raw data generated under inflation targeting is simply that the presence of a positive inflation target, by introducing a positive drift in the price level, cause the long-horizon correlation to become *hidden* in the raw data.

Second, the evidence for inflation targeting regimes suggests that Gibson's paradox had nothing to do with the Gold Standard per se. In particular, the fact that under inflation targeting ϵ_t^ϵ generates a positive long-horizon correlation between prices and the long rate exactly as under the Gold Standard naturally suggests that, in principle, other monetary regimes might also be able to generate the paradox. In the next section we will indeed see one example of such a regime.

[38] We do not report this evidence for reasons of space, but it is available upon request.

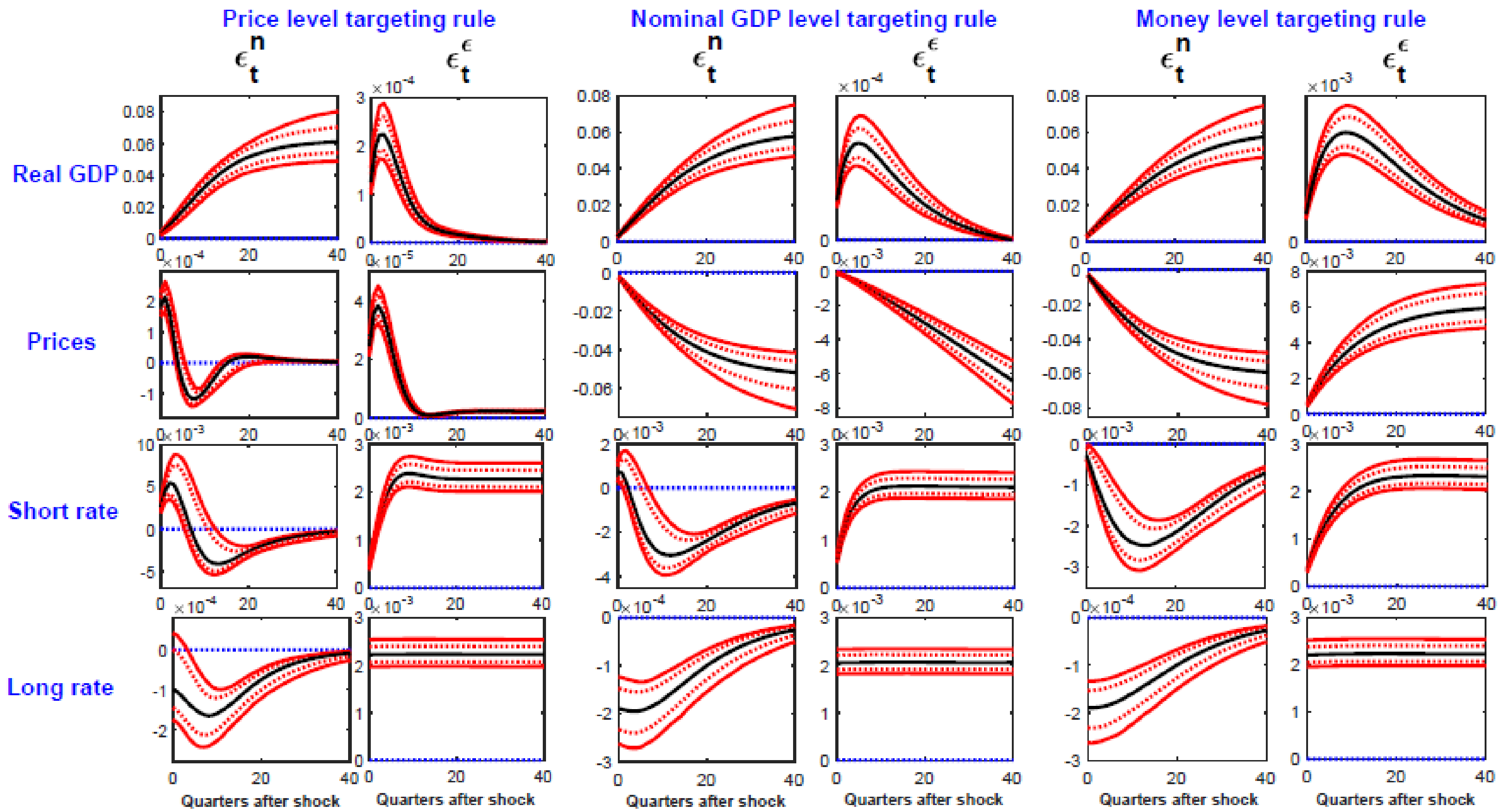

Figure 5 United Kingdom, 1993Q1-2019Q4: Impulse-response functions of the New Keynesian model to the structural shocks obtained by replacing the estimated Taylor rule with alternative calibrated monetary rules (medians of the posterior distributions, and 16-84 and 5-95 credible sets)

6 Long-Term Nominal Interest Rates and Prices Under Alternative Monetary Policy Rules

Figure 5 shows the results of the following exercise. We take the posterior distribution of the structural parameters of the New Keynesian model estimated for the United Kingdom under inflation targeting (i.e., the estimated model that produced the IRFs shown in Figure 4), and for each draw from the posterior we replace the estimated Taylor rule (32) with either the price level targeting rule

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\phi E_t p_{t+1}; \tag{33}$$

the nominal GDP level targeting rule

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\phi[y_t + p_t] = \rho_i i_{t-1} + (1 - \rho_i)\phi[y_t^N + \hat{y}_t + p_t]; \tag{34}$$

or a monetary rule keeping the logarithm of the money stock, m_t, constant,

$$m_t = \bar{m} \tag{35}$$

together with the money demand equation[39]

$$m_t - p_t = y_t - \alpha i_t \tag{36}$$

We set the policy parameters as follows. As for ρ_i in (33) and (34), for each draw from the posterior distribution we keep the estimate of ρ_i corresponding to that draw. We set ϕ in (33) and (34) to 1.5. As for the money level targeting rule we set $\bar{m} = 0$ and $\alpha = 10$. The figure only reports the IRFs to ϵ_t^n and ϵ_t^ϵ for the key variables (excluding therefore M1 velocity), because all other shocks have no persistent impact on the long rate. The full set of IRFs for all series and all shocks is however available upon request.

The evidence in Figure 5 confirms that the long-horizon correlation between prices and the long-term rate crucially depends on the nature of the monetary regime.

A money level targeting regime generates qualitatively the *same* IRFs for the two series in response to ϵ_t^ϵ that we estimated under the Gold Standard. As for ϵ_t^n, different from the Gold Standard it generates a negative response in both series. The response of prices, however, is very highly persistent, whereas that of the long rate is ultimately mean-reverting, so that in the end ϵ_t^n is much less effective at generating Gibson's paradox than ϵ_t^ϵ. The ability of a money level targeting regime to generate Gibson's paradox, and in particular the fact that the IRFs it generates in response to ϵ_t^ϵ are qualitatively the same as for the Gold Standard should in fact come as no surprise. As stressed e.g. by Bordo and Kydland (1995), one way to think of

[39] We consider Cagan's (1956) 'semi-log' functional form (36), rather than the 'Selden-Latané' form (31), because the latter—as well as Meltzer's (1963) 'log-log'—would make the model non-linear, thus unnecessarily complicating the exercise.

monetary regimes based on commodity standards is as rules designed to 'nail down' the money supply—thus subtracting it from the discretionary manipulation on the part of the government—with 'escape clauses' for extreme events such as wars. The fact that the Gold Standard and a money level targeting regime generate qualitatively the same IRFs is therefore logically to be expected.

On the other hand, regimes targeting either the price level or the level of nominal GDP produce starkly different correlations. In particular, a price level targeting regime, by making prices strongly mean-reverting, causes there to be no long-horizon correlation between the long rate and the price level. Under nominal GDP targeting, on the other hand, the response of prices to either ϵ_t^n or ϵ_t^ϵ is negative and very highly persistent, whereas the long rate persistently increases in response to ϵ_t^ϵ, and it decreases but ultimately mean-reverts to zero in response to ϵ_t^n. The implication is that in response to ϵ_t^n a nominal GDP targeting regime can produce a medium-horizon positive correlation between prices and the long rate, but at long horizons this correlation ultimately disappears.

We now turn to showing that Gibson's paradox can in fact be recovered from the data generated by inflation targeting regimes.

7 Recovering Gibson's Paradox in the Raw Data Under Inflation Targeting Regimes

As we discussed in Section 5.3.2, a plausible explanation for the fact that under inflation targeting Gibson's paradox is not apparent in the raw data as it had been under the Gold Standard is that the presence of a positive inflation target introduces drift in the price level.

Figure 6 shows evidence in support of this conjecture. The figure reports results from the following exercise. For any of the ten inflation targeting countries we estimate via OLS the VARs in levels

$$Y_t = B_0 + B_1 Y_{t-1} + ... + B_p Y_{t-p} + u_t \tag{37}$$

featuring either (1) only a long-term nominal interest rate and the logarithm of the GDP deflator, or (2) additionally, also the logarithms of hours worked, real consumption, and the consumption/GDP and investment/GDP ratios. Based on the estimated VARs we then re-run history by (i) setting the estimated VAR intercept, B_0^{OLS}, to zero, and (ii) feeding to the estimated VAR the reduced-form residuals, i.e. the u_t^{OLS}. Whereas (i) removes the deterministic drift in the price level caused by the presence of a positive inflation target,[40] (ii) makes sure that the counterfactual we are

[40] By the same token, it also removes the deterministic component of the downward drift in the long-term rate that has taken place at least since the early 1990s, due to the progressive decrease in the natural rate of interest. This downward drift in the long rate has also contributed to making Gibson's correlation to become hidden in the raw data.

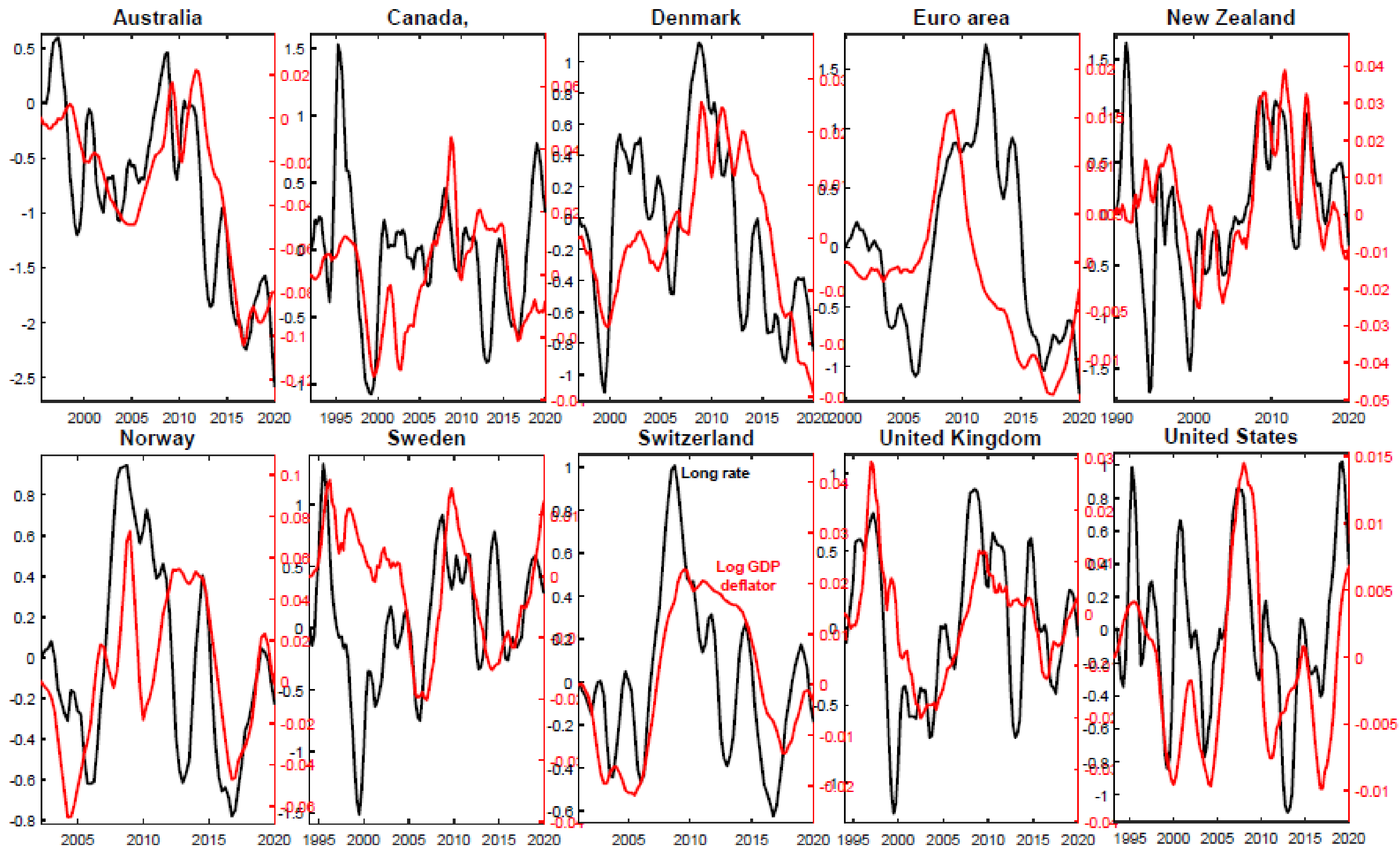

Figure 6 Recovering Gibson's paradox under inflation targeting

running is conditional on exactly the same shocks that have historically driven the economy. The two VAR specifications, (1) and (2), produce qualitatively the same and quantitatively very close results. Figure 6 reports evidence based on system (2), but the corresponding evidence based on (1) is available upon request.

Overall, evidence of a positive low-frequency correlation between prices and the long-term rate is quite clear. This is the case in particular for Australia, Denmark, New Zealand, Norway, Switzerland, and the United Kingdom, but the correlation is reasonably strong also for the remaining countries. In the light of the evidence in Figure 4 this is to be expected: since both shocks driving the natural rate of interest generate Gibson's correlation, unless monetary policy shocks, and shocks to the IS and Phillips curves play a dominant role, a positive correlation between prices and the long rate should be apparent in the raw data once removing the deterministic component of the drift in the price level.

8 Assessing the Sub-Optimality of Monetary Policy

The appearance of Gibson's paradox under either the Gold Standard or inflation targeting regimes is a clear indication of the sub-optimality of the monetary policies that had, and have been followed under these monetary frameworks. If the central bank were able to track fluctuations in the natural rate of interest, and to neutralize the impact on the economy of shocks to the natural rate, Gibson's paradox would never appear in the data, as in the Neo-Wicksellian approach to policy proposed by Woodford (2003).

A natural metric for assessing the sub-optimality of monetary policy is the fraction of the variance of macroeconomic time series that is explained by shocks to the natural rate. Figure 7 reports evidence from the following exercise. For the United Kingdom and the United States under the Gold Standard, and for the United Kingdom and Canada under inflation targeting,[41] we estimate Bayesian VARs for GDP deflator inflation, real GDP growth, and a short- and a long-term nominal interest rate.

We estimate the VARs based on the methodology proposed by Giannone, Lenza, and Primiceri (2015),[42] which is described in detail in Online Appendix G, with the only difference that in estimation we impose stationarity upon the VAR as in e.g. Cogley and Sargent (2002, 2005) and Primiceri (2005).[43] In setting up the Minnesota-type prior we proceed as follows. For inflation we set the parameter on the first lag of itself to zero, reflecting the fact that under inflation targeting regimes this series

[41] Under inflation targeting we exclude the period following the collapse of Lehman Brothers.

[42] We use the MATLAB codes found at Giorgio Primiceri's web page.

[43] So in the MCMC algorithm used for estimation, we move to iteration $i+1$ if and only if the draw for the VAR's parameters associated with iteration i is stationary. Otherwise, we redraw the parameters for iteration i.

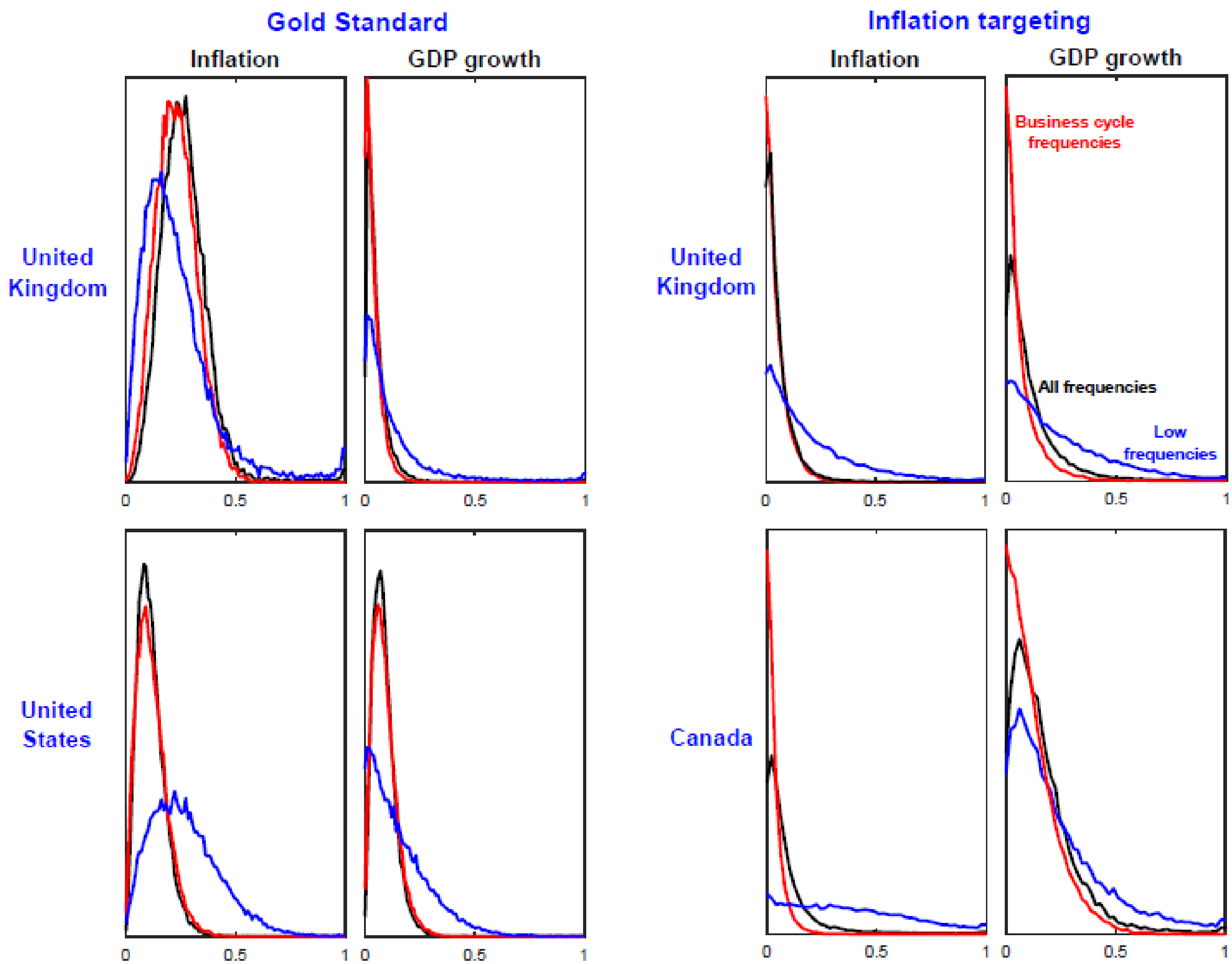

Figure 7 Posterior distributions of the fractions of variance of inflation and real GDP growth explained by shocks to the natural rate of interest by frequency band

has been near-uniformly indistinguishable from white noise. For real GDP growth we set it at 0.5, reflecting the mild extent of serial correlation of this series. For the short- and the long-term nominal rate, on the other hand, we set it to 0.75 and to 1-0.5/T, respectively, where T is the sample length. For the short rate this reflects its non-negligible extent of persistence, whereas for the long rate the fact that this series is best thought of as near unit root process.

For each draw from the posterior distribution we then identify the shock to the natural rate of interest in the frequency domain, as the disturbance explaining the maximum fraction of the variance of the long rate at the frequency $\omega = 0$.[44] Based on the previous discussion, under either monetary regime this is the natural way of identifying natural rate shocks. We then Fourier-transform the identified SVAR, and for each series we compute the fraction of its variance that is explained by natural rate shocks across either all frequencies, the business-cycle frequencies, or the low frequencies.[45] In doing so we exploit the fact that the variance of a stationary series within a specific frequency band is equal to the integral of its spectral density within that band.

Figure 7 shows the posterior distributions of the fractions of variance of inflation and real GDP growth explained by shocks to the natural rate of interest.[46] The main finding is that for all countries, and under either monetary regime, shocks to the natural rate had, and have explained non-negligible fractions of the variance of the two series. This is especially clear at the low frequencies. In the light of the long-standing criticism of the inflexibility of the Gold Standard,[47] the evidence for this regime should come as no surprise. More interesting is the fact that evidence for inflation targeting regimes is qualitatively the same, as this clearly points towards the sub-optimality of the way inflation targeting has been implemented in practice.

9 Conclusions

For more than a century a vast literature has studied Gibson's paradox, without reaching any consensus on what, exactly, had originated it. Following Friedman and Schwartz (1982), the only broad agreement appears to be that, since the paradox had only appeared under the Gold Standard, and it has instead been absent from post-WWII data, it had likely originated from the peculiar workings of monetary regimes based on commodity money.

[44] We implement this restriction via the methodology proposed in Benati (2014), which is based on the notion of working with the entire set of available rotation matrices, maximizing the relevant criterion function over the set of the corresponding rotation angles via numerical methods. Monte Carlo evidence there illustrates its extremely robust performance.

[45] Following standard conventions we define the business cycle and the low frequencies as those pertaining to fluctuations between 6 and 32 quarters, and beyond 32 quarters, respectively.

[46] The full set of results is reported in Figures A.2 and A.3 in the Online Appendix.

[47] See in particular Keynes (1925) and Eichengreen (1996).

In this paper we have advanced the view that Gibson's paradox has nothing to do with the Gold Standard or commodity-money regimes per se, and it rather originates from low-frequency variation in the natural rate of interest under certain types of monetary regimes that make inflation I(0) and approximately zero-mean. Although the Gold Standard is the only historical example of such a regime, Gibson's paradox is, in principle, a feature of a potentially wide array of monetary arrangements. Consistent with this, we have shown that Gibson's paradox can be recovered from the data generated under inflation targeting regimes once removing the deterministic component of the drift in the price level induced by the presence of a positive inflation target.

Intuitively, the mechanism underlying the emergence of Gibson's paradox under specific types of monetary regimes hinges on the interaction between the Fisher equation and an asset pricing condition determining the current value of money. The Fisher equation implies that long-horizon fluctuations in the natural rate of interest automatically map, one-for-one, into corresponding fluctuations in nominal interest rates at all maturities. The asset pricing condition, on the other hand, implies that increases in the natural rate—which is the discount factor for the determination of the current value of money—map into decreases in the expected value of future money, and therefore in the current value of money, which are obtained via corresponding increases in the price level. Long-horizon increases (decreases) in the natural rate of interest therefore map into corresponding low-frequency increases (decreases) in the price level via the asset pricing condition, and in nominal interest rates via the Fisher equation. Although this mechanism works for nominal interest rates at all maturities, in practice it is especially apparent for long rates, which behave as the long-horizon components of short rates.

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Online Appendix for:
Gibson's Paradox and the Natural Rate of Interest

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A The Data

Here follows a detailed description of the dataset.

A.1 The gold standard

Canada An annual series for the consumer price index is from Rolnick and Weber's (1995, 1997) dataset. An annual series for a nominal long-term interest rate is from Furlong (2001).

Denmark Annual series for the consumer price index and a nominal long-term interest rate are both from the dataset assembled by Kim Abildgren, which is described in detail in Abildgren (2006), and is available from his web page at: https://sites.google.com/view/kim-abildgren/historical-statistics.

France An annual series for the price level is from Rolnick and Weber's (1995, 1997) dataset. An annual series for a nominal long-term interest rate ('taux long') is from Section 7 ('Evaluation des taux de l'interet'), pp 93-96, of Saint Marc (1983).

Germany Monthly series for the price level ('Index of Sensitive Prices for Germany, Index 1913=100, Monthly, Not Seasonally Adjusted') and a nominal long-term interest rate ('Bond Yields for Germany') are from the NBER Historical database. The FRED II acronyms are M04056DEM324NNBR and M1328ADEM193NNBR, respectively.

Norway A monthly series for the wholesale price index available for the period January 1777-December 1919 is from Norway's long-run historical statistics database, which is

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available at the website of Norges Bank (Norway's central bank). The data are documented in Grytten (2013). A monthly series for a nominal long-term interest rate is from Klovland (2004).

United Kingdom All U.K. data are from version 3.1 of the Excel spreadsheet "A millennium of macroeconomic data" which is available from the Bank of England's website at: http://www.bankofengland.co.uk/statistics/research-datasets. The first version of the dataset (which was called "Three centuries of macroeconomic data") was discussed in detail in Hills and Dimsdale (2010). Details for the series plotted in Figure 1.a in the main text are as follows. In the first panel, an annual series for the consumer price index is from sheet A.47 ('Wages and prices'), whereas a series for a consol rate is from sheet A.31 ('Interest rates'). In the second panel, a monthly seasonally unadjusted series for the prices of domestic commodities from sheet M.6 ('Monthly prices and wages'), whereas a series for the yield on 3% consols is from sheet M.10 ('Monthly long-term interest rates'). The series for the prices of domestic commodities is originally from Gayer, Rostow, and Schwartz (1953). In the third panel the monthly seasonally unadjusted series for the 'Sauerbeck Statist price index including duty' if from sheet M.6 ('Monthly prices and wages'), whereas in the fourth panel the monthly seasonally unadjusted series for the 'Sauerbeck Statist index for all commodities' if from sheet M.6 ('Monthly prices and wages'). In the third and fourth panels, a series for the 'Yield on consols corrected for Goschen's conversion issues' is from sheet M.10 ('Monthly long-term interest rates'). Then, a seasonally unadjusted weekly series for 'Gold Coin and Bullion in the Bank of England's balance sheet' is from sheet W.1 ('Issue Department'), and it has been converted to the monthly frequency by taking averages within the month. Monthly series for the Bank of England's discount rate and a consol yield are from sheet M.9 ('Monthly short-term interest rates 1694-2016') and M.10 ('Monthly long-term interest rates 1753-2016'), respectively. Turning to the quarterly data used to estimate the New Keynesian model in Section 4, series for the Bank of England's discount rate, a consol yield, and the Bank Of England stock of gold have been obtained by converting to the quarterly frequency the previously mentioned corresponding monthly series, by taking averages within the quarter. As for the price level we took the monthly seasonally unadjusted series for the 'Spliced wholesale/producer price index, 1790-2015' from sheet M.6 ('Monthly prices and wages'), we seasonally adjusted it via ARIMA X-12 as implemented in *EViews*, and we converted it to the quarterly frequency by taking averages within the quarter. Finally, as for real GDP we interpolated to the quarterly frequency the annual real GDP series from sheet A.8 ('National Accounts')—i.e. the series 'Real UK

GDP at market prices, geographically-consistent estimate based on post-1922 borders, £mn, Chained Volume measure, 2013 prices'—as in Bernanke, Gertler, and Watson (1997), using as the quarterly interpolator series capturing the state of the business cycle the monthly unemployment rate series (converted to the quarterly frequency by taking averages within the quarter) from sheet M.5 ('Monthly activity'). For the purpose of performing econometric work (but not for plotting purposes), all seasonally unadjusted series have been seasonally adjusted via ARIMA X-12 as implemented in *EViews*.

United States Details for the series plotted in Figure 1.*b* in the main text are as follows. The monthly seasonally unadjusted series for the wholesale price index is from Warren and Pearson (1933), whereas the quarterly series for a long-term nominal interest rate is the series for the 'Yield on corporate bonds' from Table 2 of Balke and Gordon (1986). Turning to the quarterly series used to estimate the New Keynesian model, the long rate is the just mentioned series for the yield on corporate bonds, whereas a seasonally adjusted series for 'Real GNP in 1972 dollars' is also from Table 2 of Balke and Gordon (1986). The series for the price index has been obtained by seasonally adjusting the previously mentioned series from Warren and Pearson (1933) via ARIMA X-12 as implemented in *EViews*, and then converting the resulting series to the quarterly frequency by taking averages within the quarter. Monthly series for call money rate ('Call Money Rates, Mixed Collateral for United States, Percent, Monthly'), the stock of gold held by the monetary authority ('Gold Held in the Treasury and Federal Reserve Banks for United States, Millions of Dollars, Monthly'), and the remaining stock of gold in the economy ('Gold Outside the Treasury and Federal Reserve Banks for United States, Millions of Dollars, Monthly') are all from the NBER Historical database (the FRED II acronyms are M13001USM156NNBR, M1437AUSM144NNBR, and M1431AUSM144NNBR respectively). All of the three series have been converted to the quarterly frequency by taking averages within the quarter. Since the original gold stock series are seasonally unadjusted, we seasonally adjusted them via ARIMA X-12 as implemented in *EViews*.

A.2 Interwar period

Details for the series plotted in Figure 1.*c* in the main text are as follows.

Denmark Monthly series for the consumer price index and a nominal long-term interest rate ('Yield on long-term mortgage bonds') are both from the dataset assembled by Kim Abildgren, which is described in detail in Abildgren (2006), and is available from his web page at: https://sites.google.com/view/kim-abildgren/historical-statistics.

Norway A monthly series for the consumer price index available since January 1920 is from Norway's long-run historical statistics database, which is available at the website of Norges Bank (Norway's central bank). The data are documented in Klovland (2013). A monthly series for a nominal long-term interest rate is also from Norway's long-run historical statistics database. Details about the series are as follows. Until December 1920 it is the series for 'Norwegian long-term government bonds, monthly (1820-1920)', whereas since January 1921 it is the series ST10 from the sheet p1_c4_table_A3_Monthly ('Norwegian bond yields by maturity (average life), monthly (1921-2005)') in the spreadsheet bond_yields.xls. Both series are documented in Klovland (2004).

United Kingdom Again, the data are from version 3.1 of the Excel spreadsheet "A millennium of macroeconomic data". Specifically, a series for the wholesale price index ('Spliced wholesale/producer price index') is from sheet M.6 ('Monthly prices and wages'), whereas a series for a long-term nominal interest rate ('Long-term consols yield 1753-2015, corrected for Goschen's conversion issues') is from sheet M.10 ('Monthly long-term rates').

United States The monthly seasonally unadjusted series for the wholesale price index is from Warren and Pearson (1933). The long-term nominal interest rate series ('Yield On Long-Term United States Bonds for United States') is from the NBER Historical database (the FRED II acronym is M1333AUSM156NNBR).

A.3 Inflation-targeting regimes

Here follow details of the quarterly data for inflation-targeting regimes.

Australia Quarterly seasonally adjusted series for nominal and real GDP, and the GDP deflator, are from the Australian Bureau of Statistics. The short rate ('3-month BABs/NCDs, Bank Accepted Bills/Negotiable Certificates of Deposit-3 months; monthly average, Quarterly average, Per cent, ASX, 42767, FIRMMBAB90') is from the Reserve Bank of Australia. M1 ('M1: Seasonally adjusted, $ Millions') is from the Reserve Bank of Australia. A series for a nominal long-term interest rate has been constructed as follows. Until 2013Q1 it is the series for the 10-year yield on Australian government bonds, which is available at the Reserve Bank of Australia's website. Since then it is the series for the 10-year yield on Commonwealth government bonds.

Canada Quarterly seasonally adjusted series for nominal and real GDP, and the GDP deflator, are from Statistics Canada. A monthly seasonally unadjusted series for M1 ('v41552787, Table 176-0020: Currency outside banks, chartered bank chequable deposits, less inter-bank chequable deposits, monthly average') is from Statistics Canada. It has been seasonally adjusted via ARIMA X-12 as implemented in *EViews*, and it has then been converted to the quarterly frequency by taking averages within the quarter. Monthly series for the Bank of Canada's monetary policy rate ('Taux Officiel d'Escompte') and the 10-year rate on government bonds ('V122543 = Government of Canada benchmark bond yields - 10 year') are both from the Bank of Canada.

Denmark Quarterly seasonally adjusted series for nominal and real GDP, and the GDP deflator, are from Statistics Denmark. A monthly seasonally adjusted series for M1 ('Money stock M1, end of period, Units: DKK bn.') is from Denmark's central bank, Danmarks Nationalbank, and it has been converted to the quarterly frequency by taking averages within the quarter. A series for the central bank's discount rate is from Danmarks Nationalbank. A series for the 'Yield on long-term Danish government bonds' is from Kim Abildgren's database.

Euro area All of the quarterly data are from the European Central Bank's statistical database, which is accessible at its website.

New Zealand Quarterly seasonally adjusted series for nominal and real GDP and the GDP deflator are from Statistics New Zealand. A quarterly seasonally adjusted series for M1 is from the Reserve Bank of New Zealand. Monthly series for the 'Overnight interbank cash rate' and the 10-year yield on government bonds traded on the secondary market are both from the Reserve Bank of New Zealand, and they have been converted to the quarterly frequency by taking averages within the quarter.

Norway Quarterly seasonally adjusted series for nominal and real GDP and the GDP deflator are from Statistics Norway. A quarterly seasonally adjusted series for M1 is from Norges Bank. Monthly series for the 3-month interbank rate and the 10-year yield on government bonds are from FRED II, the Federal Reserve Bank of St. Louis' internet data portal (the acronyms are IR3TIB01NOM156N and IRLTLT01NOQ156N). Both series have been converted to the quarterly frequency by taking averages within the quarter.

Sweden Quarterly seasonally adjusted series for nominal and real GDP, the GDP deflator, and M1 are from Statistics Sweden. A monthly series for the 3-month interbank rate is from FRED II (the acronym is IR3TIB01SEM156N). A monthly series for the 10-year yield on government bonds is from Statistics Sweden. Both series have been converted to the quarterly frequency by taking averages within the quarter.

Switzerland Both M1 and the short rate ('Monetary aggregate M1, Level' and 'Switzerland - CHF - Call money rate (Tomorrow next)', respectively) are from the Swiss National Bank's internet data portal. Quarterly seasonally adjusted series for nominal and real GDP, and the GDP deflator, are from the State Secretariat for Economic Affairs (SECO) at https://www.seco.admin.ch/seco/en/home. A series for the 10-year yield on government bonds is from FRED II, the internet data portal at the Federal Reserve Bank of St. Louis website (the acronym is IRLTLT01CHM156N).

United Kingdom Quarterly seasonally adjusted series for nominal and real GDP and the GDP deflator are from the Office for National Statistics. A break-adjusted stock of M1 is from version 3.1 of the Excel spreadsheet "A millennium of macroeconomic data" (specifically, it is from sheet Q.3, 'Quarterly break-adjusted and seasonally-adjusted monetary aggregates 1870-2016'). Quarterly series for a nominal long-term rate ('Medium-term/10 year bond yield') and the Bank of England's monetary policy rate ('Bank Rate') are both from sheet M.1 ('Monthly headline series, Quarterly average of monthly series') of the spreadsheet "A millennium of macroeconomic data".

United States The short- and long-term nominal interest rates series are the Federal Funds rate and, respectively, the 10-year Treasury constant-maturity rate. Both series are from the Board of Governors of the Federal Reserve System. The original monthly data have been converted to the quarterly frequency by taking averages within the quarter. Quarterly seasonally adjusted series for nominal and real GDP and the GDP deflator are from the U.S. Department of Commerce: Bureau of Economic Analysis. Following Lucas and Nicolini (2015) and Benati, Lucas, Nicolini, and Weber (2021), the M1 aggregate has been computed as the sum of the 'standard' aggregate produced by the Board of Governors of the Federal Reserve System (the FRED II acronym is M1SL), and a series for Money Market Deposits Accounts (MMDAs) from the Federal Reserve's mainframe, which has kindly been provided by Juan-Pablo Nicolini. The rationale for augmenting the standard M1 aggregate with MMDAs, which were introduced in 1982Q4, is extensively discussed in Lucas and Nicolini (2015) and Benati, Lucas, Nicolini, and Weber (2021). In brief, MMDAs perform a function very similar to that performed by the deposits that are included in the standard definition of M1. Therefore, economic logic suggests that they should be included in a meaningful definition of a monetary aggregate for transaction purposes.

A.3.1 The sample periods

We end all samples in 2019Q4 in order to prevent our results from being distorted by the impact of the COVID pandemic. As for the starting dates, the are the following. For Canada, New Zealand, Norway, Sweden, and the United Kingdom, which introduced inflation targeting in February 1991, February 1990, March 2001, January 1993, and October 1992, respectively, they are 1991Q2, 1990Q2, 2001Q2, 1998Q1 (dictated by data availability), and 1993Q1, respectively. As for Australia, which never explicitly announced the introduction of the new regime, we follow Benati and Goodhart (2011) in taking 1994Q3 as the starting date. The rationale is that, based on the central bank's communication, during those months it became apparent that the central bank was indeed following an inflation-targeting strategy. For the Euro area, where European Monetary Union started in January 1999, we start the sample in 1999Q1. By the same token for Switzerland, for which the Swiss National Bank introduced a new 'monetary policy concept' in January 2000, we take 2000Q1 as the starting date. For the United States, although the Federal Reserve formally introduced an inflation target in 2012, we start the sample in 1992Q2, corresponding to the break in the mean of inflation identified by Levin and Piger (2003). The rationale for doing so is that, in fact, the Federal Reserve has been consistently following a strongly counter-inflationary strategy since the end of the Volcker disinflation in the early 1980s. Finally, as for Denmark, which has consistently followed a policy of pegging the Krone first to the Deutsche Mark and then to the Euro, thus importing the strong anti-inflationary stance of the Bundesbank, and then of the European Central Bank, we start the sample in 1996Q1 due to data availability.

B Details About the Approximations for the Model of the Gold Standard

The expression

$$\frac{\theta_t^m}{\theta_t^C}\frac{L_m\left(\frac{g_t^c}{P_t}\right)}{U_c(Y_t)} = \frac{i_t}{1+i_t}$$

is approximated as follows. We have that

$$\begin{aligned} & \frac{L_{mm}}{U_c}\left(\frac{g_t^c - g^c}{P} - \frac{g^c}{P^2}(P_t - P)\right) - \frac{L_m}{U_c^2}U_{cc}(Y_t - Y) + \frac{L_m}{U_c}\left(\theta_t^m - 1 - (\theta_t^c - 1)\right) \\ = \; & \frac{1}{(1+i)^2}(1 + i_t - (1+i)) \end{aligned}$$

and

$$\frac{L_m}{U_c}\frac{L_{mm}}{L_m}\frac{g^c}{P}(\hat{g}_t^c - \hat{P}_t) - \frac{L_m}{U_c}\left(\frac{U_{cc}Y}{U_c}\hat{Y}_t - \hat{\theta}_t^m + \hat{\theta}_t^c\right) = \frac{1}{(1+i)}\hat{\imath}_t$$

Therefore

$$-\epsilon_m^{-1}(\hat{g}_t^c - \hat{P}_t) + \sigma^{-1}\hat{Y}_t = \frac{1}{i}\hat{\imath}_t + \hat{\theta}_t^c - \hat{\theta}_t^m$$

and so

$$(\hat{g}_t^c - \hat{P}_t) = q_y\hat{Y}_t - q_i\hat{\imath}_t - q_\theta(\hat{\theta}_t^c - \hat{\theta}_t^m)$$

$$q_y = \frac{\sigma^{-1}}{\epsilon_m^{-1}} \qquad q_i = \frac{1}{\epsilon_m^{-1} i} \qquad q_\theta = \frac{1}{\epsilon_m^{-1}}$$

Turning to the expression

$$\frac{1}{P_t}\theta_t^m L_m\left(\frac{g_t^c}{P_t}\right) = \theta_t^g V_g(G_t - g_t^c)$$

it is approximated as follows. We have that

$$\begin{aligned} & -\left(\frac{1}{P}\right)^2 L_m(P_t - P) + \left(\frac{1}{P}\right) L_{mm}\left(\frac{g_t^c - g}{P} - \frac{g}{P^2}(P - P\right) \\ = \quad & V_{gg}(G_t - G - (g_t^c - g^c)) + V_g(\theta_t^g - 1 - (\theta_t^m - 1)) \end{aligned}$$

$$-\frac{1}{P}L_m \hat{P}_t + \frac{1}{P}L_{mm}\frac{g^c}{P}(\hat{g}_t^c - \hat{P}_t) = V_{gg}G(\hat{G}_t - s_g\hat{g}_t^c) + V_g(\hat{\theta}_t^g - \hat{\theta}_t^m)$$

$$-\hat{P}_t - \epsilon_m^{-1}(\hat{g}_t^c - \hat{P}_t) = -\epsilon_g^{-1}(\hat{G}_t - s_g\hat{g}_t^c) + \hat{\theta}_t^g - \hat{\theta}_t^m$$

in which

$$\epsilon_m = -\frac{L_m}{L_{mm}m} \qquad \epsilon_g = -\frac{V_g}{V_{gg}g} \qquad s_g = g^c/G$$

Therefore we have that

$$(\epsilon_m^{-1} - 1)\hat{P}_t = (\epsilon_m^{.1} + \epsilon_g^{-1}s_g)\hat{g}_t^c - \epsilon_g^{-1}\hat{G}_t + \hat{\theta}_t^g - \hat{\theta}_t^m$$

$$\hat{P}_t = -\vartheta_g\hat{g}_t^c + \vartheta_G\hat{G}_t - \vartheta_\theta(\hat{\theta}_t^g - \hat{\theta}_t^m)$$

where

$$\vartheta_g = \frac{(\epsilon_m^{.1} + \epsilon_g^{-1}s_g)}{(1 - \epsilon_m^{-1})} \qquad \vartheta_G = \frac{\epsilon_g^{-1}}{(1 - \epsilon_m^{-1})} \qquad \vartheta_\theta = \frac{1}{(1 - \epsilon_m^{-1})}$$

C Derivation of the Expression for the Long-Term Nominal Interest Rate

In this Appendix we detail the derivation of the expression for the long-term nominal interest rate, $i_{L,t}$, in Section 3.2.2 in the main text. As mentioned in the main text, we assume a decaying coupon structure 1, δ, δ^2, δ^3..., where

$$\delta = (1 + i)\left(1 - \frac{1}{m}\right)$$

where m is the maturity of the long-term bond expressed in quarters. Note that

$$i_{L,t} = -Q_t + \frac{\delta}{1+i}Q_t$$

and

$$Q_t = -i_t + \frac{\delta}{1+i} E_t Q_{t+1}$$

Therefore

$$i_{L,t} = i_t + \frac{\delta}{1+i}(Q_t - E_t Q_{t+1})$$

Since

$$Q_t - E_t Q_{t+1} = -i_t + \frac{\delta}{1+i} E_t Q_{t+1} - E_t Q_{t+1} = -i_t + E_t i_{L,t+1}$$

we have that

$$i_{L,t} = \frac{1+i-\delta}{1+i} i_t + \frac{\delta}{1+i} E_t i_{L,t+1}$$

Finally, note that that in a steady state $i_L = i$.

D Relationship with Barsky and Summers (1988)

Barsky and Summers (1988)'s model is similar to the one we use in Section 3.1. They assume perfect foresight and stationarity. Under these assumptions equation (4) in Section 3.1.1 implies that

$$1 + i_t = (1+r)\frac{P_{t+1}}{P_t}.$$

Since

$$1 + r_t = \frac{1}{\beta}\frac{U_c(Y_t)}{U_c(Y_{t+1})}$$

$1 + r_t = \beta^{-1}$. Equation (6) in the main text implies that

$$\frac{1}{P_t} = \frac{1}{P} = \frac{V_g(G - g^c)}{U_c(Y)} + \frac{1}{1+r}\frac{1}{P}.$$

Therefore the price level is constant and equal to

$$P = \frac{U_c(Y)}{V_g(G - g^c)}\frac{r}{1+r} \tag{C.1}$$

Since inflation is equal to zero,

$$1 + i = 1 + r.$$

On the other hand, equilibrium on the money market implies that

$$\frac{L_m\left(\frac{g^c}{P}\right)}{U_c(Y)} = \frac{r}{1+r} \tag{C.2}$$

Equations (C.1) and (C.2) are those used by Barsky and Summers (1988) in order to explain the price of gold and the relationship between the real interest rate and the price level.

E The State-Space Forms of the Models

Here follows a description of the state-space forms of the New Keynesian models for the Gold Standard and the inflation targeting regimes.

E.1 Gold Standard

The model for the Gold Standard is described by the following equations, where the notation is the same as in the main text of the paper:

$$\Delta y_t^n = \rho_n \Delta y_{t-1}^n + \epsilon_t^n, \text{ with } \epsilon_t^n \sim N(0, \sigma_n^2). \tag{E.1.1}$$

$$\Delta g_t = \mu_g + \epsilon_t^g, \text{ with } \epsilon_t^g \sim N(0, \sigma_g^2). \tag{E.1.2}$$

$$r_t^n = \epsilon_t + \sigma^{-1}[\gamma\rho_n - (1-\gamma)]\Delta y_t^n. \tag{E.1.3}$$

$$\epsilon_t = \rho_\epsilon \epsilon_{t-1} + \epsilon_t^\epsilon, \text{ with } \epsilon_t^\epsilon \sim N(0, \sigma_\epsilon^2) \tag{E.1.4}$$

$$\pi_t = \kappa\tilde{y}_t + \kappa_1\Delta\tilde{y}_t + \kappa_1\Delta y_t^n + \beta E_t\pi_{t+1} + u_t, \text{ with } u_t \sim N(0, \sigma_u^2) \tag{E.1.5}$$

$$\tilde{y}_t = \gamma E_t\tilde{y}_{t+1} + (1-\gamma)\tilde{y}_{t-1} - \sigma(\hat{\imath}_t - E_t\pi_{t+1} - r_t^n) + v_t, \text{ with } \epsilon_t^v \sim N(0, \sigma_v^2), \tag{E.1.6}$$

$$\pi_t = -\vartheta_g\Delta\hat{g}_t^c + \vartheta_G\Delta\hat{g}_t - \vartheta_\theta(\Delta\hat{\theta}_t^g - \Delta\hat{\theta}_t^m), \tag{E.1.7}$$

$$\begin{aligned} \Delta\hat{g}_t^c - \pi_t \quad &= \quad q_y(1+\rho)(\Delta y_t + \Delta y_t^n) - q_y\rho(\Delta y_{t-1} + \Delta y_{t-1}^n) + \\ &\quad -q_i(\Delta\tilde{\imath}_t + \Delta r_t^n) - q_\theta(\Delta\hat{\theta}_t^c - \Delta\hat{\theta}_t^m), \end{aligned} \tag{E.1.8}$$

$$\hat{\imath}_{L,t} = \frac{1+i-\delta}{1+i}\hat{\imath}_t + \frac{\delta}{1+i}E_t\hat{\imath}_{L,t+1}. \tag{E.1.9}$$

$$\hat{\theta}_t^x = \rho_x\hat{\theta}_{t-1}^x + \theta_t^x, \text{ with } x = g,\, c,\, m,\, \mid \rho_x \mid < 1, \text{ and } \theta_t^x \sim N(0, \sigma_{\theta^x}^2) \tag{E.1.10}$$

By defining the state vector ξ_t, the vector of forecast errors η_t, and the vector of the structural shocks ϵ_t as

$$\xi_t = [\Delta y_t^n, \Delta g_t, \Delta g_t^c, r_t^n, \epsilon_t, \pi_t, \tilde{y}_t, \hat{\imath}_t, \hat{\imath}_{L,t}, E_t\pi_{t+1}, E_t y_{t+1}, E_t\tilde{\imath}_{L,t+1}, \hat{\theta}_t^g, \hat{\theta}_t^c, \hat{\theta}_t^m, \tilde{y}_{t-1}]', \tag{E.1.11}$$

$$\eta_t = [\eta_t^\pi \eta_t^y \eta_t^{i_L}]', \tag{E.1.12}$$

and

$$\epsilon_t = [\epsilon_t^{\Delta y^n}\ \epsilon_t^g\ \epsilon_t^\epsilon\ u_t\ v_t\ \hat{\theta}_t^g\ \hat{\theta}_t^c\ \hat{\theta}_t^m]', \tag{E.1.13}$$

and augmenting the model with the identity

$$\tilde{y}_{t-1} = \tilde{y}_{t-1} \tag{E.1.14}$$

and the definition of the three forecast errors

$$\eta_t^{\pi} = \pi_t - E_{t-1}\pi_t \tag{E.1.15}$$

$$\eta_t^{y} = y_t - E_{t-1}y_t \tag{E.1.16}$$

$$\eta_t^{\tilde{\imath}_L} = \tilde{\imath}_{L,t} - E_{t-1}\tilde{\imath}_{L,t} \tag{E.1.17}$$

the model can be put into the Sims (2000) canonical form

$$\Gamma_0\xi_t = \Gamma_1\xi_{t-1} + \Psi\epsilon_t + \Pi\eta_t \tag{E.1.18}$$

where the matrices Γ_0, Γ_1, Ψ, and Π feature (convolutions of) the structural parameters.

Solving the model as in Sims (2000) produces the following representation for the state vector ξ_t:

$$\xi_t = \tilde{F}\xi_{t-1} + \tilde{A}_0\epsilon_t \tag{E.1.19}$$

It is important to stress that all of the state variables in ξ_t are stationary.

For the United States we estimate model (E.1.1)-(E.1.10) based on the six observed variables we discussed in the text: a call money rate ($\hat{\imath}_t$), a corporate bond yield ($\hat{\imath}_{L,t}$), and the logarithms of real GNP (y_t), Warren and Pearson's (1933) wholesale price index (p_t), the overall stock of gold ($\hat{g}_t$), and the stock of gold held by the monetary authority ($\hat{g}_t^c$). In order to do this we define the following state-space form, with state equation

$$\underbrace{\left[\begin{array}{c} y_t^n \\ \hat{g}_t \\ \hat{g}_t^c \\ p_t \\ \hline \xi_t \end{array}\right]}_{S_t} = \underbrace{\left[\begin{array}{c} \mu_y \\ \mu_g \\ \mu_g^c \\ 0 \\ \hline 0_{16\times 1} \end{array}\right]}_{h} + \underbrace{\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \text{Row of } \Delta y_t^n \text{ in } \tilde{F} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \text{Row of } \Delta \hat{g}_t^c \text{ in } \tilde{F} \\ 0 & 0 & 0 & 1 & \text{Row of } \pi_t \text{ in } \tilde{F} \\ \hline \multicolumn{4}{c|}{0_{16\times 4}} & \tilde{F} \end{array}\right]}_{F} \underbrace{\left[\begin{array}{c} y_{t-1}^n \\ \hat{g}_{t-1} \\ \hat{g}_{t-1}^c \\ p_{t-1} \\ \hline \xi_{t-1} \end{array}\right]}_{S_{t-1}} +$$

$$+ \underbrace{\left[\begin{array}{c} \text{Row of } \Delta y_t^n \text{ in } \tilde{A}_0 \\ 0 \quad 1 \qquad 0_{1\times 6} \\ \text{Row of } \Delta \hat{g}_t^c \text{ in } \tilde{A}_0 \\ \text{Row of } \pi_t \text{ in } \tilde{A}_0 \\ \hline \tilde{A}_0 \end{array}\right]}_{A_0} \underbrace{\left[\begin{array}{c} \epsilon_t^{\Delta y^n} \\ \epsilon_t^{g} \\ \epsilon_t^{\epsilon} \\ u_t \\ v_t \\ \hat{\theta}_t^{g} \\ \hat{\theta}_t^{c} \\ \hat{\theta}_t^{m} \end{array}\right]}_{\epsilon_t} \tag{E.1.20}$$

where ξ_t, $\tilde{F}$, and $\tilde{A}_0$ are the same objects as in equation (E.1.19), and observation equation

$$\underbrace{\begin{bmatrix} y_t \\ p_t \\ \hat{g}_t \\ \hat{g}_t^c \\ \hat{\imath}_t \\ \hat{\imath}_{L,t} \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mu_i \\ \mu_{i_L} \end{bmatrix}}_{c} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & ... & 1 & 0 & 0 & ... \\ 0 & 1 & 0 & 0 & ... & 0 & 0 & 0 & ... \\ 0 & 0 & 0 & 1 & ... & 0 & 0 & 0 & ... \\ & & & & ... & 0 & 1 & 0 & ... \\ & & & & ... & 0 & 0 & 1 & ... \\ 0 & 0 & 1 & 0 & ... & 0 & 0 & 0 & ... \end{bmatrix}}_{H'} \underbrace{\begin{bmatrix} y_t^n \\ \hat{g}_t \\ \hat{g}_t^c \\ p_t \\ \xi_t \end{bmatrix}}_{S_t} \tag{E.1.21}$$

We calibrate μ_y, μ_g, and μ_g^c to the average values taken by the log-differences of real GNP, the overall stock of gold, and the stock of monetary gold over the sample period, and we calibrate μ_i, and μ_{i_L} to the average values taken by the short- and the long-term nominal interest rates over the sample period.

For the United Kingdom we proceed in the same way, with the only difference that we only use the five observed variables discussed in the text: the Bank of England's discount rate ($\hat{\imath}_t$), a consol yield ($\hat{\imath}_{L,t}$), and the logarithms of real GDP (y_t), the wholesale price index (p_t), and the stock of gold held at the Bank of England ($\hat{g}_t^c$), so that equations (E.1.20) and (E.1.21) are modified accordingly.

E.2 Inflation targeting regimes

The model for inflation targeting regimes is described by equations (E.1.1), (E.1.3), (E.1.4), (E.1.6), (E.1.9), (E.1.15)-(E.1.17), and

$$(\pi_t - \bar{\pi}) = \kappa\tilde{y}_t + \kappa_1\Delta\tilde{y}_t + \kappa_1\Delta y_t^n + \beta E_t(\pi_{t+1} - \bar{\pi}) + u_t, \text{ with } u_t \sim N(0, \sigma_u^2) \tag{E.2.1}$$

$$V_t = \psi + \alpha\hat{\imath}_t + \lambda_t \tag{E.2.2}$$

$$\lambda_t = \rho_\lambda\lambda_{t-1} + \epsilon_{\lambda,t}, \text{ with } \epsilon_{\lambda,t} \sim N(0, \sigma_\lambda^2) \tag{E.2.3}$$

$$\hat{\imath}_t = \rho_i\hat{\imath}_{t-1} + (1-\rho_i)[\phi_\pi E_t(\pi_{t+1} - \bar{\pi}) + \phi_y E_t\hat{y}_{t+1}] + \epsilon_{i,t}, \text{ with } \epsilon_{i,t} \sim N(0, \sigma_i^2). \tag{E.2.4}$$

By defining the state vector ξ_t, the vector of forecast errors η_t, and the vector of the structural shocks ϵ_t as

$$\xi_t = [\Delta y_t^n, r_t^n, \epsilon_t, \pi_t, \tilde{y}_t, \hat{\imath}_t, \hat{\imath}_{L,t}, E_t\pi_{t+1}, E_t y_{t+1}, E_t\tilde{\imath}_{L,t+1}, \lambda_t]', \tag{E.2.5}$$

$$\eta_t = [\eta_t^\pi \eta_t^y \eta_t^{i_L}]', \tag{E.2.6}$$

and

$$\epsilon_t = [\epsilon_t^{\Delta y^n} \ \epsilon_t^\epsilon \ u_t \ v_t \ \epsilon_{i,t} \ \epsilon_{\lambda,t}]', \tag{E.2.7}$$

the model can be put into the Sims (2000) canonical form and can be solved as for the Gold Standard, producing the representation (E.1.19) for the dynamics of the state vector ξ_t.

For all countries we estimate the model based on data for a short- and a long-term nominal interest rate ($\hat{\imath}_t$ and $\hat{\imath}_{L,t}$), the velocity of M1 (V_t, computed as the ratio between nominal GDP and nominal M1), and the logarithms of real GDP and the GDP deflator (y_t and p_t). In order to do this we define the following state-space form, with state equation

$$\underbrace{\left[\begin{array}{c} y_t^n \\ p_t \\ \hline \xi_t \end{array}\right]}_{S_t} = \underbrace{\left[\begin{array}{c} \mu_y \\ \bar{\pi} \\ \hline 0_{11\times 1} \end{array}\right]}_{h} + \underbrace{\left[\begin{array}{cc|c} 1 & 0 & \text{Row of } \Delta y_t^n \text{ in } \tilde{F} \\ 0 & 1 & \text{Row of } \pi_t \text{ in } \tilde{F} \\ \hline \multicolumn{2}{c|}{0_{11\times 4}} & \tilde{F} \end{array}\right]}_{F} \underbrace{\left[\begin{array}{c} y_{t-1}^n \\ p_{t-1} \\ \hline \xi_{t-1} \end{array}\right]}_{S_{t-1}} +$$

$$+ \underbrace{\left[\begin{array}{c} \text{Row of } \Delta y_t^n \text{ in } \tilde{A}_0 \\ \text{Row of } \pi_t \text{ in } \tilde{A}_0 \\ \hline \tilde{A}_0 \end{array}\right]}_{A_0} \underbrace{\left[\begin{array}{c} \epsilon_t^{\Delta y^n} \\ \epsilon_t^{\epsilon} \\ u_t \\ v_t \\ \epsilon_{i,t} \\ \epsilon_{\lambda,t} \end{array}\right]}_{\epsilon_t} \tag{E.2.8}$$

and observation equation

$$\underbrace{\left[\begin{array}{c} y_t \\ p_t \\ \hat{\imath}_t \\ \hat{\imath}_{L,t} \\ V_t \end{array}\right]}_{Y_t} = \underbrace{\left[\begin{array}{c} 0 \\ 0 \\ \mu_i \\ \mu_{i_L} \\ \psi \end{array}\right]}_{c} + \underbrace{\left[\begin{array}{ccccccccc} 1 & 0 & ... & 1 & 0 & 0 & ... & 0 \\ 0 & 1 & ... & 0 & 0 & 0 & ... & 0 \\ 0 & 0 & ... & 0 & 1 & 0 & ... & 0 \\ & & ... & 0 & 0 & 1 & ... & 0 \\ 0 & 0 & ... & 0 & \alpha & 0 & ... & 1 \end{array}\right]}_{H'} \underbrace{\left[\begin{array}{c} y_t^n \\ p_t \\ \xi_t \end{array}\right]}_{S_t} \tag{E.2.9}$$

We calibrate μ_y and $\bar{\pi}$ to the average value taken by the log-difference of real GDP over the sample period, and to the inflation target, respectively. We also calibrate μ_i and μ_{i_L} to the average values taken by the short- and the long-term nominal interest rates over the sample period.

F Details of the Bayesian Estimation Procedure

We estimate the New Keynesian models described in Sections 3.2 and 3.3 via standard Bayesian methods exactly as in Benati (2008). Specifically, we numerically maximise the log posterior—defined as ln $L(\theta|Y)$ + ln $P(\theta)$, where θ is the vector collecting the model's structural parameters, $L(\theta|Y)$ is the likelihood of θ conditional on the data, and $P(\theta)$ is the prior—via simulated annealing. Following Goffe, Ferrier, and Rogers (1994) we implement simulated annealing via the algorithm proposed by Corana, Marchesi, Martini and

Ridella (1987), setting the key parameters to T_0=100,000, r_T=0.9, N_t=5, N_s=20, ϵ=10^{-6}, N_ϵ=4, where T_0 is the initial temperature, r_T is the temperature reduction factor, N_t is the number of times the algorithm goes through the N_s loops before the temperature starts being reduced, N_s is the number of times the algorithm goes through the function before adjusting the stepsize, ϵ is the convergence (tolerance) criterion, and N_ϵ is number of times convergence is achieved before the algorithm stops. Finally, initial conditions were chosen stochastically by the algorithm itself, while the maximum number of functions evaluations, set to 1,000,000, was never achieved.

In implementing Random-Walk Metropolis (RWM) we exactly follow An and Schorfheide (2006, Section 4.1), with the single exception of the method we use to calibrate the covariance matrix's scale factor (the parameter c below) for which we follow the methodology described in the next paragraph. Let then $\hat{\theta}$ and $\hat{\Sigma}$ be the mode of the maximised log posterior and its estimated Hessian, respectively (we compute $\hat{\Sigma}$ numerically as in An and Schorfheide, 2006). We start the Markov chain of the RWM algorithm by drawing $\theta^{(0)}$ from $N(\hat{\theta}, c^2\hat{\Sigma})$. For $s = 1, 2, ..., N$ we then draw $\tilde{\theta}$ from the proposal distribution $N(\theta^{(s-1)}, c^2\hat{\Sigma})$, accepting the jump (i.e., $\theta^{(s)} = \tilde{\theta}$) with probability min $\{1, r(\theta^{(s-1)}, \theta|Y)\}$, and rejecting it (i.e., $\theta^{(s)} = \theta^{(s-1)}$) otherwise, where

$$r(\theta^{(s-1)}, \theta|Y) = \frac{L(\theta|Y)\ P(\theta)}{L(\theta^{(s-1)}|Y)\ P(\theta^{(s-1)})}$$

A key problem in implementing Metropolis algorithms is how to calibrate the covariance matrix's scale factor in order to achieve an acceptance rate of the draws close to the ideal one (in high dimensions) of 0.23—see Gelman, Carlin, Stern and Rubin (1995). Our approach for calibrating c is based on the idea of estimating a reasonably good approximation to the inverse relationship between c and the acceptance rate by running a pre-burn-in sample. Specifically, let C be a grid of possible values for c (in what follows, we consider a grid over the interval [0.1, 1] with increments equal to 0.05). For each value of c in the grid (call it c_j) we run n draws of the RWM algorithm, storing, for each c_j, the corresponding fraction of accepted draws, f_j. We then fit a third-order polynomial to the f_j's via least squares, and letting $\hat{a}_0$, $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$ be the estimated coefficients, we choose c by solving numerically the equation $\hat{a}_0+\hat{a}_1 c+\hat{a}_2 c^2+\hat{a}_3 c^3$=0.23.

We check convergence of the Markov chain to the ergodic distribution based on Geweke's (1992) inefficiency factors of the draws for each individual parameter. The inefficiency factors are defined as the inverse of the relative numerical efficiency measure of Geweke (1992),

$$RNE \ = (2\pi)^{-1}\frac{1}{S(0)}\int_{-\pi}^{\pi} S(\omega)d\omega,$$

where $S(\omega)$ is the spectral density of the sequence of draws from RWM for the parameter of interest at frequency ω. We estimate the spectral densities via the lag-window estimator as described in chapter 10 of Hamilton (1994). We also considered an estimator based on the fast-Fourier transform, and results were very similar. For all parameters the IFs are equal to at most 10, that is, well below the values of 20-25 which are typically taken to indicate problems in the convergence of the Markov chain.

G Details of Giannone, Lenza, and Primiceri's (2015) Bayesian VAR Estimation Procedure

In Section 8 of the main text of the paper we estimate the Bayesian VARs based on the methodology proposed by Giannone, Lenza, and Primiceri (2015), with the only difference that in estimation we impose stationarity upon the VAR as in e.g. Cogley and Sargent (2002, 2005) and Primiceri (2005). Specifically, in the MCMC algorithm used for estimation we move to iteration i+1 if and only if the draw for the VAR's parameters associated with iteration i is stationary. Otherwise, we redraw the parameters for iteration i.

Let the VAR(p) model be

$$Y_t = B_0 + B_1 Y_{t-1} + ... + B_p Y_{t-p} + u_t \tag{G.1}$$

where the notation is obvious, with Y_t and u_t being $N \times 1$, and $u_t \sim N(0, \Sigma)$. By defining $\beta \equiv vec([B_0, B_1, ..., B_p]')$ and $x_t \equiv [1, Y_{t-1}', ..., Y_{t-p}']'$, equation (G.1) can be rewritten as

$$Y_t = X_t \beta + u_t \tag{G.2}$$

where $X_t \equiv I_N \otimes x_t'$. The prior distribution for the VAR coefficients is postulated to belong to the Normal-Wishart family, i.e.

$$\Sigma \sim IW(\Psi; d) \tag{G.3}$$

$$\beta|\Sigma \sim N(b; \Sigma \otimes \Omega) \tag{G.4}$$

where the elements of Ψ, d, b and Ω are functions of a lower-dimensional vector of hyper-parameters. The degree of freedom of the Inverse-Wishart distribution is set to d=N+2, which is the minimum value that guarantees the existence of the prior mean of Σ. Ψ is postulated to be a diagonal matrix with the $N \times 1$ vector of hyperparameters ψ on the main diagonal.

The conditional Gaussian prior for β is of the Minnesota type, with the only difference that since, as we discuss below, in estimation we impose stationarity upon the VAR, instead of imposing Litterman's 'random-walk prior', we postulate the following first moment:

$$E\left[(B_s)_{ij}|\Sigma\right] = \begin{array}{l} \mu \text{ if } i = j \text{ and } s = 1 \\ 0 \text{ otherwise} \end{array} \tag{G.5}$$

We set μ as follows. For inflation we set $\mu = 0$, reflecting the fact that under inflation targeting regimes this series has been near-uniformly indistinguishable from white noise. For real GDP growth we set $\mu = 0.5$, reflecting the mild extent of serial correlation of this series, in general. For the short- and the long-term nominal rate, on the other hand, we set $\mu = 0.75$ and $\mu = 1 - 0.5/T$, respectively, where T is the sample length. For the short rate this reflects its non-negligible extent of persistence, whereas for the long rate the fact that this series is best thought of as near unit root process.

As for the second moment, as in Giannone *et al.* (2015) we postulate that

$$Cov\left[(B_s)_{ij}, (B_r)_{hm}|\Sigma\right] = \begin{array}{l} \lambda^2 \frac{1}{s^2} \frac{\Sigma_{ih}}{\psi_j/(d-N-1)} \text{ if } m = j \text{ and } r = s \\ 0 \text{ otherwise} \end{array} \tag{G.6}$$

where the hyperparameter λ controls the scale of the variances and covariances, thus determining the prior's overall tightness. We set the hyperpriors for λ and ψ as in Giannone *et al.* (2015), and we estimate the VAR as discussed there, with the only difference that, as we already pointed out, we impose stationarity upon the VAR.

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Figures for the Online Appendix

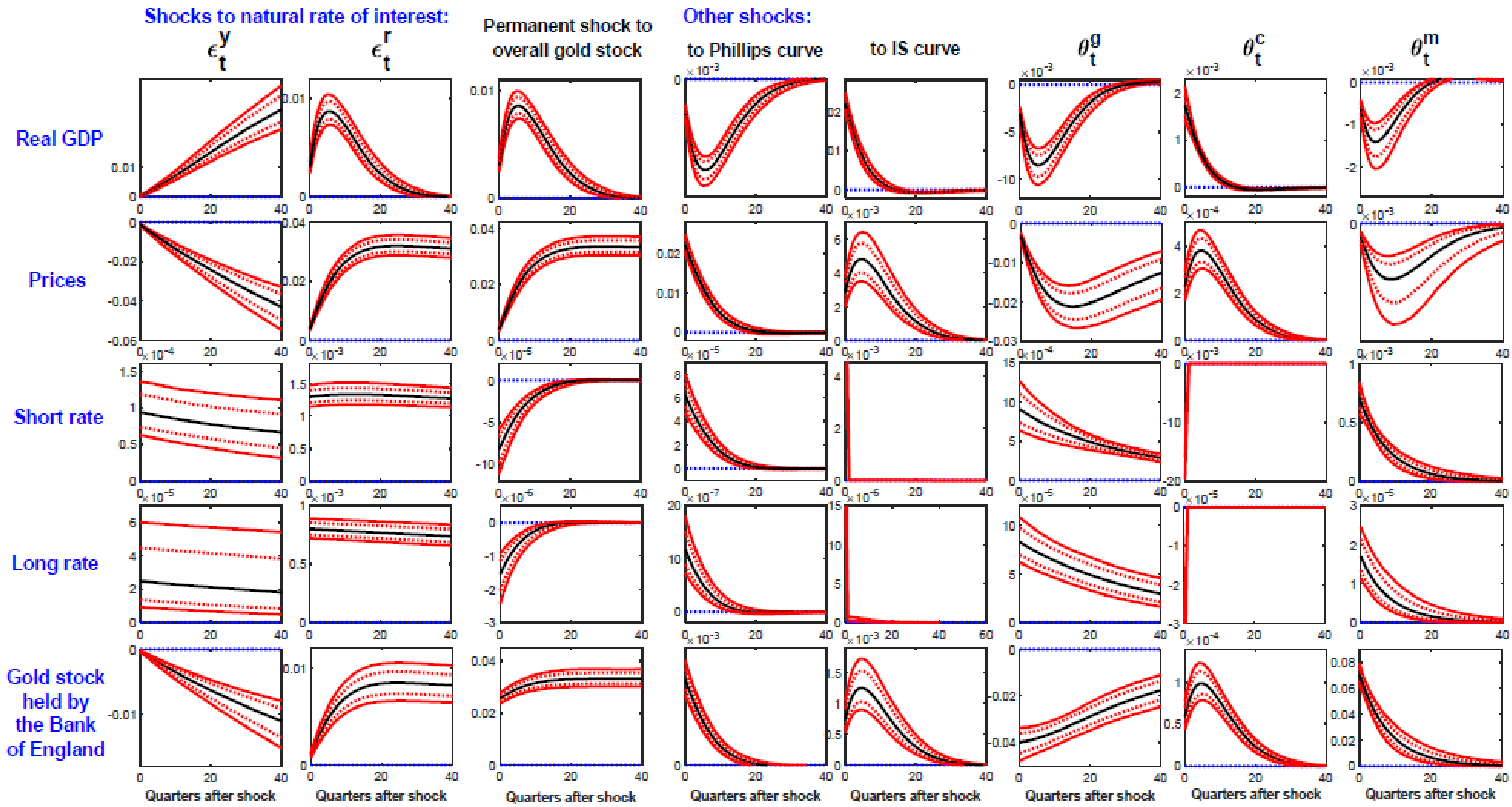

Figure A.1 United Kingdom, 1858Q1-1914Q2: Impulse-response functions of the New Keynesian model to the structural shocks (medians of the posterior distributions, and 16-84 and 5-95 credible sets)

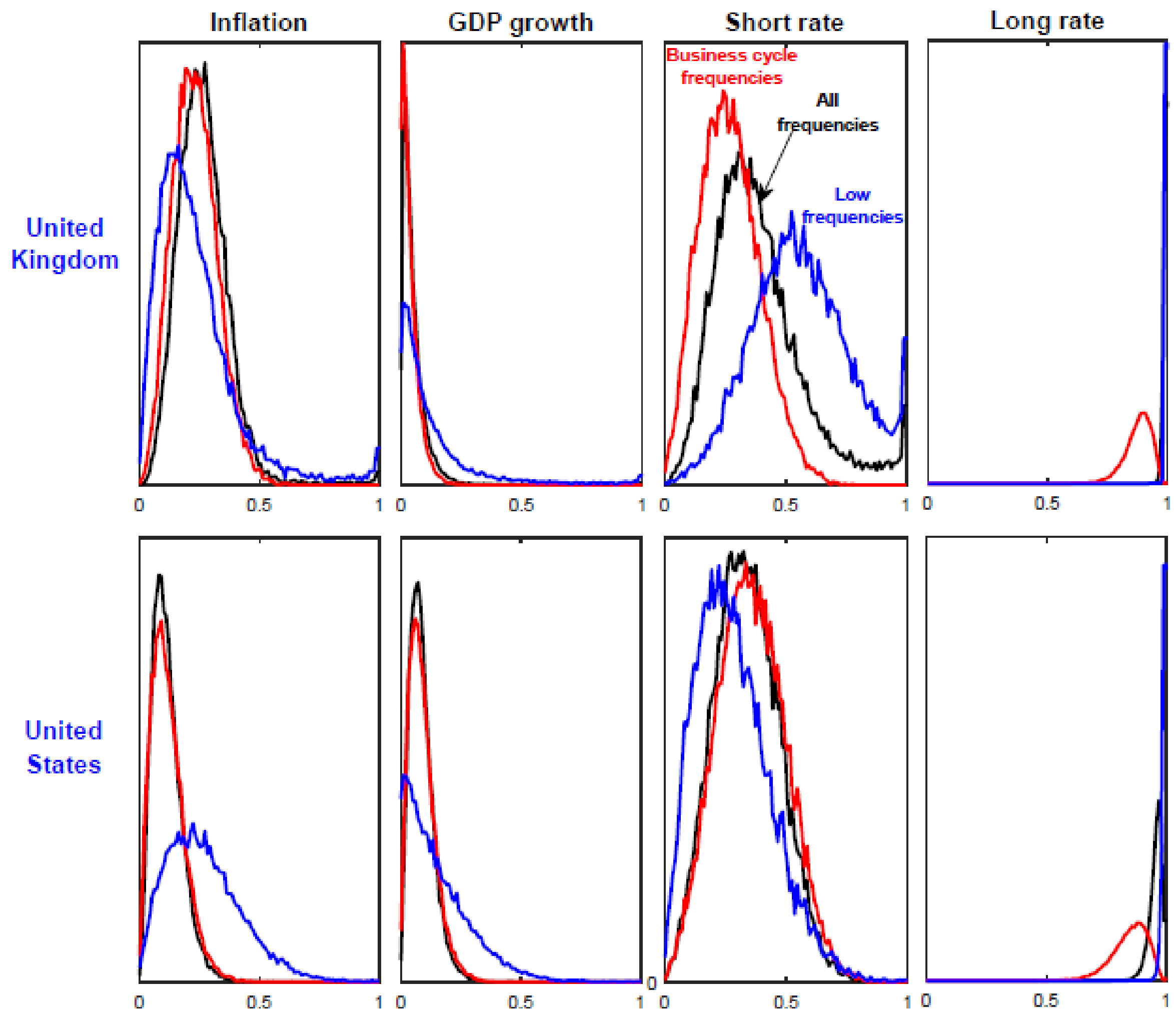

Figure A.2 United Kingdom and United States under the Gold Standard: Posterior distributions of the fractions of variance explained by shocks to the natural rate of interest by frequency band

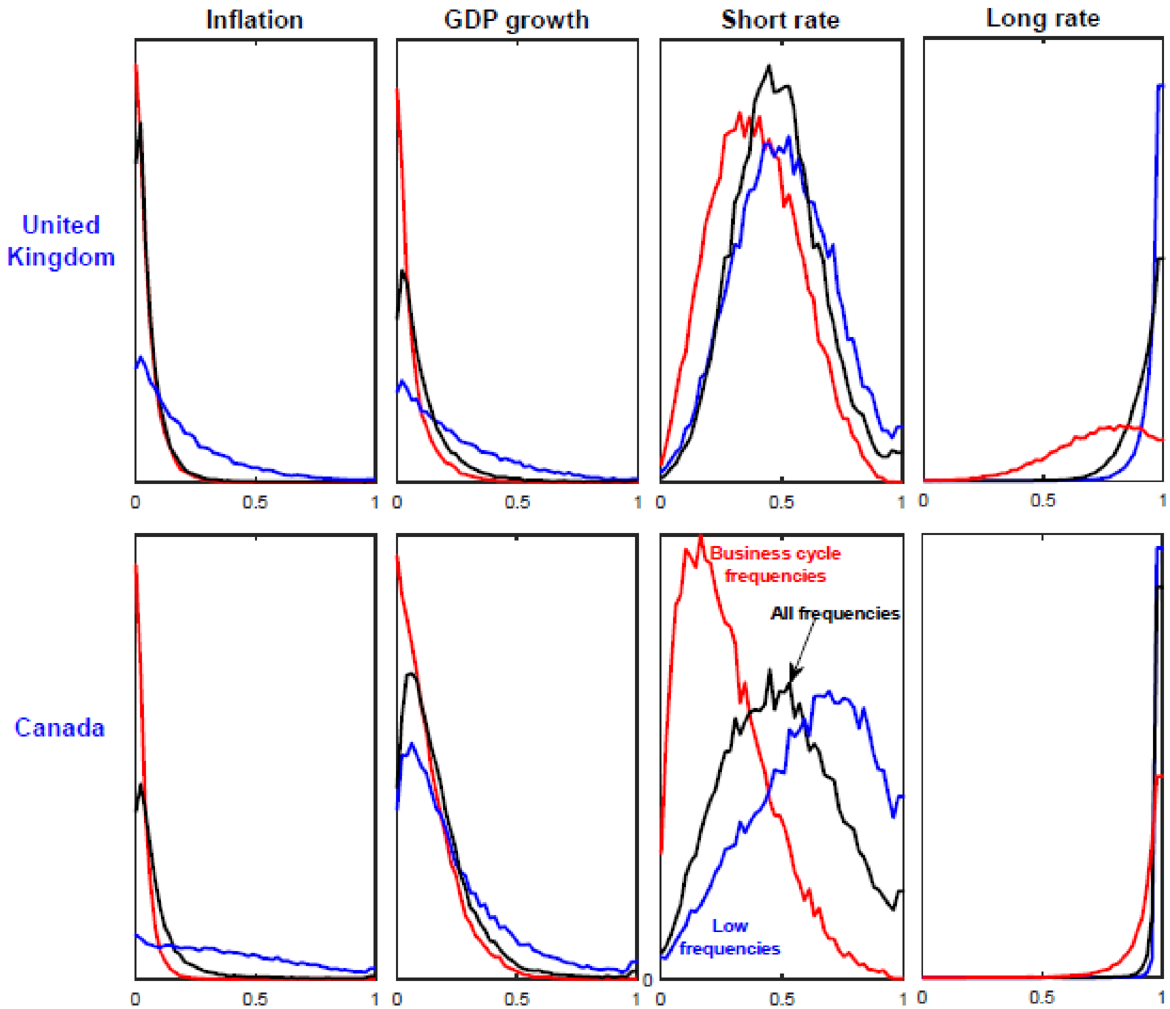

Figure A.3 United Kingdom and Canada under inflation targeting: Posterior distributions of the fractions of variance explained by shocks to the natural rate of interest by frequency band