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Optimal monetary and transfer policy in a liquidity trap

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Abstract

Optimal monetary and fiscal policy are jointly analyzed in a heterogeneous two-agents New Keynesian environment, where fiscal policy is modeled in the form of lump-sum transfers set by the government. The main result is that transfer policy does not serve as a substitute for forward guidance - as it entails consumption dispersion costs - and does not affect its optimal duration. Transfers indeed influence the length of stay at the zero lower bound through two offsetting channels: a *shortening* channel works through an initial increase in transfers that mitigates the recession (reducing the need for forward guidance), and a *lengthening* channel works through a later transfer cut that curbs the undesired expansion (making forward guidance desirable for a longer horizon). Imposing a homogeneous transfer policy across agents does not change the stabilization outcome or the effect on the duration of forward guidance, nor does so allowing for cyclical income differences.

JEL Codes: E52, E62, E63

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1 Introduction

The challenges posed by a liquidity trap to stabilization policy relate to the impossibility to fully address a demand shock by conventional monetary tools, due to the zero lower bound on the nominal interest rate. Several authors have explored policy alternatives to overcome this émpasse, within the perimeter of monetary policy or fiscal policy. The range of considered policies include forward guidance on the nominal interest rate (according to the seminal paper by Eggertsson and Woodford (2003)), quantitative easing (Gertler and Karadi (2013)), distortionary taxation (Eggertsson and Woodford (2006)), helicopter money (Benigno and Nisticò (2020)) and lump sum transfers.

This paper studies lump sum transfers from the perspective of an optimal monetary and fiscal policy problem. The setting is one with heterogeneous agents, in which transfers have different effects on consumption due to a heterogeneity in marginal propensity to consume (MPC). In this perspective, the conventional view is that transfers can be used as a substitute of forward guidance over the recessionary phase of the liquidity trap, as they can produce an expansionary stimulus (see Eggertsson and Krugman (2012), Farhi and Werning (2016), Wolf (2021)). In this paper I challenge this view by showing first that, from an optimal policy perspective, transfers are not a substitute tool for forward guidance; secondly, that their optimal use over the liquidity trap does not even influence the optimal time of the liftoff of the interest rate from the zero lower bound.

The first result relies on the fact that transfer policies create consumption variations across households, that negatively affects welfare: therefore, a government seeking to use transfers to mitigate output fluctuations in the liquidity trap faces a trade-off between stabilization and consumption dispersion. The second result relies on the presence of a *shortening* and *lengthening* role of transfer policy with respect to the duration of forward guidance, which quantitatively offset each other. Over the early stages of the liquidity trap, the government wants to transfer resources to the high MPC agents (in my model, “hand to mouth” households) in order to mitigate the drop in output: this effect alone would reduce the room for forward guidance intervention, and shorten the optimal stay of the interest rate at the zero lower bound. When the shock is over and the interest rates are still kept at zero, then the government would like to cut the transfers to the hand to mouth in order to cool down the overheating of the economy - making a delay of the interest rate liftoff more desirable, as forward guidance becomes less costly in terms of output expansion. The shortening and lengthening channels lean against each other, giving rise to an analytically ambiguous effect on the duration of forward guidance. I calibrate the model using standard parameters assumed in the liquidity trap literature, finding that quantitatively these effects offset each other, leaving no significant influence on the optimal duration of stay at the zero lower bound.

Results are robust to two important extensions. First, I consider an environment in which the government is constrained to set the same transfer for all the households (as in Wolf (2021)). While in the baseline case the transfer to the hand to mouth could be used for stabilization, and the transfer to the other households could be set to satisfy public debt solvency, instead, with homogeneous transfer policy, the transfer is set equal for all the households: therefore it needs to satisfy both the goals, creating a trade-off between stabilization and public debt solvency. My finding points out that this additional trade-off is negligible, as any necessary public debt adjustment required by government policy can be smoothed out by long run movements in transfers - so well beyond the end of the liquidity trap.

Then, I study optimal transfer policy in a general environment where I allow for cyclical income differences between the hand to mouth and the other households (see Bilbiie (2018)), other than the ones implied by transfer policy. In this case, the extent to which the hand to mouth consumes more than the other households over the trap depends both on transfers and these additional sources of income differences. I show that the optimal transfer pattern is replaced by an optimal *augmented transfer* pattern, which incorporates both the transfer and the other cyclical income differences. By setting the transfer, the government can fully control the pattern of the augmented transfer, so it can still achieve the same stabilization results of the baseline setting. The results in terms of the role of transfers and their effect on forward guidance extend also to this more general framework.

Literature This paper formulates an optimal fiscal-monetary policy problem in a liquidity trap, following in spirit Eggertsson and Woodford (2006). While they model fiscal policy as a distortionary VAT tax, I analyse lump sum transfers. The model builds on a literature exploring optimal policy in a TANK environment: Bilbiie, Monacelli, and Perotti (2020) analyse optimal monetary and transfer policy, where consumption dispersion arises from the tax scheme financing government spending; Hansen, Mano, and Lin (2020) treat instead optimal monetary policy alone in a two agents new keynesian environment. An analysis of optimal monetary policy in a TANK setting over the liquidity trap is carried out in Eggertsson and Krugman (2012) and in Benigno, Eggertsson, and Romei (2020): I contribute to these works by using the TANK framework to analyse a joint fiscal and monetary policy.
In the heterogeneous agents (HANK) literature, optimal monetary policy has been analysed in Acharya, Challe, and Dogra (2020), Nuño and Thomas (2020) and Ragot (2017) - the latter in a liquidity trap scenario; Le Grand, Martin-Baillon, and Ragot (2022) treats optimal fiscal policy, while Bhandari, Evans, Golosov, and Sargent (2021) and Wolf (2022) analyse the optimal fiscal-monetary mix. I contribute to these last two papers by studying optimal fiscal and monetary policy in a liquidity trap.
The effect of transfers on aggregate output , disentangled from an optimal policy perspective, is addressed in Farhi and Werning (2016), McKay and Reis (2013), Mehrotra (2018), Giambattista and Pennings (2017). Wolf (2021) shows an equivalence result in aggregate inflation-output stabilization between interest rate and stimulus check policies. I embed the results of this literature in my paper, by considering the role of transfers in achieving output and inflation stabilization.
This paper relates also to the analysis of the interaction between cyclical inequality and the liquidity trap (see Bilbiie (2021)), which I account for in the second extension of the model.

The paper is organized as follows: section 2 reports the model's features: section 3 illustrates the main results in terms of transfer policy over the liquidity trap. Sections 4 and 5 develop the extensions with respect to the homogeneous transfer response and cyclical income difference.

2 Model

2.1 Households

An infinite-horizon economy features unit mass of households, with a fraction $1-\lambda$ of "ricardian" and λ of "hand-to-mouth" ("HtM"). The ricardian households can access to a financial market for short term bonds, in which they can save or borrow, whereas this possibility is instead precluded to the hand to mouth. The ricardian solves the following utility maximization problem:

$$\max E_0 \sum_{t=0}^{\infty} \prod_{s=0}^{t-1} \xi_s \beta^t \left(1 - \exp(-zC_t) - \delta \exp(\eta L_t)\right)$$

s.t.

$$P_t C_t + \frac{B_t}{1+i_t} \leq W_t L_t + B_{t-1} + T_t + \frac{1-\chi}{1-\lambda} P_t \bar{D}_t \tag{2.1.1}$$

$$\lim_{s \to \infty} \beta^{s-t} \frac{B_s}{P_s} = 0 \tag{2.1.2}$$

where C_t is consumption, L_t is labor supply, B_t is bond holding, W_t is the nominal wage, P_t is the aggregate price index, T_t is a nominal transfer from the public sector, L_t is labor supply, β is the discount factor, ξ_t is an intertemporal preference shock[2], and z, δ, η are positive parameters. I assume that each period a fraction $1-\chi$ of the total amount of real firms' profits $\bar{D}_t$ is rebated evenly across the $1-\lambda$ ricardian households, and the rest to the hand to mouth. The last condition (2.1.2) is the transversality condition on bond holding. A particular remark relates to the adoption of exponential utility: it is suitable to maintain tractability in building an aggregate demand and supply for the economy in a heterogenous agents setting as the current one.

Consumption is specified by a Dixit-Stigliz aggregator of a unit mass of varieties:

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{2.1.3}$$

[2]Without loss of generality, I set $\xi_{-1} = 1$

where $C_t(j)$ is ricardian household's consumption of good of variety j and $\theta > 1$ is the elasticity of substitution between goods. First order conditions imply variety demand

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} C_t$$

whose sensitivity to the ratio between the variety price $P_t(j)$ and the price index P_t is measured by the elasticity θ - a standard result. The first order condition for labor supply implies:

$$\delta\eta\exp(\eta L_t) = z\exp(-zC_t)\frac{W_t}{P_t} \tag{2.1.4}$$

where the marginal disutility of labor is equated to the marginal utility of consumption multiplied by the real wage. The Euler equation is given by:

$$z\exp(-zC_t) = \xi_t\beta(1+i_t)E_t\left[z\exp(-zC_{t+1})\frac{P_t}{P_{t+1}}\right] \tag{2.1.5}$$

where the shock ξ_t affects the intertemporal consumption choice of the household: the higher is the realization of ξ_t, the more the household is propense to shift consumption from period t to $t+1$.

Let us now turn the attention to the hand to mouth problem. The latter writes similarly to the ricardian's one, with the notable differences that the household cannot trade in bonds. The hand to mouth receives a transfer T_t^* from the public sector- analogously to the ricardian household; moreover, a fraction χ of the total real dividend amount is rebated evenly across the λ hand to mouth households. The problem of the hand to mouth writes

$$\max E_0\sum_{t=0}^{\infty}\prod_{s=0}^{t-1}\xi_s\beta^t\left(1-\exp(-zC_t^*)-\delta\exp(\eta L_t^*)\right)$$

s.t.

$$P_tC_t^* \leq W_tL_t^* + \frac{\chi}{\lambda}P_t\bar{D}_t + T_t^* \tag{2.1.6}$$

where C_t^* and L_t^* are the consumption level and the labor supplied, respectively. I will assume for now $\chi = \lambda$, so that the share of profit levied to the hand to mouth is equal to the share of this type of household out of total population. This assumption implies that each period the hand to mouth receives the same dividend amount of the ricardian household. Assuming C_t^* to have the same Dixit-Stigliz aggregator form of (2.1.3), the hand to mouth demand for the variety of good is specular to the ricardian household case:

$$C_t^*(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} C_t^* \tag{2.1.7}$$

Taking the first order condition with respect to labor in the hand to mouth problem, we also obtain a labor supply condition analogous to the ricardian household:

$$\delta\eta \exp(\eta L_t^*) = -z \exp(-zC_t^*) \frac{W_t}{P_t} \tag{2.1.8}$$

Optimally, the budget constraint (2.1.6) hold with equality, pinning down the consumption of the hand to mouth for each period t:

$$P_t C_t^* = L_t^* W_t + T_t^* + P_t \bar{D}_t \tag{2.1.9}$$

Due to the lack of access to the bond market, the hand to mouth cannot save or borrow: therefore each period the whole sum of labor income and transfers is spent in consumption.

2.2 Firms

There is a unit mass of monopolistically competitive firms, each one producing a different variety j of good, with technology:

$$Y_t(j) = AL_t(j)$$

where $L_t(j)$ is labor demanded by firm j, and A is labor productivity. Each firm faces a probability α each period of not being able to reset its price; in that case, its price automatically increases by the steady state inflation Π. When a firm resets its price, it seeks to maximize its expected discounted sum of profits, adjusted for the probability of not being able to reoptimize in the future:

$$\max_{P_t(j)} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \prod_{s=t}^{T-1} \xi_s \Lambda_T \frac{1}{P_T} \left[\Pi^{T-t} P_t(j) \left(\frac{P_t(j)\Pi^{T-t}}{P_T} \right)^{-\theta} Y_T - \frac{W_T}{A}(1-\nu) \left(\frac{P_t(j)\Pi^{T-t}}{P_T} \right)^{-\theta} Y_T - \zeta_T \right] \tag{2.2.1}$$

where the term ν is a government subsidy on labor costs and ζ_t is a lump sum tax. Firms value future profits according to an average Λ_t of marginal utilities of the two households, weighted by the respective profit shares: $\Lambda_t = (1-\chi) z \exp(-zC_t) + \chi z \exp(-zC_t^*)$ (see Benigno et al. (2020)). The first order condition for the optimal pricing problem yields:

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1}(1-\nu) \frac{E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \prod_{s=t}^{T-1} \xi_s \Lambda_T \frac{W_T}{P_T} \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta} \frac{Y_T}{A}}{E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \prod_{s=t}^{T-1} \xi_s \Lambda_T \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta-1} Y_T} \tag{2.2.2}$$

where P_t^* is the optimal price set by the resetting firms at time t. Equation (2.2.2) shows how firms set current price price by taking into account future discounted flow of costs and revenues, weighted by the probability of not be able to reset the price in the future.
Calvo pricing implies the following standard motion for inflation:

$$P_t^{1-\theta} = (1-\alpha) P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \Pi^{1-\theta} \tag{2.2.3}$$

Or equivalently:

$$1 = (1-\alpha) \left(\frac{P_t^*}{P_t} \right)^{1-\theta} + \alpha \left(\frac{\Pi_t}{\Pi} \right)^{\theta-1} \tag{2.2.4}$$

The optimal price setting condition (2.2.2) and the law of motion (2.2.4) give rise to the usual forward-looking expression for inflation in sticky price models (New Keynesian Phillips curve):

$$\left(\frac{1 - \alpha \left(\frac{\Pi_t}{\Pi} \right)^{\theta-1}}{1-\alpha} \right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t} \tag{2.2.5}$$

where:

$$F_t = Y_t \Lambda_t + \alpha\beta\xi_t E_t \left\{ F_{t+1} \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\theta-1} \right\} \tag{2.2.6}$$

$$K_t = \frac{\theta}{\theta-1}(1-\nu)\Lambda_t \frac{W_t}{P_t}\frac{Y_t}{A} + \alpha\beta\xi_t E_t \left\{ K_{t+1} \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\theta} \right\} \tag{2.2.7}$$

2.3 Public sector

Public sector sets bond supply $\bar{B}_t$ and taxes T_t. It needs to satisfy the following flow constraint:

$$\bar{B}_t \frac{1}{1+i_t} = \bar{B}_{t-1} + (1-\lambda)T_t + \lambda T_t^* + V_t P_t - \zeta_t P_t \tag{2.3.1}$$

The resources gathered through the new debt issued $\bar{B}_t$ serves to repay the existing debt $\bar{B}_{t-1}$ and to finance the transfers to the agents T_t, T_t^*. The spending for subsidy $V_t = \nu \frac{W_t}{P_t}((1-\lambda)L_t + \lambda L_t^*)$ is exactly financed by the lump sum tax on firms ζ_t:

$$V_t = \zeta_t \tag{2.3.2}$$

Levying the lump sum fiscal burden of subsidies on firms allows to isolate the transfers T_t, T_t^* as the only lump sum fiscal instrument affecting the budget constraint of the household. I assume that transfers are set by the public sector in real terms. I will refer to these quantities by the following notation:

$$\tau_t \equiv \frac{T_t}{P_t} \quad , \quad \tau_t^* \equiv \frac{T_t^*}{P_t} \tag{2.3.3}$$

The public sector also sets the nominal interest rate i_t. The interest rate policy is constrained by a zero lower bound:

$$i_t \geq 0 \tag{2.3.4}$$

2.4 Equilibrium

The equilibrium is given by the households' optimality conditions (2.1.4),(2.1.5),(2.1.8),(2.1.9), the firms' optimality condition (2.2.5), the public sector budget constraint (2.3.1) together with the market clearing conditions:

$$Y_t(i) = (1-\lambda)C_t(i) + \lambda C_t^*(i) \quad \forall i \tag{2.4.1}$$

$$Y_t = (1-\lambda)C_t + \lambda C_t^* \tag{2.4.2}$$

$$\bar{B}_t = (1-\lambda)B_t \tag{2.4.3}$$

$$\frac{Y_t \Delta_t}{A} = (1-\lambda)L_t + \lambda L_t^* \equiv \bar{L}_t \tag{2.4.4}$$

The first two market clearing conditions above are the ones holding in the the goods market: for each variety and at the aggregate level, supply needs to be equal to the sum of the consumption levels of each household type, multiplied by the relative mass. The second condition equalizes aggregate bond supply to the aggregate demand for bonds of the ricardian households, which are the only ones who can hold them. The third condition is the market clearing condition in the labor market, displaying aggregate firms' labor demand on the left hand side - distorted by price dispersion[3] $\Delta_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta}$ - and the aggregate labor supply of the households on the right hand side ($\bar{L}_t$).

2.5 Steady state

In steady state the firm's problem (2.2.1) boils down to a static problem yielding to the real wage ω determination.

$$\omega = (1-\nu)\frac{\theta - 1}{\theta}A \tag{2.5.1}$$

The subsidy ν is set to eliminate the monopolistic distortions, yielding an undistorted steady state:

$$\omega = A \tag{2.5.2}$$

Following Benigno et al. (2020) and Wolf (2022), I also assume that the steady state distribution of transfers τ, τ^* is such that the consumption levels C and C^* are the solutions of a static Ramsey problem of the government seeking to maximise in steady state a welfare

[3] Aggregate labor demand is indeed given by the sum of all the firm-specific demands for good variety, divided by labor productivity: $L^{demand} = \int_0^1 \frac{Y_t(j)}{A} dj = \int_0^1 \frac{P_t(j)}{P_t} \frac{Y_t}{A} dj = \frac{Y_t \Delta_t}{A}$

function given by weighted average of the flow utility of the two agents[4]. This, together with the optimal subsidy to firms, implies that in steady state the first best is achieved. This assumption is made to prevent any steady state suboptimality concern from interfering with the optimal policy formulation in the dynamics of the liquidity trap.

In what follows I will assume $C = C^* = Y$, so that the government's optimum is to let the two household consume the same amount of goods in steady state: this implies, by (2.1.4) and (2.1.8), also an equal labor supply between household types $L = L^* = \bar{L}$. This assumption, together with the equal dividend split, is necessary to rule out endogenous cyclical differences in income between ricardian and hand to mouth households: asymmetries in steady state labor supply yield indeed different labor - and then income - response over the liquidity trap. I will come back to this in Section 5, when both the steady state labor-consumption equalization and the equal dividend split assumptions will be lifted.

3 The stabilization role of transfer policy

In this section I will consider a government solving a dynamic Ramsey problem of maximization of the average utility of the two household types (weighting each type as in the steady state static Ramsey problem discussed in section 2.5). In order to set up the welfare objective function of the government it suffices to take a second order expansion of the weighted sum of the utility of the two types of households around the efficient steady state (details are reported in the appendix), yielding the following object that the government aims at minimizing:

$$\min \; E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2} \frac{\theta}{\kappa} \hat{\pi}_t^2 + \frac{1}{2} \hat{y}_t^2 + \frac{1}{2} \frac{\sigma}{\phi} \lambda(1-\lambda)(\hat{c}_t - \hat{c}_t^*)^2 \right\} \tag{3.0.1}$$

where, denoting $U(C_t) \equiv 1 - \exp(-zC_t)$ and $V(L_t) \equiv \delta \exp(\eta L_t)$, we have $\phi \equiv \frac{V''(\bar{L})}{V'(\bar{L})}\bar{L}$ as the inverse Frisch elasticity of labor supply and $\sigma \equiv -\frac{U''(Y)}{U'(Y)}Y$ as the relative risk aversion, when labor and consumption are equal to the aggregate steady state levels $\bar{L}$ and Y (which, in the current case, correspond to the equal steady state labor and consumption levels of the

[4]Details about the Ramsey problem in steady state are reported in the appendix

two household types). The coefficient κ is given by $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(\phi + \sigma)$. "Hat" variables are log-linear deviations around the steady state. Since households face a concave utility function, and their utility levels are weighted equally by the government, any departure from equalized consumption $\hat{c}_t = \hat{c}_t^*$ entails welfare costs, under the form of *consumption dispersion* $(\hat{c}_t - \hat{c}_t^*)^2$. This term shows up in the loss function together with the usual output gap and inflation costs.

The linearized budget constraint of the hand to mouth - see the appendix for a detailed derivation - can be written as:

$$\hat{c}_t^* = \hat{y}_t + \frac{\phi}{\phi + \sigma}\hat{\tau}_t^* \tag{3.0.2}$$

where I define the linearized transfer $\hat{\tau}_t^*$ as $\hat{\tau}_t^* \equiv \frac{\tau_t^* - \tau^*}{Y}$. Using (3.0.2) together with the aggregate resource constraint $\hat{y}_t = (1-\lambda)\hat{c}_t + \lambda\hat{c}_t^*$, we can express consumption dispersion as a function of the HtM transfer only (see derivation in the appendix):

$$(\hat{c}_t - \hat{c}_t^*)^2 = \left(\frac{1}{1-\lambda}\right)^2 \left(\frac{\phi}{\phi + \sigma}\hat{\tau}_t^*\right)^2 \tag{3.0.3}$$

Assuming steady state consumption and labor equalization, and dividends rebated equally to each household type ($\chi = \lambda$) is key to obtain the above result of consumption dispersion as a function only of the HtM transfer; any cyclical income differences over the dynamics of the model, depending on steady state asymmetries or on uneven dividend distribution, are indeed ruled out. Therefore, the only way to have the hand to mouth consume more - or less - than the ricardian household is through a positive - or negative - change in HtM transfer $\hat{\tau}_t^*$.

Problem (3.0.1) is constrained by the *aggregate demand* equation of the economy, that is derived as follows. The linearized version of the ricardian household's Euler equation (2.1.5) writes:

$$\hat{c}_t - E_t\hat{c}_{t+1} = -\frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1} + \hat{\xi}_t) \tag{3.0.4}$$

Using the aggregate resource constraint $\hat{y}_t = (1-\lambda)\hat{c}_t + \lambda\hat{c}^*_t$ at time t and $t+1$, together with (3.0.2) and (3.0.4), we obtain the aggregate demand equation:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1} + \hat{\xi}_t) - \frac{\lambda}{1-\lambda}\frac{\phi}{\phi+\sigma}E_t\Delta\hat{\tau}^*_{t+1} \tag{3.0.5}$$

Output $\hat{y}_t$ changes over time according both on the evolution in the ricardian and hand to mouth consumption. The former is determined by the intertemporal incentives given by interest rate, inflation and preference shock; the latter is pinned down by the variation in the transfer $\Delta\hat{\tau}^*_t$. The evolution of HtM transfer affects aggregate output proportionally to the overall fraction of hand to mouth λ.

The second constraint of problem (3.0.1) is the aggregate supply equation, given by the log-linear counterpart of (2.2.5):

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta E_t\hat{\pi}_{t+1} \tag{3.0.6}$$

As a temporary simplifying assumption, let us drop constraint (3.0.6) from the government problem by imposing $\Pi = \Pi = 1\ \forall t$, implying $\hat{\pi}_t = 0\ \forall t$ [5].

Let us consider an unexpected shock $\hat{\xi}_{t_0} > 0$ hitting the economy at t_0, which then reverts to $\hat{\xi}_t = 0\ \forall t > t_0$. Let us also define the long run HtM transfer $\hat{\tau}^{*\prime} \equiv \lim_{t\to\infty}\hat{\tau}^*_t$, that is the value that the government chooses to let the transfer converge to in the limit, after that the economy is hit by the shock. In the appendix, we show that $\lim_{t\to\infty}\hat{y}_t = 0$, so that long run output converges to the initial steady state level and is policy invariant[6]. By iterating (3.0.5) forward (and taking into account $\hat{\pi}_t = 0\ \forall t$) we obtain:

$$y_{t_0} = -\frac{1}{\sigma}E_{t_0}\left[\hat{\xi}_{t_0} + \sum_{t=t_0}^{\infty}\hat{i}_t - \sigma\Theta\hat{\tau}^*_{t_0} + \sigma\Theta\hat{\tau}^{*\prime}\right] \tag{3.0.7}$$

[5]This condition can be retrieved by setting $\kappa = 0$, that in turn can be obtained by setting the fraction of non-resetting firms α to 1.

[6]This is not a trivial result: the government may indeed impose a nonzero long run transfer deviation $\hat{\tau}^{*\prime} \neq 0$: reallocating wealth between households then would affect labor supply and the output level in the limit. In the appendix I show that this effect is negligible up to a first order approximation.

where $\Theta = \frac{\lambda}{1-\lambda}\frac{\phi}{\phi+\sigma}$. The sum of the prospective interest rates $\sum_{t=t_0}^{\infty} \hat{i}_t$ affects current output, by acting on the intertemporal consumption choices of the ricardian household. On the side of transfer policy, the effect on current output can be summarized exclusively by the difference between the current transfer $\hat{\tau}^*_{t_0}$ and the long run transfer $\hat{\tau}^{*\prime}$. Increasing the current transfer $\hat{\tau}^*_{t_0}$ with respect to the long run transfer $\hat{\tau}^{*\prime}_t$ boosts current output relatively to the long run policy-invariant output level, by increasing current hand to mouth consumption.

Since setting nonzero transfers $\hat{\tau}^*_{t_0}, \hat{\tau}^{*\prime}$ implies consumption dispersion (by (3.0.3)), and cutting future nominal rates $\{\hat{i}_t\}_{t=t_0+1}^{\infty}$ produces future undesired output expansions, the only term the government can use in equation (3.0.7) to neutralize the shock without incurring in welfare costs is the current nominal rate $\hat{i}_{t_0}$, namely by setting $\hat{i}_{t_0} = -\hat{\xi}_{t_0}$. However, for a realization of $\hat{\xi}_{t_0}$ high enough, this is not feasible because it would require $\hat{i}_{t_0}$ to go below the lower bound $\hat{i}^{ZLB}$ (i.e. the log-linearized counterpart of the zero lower bound condition (2.3.4)). The government is then willing to keep $\hat{i}_t$ at the lower bound up to some period $T > t_0$, in order improve the recession mitigation at time t_0 (see Eggertsson and Woodford (2003)). Figure 3.1 illustrates in red the qualitative behavior of output when this forward guidance intervention on nominal rates is implemented, while keeping transfers $\{\hat{\tau}^*_t\}_{t=t_0}^{\infty}$ at zero. Looking at the red line first, output drops due to the shock at the onset of the trap, and then overshoots the steady state in the subsequent periods, when the nominal interest rates are still kept at the zero lower bound by forward guidance.

If the sequence of transfers $\{\hat{\tau}^*_t\}_{t=t_0}^{\infty}$ is not kept at 0, but instead set optimally, the government can achieve a better stabilization of the output gap, by setting a positive transfer $\hat{\tau}^*_{t_0} > 0$ to mitigate even more the output drop at t_0, and by setting $\hat{\tau}^*_t < 0$ afterwards in order to curb the undesired output expansions arising from keeping the interest rates at 0 for $t > t_0$. Figure 3.2 and 3.1 report in blue respectively the implied pattern of transfers and the response of output, under a policy setting transfers optimally and jointly with the interest rate.

Notice that at the optimum the government does not want to completely stabilize output, because setting nonzero transfers entails consumption dispersion costs: transfers are not a substitute of the interest rate policy, which would instead be able to fully offset the shock by setting $\hat{i}_{t_0} = -\hat{\xi}_{t_0}$, absent the zero lower bound, and without yielding consumption dispersion.

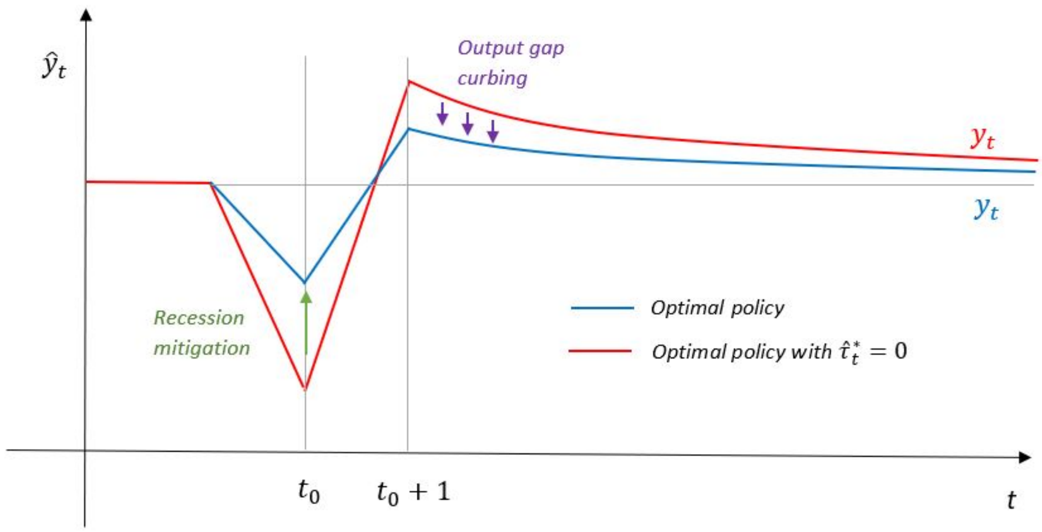

Figure 3.1: Output gap, optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$

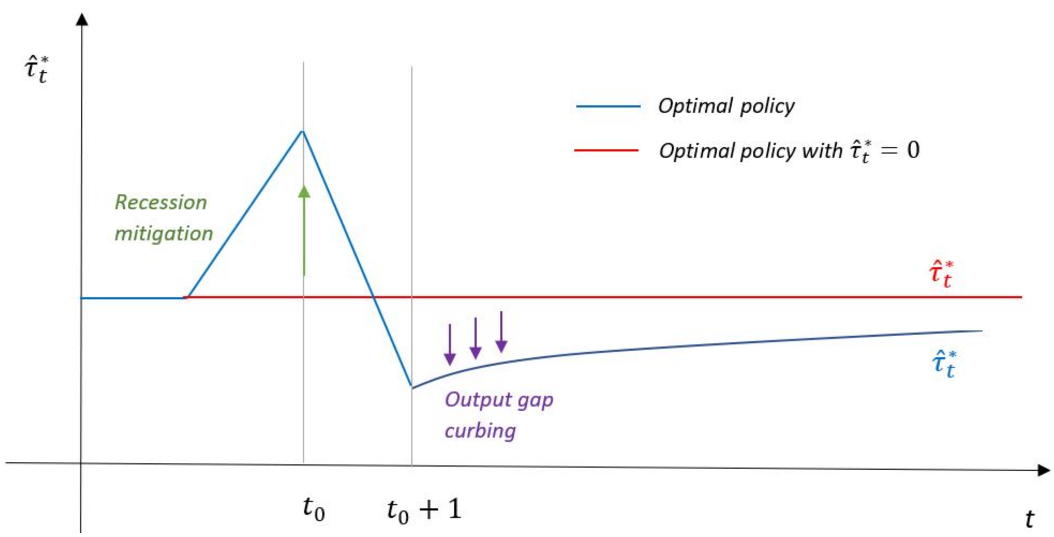

Figure 3.2: Hand to mouth transfer, optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$

Moreover, transfer policy affects the optimal duration of stay of the interest rate at the zero lower bound through two offsetting channels: at t_0 it entails a *shortening* effect on duration of the stay of the interest rate at the zero lower bound, as it exerts an additional expansionary effect on output, reducing the need for forward guidance. Afterwards, it counteracts the undesired output-boosting effect of monetary policy, so it makes the latter less costly in welfare terms: in this perspective, transfer policy plays a *lengthening* role with respect to forward guidance. The overall effect on the length of the stay of the interest rate at the zero lower bound remains ambiguous (see Figure 3.3).

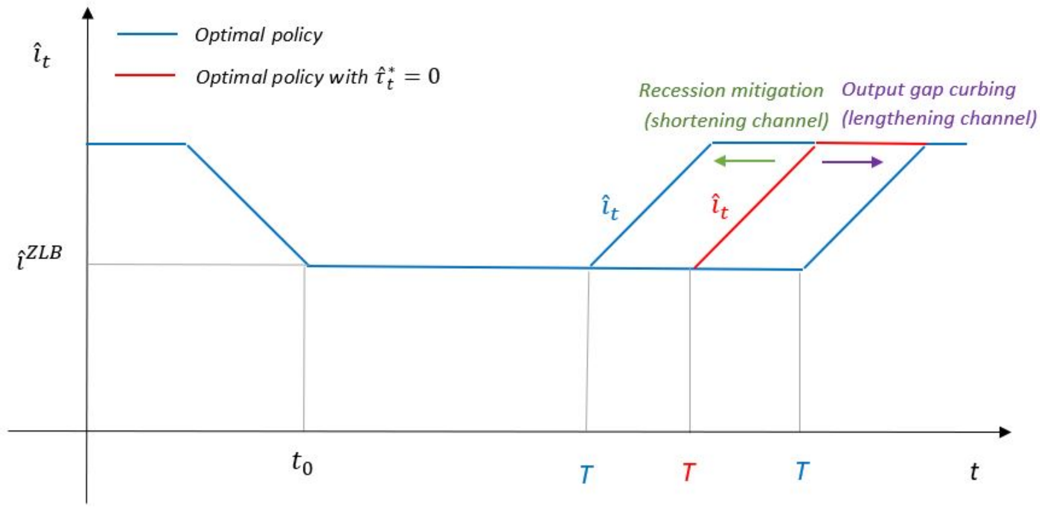

Figure 3.3: Interest rate, optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$

3.1 An interpretation through the natural rate of interest

In what follows, I will define the effective *natural interest rate* as the natural interest rate that would be faced by a hypothetical representative agent with consumption levels aggregating the ones of the optimizer and the hand to mouth:

$$r_t^n = -\xi_t - \sigma\Theta E_t \Delta\hat{\tau}_{t+1}^* \tag{3.1.1}$$

Plugging indeed the term above into the AD equation (3.0.5), the latter becomes exactly alike the one that would be found in a representative agent framework (let us recall that $\hat{\pi}_t = 0 \ \forall t$):

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{\imath}_t - r_t^n) \tag{3.1.2}$$

Equation (3.1.1) shows that the natural rate can be manipulated through he HtM transfer variation $\Delta\hat{\tau}_{t+1}^*$: it is then endogenous to fiscal policy[7]. Using again $\lim_{t\to\infty} \hat{y}_t = 0$, output gap y_{t_0} as from equation (3.0.5) (with the assumption $\hat{\pi}_t = 0 \ \forall t$) can be rewritten by forward iteration as depending on the sum of the current and future deviations of the nominal from

[7]For a liquidity trap model displaying endogenous natural rate, for different reasons from the ones of this paper, and related to debt deleveraging, see Benigno et al. (2020).

the natural interest rate, up to the liftoff period T, and from then onwards:

$$y_{t_0} = -(\hat{i}^{ZLB} - r^n_{t_0}) - \frac{1}{\sigma} E_{t_0} \sum_{t=t_0+1}^{T} (\hat{i}^{ZLB} - r^n_t) - \frac{1}{\sigma} E_{t_0} \sum_{t=T+1}^{\infty} (\hat{i}_t - r^n_t) \tag{3.1.3}$$

The shock term $\hat{\xi}_{t_0}$ is embedded into the natural rate $r^n_{t_0}$, that experiences a fall, involving a negative effect on current output. As discussed above, the government reacts by keeping the nominal rate $\hat{i}_t$ at the lower bound $\hat{i}^{ZLB}$ until time T. Therefore the future deviations up to the forward guidance horizon T, i.e. $\{\hat{i}^{ZLB} - r^n_t\}^T_{t=t_0+1}$, entail undesired future output expansions. The government can increase the current natural rate $r^n_{t_0}$ to strengthen the contemporaneous policy effect $(\hat{i}^{ZLB} - r^n_{t_0})$ and cut future natural rates $\{r^n_t\}^T_{t=t_0}$ to curb the future expansionary effects $\sum_{t=t_0+1}^{T} (\hat{i}^{ZLB} - r^n_t)$. In this way it can achieve a better output drop mitigation at t_0, at the expense of lower output expansions in the future periods. The optimal natural interest response is illustrated in Figure 3.4. Also under this interpretation, the two roles of transfer policy influence in an opposite way the duration of forward guidance. According to (3.1.1), the optimal pattern of r^n_t is produced exactly by the optimal transfer policy illustrated in Figure 3.2: the fall of the transfer at $t_0 + 1$ after the initial peak creates an upward shift the natural rate at t_0, while then increasing the transfer back to steady state pushes the natural interest rate downwards.

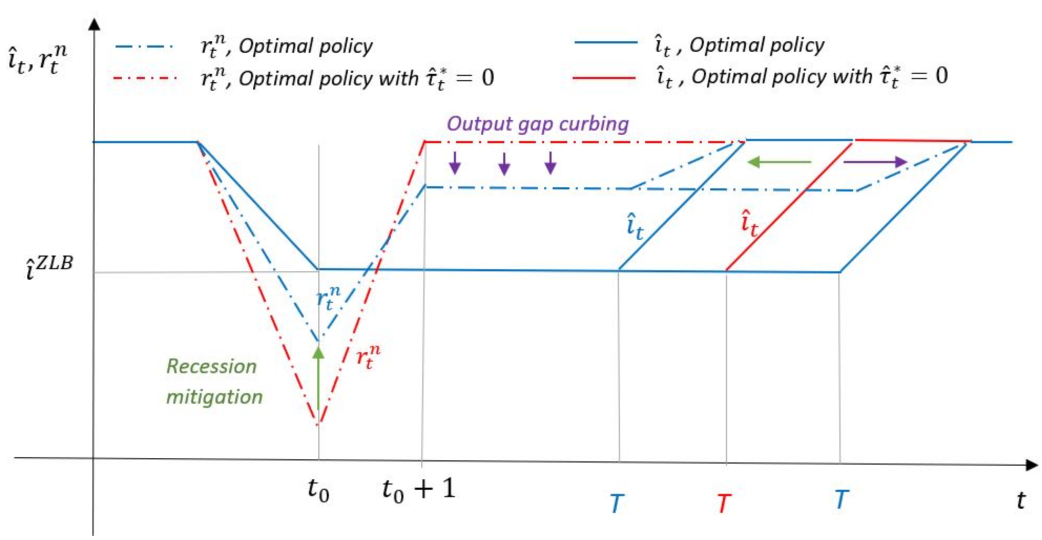

Figure 3.4: Natural and nominal interest rate, optimal policy vs. optimal policy with $\hat{\tau}^*_t = 0$

3.2 Transfer financing

In the argument outlined so far I did not yet discuss how HtM transfers are financed. Let us take a log-linear approximation of the government's budget constraint (2.3.1) (after having substituted inside for bond market clearing (2.4.3)) in this simplified environment with $\hat{\pi}_t = 0$ $\forall t$:

$$\Gamma \hat{\tau}_t^* = \frac{1-\lambda}{\lambda} \left[\beta \hat{b}_t - \hat{b}_{t-1} - \beta \hat{i}_t - \Gamma \hat{\tau}_t \right] \tag{3.2.1}$$

where $\Gamma = \frac{Y\Pi}{b} = \frac{Y}{b}$, with $\hat{b}_t$ and b the deviation and the steady state level of ricardian bond holding B_t/P_t, respectively. We can see how at any time t the transfer $\hat{\tau}_t^*$, if positive, is financed by taking resources away from ricardian household, either through an increase in its public debt holding, or through an interest rate cut, or through a direct redistribution through the transfer $\hat{\tau}_t$ (the opposite holds if $\hat{\tau}_t^*$ is negative). Taking the discounted sum with respect to the steady state discount factor β of both the right and the left hand side up to infinity, and imposing the transversality condition $\lim_{j\to\infty} \beta^j \hat{b}_{t_0+j} = 0$ and the predetermined condition $\hat{b}_{t_0-1} = 0$, we can write:

$$\Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j}^* = -\frac{1-\lambda}{\lambda} \left[\Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j} + \beta E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{i}_{t_0+j} \right] \tag{3.2.2}$$

The government can select any appropriate pattern of ricardian transfers $\{\hat{\tau}_{t_0+j}\}_{j=0}^{\infty}$ to satisfy the financing constraint above, without affecting the stabilization results (which depend uniquely on the aggregate demand determinants showing up in (3.0.7)). Of course, this will impact ricardian consumption. The latter is pinned down by the Euler equation and the ricardian intertemporal budget constraint (IBC)[8]:

$$\hat{c}_t - E_t \hat{c}_{t+1} = -\frac{1}{\sigma}(\hat{i}_t + \hat{\xi}_t) \tag{3.2.3}$$

$$\Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{c}_{t_0+j} = \Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{l}_{t_0+j} + \Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j} + \beta E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{i}_{t_0+j} \tag{3.2.4}$$

[8]The IBC of the ricardian household (3.2.4) is recovered by the infinite iteration forward of the log-linear version of (2.1.1) (the flow budget constraint), subject to the the transversality condition $\lim_{j\to\infty} \beta^j \hat{b}_{t_0+j} = 0$, the predetermined condition $\hat{b}_{t_0} = 0$, and the simplifying assumption $\hat{\pi}_t = 0$ $\forall t$

where in the IBC the real wage and dividend deviations do not show up, as they exactly offset each other (see the appendix). Transfer financing through the term $\sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j}$ has the effect of shifting up or down the whole discounted sum of ricardian consumption $\sum_{j=0}^{\infty} \beta^j \hat{c}_{t_0+j}$[9]. Therefore, while redistribution has a direct effect on current hand to mouth consumption, it can only affect ricardian consumption only with respect to its total discounted amount. The effectiveness of current HtM transfer movements in stabilizing output is not jeopardized by the financing scheme.

3.3 The general case

With the above considerations in mind, we can now consider the general case in which prices are not fully rigid and solve for the optimal policy problem of the government, which seeks to set jointly the pattern of nominal interest rates and transfers. In this perspective I will lift the zero inflation assumption of the previous simplified setting; moreover, I will substitute the consumption dispersion term $(\hat{c}_t - \hat{c}_t^*)^2$ as a function of the transfer $\hat{\tau}_t^*$ using (3.0.3). The problem writes:

$$\min_{\{\hat{\pi}_t\}_{t_0}^{\infty},\{\hat{y}_t\}_{t_0}^{\infty},\{\hat{\tau}_t^*\}_{t_0}^{\infty},\{\hat{i}_t\}_{t_0}^{\infty}} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2}\frac{\theta}{\kappa}\hat{\pi}_t^2 + \frac{1}{2}\hat{y}_t^2 + \frac{1}{2}\frac{\phi\sigma}{(\phi+\sigma)^2}\frac{\lambda}{1-\lambda}\hat{\tau}_t^{*2} \right\} \tag{3.3.1}$$

s.t

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \hat{\pi}_{t+1} + \hat{\xi}_t) - \Theta E_t \Delta\hat{\tau}_{t+1}^* \tag{3.3.2}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta E_t\hat{\pi}_{t+1} \tag{3.3.3}$$

$$\hat{i}_t \geq \hat{i}_{ZLB} \tag{3.3.4}$$

where (3.3.2) is the previously derived AD equation; constraints (3.3.3) and (3.3.4) are the log-linearized versions of the New Keynesian Phillips curve (2.2.5) and of the zero lower bound on the interest rate (2.3.4). The consumption deviations $\hat{c}_t$ and $\hat{c}_t^*$ can be determined residually by using the aggregate resource constraint $\hat{y}_t = (1-\lambda)\hat{c}_t + \lambda\hat{c}_t^*$ and the hand to

[9] I the current simplified case with rigid prices, it can be shown that the effect on ricardian consumption is null due to the adjustment in the labor sequence $\{\hat{l}_{t_0+j}\}_{j=0}^{\infty}$. This limit case is not further discussed here.

mouth budget constraint (3.0.2).

The government needs to satisfy a solvency requirement (the generalized version of constraint (3.2.2)), obtained by iterating forward the log-linearized counterpart of the public sector budget constraint (2.3.1) and imposing the transversality condition $\lim_{j\to\infty}\beta^j\hat{b}_{t_0+j}=0$:

$$\Gamma E_{t_0}\sum_{j=0}^{\infty}\beta^j\hat{\tau}^*_{t_0+j}=-\frac{1-\lambda}{\lambda}\left[\Gamma E_{t_0}\sum_{j=0}^{\infty}\beta^j\hat{\tau}_{t_0+j}+\beta E_{t_0}\sum_{j=0}^{\infty}\beta^j\hat{i}_{t_0+j}-E_{t_0}\sum_{j=0}^{\infty}\beta^j\hat{\pi}_{t_0+j}\right] \tag{3.3.5}$$

As discussed previously, the government can always choose one of the infinite possible appropriate sequences $\{\hat{\tau}_{t_0+j}\}_{j=0}^{\infty}$ to satisfy (3.3.5): as a consequence, condition (3.3.5) is not included among the constraints of the problem.

The system allows to analytically identify the trade-off between aggregate stabilization and consumption dispersion, with respect to transfer policy: taking the first order condition of problem (3.3.1)-(3.3.4) with respect to $\hat{\tau}^*_t$, we obtain:

$$\frac{\sigma}{\phi+\sigma}\hat{\tau}^*_t+\frac{1}{\beta}\nu^{AD}_{t-1}-\nu^{AD}_t=0 \tag{3.3.6}$$

The zero consumption dispersion solution $\hat{\tau}^*_t=0$ is not achieved over the dynamics, since $\hat{\tau}^*_t$ is valuable for aggregate stabilization: this value is embedded in the difference $\frac{1}{\beta}\nu^{AD}_{t-1}-\nu^{AD}_t$, where $\hat{\nu}^{AD}_t, \hat{\nu}^{AD}_{t-1}$, are the multipliers of the aggregate demand equation at time t and $t-1$, respectively. When the multiplier displays sizable variation over time, the further output is from steady state, and the more the government leans towards aggregate stabilization rather than consumption dispersion. In the simple case with $\hat{\pi}_t=0$, condition (3.3.6) boils down to the following rule:

$$\hat{\tau}^*_t=-\frac{\phi+\sigma}{\sigma}\hat{y}_t \tag{3.3.7}$$

where I substituted for the multipliers using the first order condition on $\hat{y}_t$. In this case transfer reacts linearly to deviations in output. The higher is σ, the more concave is the utility function of the households, and the more relevant are consumption dispersion costs, calling for a weaker transfer reaction.

3.4 Simulation

In order to simulate the model under optimal policy, I adopt the following calibration: I set η and z such that $\phi \equiv \frac{V''(\bar{L})}{V'(\bar{L})}\bar{L} = 0.47$, and $\sigma \equiv -\frac{U''(Y)}{U'(Y)}Y = 2$, where $\bar{L} = (1-\lambda)L + \lambda L^*$. In this way the impulse response functions of the model replicate exactly the ones that would be yielded by assuming a standard utility function of the type $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$ and $V(L) = \frac{L^{1+\phi}}{1+\phi}$, with $\sigma = 2$ and $\phi = 0.47$ (see Eggertsson and Woodford (2006)). I set κ to 0.02 (see Benigno et al. (2020)). I assume $\lambda = 0.33$, according to the observation of Kaplan and Violante (2014) about hand-to-mouth the households in the Survey of Consumer Finance being approximately one third of the total amount of surveyed households. The discount factor $\beta = 0.9987$ and the steady state inflation rate $\Pi = 1.005$ implies a 2% inflation at the annualized level and a 2.5 % steady state nominal interest rate. I assume the ratio $\frac{b}{Y}$ equal to 4 (translating a steady state debt-GDP annualized ratio of 1 in quarterly terms): this last calibrated value is not relevant for this setting but it will become so in the extension of the model developed in the next section.

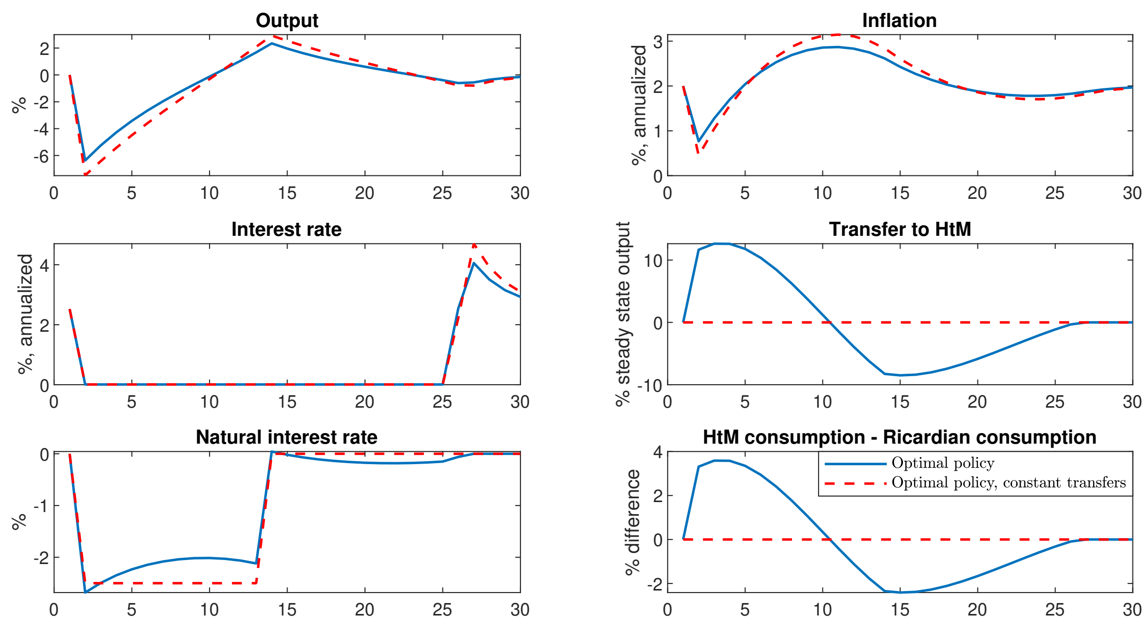

Figure 3.5: Optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$

Figure 3.5 reports the impulse response functions for the economy when hit by an unexpected shock at $t_0 = 1$, bringing $\hat{\xi}_t$ to 0.025 and lasting 12 periods. It compares optimal policy to an optimal policy when the HtM transfer does not vary. All the variables are in percentage deviation terms (let's recall here that transfer deviations are expressed in percentage of steady state output); inflation and nominal interest rate are annualized.

The shock is high enough to bring the interest rate to the zero lower bound. The duration of the stay at the zero lower bound is long and up to quarter 25, i.e. double the time span of the shock (which ends at quarter 13). The output and inflation drop in the early stages of the liquidity trap is mitigated at the expense of an output and inflation expansion later. At the onset of the shock, the government sets positive transfers for the hand to mouth to alleviate the output drop; as output increases, transfers are reduced, up until the former becomes positive; then, the government starts setting negative transfers to curb the expansion. The transfer deviation are sizable: the hand to mouth enjoys a rebate up to 12% of its steady state income over the recession and a down to -8% over the recovery. The pattern of transfers involve a manipulation of the natural interest rate: the transfer decreases gradually - after the initial peak - over the recession, implying an upward pressure on the natural rate, that has been dragged down by the shock. Then, over the recovery, the increasing pattern of transfers implies a downward shift in the natural rates, which allow to curb the expansion.

The introduction of the optimal response of transfer policy allows to achieve a better stabilization of output gap and inflation: the output trough of the constant transfer policy at the onset of the shock is reduced by nearly one fourth, and the same holds for the peak over the recovery. Also inflation fluctuates significantly less in the optimal policy with respect to the constant transfer policy scenario. This stabilization outcome is achieved at the expense of consumption dispersion costs: so transfers cannot perfectly substitute for the stabilization power of monetary policy that is foregone because of the zero lower bound. The HtM consumes more than the Ricardian when the transfer deviation is positive and less when it is negative. Remarkably, consumption difference between households moves less strongly than the the transfer: this is because agents can partially compensate the positive (or negative) transfer deviation by adjusting labor supply.

Notice that the impact of transfer policy on the duration of forward guidance is null: the nominal interest rate remains at zero until quarter 25 both in the constant transfer policy and in the optimal policy. This is due to the interaction of the two opposite roles of transfers with respect to monetary policy: on one side they call for a lower forward guidance horizon, when they mitigate the early recession; on the other side, they make more desirable an extension of the stay at the zero lower bound by counteracting the undesired output expansion of forward guidance. As discussed previously, these two effects act oppositely on the duration of stay of the rates at the zero lower bound. This can be seen in Figure 3.6, where the optimal transfer policy is decomposed into the optimal policy constrained by $\hat{\tau}_t^* \geq 0$ and the optimal policy constrained by $\hat{\tau}_t^* \leq 0$. In the first benchmark case, only positive transfer deviations are allowed, so only the recession mitigation and then the shortening role of transfers is active, and this implies a decrease in the horizon of forward guidance with respect to the optimal policy: the interest rate is kept at 0 for a quarter less. In the second case, only the later curbing of the expansion and then the lengthening role of transfers is in place - as only negative transfer deviations are allowed, and this drives the government to keep the interest rates at 0 for one quarter more.

The result of a null effect of transfer policy on the duration of forward guidance is robust to sensitivity analysis carried out on different parameters. Specifically, in the appendix I consider lower and higher values - with respect to the current calibration - for λ, σ, ϕ, which are the parameters determine the effect of transfers on the economy (through stabilization via aggregate demand, or through the impact on consumption dispersion costs). I also consider lower and higher values for the shock ε_ξ and the parameter β, which determine the severity of the zero lower bound constraint[10]. Also in this case the alternative parametrizations lead still to the same finding.

A key parameter to assess the stabilization power of transfer policy is the fraction of hand to mouth households λ. As discussed previously, the higher is the fraction of hand to mouth in the economy, the stronger is the stabilizing effect on output of rebating them more resources, as λ enters in the AD curve (3.3.2) under the form of the coefficient $\Theta = \frac{\lambda}{1-\lambda}$. However, the same expression $\frac{\lambda}{1-\lambda}$ also appears in the coefficient of the consumption dispersion term

[10]Consider that the steady state nominal interest rate is given by $i = \frac{\Pi}{\beta} - 1$. The higher is β, the closer is this value to 0, and the more binding will be the zero lower bound when the demand shock hits the economy

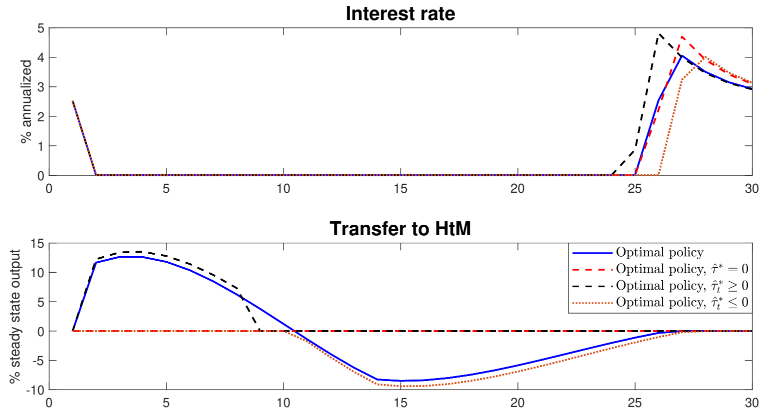

Figure 3.6: Optimal policy vs optimal policy with $\hat{\tau}_t^* \geq 0$ and $\hat{\tau}_t^* \leq 0$

in the government objective (3.3.1): this is because redistributing resources to or away from the hand to mouth impacts consumption dispersion more heavily the higher is their relative weight in the population. However, the effect of λ on output stabilization enters quadratically in the welfare objective (as the government draws disutility from output deviation squared), whereas it enters only linearly in the consumption dispersion coefficient. Therefore increasing λ entail better aggregate stabilization results at the expense of lower consumption dispersion: this can be seen by comparing Figure 3.5 (where λ is equal to 0.33) to Figure 3.7 - where we have instead $\lambda = 0.5$, implying half of the population being hand to mouth.

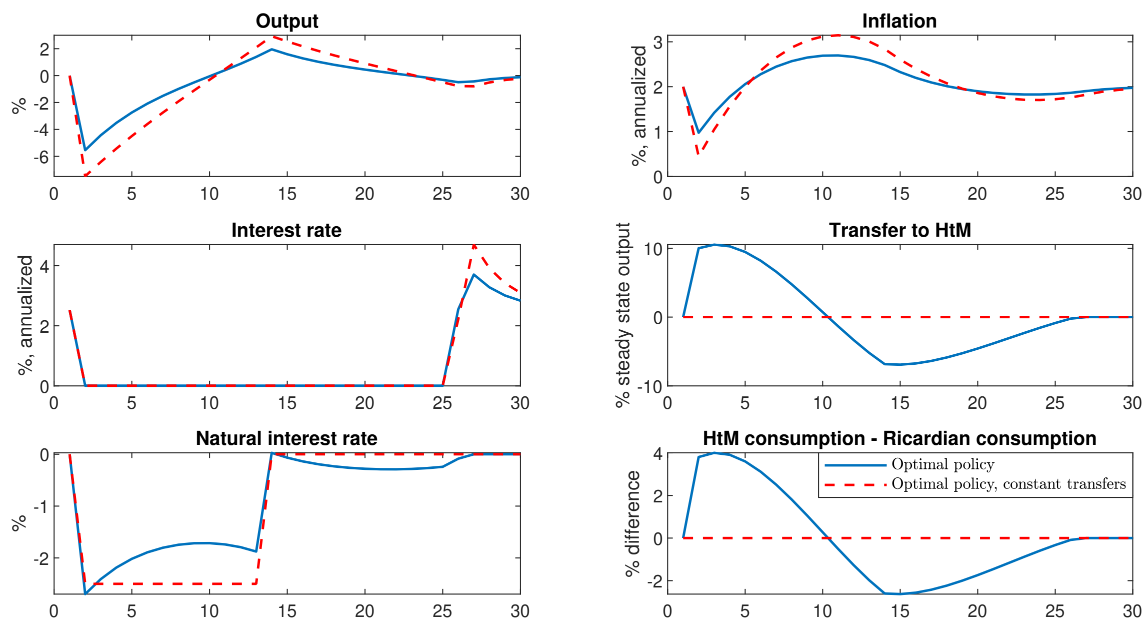

Figure 3.7: Optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$, $\lambda = 0.5$ case

4 Optimal policy under a homogeneous tax response

So far I assumed that the government was able to freely differentiate lump sum taxation between Ricardians and hand to mouth. While using HtM tax $\hat{\tau}_t^*$ to stabilize output and inflation over the liquidity trap, the government could select any of the infinite possible sequences of ricardians' transfers $\{\hat{\tau}_{t_0+j}\}_{j=0}^{\infty}$ appropriate to guarantee the solvency constraint to hold (equation (3.3.5)).

However, due to political constraints, a government could have hard time in implementing a heterogeneous tax response across households. In this section I explore to what extent the results in terms of the stabilizing effect of optimal transfers, as well as their imperfect substitutability with interest rate policy and their null effect on the duration of the stay at the zero lower bound, carry over to a case in which a unique stimulus check is rebated to all households in the economy (following in spirit Wolf (2021)). The additional constraint that I am setting is:

$$\tau_t - \tau = \tau_t^* - \tau^* \quad \forall t \tag{4.0.1}$$

Constraint (4.0.1) imposes that the the same increment of transfers with respect to steady state is set for the whole cross-section of households. This also implies that the transfer deviation terms - defined in output terms as previously - are equal:

$$\hat{\tau}_t = \frac{\tau_t - \tau}{Y} = \frac{\tau_t^* - \tau^*}{Y} = \hat{\tau}_t^* \tag{4.0.2}$$

I will thereafter call $\hat{\tau}_t^{**}$ the unique transfer deviation set on both ricardian and hand to mouth households.

Let us restate the solvency constraint (3.3.5), with only the unique transfer $\hat{\tau}_t^{**}$ available:

$$\Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j}^{**} = -(1-\lambda)\beta E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{i}_{t_0+j} + (1-\lambda) E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\pi}_{t_0+j} \tag{4.0.3}$$

Now the transfer instrument used for aggregate stabilization $\hat{\tau}^{**}$ in (3.3.2) is the same that is used to guarantee solvency (4.0.3). It appears that the use of transfers now implies a trade-off not only between aggregate stabilization and consumption dispersion, but also with respect to public debt management.

Let us for now consider only the trade-off between aggregate stabilization and public debt management, leaving aside consumption dispersion concerns. Notice that when the prospective sequences of nominal interest rates and inflation change as a consequence of the preference shock hitting the economy, transfers have to adjust as well to guarantee solvency (4.0.3). However, since condition (4.0.3) is satisfied by setting an appropriate *sum* of discounted transfers, the government is free to smooth out the required fiscal response over time. In particular, given $\sum_{j=0}^{\infty} \beta^j \hat{\tau}^*_{t_0+j}$ being the sum of discounted hand to mouth transfers generated by optimal policy in the benchmark heterogeneous transfer scheme of last section (either with or without the constant transfer constraint $\hat{\tau}^*_t = 0$), we can obtain the required total discounted transfer amount for public debt management in the current setting, $\sum_{j=0}^{\infty} \beta^j \hat{\tau}^{**}_{t_0+j}$, by increasing every period $\hat{\tau}^*_t$ by a fixed amount Δ_{τ^*}:

$$E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\tau}^{**}_{t_0+j} = E_{t_0} \sum_{j=0}^{\infty} \beta^j (\hat{\tau}^*_{t_0+j} + \Delta_{\tau^*}) \tag{4.0.4}$$

In this way we can obtain exactly the same aggregate output and inflation dynamics $\{\hat{y}_t\}_{t=t_0}^{\infty}$, $\{\hat{\pi}_t\}_{t=t_0}^{\infty}$ as in the benchmark case: the effect of the Δ_{τ^*} increase indeed cancels out in the output determination equation:

$$y_{t_0} = -\frac{1}{\sigma} E_{t_0} \left[\hat{\xi}_{t_0} + \sum_{t=t_0}^{\infty} \hat{i}_t - \sum_{t=t_0+1}^{\infty} \hat{\pi}_t - \sigma\Theta(\hat{\tau}^{**}_{t_0} + \Delta_{\tau^*}) + \sigma\Theta(\hat{\tau}^{**\prime} + \Delta_{\tau^*}) \right] = \tag{4.0.5}$$

$$= -\frac{1}{\sigma} E_{t_0} \left[\hat{\xi}_{t_0} + \sum_{t=t_0}^{\infty} \hat{i}_t - \sum_{t=t_0+1}^{\infty} \hat{\pi}_t - \sigma\Theta\hat{\tau}^{**}_{t_0} + \sigma\Theta\hat{\tau}^{**\prime} \right] \tag{4.0.6}$$

The effect of the increase in current transfer $\hat{\tau}^*_{t_0}$ on output is indeed exactly offset by the increase in the limit transfer $\hat{\tau}^{*\prime}$. A rise in the latter implies indeed that resources are systematically rebated away from the ricardian budget constraint in the final steady state, making it poorer and forcing it to cut its current consumption.

From the argument developed so far, we can infer that there is no trade-off between aggregate stabilization and public debt management: this result follows from the fact that output at any time t is affected by the *difference* between current transfer $\hat{\tau}^{**}_t$ and long run transfer

$\hat{\tau}^{**\prime}$; while public debt instead is determined by the *size* of transfers per se. Therefore the government is able to conduce a transfer policy that disentangles aggregate stabilization from public debt management. Intuition is that in the limit steady state, additional Δ_{τ^*} resources are rebated away from ricardian's consumption, which shrinks accordingly also its current consumption by an amount Δ_{τ^*}, offsetting the additional expansionary effect on output that passes through the current hand to mouth transfer $\hat{\tau}^{**}_{t_0} + \Delta_{\tau^*}$: output response remains therefore unchanged with respect to the baseline setting at time t_0.

Rearranging equation (4.0.4), we can back out Δ_{τ^*} as a function of the difference between the discounted sum of transfers in the homogeneous transfer scheme and the one in the heterogeneous transfer scheme:

$$\Delta_{\tau^*} = (1-\beta)E_{t_0}\left[\sum_{j=0}^{\infty}\beta^j\hat{\tau}^{**}_{t_0+j} - \sum_{j=0}^{\infty}\beta^j\hat{\tau}^{*}_{t_0+j}\right] \tag{4.0.7}$$

The extra-fiscal deficit (or surplus) needed is multiplied by a coefficient $1-\beta$ to give rise to the required Δ_{τ^*}, that therefore turns out to be *small* - since β is close to 1. The government, by shifting the whole transfer sequence, can indeed smooth out the fiscal surplus/deficit over time.

As showed above, shifting all the transfers by a quantity Δ_{τ^*} does not interfere with aggregate stabilization; however it does affect consumption dispersion, as the latter is related with the squared size of the transfers:

$$(\hat{c}_t - \hat{c}^*_t)^2 = \left(\frac{1}{1-\lambda}\right)^2\left(\frac{\phi}{\phi+\sigma}\hat{\tau}^{**}_t\right)^2 = \left(\frac{1}{1-\lambda}\right)^2\left(\frac{\phi}{\phi+\sigma}(\hat{\tau}^*_t + \Delta_{\tau^*})\right)^2 \tag{4.0.8}$$

However, since the term Δ_{τ^*} is small in size, as showed above, the optimal solution of the government will not significantly deviate from the parallel shift of the whole HtM transfer sequence, nor it will display significant departures of output and inflation from the baseline heterogeneous transfer case. Figure 4.1 compares the impulse response functions of the economy in the case where the unique transfer $\hat{\tau}^{**}_t$ is set optimally to the case in which it is set constant to the level satisfying solvency (4.0.3), keeping the same calibration of parameters and specification of the shock as in section 3.

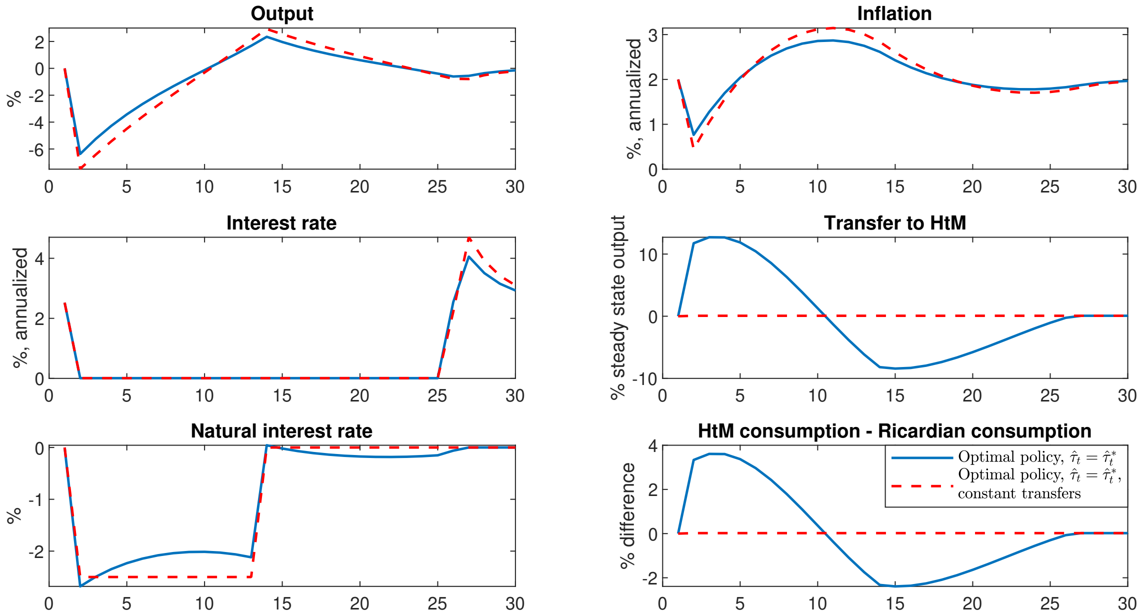

Figure 4.1: Optimal policy vs. optimal policy with constant transfers, $\tau = \tau^*$ case

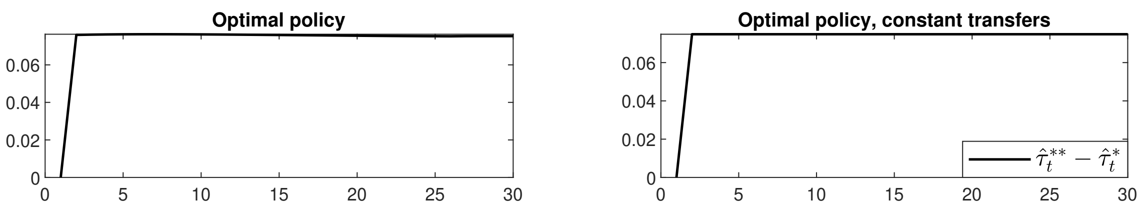

Figure 4.2: Optimal policy vs. optimal policy with homogeneous transfers: difference in HtM transfer

By equation (4.0.3) we can see that when debt dynamics are characterized by a low interest rate time span (a fall in the term $\beta \sum_{j=0}^{\infty} \beta^j \hat{i}_{t_0+j}$) which is a *wealth gain* for the government, that needs to be offset by an increase in transfers $\{\hat{\tau}_t^{**}\}_{t=t_0}^{\infty}$. According to the argument made above, the whole sequence of transfers need to be shifted (upward, in this case) with respect to the baseline path $\{\hat{\tau}_t\}_{t=t_0}^{\infty}$: in this way solvency is satisfied, and both optimal and constant transfer policy succeed in generating the same response of the economy as in the baseline setting (it can be indeed seen by comparing Figure 4.1 to Figure 3.5). The difference between the sequence of HtM transfers in the heterogeneous transfer scheme and in the homogeneous transfer response, reported in Figure 4.2, is of a negligible degree of magnitude - as it is smoothed out in the long run.

Since all the variables' responses to the shock track closely the ones of the baseline setting, we can conclude that the results in terms transfers being imperfect substitutes for interest rate policy - as they generate consumption dispersion - and entailing a zero effect on the length of the stay at the zero lower bound, carry over to the homogeneous transfer response case.

5 Optimal policy under cyclical income differences

So far, cyclical income differences (unrelated to transfer policy) have been shut down through two assumptions: the equal dividend split $\chi = \lambda$ and the equalization of steady state consumption and labor levels $C = C^*$, $L = L^*$. In this section I relax these two assumptions and explore the implications for the formulation of an optimal monetary and transfer policy. The relevance of these forces in affecting hand to mouth consumption can be analytically identified by considering the HtM budget constraint, once these assumptions are lifted (see the appendix for the derivation):

$$\frac{C^*}{Y}\hat{c}_t^* = \Phi\hat{y}_t + \hat{y}_t + \frac{\phi}{\phi+\sigma}\hat{\tau}_t^* \tag{5.0.1}$$

Hand to mouth consumption, standardized by the steady state consumption share $\frac{C^*}{Y}$, is determined by the same term of the baseline framework, $\hat{y}_t + \frac{\phi}{\phi+\sigma}\hat{\tau}_t^*$, plus an additional component $\Phi\hat{y}_t$, such that:

$$\Phi = \phi\left(\frac{L^*}{\bar{L}} - \frac{\chi}{\lambda}\right) \tag{5.0.2}$$

The sensitivity of HtM income to aggregate output $(1+\Phi)$ depends on both the steady state heterogeneity - through the term $\frac{L^*}{\bar{L}}$ - and on the dividend split rule $\frac{\chi}{\lambda}$. The higher is L^* with respect to aggregate labor supply $\bar{L}$, the stronger HtM labor supply varies with the aggregate output, making hand to mouth consumption more cyclical. The higher is the fraction of dividends $\frac{\chi}{\lambda}$ allocated to each hand to mouth household, the more countercyclical is HtM consumption instead. The latter feature is due to the unrealistic countercyclical nature of dividends in the New Keynesian models with stickiness in firms' price setting. I will nevertheless not take a stance on the sign and size of the whole cyclical coefficient Φ, but instead I will hereafter incorporate this quantity together with the transfer $\hat{\tau}_t^*$ into a single term $\tilde{\tau}_t^*$ - *the augmented transfer* - which consists in an endogenous cyclical component and a policy-driven component provided by the transfer:

$$\tilde{\tau}_t^* \equiv \frac{\phi+\sigma}{\phi}\Phi\hat{y}_t + \hat{\tau}_t^* \tag{5.0.3}$$

So that we can rewrite the hand to mouth budget constraint (5.0.1) as:

$$\tilde{c}_t^* = \hat{y}_t + \frac{\phi}{\phi+\sigma}\tilde{\tau}_t^* \tag{5.0.4}$$

Where $\tilde{c}_t^* \equiv \frac{C^*}{Y}\hat{c}_t$ is the hand to mouth consumption deviation standardized by its steady state consumption share. Notice that the government can always freely choose the augmented transfer level $\tilde{\tau}_t^*$, thanks to the degree of freedom provided by the transfer term $\hat{\tau}_t^*$ in equation (5.0.3). The augmented transfer affects the economy through the exact same channel of the transfer in the baseline model: by boosting hand to mouth consumption though its budget constraint. We can then reformulate the problem of the government in reaction to the shock ξ_{t_0} as an optimal policy setting jointly the interest rate and the augmented transfer sequence $\{\hat{i}_t, \tilde{\tau}_t^*\}_{t=t_0}^{\infty}$:

$$\min_{\{\hat{\pi}_t\}_{t_0}^{\infty},\{\hat{y}_t\}_{t_0}^{\infty},\{\tilde{\tau}_t^*\}_{t_0}^{\infty},\{\hat{i}_t\}_{t_0}^{\infty}} E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\frac{1}{2}\frac{\theta}{\kappa}\hat{\pi}_t^2 + \frac{1}{2}\hat{y}_t^2 + \frac{1}{2}\frac{\phi\sigma}{(\phi+\sigma)^2}\frac{\lambda}{1-\lambda}\tilde{\tau}_t^{*2}\right\} \tag{5.0.5}$$

s.t

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \hat{\pi}_{t+1} + \hat{\xi}_t) - \Theta E_t\Delta\tilde{\tau}_{t+1}^* \tag{5.0.6}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta E_t\hat{\pi}_{t+1} \tag{5.0.7}$$

$$\hat{i}_t \geq \hat{i}_{ZLB} \tag{5.0.8}$$

The problem is exactly equivalent to the baseline problem (3.3.1)-(3.3.4), with transfers $\{\hat{\tau}_t^*\}_{t=t_0}^{\infty}$ replaced by augmented transfers $\{\tilde{\tau}_t^*\}_{t=t_0}^{\infty}$; therefore it gives rise to exactly the same optimal impulse response for the augmented transfer as for transfers in the setting without cyclical income differences; output, inflation, and consumption dispersion are also generated by the equivalent problem, so track exactly the ones produced in the case with no cyclical income difference. Consumption dispersion, in particular, is here given in terms of the consumption deviations of the households, standardized for the steady state consumption shares, that is expressed as a function of the augmented transfer:

$$(\tilde{c}_t - \tilde{c}_t^*)^2 = \left(\frac{1}{1-\lambda}\right)^2 \left(\frac{\phi}{\phi+\sigma}\tilde{\tau}_t^*\right)^2 \tag{5.0.9}$$

The sequence of transfers that the government needs to engineer to produce the desired sequence of the augmented transfer $\tilde{\tau}_t^*$ is now reliant on the sensitivity of HtM income to aggregate output (see (5.0.3)). If $\Phi > 0$, an endogenous cyclical component is introduced into hand to mouth consumption, dragging it downward over the recession and upwards during the boom. Optimal augmented transfer policy aims at achieving the opposite pattern (boosting HtM consumption initially and then curbing it), so transfers need to compensate for this effect: they will be raised more during the trough and cut more during the expansionary phase, with respect to the no-cyclical income difference scenario. By the same logic, transfers will display milder fluctuations with respect to the no-cyclical income difference case if $\Phi < 0$, i.e. if endogenous inequality boosts HtM consumption when output drops and curbs it when output expands.

Figure 5.1 compares aggregate stabilization outcomes in the case $\Phi = 0$, with the case $\Phi = 0.094$ and $\Phi = -0.094$ - corresponding to a calibration where χ is kept equal to λ, and the steady state hand to mouth's hours worked are 20% more and less than the economywide labor supply $\bar{L}$, respectively. In the former case, the HtM supplies more labor to make partially up for a steady state consumption lower than average ($C^* < Y$), in the latter, it affords working less by a consumption advantage ($C^* > Y$). This consumption and labor steady state asymmetry arises from an uneven transfer distribution τ, τ^*, favouring the Ricardian in the former case and the hand to mouth in the latter. The shock process considered is the same as in section 3.3. In the case $\Phi = 0$, we are back to exactly the same impulse response as in Figure 3.5, since cyclical income difference is shut down. In the case $\Phi > 0$, the high steady state labor supply of the hand to mouth implies a higher cyclicality of its consumption, which calls for a more massive use of transfers over both the recession and the boom. In the case $\Phi < 0$ consumption of the hand to mouth is instead more countercyclical, and this feature substitute partially for the transfer intervention: the latter display then less sharp fluctuations over the trap. Overall, the government succeeds

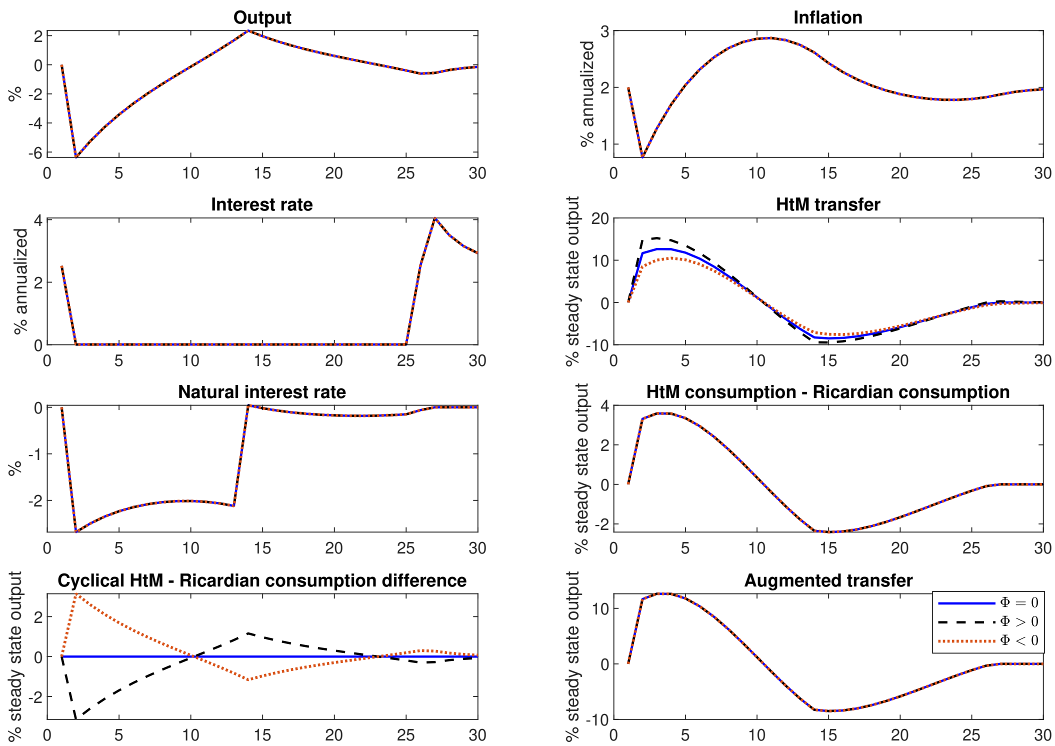

Figure 5.1: Optimal policy. $\Phi = 0$: no cyclical income difference; $\Phi > 0$: procyclical HtM income; $\Phi < 0$: countercyclical HtM income. The bottom left graph relates to cyclical income differences unrelated to transfer policy.

in making the augmented transfer term $\tilde{\tau}_t$ follow exactly the same optimal pattern in all the three cases, and that guarantees the same outcomes in terms of aggregate inflation-output stabilization and consumption dispersion.

The use of transfers affects consumption dispersion costs (together with cyclical income difference): then, in line with the results of in the baseline setting, the transfer instrument is not a substitute for monetary policy. Optimal transfers rise over the recession to mitigate the output drop, and fall over the later stages of the trap, to curb the expansion: these two forces once again imply countervailing effects on forward guidance duration, yielding an overall null impact on the duration of stay at the zero lower bound, as reported in Figure 5.2 and 5.3, and analogously to the baseline setting's results.

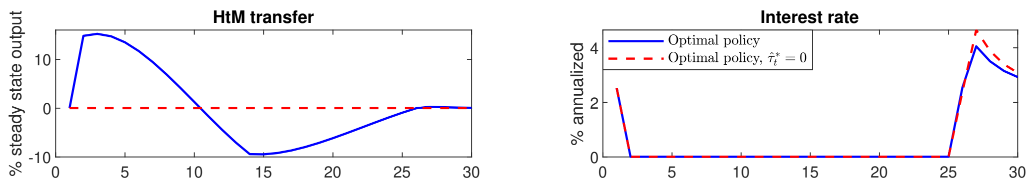

Figure 5.2: Optimal policy vs optimal policy with $\hat{\tau}_t^* = 0$, $\Phi < 0$ case

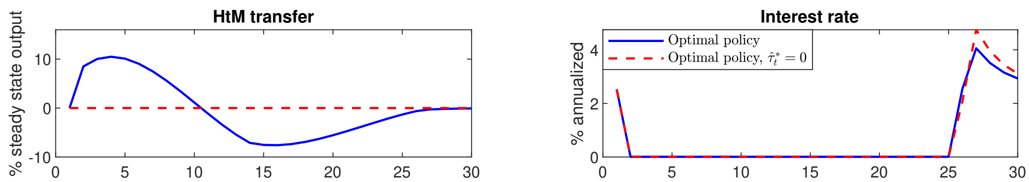

Figure 5.3: Optimal policy vs optimal policy with $\hat{\tau}_t^* = 0$, $\Phi > 0$ case

6 Conclusion

In this paper I formulate an optimal monetary and fiscal policy problem in a heterogeneous agents economy facing a shock that brings it to liquidity trap, where fiscal policy is modelled as transfer policy. Transfers are used by the government to manipulate the natural interest rate in the economy, at the expense of consumption dispersion, which prevent them from being an effective substitute of the foregone stabilization power of monetary policy. During the early stages of the liquidity trap, transfer policy is used jointly with monetary policy to mitigate the recession, while later it is used to curb the undesired output expansion implied by forward guidance. These two forces impact oppositely on the optimal duration of stay of nominal rates at the zero lower bound, with an overall impact that is negligible. The findings are robust to both restrictions imposing homogeneous transfer responses between household types, and to a broader framework allowing for cyclical income difference.

Remarkably, the optimal fiscal-monetary policy prescriptions of the paper do not call for a relaxation of treasury - central bank separation. Since the duration of forward guidance is not affected by transfers - when the latter are introduced in an optimal fashion - the optimal fiscal-monetary mix can be implemented with treasury observing the planned path of interest rates and setting transfers accordingly.

This paper opens up several avenues of extension: taking into account shocks triggering a liquidity trap through the hand to mouth side, as a deleveraging shock, would change the optimal transfer policy implication, introducing different trade-offs between aggregate stabilization and consumption dispersion. Also modeling the effect of dividends and stock market fluctuations on inequality can have relevant implications, as foreshadowed by the results of the last section . Finally, extending the model to a full HANK-type environment would allow to refine the analysis of the quantitative effects of transfer policy over the liquidity trap.

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A Appendix

A.1 Derivation of the linearized labor supply conditions of the ricardian and hand to mouth

In the following, take into account the following definitions:

$$\sigma = -\frac{U''(Y)}{U'(Y)}Y = zY \tag{A.1.1}$$

$$\phi = \frac{V''(L)}{V'(L)}\bar{L} = \eta\bar{L} \tag{A.1.2}$$

Where $\bar{L} = (1-\lambda)L + \lambda L^*$. Taking a log-linear approximation of the Ricardian and HtM labor supply, we obtain:

$$\eta L\hat{l}_t = -zC\hat{c}_t + \hat{\omega}_t \tag{A.1.3}$$

$$\eta L^*\hat{l}_t^* = -zC^*\hat{c}_t^* + \hat{\omega}_t \tag{A.1.4}$$

Where $\hat{\omega}_t$ is real wage deviation. Aggregating up (A.1.3) and (A.1.4), and using $Y = A((1-\lambda)L + \lambda L^*) = \omega((1-\lambda)L + \lambda L^*)$, we obtain:

$$\eta\frac{Y}{\omega}\hat{y}_t = -zY\hat{y}_t + \hat{\omega}_t \tag{A.1.5}$$

Then:

$$\hat{\omega}_t = \left(\frac{\eta}{\omega}Y + zY\right)\hat{y}_t = (\phi + \sigma)\hat{y}_t \tag{A.1.6}$$

Therefore, plugging (A.1.1), (A.1.2) and (A.1.6), into (A.1.3) and (A.1.4), we get:

$$\frac{L}{\bar{L}}\hat{l}_t = -\frac{\sigma}{\phi}\frac{C}{Y}\hat{c}_t + \frac{\phi+\sigma}{\phi}\hat{y}_t \tag{A.1.7}$$

$$\frac{L^*}{\bar{L}}\hat{l}^*_t = -\frac{\sigma}{\phi}\frac{C^*}{Y}\hat{c}^*_t + \frac{\phi+\sigma}{\phi}\hat{y}_t \tag{A.1.8}$$

Which, in the baseline case with $C = C^* = Y$ and $L = L^* = \bar{L}$, boils down to:

$$\hat{l}_t = -\frac{\sigma}{\phi}\hat{c}_t + \frac{\phi+\sigma}{\phi}\hat{y}_t \tag{A.1.9}$$

$$\hat{l}^*_t = -\frac{\sigma}{\phi}\hat{c}^*_t + \frac{\phi+\sigma}{\phi}\hat{y}_t \tag{A.1.10}$$

A.2 Derivation of the linearized hand to mouth budget constraint

Let us take a log-linear approximation of the hand to mouth budget constraint (2.1.9) (in real terms) around the steady state:

$$C^*\hat{c}^*_t = L^*\omega(\hat{l}^*_t + \hat{\omega}_t) + Y\hat{\tau}^*_t + \frac{\chi}{\lambda}Y\hat{d}_t \tag{A.2.1}$$

Were I define the HtM transfer deviation $\hat{\tau}^*_t = \frac{\tau_t - \tau^*}{Y}$, and the dividend deviation $\hat{d}_t$ as $\frac{\bar{D}_t - D}{Y}$. Aggregate real dividend $\bar{D}_t$ consists of aggregate output net of the labor cost, corrected for the subsidy and net of the lump sum tax ζ_t (which is given by $\zeta_t = ((1-\lambda)L_t + \lambda L^*_t)\omega_t\nu$):

$$\bar{D}_t = Y_t - ((1-\lambda)L_t + \lambda L^*_t)\omega_t(1-\nu) - ((1-\lambda)L_t + \lambda L^*_t)\omega_t\nu \tag{A.2.2}$$

In log linearized terms:

$$Y\hat{d}_t = Y\hat{y}_t - (1-\lambda)L\omega\hat{l}_t + \lambda L^*\omega\hat{l}^*_t - ((1-\lambda)L + \lambda L^*)\omega\hat{\omega}_t = -Y\hat{\omega}_t \tag{A.2.3}$$

Plugging (A.1.6),(A.1.8) and (A.2.3) into (A.2.1), we obtain the expression:

$$\frac{C^*}{Y}\hat{c}_t^* = \Phi\hat{y}_t + \hat{y}_t + \frac{\phi}{\phi+\sigma}\hat{\tau}_t^* \tag{A.2.4}$$

Where $\Phi = \phi\left(\frac{L^*}{L} - \frac{\chi}{\lambda}\right)$. This is the general form for the hand to mouth budget constraint, reported in Section 5 (equation (5.0.1)). Setting $L = L^*$ and $\chi = \lambda$, we recover instead the budget constraint in the baseline case of Section 3 (equation (3.0.2)).

A.3 Proof of $\lim_{t\to\infty} \hat{y}_t = 0$

We do not restrict our analysis to optimal transfer policy, but account for any possible sequence of HtM transfer $\{\hat{\tau}_t^*\}_{t=t_0}^{\infty}$. Let us take a first order approximation of the aggregate resource constraint $(1-\lambda)C + \lambda C^* = Y_t$ around the initial steady state (namely, the state of the economy before the shock $\hat{\xi}_{t_0}$ hits). This approximation spans all the possible steady state to which the economy converges after the liquidity trap[11]:

$$(1-\lambda)C\hat{c} + \lambda C^*\hat{c}^* = (1-\lambda)L\hat{l} + \lambda L^*\hat{l}^* \tag{A.3.1}$$

Then, using (A.1.7) and (A.1.8), we get:

$$(1-\lambda)C\hat{c} + \lambda C^*\hat{c}^* = (1-\lambda)L\left(-\frac{zC}{\eta L}\hat{c} + \frac{1}{\eta L}\hat{\omega}\right) + \lambda L^*\left(-\frac{zC^*}{\eta L^*}\hat{c}^* + \frac{1}{\eta L^*}\hat{\omega}\right) \tag{A.3.2}$$

In steady state the real wage ω is fixed to the stationary level A (see section 2.5), then $\hat{\omega} = 0$. Then we can rearrange and simplify the equation above as:

$$(1-\lambda)C\hat{c} = -\lambda C^*\hat{c}^* \tag{A.3.3}$$

That implies

$$Y\hat{y}_t = (1-\lambda)C\hat{c} + \lambda C^*\hat{c}^* = 0 \tag{A.3.4}$$

[11]The price dispersion term Δ_t is not considered up to a first order approximation

Therefore steady state output is not affected by the crossectional distribution of consumption up to a first order approximation. So any long run HtM transfer $\hat{\tau}^{*\prime} \neq 0$ set by the government in the limit is not driving $\lim_{t\to\infty} \hat{y}_t$ away from 0.

A.4 Expressing consumption dispersion as a function of the the HtM transfer only

Using the aggregate resource constraint $Y\hat{y}_t = (1-\lambda)C\hat{c}_t + \lambda C^*\hat{c}^*_t$ and the hand to mouth budget constraint (A.2.4), we can derive:

$$
\begin{aligned}
Y\hat{y}_t &= C(1-\lambda)\hat{c}_t + C^*\lambda\hat{c}^*_t = && \text{(A.4.1)}\\
&= (1-\lambda)(C\hat{c}_t - C^*\hat{c}^*_t) + C^*\hat{c}^*_t = && \text{(A.4.2)}\\
&= (1-\lambda)(C\hat{c}_t - C^*\hat{c}^*_t) + Y\Phi\hat{y}_t + Y\hat{y}_t + Y\frac{\phi}{\phi+\sigma}\hat{\tau}^*_t && \text{(A.4.3)}
\end{aligned}
$$

Rearranging the equation above, we obtain:

$$
\frac{C}{Y}\hat{c}_t - \frac{C^*}{Y}\hat{c}^*_t = -\frac{\phi}{\phi+\sigma}\frac{1}{1-\lambda}\tilde{\tau}^*_t \tag{A.4.4}
$$

Where $\tilde{\tau}^*_t \equiv \frac{\phi+\sigma}{\phi}\Phi\hat{y}_t + \hat{\tau}^*_t$. And, squaring both sides, we obtain:

$$
\left(\frac{C}{Y}\hat{c}_t - \frac{C^*}{Y}\hat{c}^*_t\right)^2 = \left(\frac{1}{1-\lambda}\right)^2\left(\frac{\phi}{\phi+\sigma}\tilde{\tau}^*_t\right)^2 \tag{A.4.5}
$$

Notice that, if $L = L^*$ and $\chi = \lambda$, then$\Phi = 0$, $C = C^* = Y$, and $\hat{\tau}_t = \tilde{\tau}_t$, so we recover the formulation for consumption dispersion in the baseline setting without cyclical income difference (equation (3.0.3)).

A.5 The optimal steady state transfer problem

In what follows, we will approach the government Ramsey problem of optimal steady state transfer selection τ, τ^* in two steps: first, through a *social planner* problem, which selects the optimal steady state consumption and labor levels C, C^*, L, L^*; then, we will find the

transfers τ, τ^* which implement this solution in the decentralized equilibrium. the formulation of a full social planner problem is possible as the steady state is not distorted thanks to the optimal labor cost subsidy ν, that guarantee the achievement of Pareto-efficiency.
The social planner problem is a utilitarian maximization of a linear combination of the utility of the households, according to some weights ψ, ψ^*, and subject to the aggregate resource constraint of the economy:

$$\max_{C,C^*,L,L^*} \psi(1-\lambda)\left(1-\exp(-zC)-\delta\exp(\eta L)\right)+\psi^*\lambda\left(1-\exp(-zC^*)-\delta\exp(\eta L^*)\right) \tag{A.5.1}$$

$$\text{s.t. } (1-\lambda)C+\lambda C^*=(1-\lambda)AL+\lambda AL^*$$

The first order conditions of the problem yield the following optimality conditions:

$$(1-\lambda)C+\lambda C^*=(1-\lambda)AL+\lambda AL^* \tag{A.5.2}$$

$$\delta\eta\exp(\eta L)=z\exp(-zC)A \tag{A.5.3}$$

$$\delta\eta\exp(\eta L^*)=z\exp(-zC^*)A \tag{A.5.4}$$

$$\psi z\exp(-zC)=\psi^* z\exp(-zC^*) \tag{A.5.5}$$

Optimally, the social planner equates the marginal disutility from labor to the marginal utility of consumption times the productivity, for each agent. Moreover, the household weighted more in the welfare function has lower marginal utility of consumption than the other one (and then , by (A.5.3) and (A.5.4), lower marginal disutility of labor as well).
Turning the attention to the decentralized equilibrium, the consumption levels found through the social planner problem can be decentralized by setting appropriate transfers. The steady state budget constraints of the ricardian and the hand to mouth indeed write:

$$C=\omega L+\tau+\mathcal{B}\left(\frac{1}{\Pi}-\beta\right) \tag{A.5.6}$$

$$C^*=\omega L^*+\tau^* \tag{A.5.7}$$

Where $\mathcal{B}$ is steady state aggregate real bond quantity. Notice that aggregate dividends $\bar{D}$ are zero in steady state by (A.2.2), so they do not show up in the households' budget constraint. Equations (A.5.6) and (A.5.7) pin down the optimal steady state transfers τ, τ^*, given the optimal levels C, C^*, L, L^* and the aggregate bond real quantity[12].

A.6 Derivation of the welfare objective of the government (3.3.1)

Let us restate the flow welfare function of the government ((A.5.1)):

$$U_t = \psi(1-\lambda)\left(1 - \exp(-zC_t) - \delta \exp(\eta L_t)\right) + \psi^* \lambda \left(1 - \exp(-zC_t^*) - \delta \exp(\eta L_t^*)\right) \quad \text{(A.6.1)}$$

Taking a second order approximation of the expression above around the steady state U_{SS} yields:

$$\begin{aligned}
U_t \approx\; & U_{SS} + \psi(1-\lambda) z \exp(-zC) C \frac{C_t - C}{C} + \psi^* \lambda z \exp(-zC^*) C^* \frac{C_t^* - C^*}{C^*} + \\
& - \psi(1-\lambda)\delta\eta \exp(\eta L) L \frac{L_t - L}{L} - \psi^* \lambda \delta \eta \exp(\eta L^*) L^* \frac{L_t^* - L^*}{L^*} + \\
& - \frac{1}{2}\psi(1-\lambda) z^2 \exp(-zC) C^2 \left(\frac{C_t - C}{C}\right)^2 - \frac{1}{2}\psi^* \lambda z^2 \exp(-zC^*) C^{*2} \left(\frac{C_t^* - C^*}{C^*}\right)^2 + \quad \text{(A.6.2)} \\
& - \frac{1}{2}\psi(1-\lambda)\delta\eta^2 \exp(\eta L) L^2 \left(\frac{L_t - L}{L}\right)^2 - \frac{1}{2}\psi^* \lambda \delta \eta^2 \exp(\eta L^*) L^{*2} \left(\frac{L_t^* - L^*}{L^*}\right)^2 \quad \text{(A.6.3)}
\end{aligned}$$

Using results (A.5.3)-(A.5.5), we can factor out some constant terms:

$$\begin{aligned}
U_t \approx U_{SS} + \psi z \exp(-zC)\omega \Bigg[& (1-\lambda)\frac{1}{\omega} C \frac{C_t - C}{C} + \lambda \frac{1}{\omega} C^* \frac{C_t^* - C^*}{C^*} - (1-\lambda) L \frac{L_t - L}{L} - \lambda L^* \frac{L_t^* - L^*}{L^*} + \\
& - \frac{1}{2}(1-\lambda)\frac{1}{\omega} z C^2 \left(\frac{C_t - C}{C}\right)^2 - \frac{1}{2}\lambda \frac{1}{\omega} z C^{*2} \left(\frac{C_t^* - C^*}{C^*}\right)^2 - \frac{1}{2}(1-\lambda)\eta L^2 \left(\frac{L_t - L}{L}\right)^2 - \frac{1}{2}\lambda \eta L^{*2} \left(\frac{L_t^* - L^*}{L^*}\right)^2 \Bigg]
\end{aligned}$$

[12] If I allowed the size of real debt to be chosen by the planner, that would have provided an additional and not necessary degree of freedom to implement the optimal allocation.

Using the aggregate resource constraint $Y_t = (1-\lambda)C_t + \lambda C_t^*$, the expression above becomes:

$$U_t \approx U_{SS} + \psi z \exp(-zC)\omega \Bigg[\frac{Y}{\omega}\frac{Y_t - Y}{Y} - (1-\lambda)L\frac{L_t - L}{L} - \lambda L^* \frac{L_t^* - L^*}{L^*} + \\ - \frac{1}{2}(1-\lambda)\frac{1}{\omega}zC^2 \left(\frac{C_t - C}{C}\right)^2 - \frac{1}{2}\lambda\frac{1}{\omega}zC^{*2}\left(\frac{C_t^* - C^*}{C^*}\right)^2 - \frac{1}{2}(1-\lambda)\eta L^2 \left(\frac{L_t - L}{L}\right)^2 - \frac{1}{2}\lambda\eta L^{*2}\left(\frac{L_t^* - L^*}{L^*}\right)^2 \Bigg] \tag{A.6.4}$$

A first order approximation of the market clearing condition (2.4.4) yields:

$$\frac{Y\Delta}{A}\frac{Y_t - Y}{Y} + \frac{Y\Delta}{A}\frac{\Delta_t - \Delta}{\Delta} = (1-\lambda)L\frac{L_t - L}{L} + \lambda L^* \frac{L_t^* - L^*}{L^*} \tag{A.6.5}$$

Where $\Delta = 1$. Recalling that $\omega = A$, and substituting for the above expression into (A.6.4) yields:

$$U_t \approx U_{SS} + \psi z \exp(-zC)\omega \Bigg[-\frac{Y}{\omega}\frac{\Delta_t - \Delta}{\Delta} - \frac{1}{2}(1-\lambda)\frac{1}{\omega}zC^2\left(\frac{C_t - C}{C}\right)^2 - \frac{1}{2}\lambda\frac{1}{\omega}zC^{*2}\left(\frac{C_t^* - C^*}{C^*}\right)^2 + \\ - \frac{1}{2}(1-\lambda)\eta L^2\left(\frac{L_t - L}{L}\right)^2 - \frac{1}{2}\lambda\eta L^{*2}\left(\frac{L_t^* - L^*}{L^*}\right)^2 \Bigg] \tag{A.6.6}$$

Consider for any variable x_t the second order approximations $\frac{x_t - x}{x} \approx \hat{x}_t + \frac{1}{2}\hat{x}_t^2$ and $\left(\frac{x_t - x}{x}\right)^2 \approx \hat{x}_t^2$ where $\hat{x}_t$ is the log-deviation. Let us take also into account that $\hat{\Delta}_t^2 = 0$ up to a second order approximation. Then we can write the expression above as follows:

$$U_t \approx U_{SS} + \psi z \exp(-zC)\omega \left[-\frac{Y}{\omega}\hat{\Delta}_t - \frac{1}{2}(1-\lambda)\frac{1}{\omega}zC^2\hat{c}_t^2 - \frac{1}{2}\lambda\frac{1}{\omega}zC^{*2}\hat{c}_t^{*2} - \frac{1}{2}(1-\lambda)\eta L^2\hat{l}_t^2 - \frac{1}{2}\lambda\eta L^{*2}\hat{l}_t^{*2} \right] \tag{A.6.7}$$

By the aggregate resource constraint $Y\hat{y}_t = (1-\lambda)C\hat{c}_t + \lambda C^*\hat{c}_t^*$ and (A.6.5), notice the following first order equivalences (where the second order term $\hat{\Delta}_t$ does not show up):

$$C\hat{c}_t = Y\hat{y}_t + \lambda(C\hat{c}_t - C^*\hat{c}_t^*) \tag{A.6.8}$$

$$C^*\hat{c}_t^* = Y\hat{y}_t - (1-\lambda)(C\hat{c}_t - C^*\hat{c}_t^*) \tag{A.6.9}$$

$$\tag{A.6.10}$$

$$L\hat{l}_t = \frac{Y}{\omega}\hat{y}_t + \lambda(L\hat{l}_t - L^*\hat{l}_t^*) \tag{A.6.11}$$

$$L^*\hat{l}_t^* = \frac{Y}{\omega}\hat{y}_t - (1-\lambda)(L\hat{l}_t - L^*\hat{l}_t^*) \tag{A.6.12}$$

Moreover, using (A.1.7) and (A.1.8) we can rewrite (A.6.11) and (A.6.12) as:

$$L\hat{l}_t = \frac{Y}{\omega}\hat{y}_t - \lambda\frac{z}{\eta}(C\hat{c}_t - C^*\hat{c}_t^*) \tag{A.6.13}$$

$$L^*\hat{l}_t^* = \frac{Y}{\omega}\hat{y}_t + (1-\lambda)\frac{z}{\eta}(C\hat{c}_t - C^*\hat{c}_t^*) \tag{A.6.14}$$

using (A.6.8), (A.6.9), (A.6.13) and (A.6.14), we can rewrite (A.6.7) as follows:

$$\begin{aligned}
U_t \approx U_{SS} + \psi z \exp(-zC)\omega\Bigg[& -\frac{Y}{\omega}\hat{\Delta}_t + \\
& -\frac{1}{2}(1-\lambda)\frac{z}{\omega}\left[Y^2\hat{y}_t^2 + \lambda^2(C\hat{c}_t - C^*\hat{c}_t^*)^2 + 2\lambda Y\hat{y}_t(C\hat{c}_t - C^*\hat{c}_t^*)\right] + \\
& -\frac{1}{2}\lambda\frac{z}{\omega}\left[Y^2\hat{y}_t^2 + (1-\lambda)^2(C\hat{c}_t - C^*\hat{c}_t^*)^2 - 2(1-\lambda)Y\hat{y}_t(C\hat{c}_t - C^*\hat{c}_t^*)\right] + \\
& -\frac{1}{2}(1-\lambda)\eta\left[\left(\frac{Y}{\omega}\right)^2\hat{y}_t^2 + \lambda^2\left(\frac{z}{\eta}\right)^2(C\hat{c}_t - C^*\hat{c}_t^*)^2 - 2\lambda\frac{z}{\eta}\frac{Y}{\omega}\hat{y}_t(C\hat{c}_t - C^*\hat{c}_t^*)\right] + \\
& -\frac{1}{2}\lambda\eta\left[\left(\frac{Y}{\omega}\right)^2\hat{y}_t^2 + (1-\lambda)^2\left(\frac{z}{\eta}\right)^2(C\hat{c}_t - C^*\hat{c}_t^*)^2 + 2(1-\lambda)\frac{z}{\eta}\frac{Y}{\omega}\hat{y}_t(C\hat{c}_t - C^*\hat{c}_t^*)\right]\Bigg]
\end{aligned} \tag{A.6.15}$$

Rearranging the expression above, we get:

$$U_t \approx U_{SS} + \psi z \exp(-zC)Y\left[-\hat{\Delta}_t - \frac{1}{2}\left(z + \frac{\eta}{\omega}\right)Y\hat{y}_t^2 - \frac{1}{2}\lambda(1-\lambda)z\frac{\omega}{\eta}Y\left(z + \frac{\eta}{\omega}\right)\left(\frac{C}{Y}\hat{c}_t - \frac{C^*}{Y}\hat{c}_t^*\right)^2\right] \tag{A.6.16}$$

Consider a recursive formulation for price dispersion:

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\Pi}\right)^\theta \Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}} \tag{A.6.17}$$

Taking a second order approximation of the equation above and summing through time yields

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\hat{\Delta}_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\theta\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\hat{\pi}_t^2 \tag{A.6.18}$$

As standard in the literature. Taking the infinite discounted sum of (A.6.16), we can substitute for the result above, obtaining the government's loss function:

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{1}{2}\left(z+\frac{\eta}{\omega}\right)Y\frac{\theta}{\kappa}\hat{\pi}_t^2+\frac{1}{2}\left(z+\frac{\eta}{\omega}\right)Y\hat{y}_t^2+\frac{1}{2}z\frac{\omega}{\eta}\left(z+\frac{\eta}{\omega}\right)Y\lambda(1-\lambda)\left(\frac{C}{Y}\hat{c}_t-\frac{C^*}{Y}\hat{c}_t^*\right)^2\right] \tag{A.6.19}$$

where $\kappa=\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(z+\frac{\eta}{\omega})Y$. Equivalently, using (A.1.1), (A.1.2) and $\bar{L}=\frac{Y}{\omega}$, we can write the loss function as:

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{1}{2}\frac{\theta}{\kappa}\hat{\pi}_t^2+\frac{1}{2}\hat{y}_t^2+\frac{1}{2}\frac{\sigma}{\phi}\lambda(1-\lambda)\left(\frac{C}{Y}\hat{c}_t-\frac{C^*}{Y}\hat{c}_t^*\right)^2\right] \tag{A.6.20}$$

Notice that, setting $C=C^*=Y$, the welfare objective is the one of section 3.3.

A.7 Sensitivity analysis

A key quantitative result of the paper is the mutual offsetting nature of the lengthening and shortening channels of optimal transfer policy with respect to the duration of forward guidance, which gives rise to a null effect on the optimal time of the liftoff of the nominal interest rate from the zero lower bound. In this section I perform numerical robustness analysis on this result, by considering a range of alternative parametrizations. I take into account the three parameters that show up in the aggregate demand equation (3.3.2) and in the coefficient of the consumption dispersion term showing up in the welfare objective of the government ((3.3.1)), i.e. the fraction of hand to mouth households λ, the relative risk aversion coefficient σ, and inverse Frisch elasticity of labor supply ϕ. These parameters determine the effect of transfers on the economy, either through the stabilization via aggregate demand, or through the impact on consumption dispersion. For each of these parameters, I select a couplet of alternative parametrizations, one higher and the other lower than the value used in the paper - see Table 1. I also perform a sensitivity analysis with respect to the size of the shock ε_ξ and the discount factor β. Both these parameters indeed determine the extent to which the zero lower bound is binding during the liquidity trap (β in particular pins down the steady state value of the nominal interest rate $i=\frac{\Pi}{\beta}-1$, so the proximity of the latter to the zero lower bound). Also in this case I take into account a lower and a higher value with respect to the parametrization of the paper, which are reported as well in Table 1.

All these alternative simulations are carried out moving one parameter at a time. Results are summarized in Figure .1. In all the alternative configurations we can highlight the presence of the lengthening and shortening effects of transfer policy with respect to forward guidance; these effect offset each other, leaving the duration of the stay of the interest rate at the zero lower bound unchanged with respect to the baseline case with constant transfers.

Parameter	Low value	Paper	High value
λ	0.2	0.33	0.5
σ	1	2	3
ϕ	0.2	0.47	1
β	0.995	0.9987	0.999
ε_ξ	0.02	0.025	0.07

Table 1: Alternative parametrizations

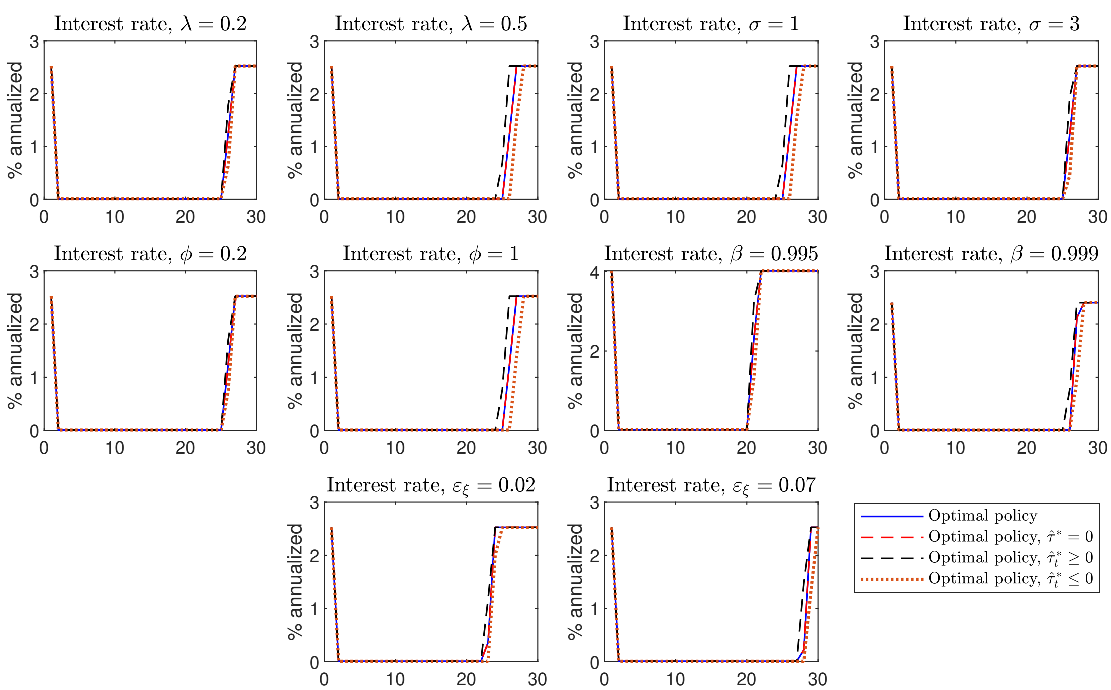

Figure A.1: Sensitivity analysis, nominal interest rate