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# A New Approach to Estimating the Natural Rate of Interest

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# DISCUSSION PAPERS

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# A New Approach to Estimating the Natural Rate of Interest*

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**Abstract**

Building upon the insight that M1 velocity is the permanent component of nominal interest rates—see Benati (2020)—I propose a novel, and straightforward approach to estimating the natural rate of interest, which is conceptually related to Cochrane's (1994*a*) proposal to estimate the permanent component of GNP by exploiting the informational content of consumption. Under monetary regimes (such as inflation-targeting) making inflation I(0), the easiest way to implement the proposed approach is to (*i*) project the monetary policy rate onto M1 velocity—thus obtaining an estimate of the *nominal* natural rate—and then (*ii*) subtract from this inflation's sample average (or target), thus obtaining the *real* natural rate. More complex implementations based on structural VARs produce very similar estimates. Compared to existing approaches, the one proposed herein presents two key advantages: (1) under regimes making inflation I(0), M1 velocity is equal, up to a linear transformation, to the real natural rate, so that the natural rate is, in fact, *observed*; and (2) based on a high-frequency estimate of nominal GDP, the natural rate can be computed at the monthy or even weekly frequency. In the U.S., Euro area, and Canada the natural rate dropped sharply in the months following the collapse of Lehman Brothers. Likewise, the 1929 stock market crash was followed in the U.S. by a dramatic decrease in the natural rate.

*Keywords:* Natural rate of interest; money velocity; structural VARs; unit roots; cointegration.

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# 1 Introduction

Since the outbreak of the financial crisis, the natural rate of interest has been one of the most intensely discussed issues in both policymaking circles and academia. Currently, there are two approaches to estimating the natural rate. In the first, which was originally proposed by Laubach and Williams (2003), the natural rate is modelled as an I(1) process, usually a pure unit root;[1] it is embedded within a semi-structural framework also featuring a Phillips curve; and it is extracted from the data *via* the Kalman filter. In the second approach the natural rate is instead estimated based on a fully-specified DSGE model.[2]

In this paper I illustrate a novel, and straightforward method to estimate the natural rate of interest, which in line with the recent non-DSGE literature I define as a pure unit root process, specifically as the permanent component of the *ex post* real short-term (monetary policy) rate. The approach is conceptually related to Cochrane's (1994*a*) proposal to estimate the permanent component of GNP by exploiting the informational content of consumption, and builds upon the insight that M1 velocity[3] is, to a close approximation, the permanent component of nominal interest rates (see Benati, 2020). This suggests that in the same way as, as argued by Cochrane (1994*a*), consumption can be treated as a good estimate of permanent GNP, M1 velocity can be regarded as a reliable estimate of the permanent component of the nominal short-term rate, $R_t^P$, i.e. of the *nominal natural rate*.

Further, basic economic logic suggests that $R_t^P$ is driven by ($i$) permanent inflation shocks (*via* the Fisher effect), and ($ii$) permanent shocks to the *real natural rate* of interest, i.e., $R_t^P = \pi_t^P + r_t^N$, where $\pi_t^P$ is the permanent component of inflation, and $r_t^N$ is the real natural rate. This implies that under monetary regimes, such as inflation-targeting, that cause inflation to be I(0)[4]—so that $\pi_t^P$=0—permanent shifts in M1 velocity, $V_t$, *uniquely* reflect permanent fluctuations in the natural rate of interest, so that, e.g., $V_t = \alpha + \beta r_t^N + \eta_t$, where $\eta_t$ is a 'small'[5] I(0) component, and the rest of the notation is obvious. Under these regimes, the easiest way to implement the proposed approach is to

(1) project the monetary policy rate onto M1 velocity—thus obtaining an estimate of the nominal natural rate—and then

(2) subtract from this inflation's sample average (or target), thus obtaining the real natural rate.

[1] See e.g. Holston, Laubach and Williams (2017), and Fiorentini, Galesi, Pérez-Quirós and Sentana (2018). To be precise, in these papers the natural rate is modelled as the sum of two pure random walks (see Holston *et al.*'s equations 6, 8 and 9, and Fiorentini *et al.*'s equations 3, 5 and 6), one being trend GDP growth, and the other an additional 'catch-all' factor.

[2] See e.g. Del Negro, Giannone, Giannoni, and Tambalotti (2017).

[3] Defined as the ratio between nominal GDP and nominal M1, i.e. as the inverse of M1 balances expressed as a fraction of GDP.

[4] See Benati (2008).

[5] In the sense of explaining close to *nil* of fluctuations in velocity.

More complex implementations based on structural VARs produce very similar estimates. If, on the other hand, over the sample period inflation had been I(1), so that $\pi_t^P \neq 0$, in order to compute the real natural rate it is necessary to purge the nominal natural rate of permanent inflations shocks. This can be accomplished, e.g., based on a cointegrated SVAR for M1 velocity, the short rate and inflation (and possible other series).

Compared to existing approaches, the one proposed herein presents two advantages. First, since under regimes making inflation I(0) M1 velocity is equal, up to a linear transformation, to the real natural rate, this implies that, under such regimes, *the real natural rate of interest is observed*. This is of obvious interest to policymakers, and (e.g.) it implies that a consistent decrease in M1 velocity under such a regime—such as the progressive fall that has been going on in several inflation-targeting countries since the early 1990s—provides *direct evidence* of a fall in the natural rate. Second, since M1 is observed (at least) at the weekly frequency, and interest rates are observed on a continuous basis, based on a high-frequency estimate of nominal GDP the natural rate can in principle be computed at the monthy, or even weekly frequency. In an application based on monthly data I show that in the U.S., Euro area, and Canada the natural rate fell sharply in the months following the collapse of Lehman Brothers. More generally, my evidence suggests that in all of the countries I analyze herein the real natural rate has been declinining at least since the early 1990s, and that at the end of the sample, in 2019, it had highly likely been negative in several of them.

The paper is organized as follows. The next section discusses the data, whereas Section 3 discusses the close conceptual similarity between the present work and Cochrane (1994*a*). Section 4 estimates the nominal natural rate of interest, whereas Section 5 explores the integration properties of inflation by monetary regime. Section 6 estimates the real natural rate, whereas Section 7 discusses the advantages of the proposed approach compared to existing alternatives, and computes monthly natural rate estimates for the U.S., the Euro area, the U.K., and Canada. Section 8 discusses two applications of the proposed methodology, pertaining to the evolution of the natural rate during the Great Depression, and to the impact of the COVID pandemic. Section 9 concludes.

## 2 The Data

Online Appendix A describes the data and their sources in detail. In brief, nearly all of data are from the datasets assembled by Benati (2020) and Benati, Lucas, Nicolini and Weber (2021), which for the post-WWII period I have updated to 2019Q4.[6] All of the series are standard, with the single exception that, following Lucas and

[6] With the exception of the exercise in Section 8.2 I exclude the year 2020 from all samples, in order to avoid that my results could be distorted by the impact of the COVID pandemic.

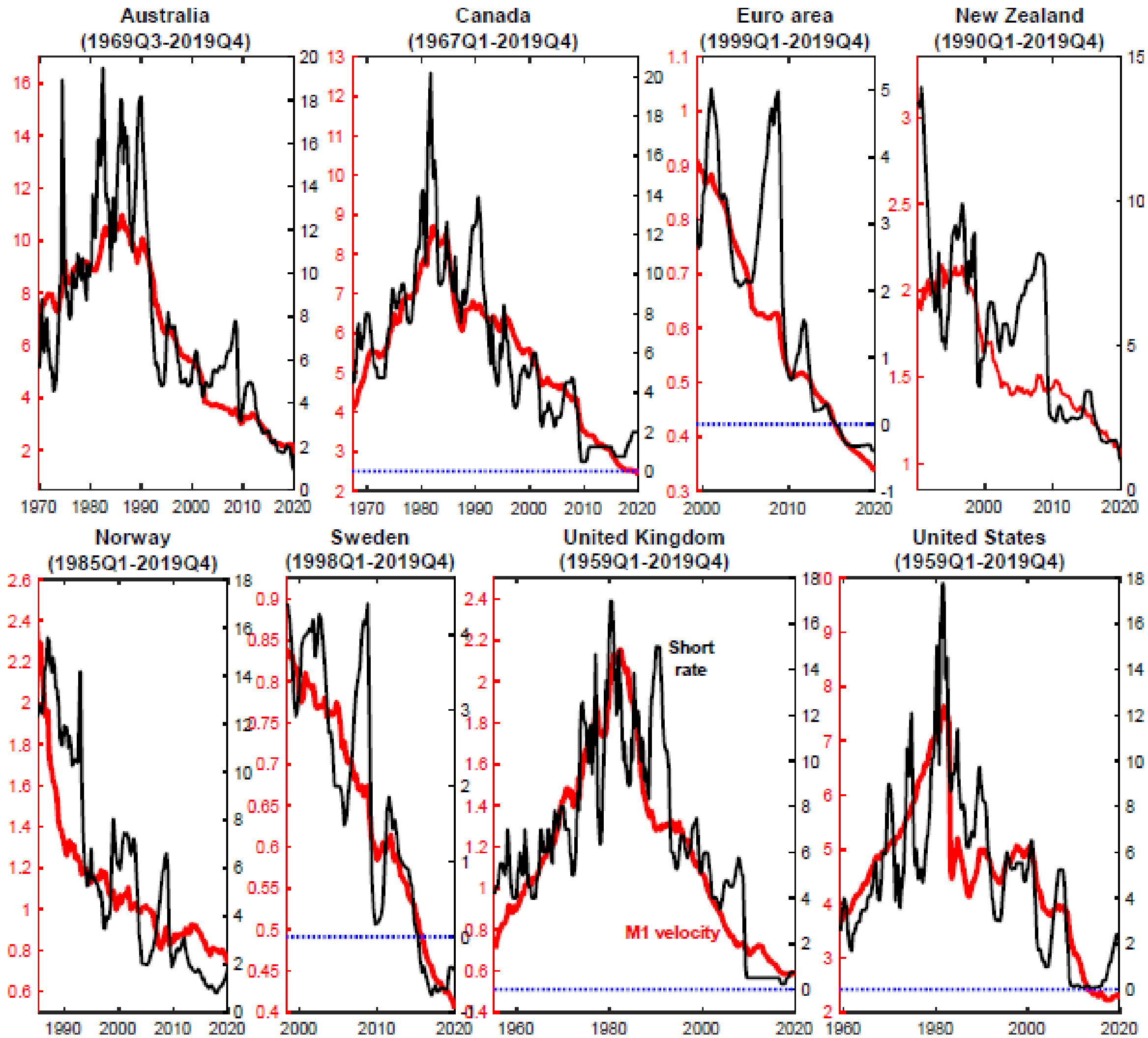


Figure 1a M1 velocity and a nominal short-term interest rate over the post-WWII period

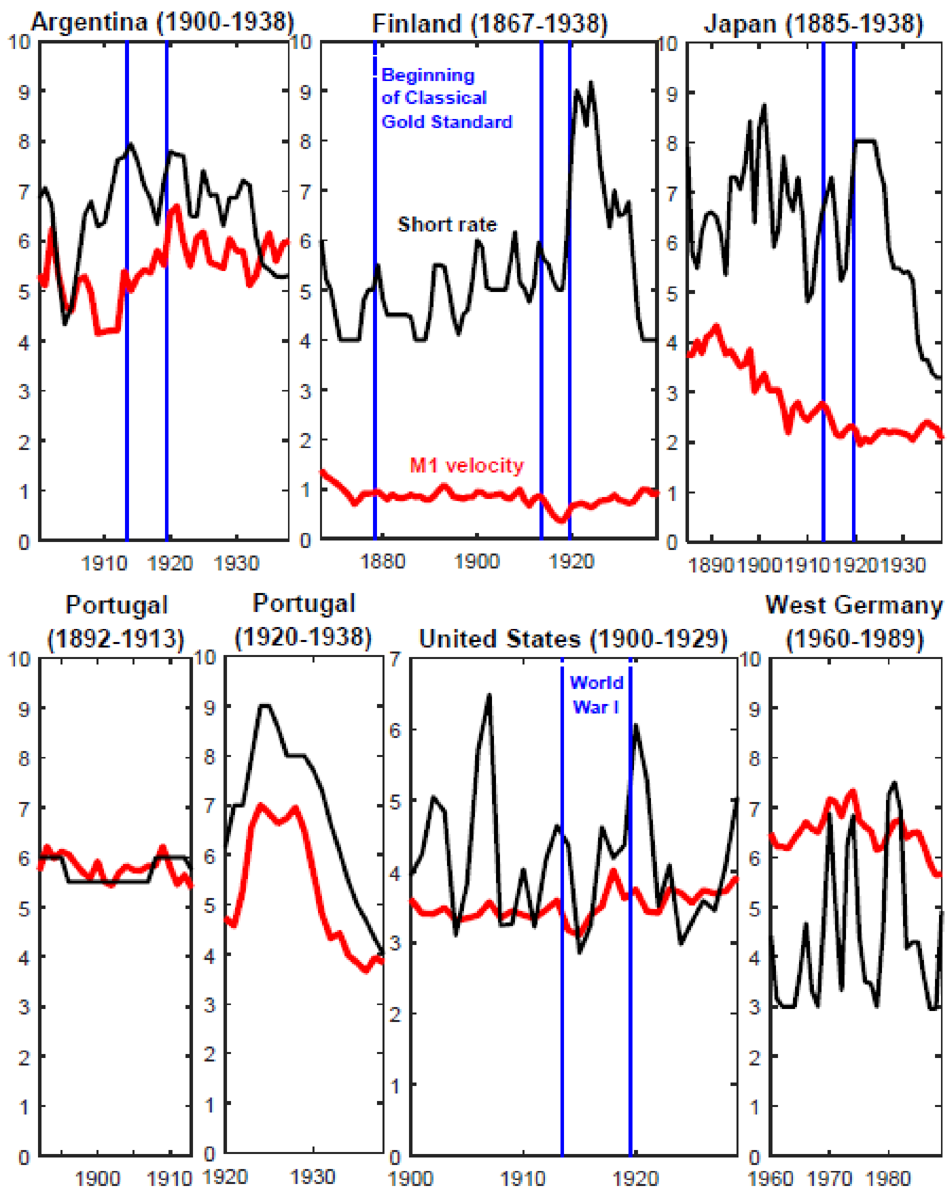


Figure 1b M1 velocity and a nominal short-term interest rate for West Germany and selected pre-World War II samples

Nicolini (2015), Benati (2020), and Benati *et al.* (2021), for the United States I consider, instead of the standard M1 aggregate produced by the Federal Reserve, one of the modifications that had originally been suggested by Goldfeld and Sichel (1990, pp. 314-315) in order to restore the stability of the long-run demand for M1. This alternative M1 series—which Lucas and Nicolini (2015) label as 'New M1'—is obtained by adding to the standard M1 aggregate Money Market Deposits Accounts (MMDAs). The rationale for doing so is that MMDAs perform an economic function very similar to that of the 'checkable deposits' included in the standard M1 series (on this see the discussion in Lucas and Nicolini, 2015).

Figure 1*a* shows, for the eight countries analyzed herein, M1 velocity and a short-term nominal interest rate over the post-WWII period. Visual impression suggests the following three facts, which as shown by Benati (2020) are indeed confirmed by a proper econometric analysis: (*i*) M1 velocity and the short rate are both I(1); (*ii*) the two series are cointegrated; and, crucially, (*iii*) up to a linear transformation, M1 velocity is, essentially, the stochastic trend of the short rate. Figure 1*b* shows the same type of evidence for selected pre-World War II samples and for West Germany. The evidence for Portugal during the interwar period, with the hump-shaped fluctuation in the short rate being mirrored by a corresponding fluctuation in velocity, is qualitatively in line with the post-WWII evidence in Figure 1*a*. This is also the case, although to a lesser extent, for interwar Japan, with both series exhibiting an overall downward trend. For all other countries, however, the lack of any discernible trend in the short rate is mirrored by the broad flatness of M1 velocity. This is especially the case for Finland, Portugal (1892-1913), the United States, and to a lesser extent for Argentina and West Germany.

This evidence naturally suggests that the large fluctuations in M1 velocity that have characterized the post-WWII period have been caused, under a stable demand for M1 balances as a fraction of GDP, by permanent fluctuations in both inflation and the real natural rate of interest injecting a unit root in nominal short-term interest rates. On the other hand, as the evidence in Figure 1*b* shows, when nominal interest rates do not exhibit any trend, M1 velocity is likewise essentially flat. In turn, this suggests that, to the extent that (*i*) inflation will remain under the control of the monetary authority, and therefore I(0), and (*ii*) the decline in the real natural rate of interest will ultimately stop, the fall in velocity that has been going on since the early 1980s (see Figure 1*a*) will also cease.

I now turn to discussing the conceptual similarity between the present work and Cochrane's (1994*a*) analysis for consumption and output.

# 3 Conceptual Similarity With Cochrane (1994*a*)

The best way to illustrate the approach I am advocating herein is to highlight its close conceptual similarity with Cochrane's (1994*a*) proposal to estimate the permanent component of GNP by exploiting the informational content of consumption.

## 3.1 Cochrane (1994*a*): consumption is the permanent component of GNP

In his investigation of the permanent income hypothesis (PIH) based on cointegrated structural VARs (SVARs) for consumption and GNP, Cochrane (1994*a*) documented how consumption is, to a close approximation, the permanent component of GNP. The first two panels of Figure 2 report evidence in line with Cochrane's for the United States for the period 1947Q1-2019Q4, based on a cointegrated SVAR for the logarithms of real consumption and real GDP identified *via* long-run restrictions.[7] The two panels report the fractions of forecast error variance (FEV) of consumption and GDP, respectively, explained by the common permanent shock, which is identified as the only shock having a permanent impact on the two series. In line with Cochrane's (1994*a*) Table I.2, the shock explains (nearly) 100 per cent of the FEV of consumption at all horizons up to 10 years ahead, whereas GDP contains, at the short horizons, a sizeable transitory component.[8] As pointed out by Cochrane (1994*a*),

> 'It is natural to interpret these features of the data via the simple permanent income model. The model predicts that consumption is a random walk and that consumption and total income are cointegrated. If consumption does not change, consumers must think any fluctuation in GNP is transitory. [...] by observing consumption, we separate GNP into permanent and transitory components, as viewed by consumers. [...] Thus, consumption provides a good measure of the trend in GNP, since it measures consumers' expectations of long-run GNP.'

As stressed by Cochrane (1994*b*), this is because

> 'Each person has information about his own prospects, most of which is idiosyncratic. Total consumption aggregates all this information about aggregate activity.'

## 3.2 M1 velocity is the permanent component of the short rate

As documented by Benati (2020), since World War I M1 velocity has been, to a close approximation, the permanent component of the short-term nominal interest

[7] The data are described in Online Appendix A.2.9. Elliot, Rothenberg, and Stock (1996) unit root tests with an intercept and a time trend strongly suggest that both series are I(1), with $p$-values bootstrapped as in Diebold and Chen (1996) ranging between 0.4693 and 0.6659 for GDP, and between 0.9514 and 0.9912 for consumption. By the same token, Johansen's maximum eigenvalue test, bootstrapped as in Cavaliere *et al.* (2012), provides clear evidence of cointegration, with a $p$-value equal to 2.0e-4.

[8] In Cochrane's (1994*a*) Table I.2 the permanent ('consumption') shock explains 97 per cent of the variance of $\Delta \ln C_t$, whereas it explains only 30 per cent of the variance of $\Delta \ln Y_t$, where $C_t$ and $Y_t$ are consumption and GDP, respectively.

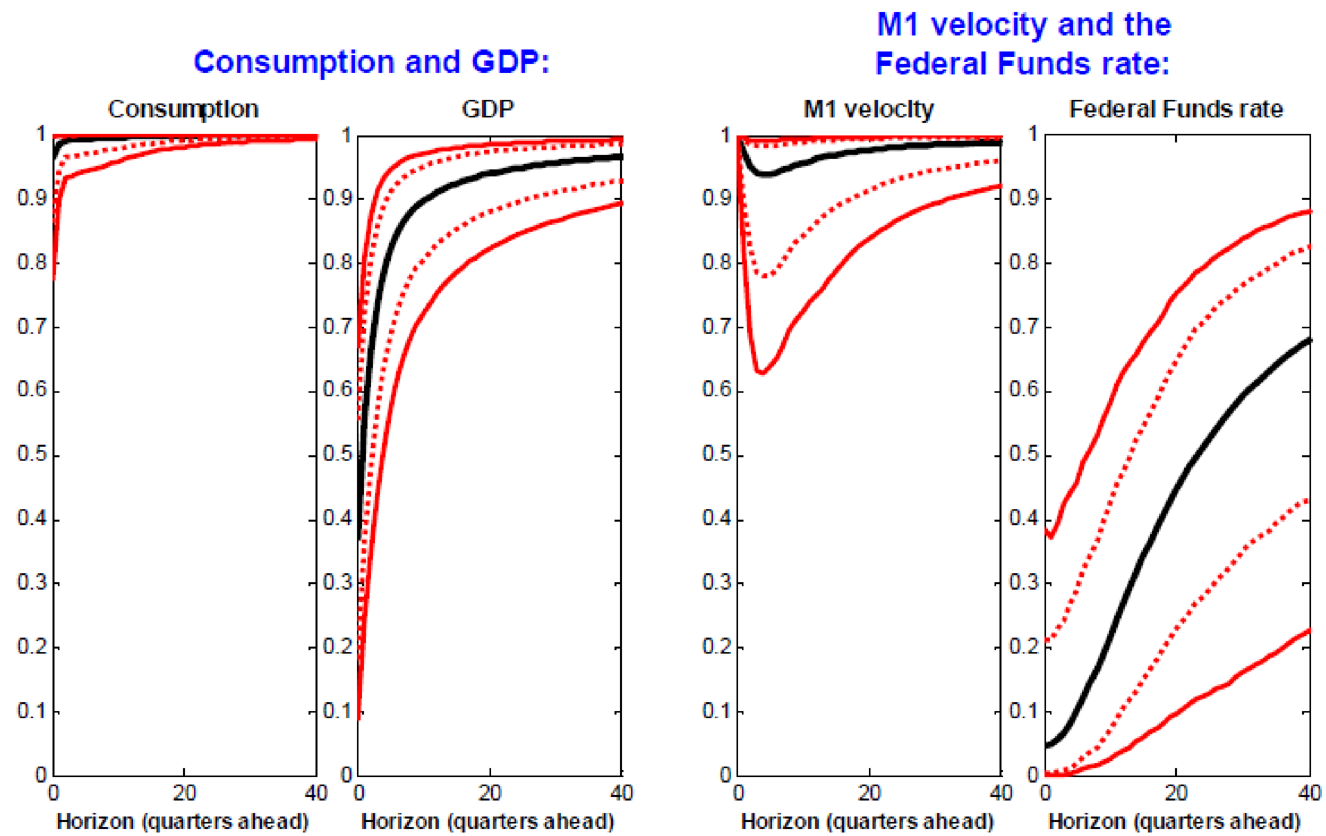


Figure 2 United States: fractions of forecast error variance explained by the permanent shock, based on cointegrated VARs featuring either (i) consumption and GDP or (ii) M1 velocity and the Federal Funds rate (with 1- and 2-standard deviations bootstrapped confidence bands)

rate, so that the time-series relationship between the two series has been *exactly* the same as that between consumption and GDP.[9] The last two panels of Figure 2 report, for the United States for the period 1959Q1-2019Q4, the fractions of FEV explained by the common permanent shock based on a cointegrated SVAR for M1 velocity and the Federal Funds rate identified *via* long-run restrictions. Once again, the permanent shock is identified as the only shock having a permanent impact on the two series. In line with the previously discussed evidence for consumption and GDP, the permanent shock explains (close to) 100 per cent of the FEV of M1 velocity at all horizons, whereas the Federal Funds rate contains a sizeable transitory component, which is in fact dominant at all horizons up to about five years ahead. Benati's (2020) Figure 1*b* reports the corresponding evidence for the post-WWII United Kingdom, whereas Figures 3*a* and 4*a* report evidence for eight additional countries for the post-WWII period, and for ten countries since World War I, respectively. With the single exception of Taiwan, and to a lesser extent of Japan, the evidence there is in line with that reported in Figure 2 in the present work, with the permanent shock explaining (close to) 100 per cent of the FEV of M1 velocity at all horizons. Short-term nominal interest rates, on the other hand, consistently feature a sizeable, and often dominant transitory component.

Figure A.11 in the Online Appendix reports, for the eight countries I analyze in the present work, the corrisponding evidence based on SVARs in levels. Consistent with the notion that the two series are cointegrated, the common shock is identified as the shock maximizing their long-horizon (i.e., 10-year ahead) conditional covariance (for details, see Benati, 2014). Figure A.11 reports the fractions of FEV of M1 velocity and the short rate explained by the identified shock. This evidence is qualitatively the same as that in Figure 2 in the present work (for the United States), and in Benati's (2020) Figures 1*b*, 3*a* and 4*a* (for the other countries) based on cointegrated SVARs, and it shows that the identified shock explains (close to) 100 per cent of the FEV of M1 velocity at all horizons, whereas it explains markedly smaller fractions of the FEV of the short rate. Once again, the implication is that, to a close approximation, M1 velocity is the long-horizon component of the short rate.

[9] The unit root properties of M1 velocity and the short rate are discussed in Appendix A. In short, Elliot *et al.* (1996) tests (with an intercept, but no time trend) strongly suggest that both series are I(1) in all samples (quite obviously, this evidence is compatible with the notion that the series are in fact near-unit root processes). Table B.1 in Appendix B reports, for bivariate systems featuring M1 velocity and a short-term rate, results from either (*i*) Johansen's maximum eigenvalue test of the null hypothesis of 0 *versus* 1 cointegration vectors, or (*ii*) Wright's (2000) test of the null hypothesis that the series are cointegrated, which is designed to perform equally well when they feature either exact or near unit roots. In short, consistent with Benati (2020) and Benati *et al.* (2021), the evidence there suggests that the two series are cointegrated in all sample. Finally, Online Appendix B.4 reports results from Hansen and Johansen's (1999) tests for stability in either the cointegration vector, or the loading coefficients, in the estimated VECMs for velocity and the short rate. For all countries no break is detected in either of the two features.

### 3.2.1 Interpretation

This evidence suggests that the bivariate relationship between M1 velocity and the short rate is well captured, to a first approximation, by a simple model in which (1) the short rate, $R_t$, is the sum of two components, a random walk, $R_t^P$, and a stationary AR($p$) process, $R_t^T$,

$$R_t = R_t^P + R_t^T \tag{1}$$

$$R_t^P = R_{t-1}^P + u_t \tag{2}$$

$$R_t^T = \phi_1 R_{t-1}^T + ...\phi_p R_{t-p}^T + v_t \tag{3}$$

—with, just for the sake of simplicity, $u_t$ and $v_t$ being orthogonal white noise processes— and (2) M1 velocity is a linear function of the permanent component of the short rate, i.e.

$$V_t = \alpha + \beta R_t^P + \eta_t \tag{4}$$

where $\eta_t$ is a 'small' (in the sense of explaining close to *nil* of the FEV of velocity) stationary component, and the rest of the notation is obvious.

Expression (4) is obtained by replacing the short rate with its permanent component within the money demand specification originally proposed by Selden (1956) and Latané (1960), i.e.[10]

$$V_t = \alpha + \beta R_t + \eta_t \tag{5}$$

As discussed by Benati *et al.* (2021), for several low-inflation countries—first and foremost, the U.S. and the U.K.—the data seem to quite clearly prefer the Selden-Latané specification to the traditional log-log and semi-log ones.[11]

An important point to stress is that, as long as $\eta_t$ in expression (4) is 'small', $V_t \simeq \alpha + \beta R_t^P$, so that $R_t^P \simeq (V_t - \alpha)/\beta$. This implies that, under these circumstances, the *nominal* natural rate is *always observed*.[12]

Expression (4) implies the following cointegrated VECM representation for $\Delta V_t$ and $\Delta R_t$:

$$\begin{bmatrix} \Delta V_t \\ \Delta R_t \end{bmatrix} = \text{Constants} + \underbrace{\begin{bmatrix} 0 \\ \frac{1-\rho}{\beta} \end{bmatrix}}_{\text{Loadings}} \underbrace{\begin{bmatrix} 1 & -\beta \end{bmatrix}}_{\text{Cointegration vector}} \begin{bmatrix} V_{t-1} \\ R_{t-1} \end{bmatrix} + \text{Shocks} \tag{6}$$

In plain English, this representation implies that the system's adjustment towards its long-run equilibrium takes place *via* movements in the short rate, with no reaction

[10] As shown in Benati's (2020) Online Appendix B, the Selden-Latané specification is a special case of the 'money in the utility function' framework pioneered by Miguel Sidrauski (1967*a*, 1967*b*). By the same token, Benati *et al.* (2021) derive (5) within a generalized Baumol-Tobin framework in which agents are subject to an upper limit on how much they can borrow.

[11] This is also discussed in this paper's Online Appendix B.3.

[12] Notice, once again, the close similarity with consumption and GDP: as pointed out by Cochrane (1994*a*) in the previous quotations, consumption is, likewise, the (observed) stochastic trend of GDP.

of M1 velocity to disequilibria. This is because in the same way as (rescaled) consumption is, to a first approximation, the common stochastic trend in the bivariate system for GDP and consumption, (rescaled) M1 velocity is, likewise, the common stochastic trend in the system for velocity and the short rate.[13]

#### 3.2.2 Implications

Since M1 velocity is the inverse of the demand for M1 balances as a fraction of GDP, the fact that, to a first approximation, it only reacts to permanent shocks to the short-term nominal rate implies that economic agents, in allocating their wealth between non interest-bearing M1 and interest-bearing assets, react almost exclusively to permanent shocks to the opportunity cost of M1, whereas they essentially ignore transitory shocks. The implication is that, in the same way as consumers disentangle permanent and transitory income shocks and only react to the former, economic agents perform a *permanent-transitory decomposition of nominal interest rates*, and only react to the permanent component.[14]

### 3.3 Estimating the natural rate of interest

This suggests that, defining the real natural rate of interest as the unit root component of the *ex post* real short-term rate (see below), there is in fact a simple and straightforward way of estimating it.

**Estimating the nominal natural rate** First, we need to estimate the nominal natural rate by exploiting the fact that M1 velocity is, up to a scale factor, its stochastic trend. This can be accomplished, e.g., in the same way as Cochrane (1994*a*) estimated the permanent component of GNP (see his Figure III), i.e. based on a cointegrated VAR for the two series identified *via* long-run restrictions. An alternative, and much simpler approach involves projecting the short-term nominal (monetary policy rate) onto M1 velocity *via* a simple OLS regression.[15] The rationale for this is

[13] As pointed out by a referee, *'[m]any readers will no doubt have a strong prior that financial innovation has generated a second stochastic trend in M1 velocity beyond that induced by a stochastic trend in interest rates.'* In fact, the notion that financial innovation may have generated a second stochastic trend in M1 velocity—beyond that induced by the unit root in interest rates—is refuted by the data. If this notion were true there would be no cointegration between velocity and the short rate. The fact that—as shown by Benati (2020), Benati, Lucas, Nicolini, and Weber (JME, 2021), and the present work—M1 velocity and the short rate are indeed cointegrated logically refutes the notion that financial innovation may have injected a second unit root in velocity.

[14] No existing model of money demand exhibits this feature. In fact, no model of money demand—from the classic analyses of Baumol and Tobin on—even distinguishes between permanent and transitory variation in the opportunity cost of money.

[15] Another possibility is to use the DOLS estimator proposed by Stock and Watson (1993). Although in this paper I only report and discuss results based on the simple OLS estimator, a very similar set of results based on the DOLS estimator is available upon request.

that, since $R_t = R_t^P + R_t^T$ and $V_t = \alpha + \beta R_t^P + \eta_t$, the OLS regression

$$R_t = a + bV_t + e_t, \tag{7}$$

with $e_t$ being the residual, is a cointegrating regression, which implies that the estimator of $b$ is super-consistent. In turn this implies that, for samples of typical size, $b$ (and therefore also $a$) are likely reliably estimated, which is of obvious, paramount importance within a policy context.

In practice, the resulting estimate of the nominal natural rate,

$$\hat{R}_t^N = \hat{a}_{OLS} + \hat{b}_{OLS} V_t \tag{8}$$

is typically close to that produced by the alternative approach based on cointegrated SVARs identified *via* long-run restrictions. In what follows I will therefore only report in the main text the evidence based on the simpler approach, whereas the corresponding evidence based on cointegrated SVARs is reported in the Online Appendix.

Simple evidence on the reliability of this approach is provided in Appendix C (see in particular Figure C.1), where I compute a transitory component of post-WWII U.S. GDP by projecting log real GDP onto log real consumption[16]. Two main findings emerge from this exercise. First, the estimated transitory component of GDP captures remarkably well the peaks and troughs of the post-WWII U.S. business cycle as established by the NBER Business-Cycle Dating Committee. Second, this methodology interprets a sizeable portion of the fall in output associated with the Great Recession as permanent. As I discuss in Appendix C, simple but powerful corroborating evidence that this may in fact had been the case is provided by a comparison between the actual evolution of GDP and consumption, which since 2012 have closely tracked each other, and the forecast of the Hodrick-Prescott GDP trend computed based on data up to the end of 2004: at the end of 2019 for both series the shortfall had been equal to about 12 per cent.[17]

**Estimating the real natural rate** Since conventional monetary policy involves the manipulation, on the part of the central bank, of a nominal short-term interest rate, having a reliable estimate of the nominal natural rate is of obvious interest in itself. On the other hand, the focus of much of the recent macroeconomic debate has been on the *real* natural rate, due, first and foremost, to concerns about the limitations imposed by the Zero Lower Bound (ZLB) on conventional monetary policy; secular stagnation; and the future evolution of income inequality. Based on an estimate of the nominal natural rate, in order to obtain the corresponding estimate

[16]I.e., I estimate (7) with velocity and the short rate replaced by log real consumption and log real GDP. The two series are the same discussed in footnote 7.

[17]The fact that this has equally held for *both* GDP *and* consumption logically suggests that the shortfalls are permanent: otherwise, by the Permanent Income Hypothesis, consumption would be close to the HP trend.

of the real natural rate it is necessary to take a stand on the integration properties of inflation. Basic economic logic suggests indeed that, in general, $R_t^P$ should be driven by (*i*) permanent inflation shocks (*via* the Fisher effect) and (*ii*) permanent shocks to the real natural rate of interest, that is,

$$R_t^P = \pi_t^P + r_t^N, \tag{9}$$

where $\pi_t^P$ is the permanent component of inflation, and $r_t^N$ is the real natural rate of interest. Expression (9), together with (1) and the corresponding permanent-transitory decomposition for inflation, i.e. $\pi_t = \pi_t^P + \pi_t^T$, implies that $R_t - \pi_t = r_t^N + (R_t^T - \pi_t^T)$, so that the real natural rate of interest is the permanent component of the *ex post* real short-term rate.

Further, expression (9) logically implies that, under monetary regimes that had made, or make inflation I(0)—such as those based on metallic standards, or inflation-targeting regimes (see Benati, 2008)—so that $\pi_t^P$=0, permanent shifts in M1 velocity should *uniquely* reflect permanent fluctuations in the natural rate of interest, so that, e.g., $V_t = \alpha + \beta r_t^N + \eta_t$. Under these regimes, an estimate of the real natural rate can therefore be obtained simply by subtracting from the estimated nominal natural rate either inflation's sample average, or the inflation target.[18] If, on the other hand, over the sample period inflation has been I(1),[19] so that $\pi_t^P \neq$0, in order to compute the real natural rate it is necessary to purge the nominal natural rate of permanent inflations shocks. This can be accomplished (e.g.) based on a cointegrated SVAR for M1 velocity, the short rate, and inflation (and possible other series) identified *via* long-run restrictions. Although this is in principle straightforward, in what follows I will not pursue this avenue since, as shown by Benati (2008), and as I will confirm in Section 5 based on samples extending up to 2019Q4, under the current monetary regimes, that were introduced in the 1990s,[20] inflation has been I(0).

**Robustness to alternative assumptions about the stochastic properties of the series** As discussed briefly in footnote 9, and more extensively in Appendix B, the evidence from Elliot et al.'s (1996) tests for M1 velocity and the short rate strongly suggest that the two series are either exact, or near unit root processes. Although the entire analysis in the present work is based on the assumption that the two series feature *exact* unit roots, in fact qualitatively the same, and numerically either identical or very close results are obtained based on the alternative assumption that they are *near* unit root processes. In particular, in this case the methodology for computing

[18] Which of the two is the most appropriate depends on the credibility of the central bank's inflation target. If it is very credible, it is more appropriate to subtract the target, rather than inflation's sample average.

[19] Benati (2008) shows that, historically, this has been the case only for sample periods dominated by the Great Inflation episode.

[20] As I discuss in Section 4, for the United States I consider the period following the break in the mean of inflation identified by Levin and Piger (2004), in 1992Q2.

*point estimates* of the nominal and real natural rate is the same I previously discussed in this section, so that, by definition, the point estimates are also the same. As for the confidence bands, whereas (as I discuss in Section 4) based on the assumption that the two series feature exact unit roots they are obtained by bootstrapping the estimated cointegrated VECM as in Cavaliere *et al.* (2012), here they are computed by bootstrapping the corresponding near-unit root VAR as detailed in Benati *et al.* (2021, Section 4.2.1., 'Issues pertaining to bootstrapping').[21] The confidence bands obtained by bootstrapping the near-unit root VAR are virtually identical to those reported in the present work (this alternative set of results is available upon request).

**A comparison with Laubach and Williams' (2003) approach** It is worth spending a few words to compare the proposed approach to the unobserved-components methodology pioneered by Laubach and Williams (2003). In the latter approach,[22] the individual unobserved components are postulated to be orthogonal to one another. For example, in Holston *et al.* (2017) this is the case for (*i*) the permanent component of log real GDP ($y_t^*$), which is modelled as a random-walk with a time-varying drift; (*ii*) the drift itself ($g_t$), which is postulated to be a random-walk; and (*iii*) an additional random-walk component of the natural rate ($z_t$): all of these components are postulated to be orthogonal to one another. The assumption of orthogonality might be correct, but if it is not, imposing it upon the data is likely going to distort the estimates. The approach proposed herein, on the other hand, does not impose any such assumption, and it rather uniquely hinges on the fact that, as discussed in Section 3.2, M1 velocity is the long-horizon component of the short rate.

**Testing for stability of the projection regression (7)** Online Appendix D reports results from tests for a joint break in $a$ and $b$ in the projection regression (7) based on the tests proposed by Perron and Yabu (2009). The methodology is exactly the same I used in Benati (2014), and it is described in detail in the Online Appendix D. A key reason for using Perron and Yabu's tests is that they do not require to make any assumption on whether the residual from the regression (i.e. $e_t$) is I(0) or I(1), thus implying that the tests are equally valid for either exact or near unit roots. I bootstrap the test statistics as previously detailed when discussing robustness to alternative assumptions about the stochastic properties of the series. Specifically, based on the assumption that the two series feature *exact* unit roots I bootstrap the estimated cointegrated VECM as in Cavaliere *et al.* (2012). For the alternative

[21] As discussed there (see pages 55-56): *'Based on the cointegrated VECM estimated under the null of one cointegration vector, we compute the implied VAR in levels, which by construction features one, and only one, eigenvalue equal to 1. Bootstrapping this VAR would obviously be equivalent to bootstrapping the underlying cointegrated VECM; that is, it would be correct if the data featured exact unit roots. Since, on the other hand, here we want to bootstrap under the null of a near unit root DGP, we turn such an exact unit root VAR in levels into its corresponding near unit root by shrinking down the single unitary eigenvalue to* $\lambda$*= 1-0.5 (1/ T ), where T is the sample length.'*

[22] Beyond Laubach and Williams (2003), see Holston, Laubach, and Williams (2017).

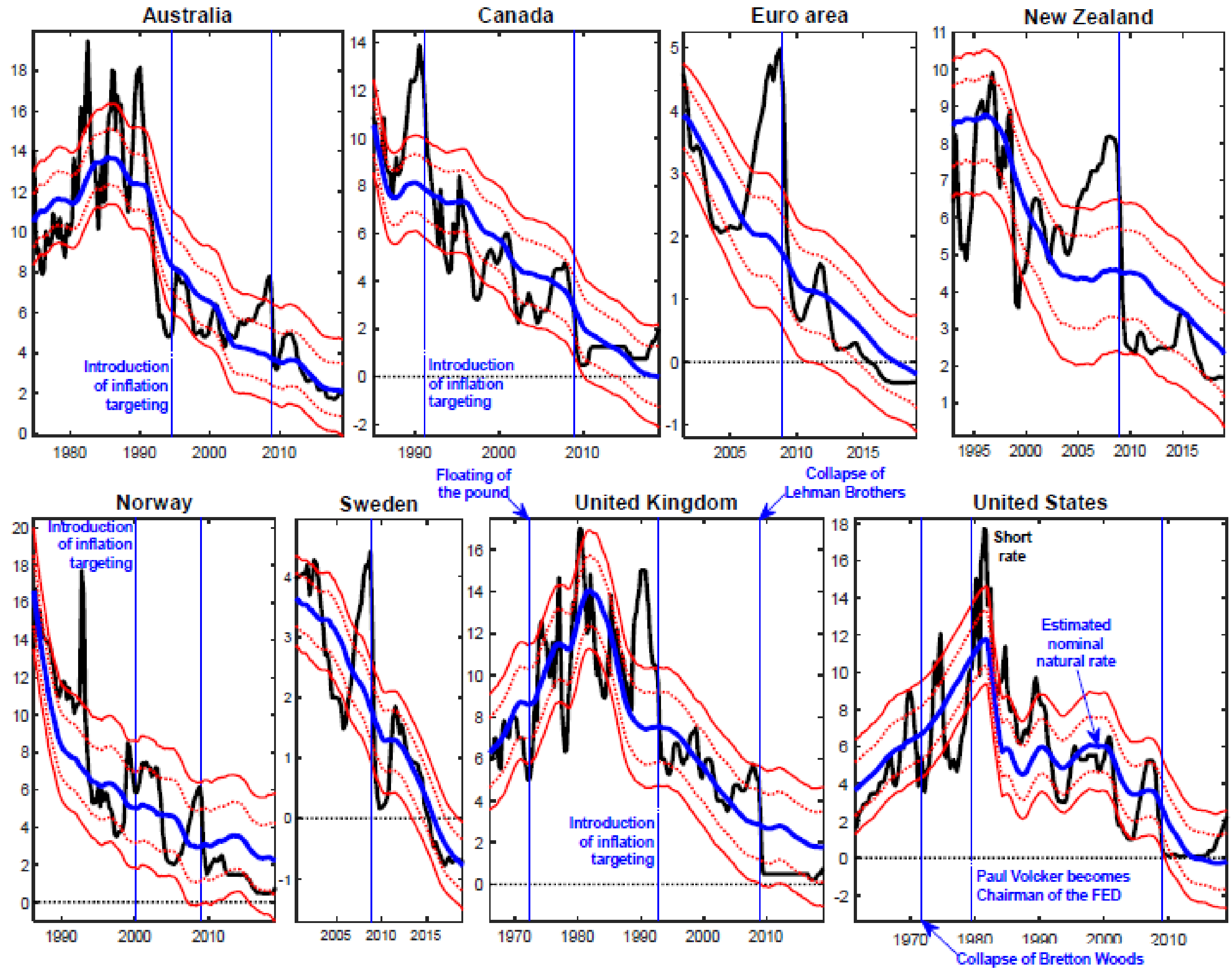


Figure 3 Estimates of the nominal natural rate computed by projecting the short rate on M1 velocity (with 1- and 2-standard deviations bootstrapped confidence bands)

possibility that they feature *near* unit roots, I bootstrap the corresponding near-unit root VAR as detailed in Benati *et al.* (2021, Section 4.2.1., 'Issues pertaining to bootstrapping'). Based on either bootstrapping methodology, not in a single case I detect breaks in $a$ and $b$, thus pointing towards stability of the projection regression. For the coefficient $b$ this is in line with the fact that, as mentioned in footnote 9, Hansen and Johansen's tests do not detect any break in the cointegration vector.

I now turn to discussing the estimates of the nominal natural rate obtained by projecting the short rate onto M1 velocity *via* a simple OLS regression.

# 4 Estimating the Nominal Natural Rate

Figure 3 shows the simple estimate of the nominal natural rate obtained by projecting the short-term nominal (monetary policy) rate onto M1 velocity—i.e. $\hat{R}_t^N$ in expression (8), which is computed by estimating equation (7) *via* OLS—together with 1- and 2-standard deviations bootstrapped confidence bands. Figure 4 shows the simple estimate of the deviation of the short rate from the nominal natural rate together with the detrended unemployment rate,[23] whereas Figures A.2-A.4 in the Online Appendix show, respectively, the deviation of the short rate from the nominal natural rate with bootstrapped confidence bands; the fraction of bootstrap replications for which the deviation has been negative; and the fraction of bootstrap replications for which the nominal natural rate is estimated to have been negative (together with the corresponding fractions for the real natural rate we will discuss in Section 6).[24] Figure A.5 in the Online Appendix reports, based on Wu and Xia's (2016) 'shadow rates', the same evidence as in Figures A.2-A.3 for the Euro area, the U.K., and the U.S.. The corresponding evidence based on cointegrated SVARs is reported in Figure A.6-A.10 in the Online Appendix.[25] In both sets of figures, confidence bands have been computed by bootstrapping as in Cavaliere *et al.* (2012) the cointegrated VECM for M1 velocity and the short rate (or the shadow rate) estimated *via* Johansen's estimator (as described in Hamilton, 1994) imposing one cointegration vector. I set the number of bootstrap replications to 10,000. Based on each bootstrapped, artificial sample $j$, with $j = 1, 2, ..., 10{,}000$, I then perform exactly the same operations I previously performed based on the actual data. When estimating the nominal natural rate by projecting the short rate onto M1 velocity, I therefore estimate (7) based on

[23] I detrended the unemployment rate *via* the band-pass filter proposed by Christiano and Fitzgerald (2003), by removing all components associated with cycles slower than 30 years. In doing this I consider for each country, the longest sample for which the unemployment rate has been available: e.g., although for the Euro area my analysis focuses on the period since the start of European Monetary Union, in January 1999, I detrend the unemployment rate based on data since 1970Q1.

[24] In what follows I will interchangeably refer to such fractions as the 'probabilities that the natural rates had been negative'.

[25] In all figures, estimates have been smoothed *via* a centered 4-quarters moving-average in order to remove some high-frequency noise.

the bootstrapped short rate and bootstrapped velocity, i.e. I run the OLS regression $R_t^{B,j} = a + bV_t^{B,j} + e_t^{B,j}$, where $R_t^{B,j}$ and $V_t^{B,j}$ are the bootstrapped short rate and velocity for replication $j$. This produces an estimate of the nominal natural rate for bootstrap replication $j$, i.e. $\hat{R}_t^{N,B,j} = \hat{a}_{OLS}^{B,j} + \hat{b}_{OLS}^{B,j} V_t^{B,j}$, and of the associated transitory component, $\hat{R}_t^{T,B,j} = \hat{R}_t^{N,B,j} - \hat{a}_{OLS}^{B,j} - \hat{b}_{OLS}^{B,j} V_t^{B,j}$. When working with cointegrated VARs identified *via* long-run restrictions, on the other hand, I compute $\hat{R}_t^{T,B,j}$ as in Blanchard and Quah (1989), i.e. by re-running history only conditional on transitory shocks. In this way, based on either approach I build up the bootstrapped distribution of the transitory component of the short (or shadow) rate, which I then use in order to compute confidence bands for the transitory component, and therefore, as a result, also for the permanent component.

The following main results emerge from the two sets of figures:

($i$) as already mentioned, the simple, projection-based methodology produces results that are qualitatively the same, and quantitatively close to those produced by the alternative approach based on cointegrated SVARs. For reasons of simplicity and especially robustness,[26] in what follows I will uniquely focus on the results produced by the simpler approach.

($ii$) Whereas for the U.S. using shadow rates does not produce materially different estimates, for the U.K. and especially the Euro area this is not the case (see Figures A.5 and A.9 in the Online Appendix). This reflects the fact that, for the latter two countries, the difference between the shadow rate and the official monetary policy rate has been significantly greater than for the U.S.. For these three countries, in what follows I will exclusively focus on the results based on the official monetary policy rate, but the full sets of results based on the shadow rates are available upon request.

($iii$) As one would expect, for all countries the estimated nominal natural rate behaves as a very low-frequency component of the short-term rate.

($iv$) As shown in Figure 4, the nominal rate gap—defined as the difference between the short rate and the nominal natural rate, i.e. as $R_t - \hat{R}_t^N$ in (8)—exhibits a strong negative contemporaneous correlation with the detrended unemployment rate. This is in line, e.g., with the evidence in King and Watson (1996, pp. 38-39 and Figure 2) that '[the band-pass filtered cyclical components of] nominal interest rates and output are positively correlated', and it has a straightforward interpretation in terms of counter-cyclical monetary policy.[27]

[26] If M1 velocity were *exactly* equal to the nominal natural rate, the projection-based approach would exactly capture the latter. Under these conditions the SVAR-based approach could not improve upon this estimate, because the projection-based approach would rely on an observed linear transformation of the natural rate. To the extent that, as documented by Benati (2020), in fact we are close to such ideal situation, this argument approximately holds. On the other hand, a permanent-transitory decomposition based on a cointegrated SVAR is significantly more complex than a simple OLS regression, and as such the results it produces are likely more sensitive to issues such as lag order selection, and initial conditions (i.e., when the sample starts).

[27] Interestingly, for the U.S. the relationship between the nominal rate gap and the detrended

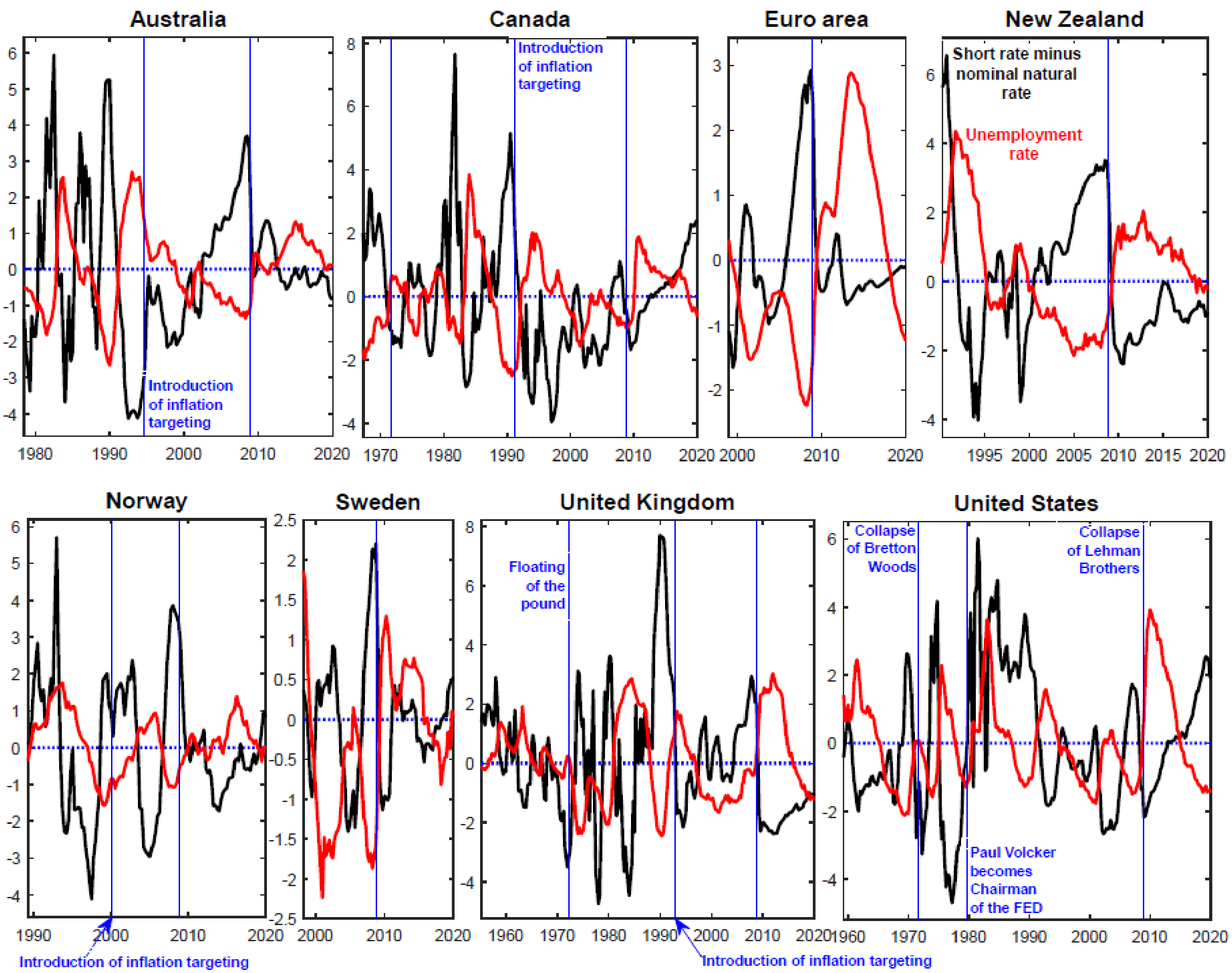


Figure 4 The nominal rate gap and the detrended unemployment rate

($v$) Before the collapse of Lehman Brothers, the probability that the nominal natural rate had been negative had consistently been (close to) *nil*. Since then, however, it has materially increased in Canada, the Euro area, Sweden, and the U.S., whereas it has exhibited little variation in the remaining countries. In particular, at the end of the sample the probability was equal to 95 per cent in Sweden, 50 per cent in Canada, and around 60 per cent in both the Euro area and the U.S..

I now turn to discussing the integration properties of inflation.

# 5 Monetary Regimes and the Stochastic Properties of Inflation

Table C.1 in Online Appendix C reports results from tests for multiple breaks at unknown points in the sample in the mean of inflation based on the methodology proposed by Bai and Perron (1998, 2003).[28] For Australia, Canada, New Zealand, Norway, Sweden, and the United Kingdom I focus on the sample period since the introduction of inflation targeting;[29] for the Euro area I consider the period since the start of European Monetary Union, in January 1999; and for the United States the period following the break in the mean of inflation identified by Levin and Piger (2004), in 1992Q2. The null hypothesis of no breaks in the mean of inflation cannot be rejected for any country.

Table 1 reports bootstrapped $p$-values[30] for Elliot, Rothenberg, and Stock (1996) unit root tests for inflation, together with Hansen (1999) 'grid bootstrap' median-unbiased (MUB) estimates of the sum of the autoregressive coefficients ($\rho$) in AR($p$) representations for inflation.[31] In both cases I set the number of bootstrap replications

unemployment rate had been put temporarily off kilter by the introduction of Money Market Deposits Accounts (MMDAs) in 1982Q4. After a brief period of adjustment, however, the relationship strongly reasserted itself since the second half of the 1980s. This confirms the meaningfulness of working with Lucas and Nicolini's (2015) 'New M1' aggregate: what it suggests is that in fact New M1 is the equivalent, for the period since the 1980s, of the standard M1 aggregate for the previous period. Benati (2021*b*) presents additional evidence on this based on a comparison between the evolution of M1 velocity and of long-term interest rates.

[28] In performing the tests I exactly follow the recommendations of Bai and Perron (2003), with the only difference that, instead of relying on the asymptotic critical values tabulated in Bai and Perron (1998), I bootstrap the $p$-values *via* the procedure proposed by Diebold and Chen (1996), setting the number of bootstrap replications to 10,000.

[29] In Canada, New Zealand, Norway, Sweden, and the United Kingdom inflation targeting was was introduced, respectively, in February 1991, February 1990, March 2001, January 1993, and October 1992. As for Australia, which never formally announced an inflation target, I consider the period since mid-1994 (specifically, since 1994Q3), when the *Reserve Bank of Australia* started to target inflation *de facto*.

[30] $p$-values have been computed by bootstrapping 10,000 times estimated ARIMA($p$,1,0) processes.

[31] For Hansen's (1999) procedure, I select the lag order $p$ as the maximum between the lag orders selected by the Schwartz and Hannan-Quinn criteria, and I set the 'step' in the grid of possible values for $\rho$ to 0.01.

to 10,000. As for the sample periods, I consider both the previously mentioned monetary regimes featuring clearly-defined nominal anchors,[32] and, depending on data availability, the previous periods.

**Table 1 Exploring inflation persistence by monetary regime**

| *Country* | *Period* | Bootstrapped p-values for $p$=2 | $p$=4 | $p$=6 | $p$=8 | Hansen MUB estimate of $\rho^b$ |
|---|---|---|---|---|---|---|
| | | *I: Regimes with clearly-defined nominal anchors* | | | | |
| Australia | 1994Q3-2019Q4 | 0.0012 | 0.0148 | 0.0908 | 0.3618 | 0.28 [0.05 0.51] |
| Canada | 1991Q1-2019Q4 | 0.0000 | 0.0002 | 0.0002 | 0.0009 | -0.08 [-0.35 0.19] |
| Euro area | 1999Q1-2019Q4 | 0.1015 | 0.2230 | 0.1919 | 0.3032 | 0.66 [0.46 0.88] |
| New Zealand | 1990Q1-2019Q4 | 0.0000 | 0.0001 | 0.0060 | 0.0029 | -0.21 [-0.51 0.10] |
| Norway | 2001Q2-2019Q4 | 0.0016 | 0.0047 | 0.0616 | 0.1582 | 0.22 [-0.09 0.52] |
| Sweden | 1998Q1-2019Q4 | 0.0000 | 0.0210 | 0.0640 | 0.1115 | 0.30 [-0.15 0.78] |
| United Kingdom | 1992Q4-2019Q4 | 0.0000 | 0.0001 | 0.0030 | 0.0060 | -0.40 [-0.66 -0.16] |
| United States | 1992Q2-2019Q4 | 0.0081 | 0.0501 | 0.0704 | 0.0090 | 0.49 [0.35 0.64] |
| | | *II: Previous periods* | | | | |
| Australia | 1972Q2-1994Q2 | 0.1627 | 0.7399 | 0.5881 | 0.2010 | 1.01 [0.78 1.07] |
| Canada | 1967Q2-1990Q4 | 0.2220 | 0.4366 | 0.3495 | 0.2957 | 0.90 [0.73 1.03] |
| Euro area | 1970Q2-1998Q4 | 0.4598 | 0.4598 | 0.7095 | 0.8493 | 1.01 [0.92 1.04] |
| Norway | 1985Q2-2001Q1 | 0.0050 | 0.0711 | 0.2865 | 0.2605 | 0.34 [-0.04 0.79] |
| United Kingdom | 1955Q2-1992Q3 | 0.0410 | 0.1942 | 0.1752 | 0.2245 | 0.87 [0.74 1.02] |
| United States | 1959Q2-1992Q1 | 0.3491 | 0.3266 | 0.3062 | 0.3316 | 0.92 [0.85 0.99] |

[a] Based on 10,000 bootstrap replications. [b] With 90% bootstrapped confidence interval.

The evidence in Table 1 confirms the findings in Benati (2008). In particular, for regimes with clearly-defined nominal anchors,

($i$) the point estimates of $\rho$ produced by Hansen's procedure range between -0.40 and 0.66, and the upper limits of their bootstrapped 90%-coverage confidence interval range between -0.16 and 0.88: based on Hansen's procedure there is no evidence that, under these regimes, inflation may have been I(1).

($ii$) By the same token, based on Elliot *et al.*'s tests a unit root in inflation is strongly rejected for Canada, New Zealand, the United Kingdom, and the United States, and for Australia, Norway, and Sweden it is rejected, at the 10 per cent level, for all lag orders except $p$=8. Only for the Euro area a unit root cannot be rejected for any lag order.

For the previous periods, which had been largely dominated by the Great Inflation episode, the opposite is true. Starting from Hansen's MUB estimates of $\rho$, the point

[32] Strictly speaking, the U.S. Federal Reserve introduced an inflation target only in January 2012. In what follows I consider the entire period since 1992Q2 because, even before the introduction of a formal target, the Fed's monetary policy had been characterized since the end of the Volcker disinflation by a strong, although generic committment to price stability.

estimate is borderline explosive for Australia and the Euro area, and for four countries (Australia, Canada, Euro area, and United Kingdom) the 90 per cent confidence interval includes 1, whereas for the United States, with an upper bound equal to 0.99, this is almost the case. Likewise, based on Elliot *et al.*'s tests the null of a unit root cannot be rejected for any lag order for Australia, Canada, the Euro area, and the United States, whereas evidence is mixed for Norway, and for the United Kingdom it can be rejected only for $p$=2.

These results confirm Benati's (2008) main finding that whereas for sample periods dominated by the Great Inflation experience it is typically not possible to reject the null hypothesis of a unit root in inflation, under monetary regimes, such as inflation targeting, featuring a clearly-defined nominal anchor (or, in the case of the United States before the introduction of an inflation target, a generic, but strong and credible committment to keeping inflation low and stable), inflation has consistently been I(0). Under this respect, the results from Elliot *et al.*'s tests for the Euro area should be quite heavily discounted for two reasons. First, the visual evidence in Figure A.1 in the Online Appendix clearly suggests that the collapse of Lehman Brothers, which unleashed the most violent phase of the Great Recession, was associated with a dramatic, highly persistent, but ultimately transitory fall in Euro area inflation, from an average of 2.01 per cent over the period 1999Q1-2008Q3,[33] to 1.05 per cent over the period 2008Q4-2017Q3. Over the subsequent period inflation has progressively converged towards 2 per cent. A possible, and (I would argue) plausible interpretation of the lack of rejection of a unit root for the period 1999Q1-2019Q4 is therefore that it is the figment of a very large negative transitory shock, which in a small sample can easily be confused for a permanent one. Second, in spite of such persistent downard shift in inflation, inflation expectations (as measured by the ECB's Survey of Professional Forecasters) have remained well-anchored,[34] thus suggesting that agents have correctly interpreted the shift as temporary.

In what follows I will therefore work under the assumption that, for the sample periods reported in Table 1 as 'regimes with clearly-defined nominal anchors', inflation has consistently been I(0). I now turn to discussing the estimates of the real natural rate for such monetary regimes.

[33] In fact, in line with Benati (2008), for the period 1999Q1-2008Q3 the bootstrapped $p$-values for Elliot *et al.*'s tests are equal to 0.1077, 0.0416, 0.0249, and 0.0134, and Hansen's (1999) MUB estimate of $\rho$ is 0.41 [0.02 0.82].

[34] See in particular Figure A.3 in the Online Appendix of Benati (2020). The figure shows the inflation forecasts from the ECB's Survey of Professional Forecasters at three alternative horizons, 1-, 2-, and 5-years ahead. Over the entire period since 1999Q1, the 5-years ahead forecast has fluctuated between 1.8 and 2.0 per cent.

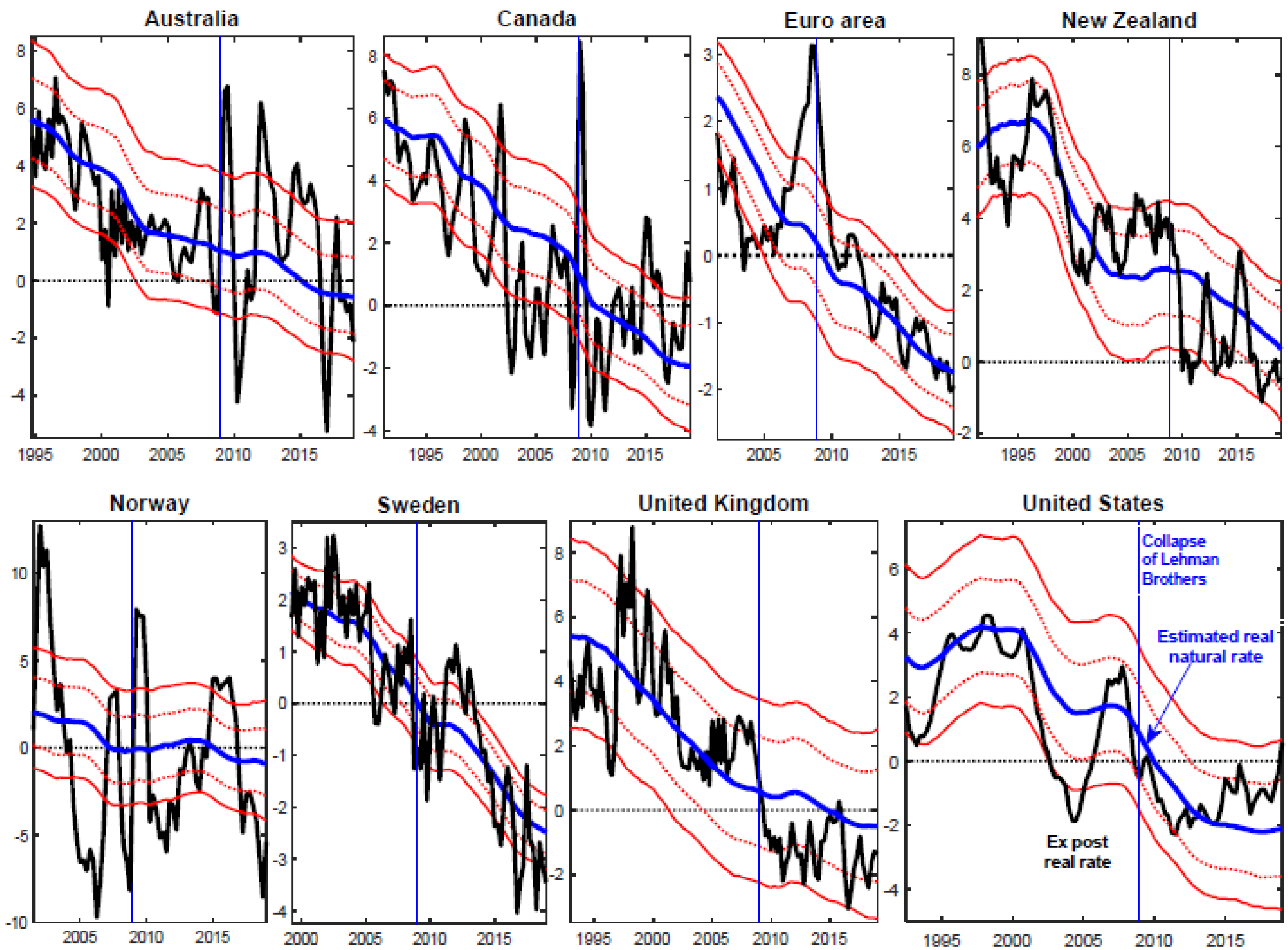


Figure 5 Estimates of the real natural rate for monetary regimes making inflation I(0) (with 1- and 2-standard deviations bootstrapped confidence bands) computed by projecting the short rate on M1 velocity

# 6 Estimating the Real Natural Rate

Figure 5 shows, for monetary regimes with clearly-defined nominal anchors,[35] the *ex post* short-term real rate, computed as the difference between the short-term nominal rate and inflation, together with the estimated real natural rate, which has been computed by subtracting inflation's sample average from the nominal natural rate estimates shown in Figure 3.[36] Figure A.10 in the Online Appendix shows the corresponding estimates based on cointegrated SVARs identified *via* long-run restrictions. For all countries, the estimated real natural rate behaves, as expected, as a very low-frequency component of the *ex post* short-term real rate. The following main results emerge from the two figures:

(*i*) consistent with both conventional wisdom, and previous evidence—see in particular Holston *et al.* (2017), and Fiorentini *et al.* (2018)—in all countries natural rate estimates have been consistently trending downwards over the entire sample period. The decrease has been especially marked for Australia, New Zealand, the U.K., and Canada: since the first half of the 1990s, the point estimate of the natural rate has fallen by about 6 per cent for the first three countries, and by about 8 per cent for the fourth. By the same token, in the U.S. it has fallen by about 6 percentage points since the peak of 4.1 per cent reached in the second half of the 1990s around the time of the 'New Economy', whereas in both the Euro area and Sweden the decrease since the start of the new millennium has been equal to about 4 percentage points.

(*ii*) Different from the corresponding results for the nominal natural rates discussed in point (*v*) of Section 4, in several countries the probability that the real natural rate had been negative had already been increasing before the collapse of Lehman Brothers. This is the case especially for the Euro area, Norway, and the U.K.. Following Lehman's collapse the probability has markedly increased in all countries except Norway. In particular, at the end of the sample the probability was equal to 100 per cent in both the Euro area and Sweden, whereas in Canada and the U.S. it was slightly greater than 90 per cent. This evidence provides support to the conjecture, first advanced by Summers (1991), that following large negative shocks the natural rate might fall below zero.

(*iii*) Further, in several countries the estimates are quite sobering, especially towards the end of the sample. In the U.S., for example, the point estimate has been equal to about -2 per cent since 2014, whereas in Canada, the Euro area and Sweden it has reached, in 2019, -1.9, -1.7, and -2.4 per cent, respectively. The only two countries for which in 2019 the point estimate was still (barely) positive were New Zealand and Norway.

These estimates are very similar to those produced by Fiorentini *et al.* (2018) based on a modified (and, they argue, superior) version of the methodology originally

[35] The sample periods are the same reported in Table 1.

[36] Once again, in order to eliminate some low-frequency variation, all series have been smoothed *via* a 4-quarter centered moving average.

proposed by Laubach and Williams (2003), and more recently used, e.g., by Laubach and Williams (2016) and Holston *et al.* (2017). For example, in Fiorentini *et al.*'s (2018) Figure 10, the U.S. natural rate decreased from 2.5-3 per cent in the second half of the 1990s to about -2 per cent in 2016, whereas that for the Euro area fell from 2 per cent in 2000 to about -1 per cent in 2016. These figures are very close to those in Figure 5 in the present work. On the other hand, my estimates are lower than those found in Holston *et al.* (2017), but based on Fiorentini *et al.*'s (2018) arguments those estimates should be regarded as less reliable. My estimates for the U.S. are also materially lower than the range of estimates reported in Williams' (2017) Figure 1, which for 2016 was between about 0 and about 1 per cent.

Estimates of the real natural rate as low as those in Figure 5, as well as in Fiorentini *et al.* (2018), raise an obvious question: Are they plausible? Could the real natural rate truly sink *that* low? This question is best addressed by focusing on ($i$) the relationship between the natural rate and the *ex post* real rate, and ($ii$) the behavior of GDP and inflation over the sample period. Let us consider for example Sweden, with an estimated natural rate of -2.5 per cent at the end of 2019. At first sight, this number might appear to some researchers as manifestly absurd. It becomes however much less absurd, and much more plausible, when one considers that since Lehman's collapse (and in fact since the beginning of the millennium) the natural rate has closely tracked the dramatic decrease in the *ex post* real rate: this suggests that *on average* the *Riksbank*'s monetary policy has been broadly neutral, and that the fall in the *ex post* real rate it has engineered by decreasing the monetary policy rate was simply a reaction to the progressive fall in the natural rate. The evolution of prices and output is consistent with this: since the 2008-2009 recession annual inflation and GDP growth have both been broadly stable, the former slowly increasing from about 1 per cent in early 2010 to 2.5 per cent at the end of 2019, and the latter fluctuating around an average of about 2 per cent. A very similar argument can be made for the remaining countries. This suggests that central banks have been broadly tracking the natural rate, and that the progressive decreases in *ex post* real rates across the board have simply reflected the underlying fall in the natural rates. In turn, this suggests that the estimates in Figure 5 are likely plausible.[37]

Finally, it is worth highlighting how, in line with Taylor (2008, 2009), for the U.S. a comparison between the *ex post* real rate and the estimated natural rate suggests that monetary policy had been highly expansionary during the years immediately preceding the outbreak of the financial crisis. In particular, in 2004 the *ex post* real rate had been below the natural real rate, on average, by about 300 basis points.

I now turn to discussing the advantages of the methodology I am advocating compared to existing approaches.

[37] It is also worth recalling that DSGE-based estimates are often much more volatile. For example, in Barsky *et al.*'s (2014) Figure 1 the U.S. natural rate has fluctuated, since the early 1990s, between about -7 and about 12 per cent, i.e. over a range of *nearly 20 percentage points*.

# 7 Advantages of the Proposed Approach

Compared to existing approaches to the estimation of the natural rate, the one proposed herein features two advantages, which I discuss in turn.

## 7.1 Under monetary regimes making inflation I(0) the real natural rate is *observed*

As discussed in Section 3.3, under monetary regimes making inflation I(0)—so that, in (9), $\pi_t^P$=0—permanent shifts in M1 velocity *uniquely* reflect, to a first approximation, permanent fluctuations in the real natural rate of interest. In fact, as long as $\eta_t$ in expression (4) is 'small', $V_t \simeq \alpha + \beta r_t^N$, so that $r_t^N \simeq (V_t - \alpha)/\beta$: in plain English, under such regimes the real natural rate of interest is, up to a linear transformation, *observed*. An immediate implication is that a consistent decrease in M1 velocity under a monetary regime causing inflation to be I(0)—such as the protracted fall in velocity that has been going on in several inflation-targeting countries since the early 1990s—provides *direct evidence* of a fall in the real natural rate of interest.

The fact that the approach I am advocating herein relies on a series that, under monetary regimes making inflation I(0), is essentially a linear transformation of the real natural rate highlights a stark difference with existing approaches (either DSGE- or non-DSGE based), none of which exploits a series with such a strong informational content for the real natural rate.

## 7.2 Computing high-frequency estimates of the natural rate

Since interest rates are observed on a continuous basis, and M1 is observed (at least) at the weekly frequency, all a researcher needs in order to compute high-frequency estimates of the nominal and real natural rates of interest is a corresponding high-frequency estimate of nominal GDP. Interpolating quarterly GDP to the monthly frequency[38] is routinely done in the literature (for the United States, see e.g. Bernanke, Gertler, and Watson, 1997, and Stock and Watson, 2012). The recent work of (e.g.) Lewis, Mertens, and Stock (2020) about tracking the economic impact of the COVID pandemic has shown how to perform a similar interpolation at the weekly frequency.[39] Based on a weekly estimate of nominal GDP, and weekly observations for M1 and nominal interest rates, a central bank could therefore, in principle, produce weekly estimates of nominal and real natural rates.

Figure 6 presents estimates of nominal and real natural rates at the monthly frequency for Canada, the Euro area, the United Kingdom and the United States,

[38] To the very best of my knowledge, Canada and the U.K. are the only countries producing official monthly estimates of real GDP. U.K. estimates start however in 1997, so that in the present work I have relied on the unofficial estimates from NIESR (for details, see Online Appendix A).

[39] The be precise, Lewis *et al.* (2020) focus on real GDP, but their methodology can obviously also be applied to nominal GDP.

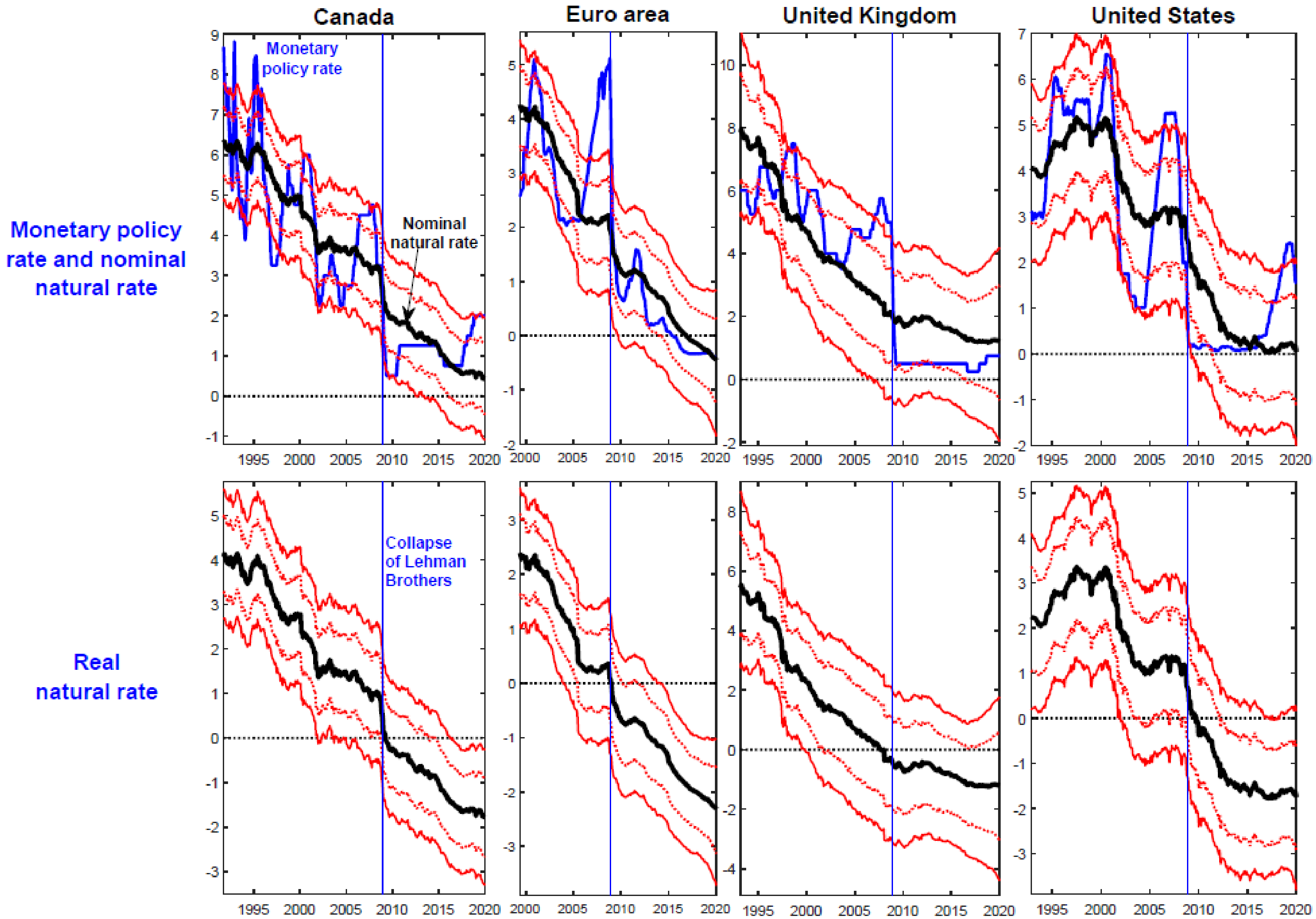


Figure 6 Monthly estimates of the nominal and real natural rate of interest for monetary regimes making inflation I(0), computed by projecting the short rate on M1 velocity (with 1- and 2-standard deviations bootstrapped confidence bands)

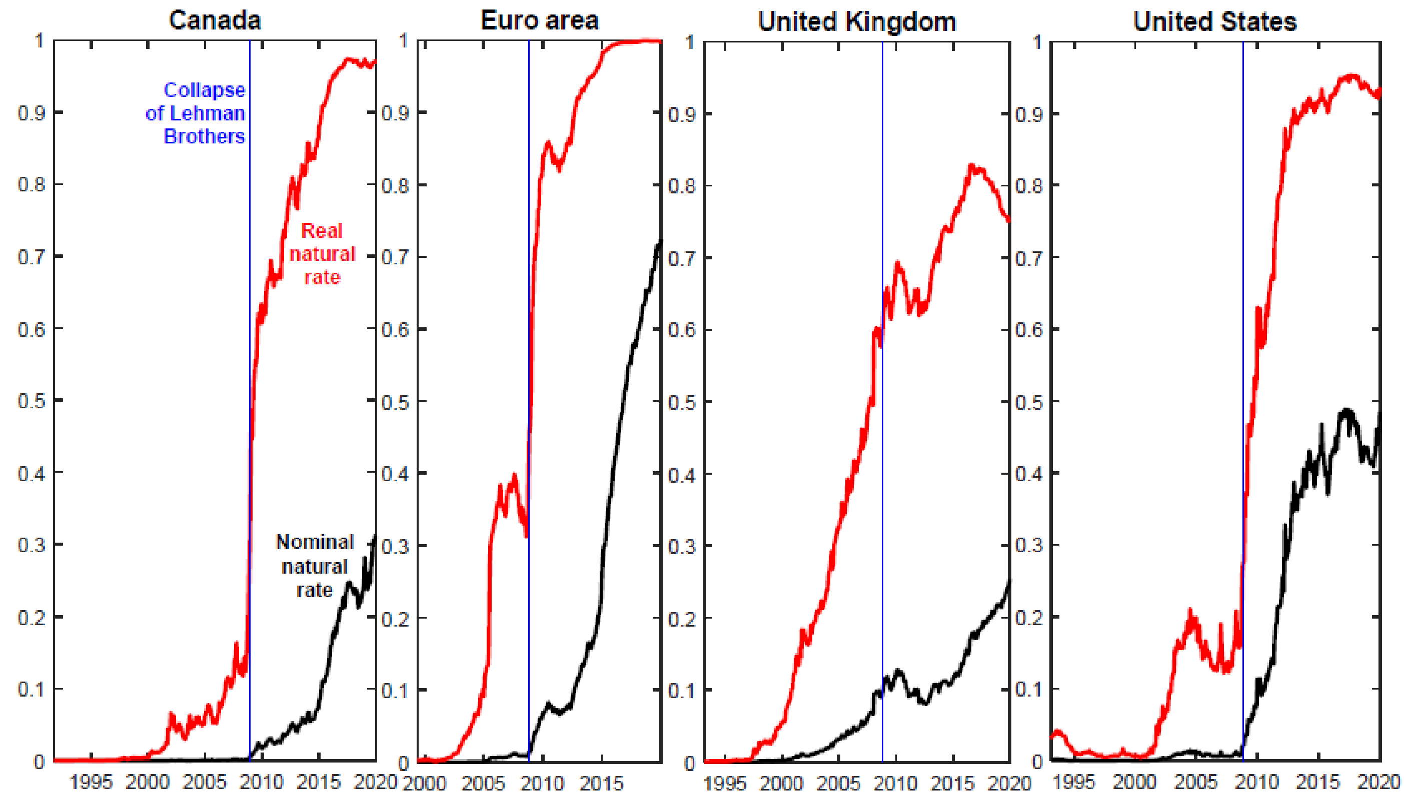


Figure 7 Evidence based on monthly data: fractions of bootstrap replications for which the nominal and the real natural rates of interest are estimated to have been negative

whereas Figure 7 reports the fractions of bootstrap replications for which the nominal and real natural rates are estimated to have been negative.[40] Table 2 reports, for selected months around the collapse of Lehman Brothers, point estimates of the real natural rate, together with the fractions of bootstrap replications for which the real natural rate is estimated to have been negative. The methodology is exactly the same used in order to produce the estimates reported in Figures 3-5. Different from those estimates, however, those in Figure 6-7 have not been smoothed in any way. On the one hand, this causes the estimates to retain a small extent of high-frequency variation, which is obviously sub-optimal from a monetary policy perspective. On the other hand, however, this highlights in an especially stark way how, following the collapse of Lehman Brothers on September 15, 2008, in three countries out of four (Canada, the Euro area, and the United States) both the nominal and the real natural rates experienced sharp and sudden declines.

**Table 2 Point estimates of the real natural rate, and probability that it had been negative, in the months around the collapse of Lehman Brothers**

| | Point estimates of the real natural rate | | | | Probability that the real natural rate had been negative[a] | | | |
|---|---|---|---|---|---|---|---|---|
| | CA | EA | UK | US | CA | EA | UK | US |
| August 2008 | 0.90 | 0.35 | -0.28 | 1.09 | 0.15 | 0,32 | 0.57 | 0.18 |
| October 2008 | 0.60 | 0.08 | -0.34 | 0.67 | 0.23 | 0,46 | 0.59 | 0.28 |
| December 2008 | 0.09 | -0.02 | -0.50 | 0.38 | 0.46 | 0,51 | 0.63 | 0.37 |
| June 2009 | -0.12 | -0.44 | -0.45 | 0.11 | 0.56 | 0,73 | 0.62 | 0.46 |
| December 2009 | -0.23 | -0.67 | -0.68 | -0.38 | 0.61 | 0,83 | 0.68 | 0.63 |

CA = Canada; EA = Euro area; UK = United Kingdom; US = United States. [a] Fraction of bootstrap replications for which the real natural rate is estimated to have been negative.

Focusing on the real natural rate, the decline is clearly apparent both from the estimates shown in the bottom row of Figure 6, and especially from the fractions of bootstrap replications reported in Figure 7, and from the figures in Table 2. In the *two months* from August to October 2008 —i.e., from one monthy before to one month after Lehman's collapse—the real natural rate declined in Canada, the Euro area, and the U.S. by -0.30, -0.27, and -0.42 per cent: these are quite remarkable decreases, corresponding to -1.80, -1.62, and -2.52 per cent on annual basis. For the U.K. the decrease, equal to -0.06 from August to October, was comparatively minor, but it still corresponded to a fall by -0.36 per cent on an annual basis. For Canada, the Euro area, and the U.S. the decrease from August to December 2008 was equal to -0.81, -0.37, -0.71 per cent, whereas the corresponding figures for the period up to

[40]The monthly data are discussed in Appendix C, and more extensively in Online Appendix A.3.

December 2009 had been respectively equal to -1.13, -1.02, and -1.47 per cent. Even for the comparatively less affected U.K., the decline from August 2008 to December 2009 had been equal to -0.40 per cent.

By the same token, the fractions of bootstrap replications for which the real natural rate is estimated to have been negative literally skyrocketed in October 2008 for both Canada and the Euro area, and it increased very sharply, although less dramatically, for the U.S.. For the U.K., on the other hand, the increase has been continuous over the entire sample period, and it had apparently been unaffected by Lehman's collapse.

A *caveat* to these results[41] is that the higher the frequency of the data that is being considered, the more difficult it is for economic agents to effectively perform the permanent-transitory decomposition that form the basis of the approach proposed herein. Intuitively, whereas it is by no means unreasonable to believe that, within the space of a quarter, agents may be able to effectively disentangle the permanent and transitory components of short-term nominal rates, this assumption becomes much less credible, e.g., at the weekly frequency.

I now turn to two additional applications of the proposed methodology.

# 8 Two Additional Applications

In this section I present two additional applications of the proposed methodology which should be regarded with some caution: the first because of the comparatively lower quality of pre-WWII data, and the second because of the idiosyncratic nature of the COVID shock.

## 8.1 The evolution of the natural rate during the Great Depression

Figure 8 shows the evolution of the nominal and real U.S. natural rates during the interwar period, based on the same methodology underlying the estimates reported in Figures 3-7.[42] The sample period, 1920Q1-1941Q3 is bookended by the end of World War I, and by Japan's attack on Pearl Harbor. With the partial exception of a temporary downward fall during the deep recession of 1921, the real rate of interest had been broadly stable during the entire decade of the 1920s. Between the October 1929 stockmarket crash and Roosevelt's inauguration in March 1933,

[41] I wish to thank a referee for pointing this out.

[42] The data are described in detail in Online Appendix A.2.8. In short, the M1 aggregate is from Friedman and Schwartz (1963), the monetary policy rate is the Federal Reserve Bank of New York's discount rate, and nominal GNP and the GNP deflator are from Balke and Gordon (1986). Bootstrapped $p$-values for Elliot *et al.* tests for inflation for $p = 2, 4, 6, 8$ are equal to 0.0015, 0.0425, 0.0708, 0.0814, thus strongly rejecting the null of a unit root, so that the same logic used in Sections 6 and 7 can also be applied for the interwar period.

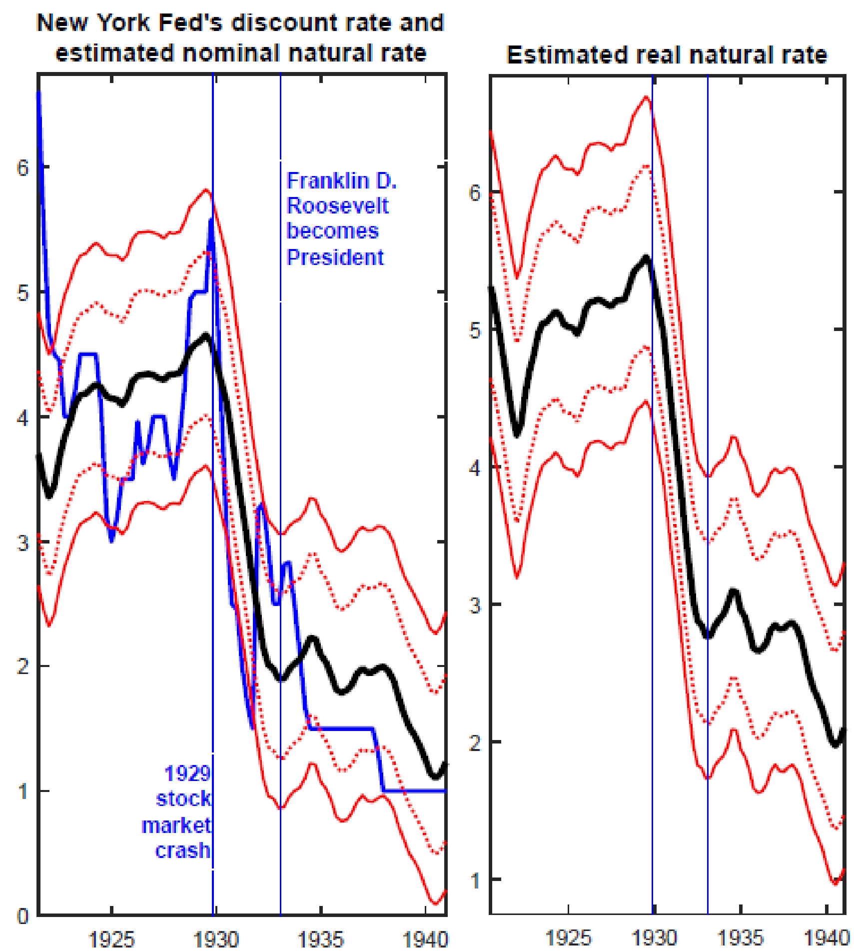


Figure 8 The evolution of the U.S. natural rate during the Great Depression

however, it collapsed from 5.6 to 2.8 per cent. Although starting in 1933 the natural rate temporarily stabilized around 2.8-3.0 per cent, following (the mistake of) 1938 it fell by nearly an additional percentage point in the period immediately preceding the United States' entry into World War II.

These results naturally lend themselves to an admittedly imperfect comparison with Eggertsson (2008). There are two key tenets of Eggertsson's analysis: first, the onset of the Great Depression was caused by a dramatic fall in the real natural rate of interest; second, the recovery that followed Roosevelt's inauguration was not caused by a rebound of the natural rate around its previous level, but rather by the radical change in monetary and fiscal policies associated with the New Deal. The estimates in the right hand-side panel of figure 8 accord well with Eggertsson's analysis. On the one hand, between the crash of 1929 and Roosevelt's inauguration the natural rate decreased by about 2.8 percentage points. On the other hand, after March 1933 the natural rate did not increase, but it rather stabilized until 1938, and then it further collapsed.

## 8.2 Estimating the impact of the COVID shock on the natural rate

In order to avoid that the analyses of Sections 4, 6, and 7.2 be possibly distorted by the highly idiosyncratic nature of the COVID shock, I ended all of the samples there in 2019. In this section, on the other hand, I (very tentatively) attempt to apply the framework proposed herein to estimate the impact of the COVID shock on the natural rate. Although the unprecedented (in about a century) nature of the shock suggests to treat these results with significant *caveats*, on the other hand we have no reason to believe that fundamental laws of economics should somehow become 'suspended' during a pandemic. It is therefore of interest to see what the approach I am advocating has to say about the impact of COVID.

**Table 3 Point estimates of the real natural rate for the months around the outbreak of COVID**

| | CA | EA | UK | US |
|---|---|---|---|---|
| January 2020 | -1.63 | -2.28 | -1.99 | -1.64 |
| February | -1.69 | -2.33 | -2.03 | -1.68 |
| March | -2.08 | -2.50 | -2.34 | -2.04 |
| April | -2.50 | -2.74 | -2.88 | -2.63 |
| May | -2.45 | -2.76 | -2.85 | -2.59 |
| June | -2.39 | -3.03 | -2.43 | -2.48 |

CA = Canada; EA = Euro area; UK = United Kingdom; US = United States.

Table 3 reports, for the same four countries in Table 2, point estimates of the real natural rate for the months around the outbreak of the pandemic. The main finding emerging from the table is that the impact of COVID on the natural rate has been broadly comparable to that of the collapse of Lehman Brothers. For the U.S., for example, the decrease between January 2020 (one month before the outbreak) and June had been equal to -0.84 per cent, whereas for Canada, the Euro area and the U.K. it had been equal, respectively, to -0.76, -0.75, and -0.44 per cent. Once again, it is important to stress the very tentative nature of these results and the significant *caveats* they are subject to. At the same time, taken at face value they suggest that the two crises had a very similar impact on the natural rate.

# 9 Conclusions

Since the early 1980s it has been conventional wisdom among macroeconomists and policymakers that monetary aggregates contain little useful information for monetary policy. In this paper I have shown that, in fact, a specific transformation of a monetary aggregate, the velocity of M1, contains crucial information about the evolution of the real natural rate of interest. Building upon the the insight that M1 velocity is the permanent component of nominal interest rates (see Benati, 2020), I have proposed a new and straightforward approach to estimating the natural rate of interest, which is conceptually related to Cochrane's (1994*a*) proposal to estimate the permanent component of GDP by exploiting the informational content of consumption. Under monetary regimes (such as inflation-targeting) making inflation I(0), the easiest way to implement the proposed approach is to (*i*) project the monetary policy rate onto M1 velocity—thus obtaining an estimate of the *nominal* natural rate—and then (*ii*) subtract from this inflation's sample average (or target), thus obtaining the *real* natural rate. More complex implementations based on structural VARs produce very similar estimates. Compared to existing approaches, the one proposed herein presents two key advantages: (1) under regimes making inflation I(0), M1 velocity is equal, up to a linear transformation, to the real natural rate, so that the natural rate is, in fact, *observed*; and (2) based on a high-frequency estimate of nominal GDP, the natural rate can be computed at the monthy or even weekly frequency. In the U.S., Euro area, and Canada the natural rate dropped sharply in the months following the collapse of Lehman Brothers. Likewise, the 1929 stock market crash was followed in the U.S. by a dramatic decrease in the natural rate.

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# A Results from unit root tests

Table A.1 reports bootstrapped $p$-values for Elliot, Rothenberg, and Stock (1996) unit root tests (with an intercept, but no time trend) for M1 velocity and a short-term nominal interest rate. The $p$-values have been computed by bootstrapping estimated ARIMA($p$,1,0) processes *via* the procedure proposed by Diebold and Chen (1996), setting the number of bootstrap replications to 10,000.

**Table A.1 Bootstrapped $p$-values for Elliot, Rothenberg, and Stock unit root tests[a] for velocity and a short rate**

| *Country* | *Period* | $p$=2 | $p$=4 | $p$=6 | $p$=8 |
|---|---|---|---|---|---|
| | | M1 velocity | | | |
| Australia | 1972Q1-2019Q4 | 0.9557 | 0.9573 | 0.9508 | 0.9841 |
| Canada | 1982Q3-2019Q4 | 0.2122 | 0.3114 | 0.3555 | 0.1592 |
| Euro area | 1970Q1-2019Q4 | 0.0733 | 0.3247 | 0.2133 | 0.1701 |
| New Zealand | 1990Q1-2019Q4 | 0.8679 | 0.8850 | 0.8748 | 0.7521 |
| Norway | 1985Q1-2019Q4 | 0.0044 | 0.0045 | 0.0011 | 0.0012 |
| Sweden | 1998Q1-2019Q4 | 0.7846 | 0.7036 | 0.7687 | 0.6471 |
| United Kingdom | 1963Q1-2019Q4 | 0.9884 | 0.9647 | 0.9436 | 0.9268 |
| United States | 1959Q1-2019Q4 | 0.8686 | 0.8662 | 0.8576 | 0.8340 |
| | | Short rate | | | |
| Australia | 1972Q1-2019Q4 | 0.4980 | 0.5122 | 0.5269 | 0.4098 |
| Canada | 1982Q3-2019Q4 | 0.0792 | 0.4167 | 0.4657 | 0.5730 |
| Euro area | 1970Q1-2019Q4 | 0.5228 | 0.6106 | 0.6629 | 0.5678 |
| New Zealand | 1990Q1-2019Q4 | 0.1123 | 0.0809 | 0.0258 | 0.0972 |
| Norway | 1985Q1-2019Q4 | 0.0431 | 0.3585 | 0.4475 | 0.4890 |
| Sweden | 1998Q1-2019Q4 | 0.3808 | 0.3850 | 0.4061 | 0.5515 |
| United Kingdom | 1963Q1-2019Q4 | 0.5847 | 0.4865 | 0.5016 | 0.5134 |
| United States | 1959Q1-2019Q4 | 0.3162 | 0.2854 | 0.2914 | 0.2031 |

[a] Based on 10,000 bootstrap replications.

In nearly all cases, evidence of a unit root for either series is very strong. The only exceptions are the short rate for New Zealand and M1 velocity for Norway, for which the null of unit root is rejected. In what follows I will proceed under the assumption that all series have been I(1) over the sample periods analized herein,[43] and that the two rejections of the null of a unit root are statistical fluke, possibly due to small-sample issues. There are two reasons for doing so. First, even a perfectly sized test, by definition, incorrectly rejects the null hypothesis $x$ per cent of the time at the $x$

[43]This is an important qualification. Under metallic standards—for which inflation had been uniformly I(0), and in fact most of the time statistically indistinguishable from white noise (see Benati, 2008)—it is often possible to reject the null hypothesis of a unit root in short-term interest rates, as one would logically expect if the natural real rate featured comparatively little variation (this evidence is available upon request).

per cent level. When performing many statistical tests, such as in the present case, a certain fraction of 'fluke rejections' of the null should therefore be logically expected. Sure enough, the data I am using herein have not been randomly generated as part of a Monte Carlo experiment,[44] but the basic logic of this argument still holds. For example, taking the argument literally—i.e., as if we were here dealing with a Monte Carlo experiment featuring *independent* random draws—the ten rejections (at the 10 per cent level) of the null of a unit root reported in Table B.1 represent 15.6 per cent of the overall number of tests reported in the table, i.e. just 5.6 per cent in excess of the 10 per cent of 'fluke rejections' we would expect from a perfectly sized test with independent Monte Carlo artificial samples. Second, visual evidence (see Figure 1*a*) strongly suggests that all series have been non-stationary over the sample periods analized herein.

# B Evidence on Cointegration Between M1 Velocity and the Short Rate

Table B.1 reports, for bivariate systems featuring M1 velocity and a short-term rate, (*i*) bootstrapped *p*-values for Johansen's maximum eigenvalue tests of the null hypothesis of 0 *versus* 1 cointegration vectors, and (*ii*) 90%-coverage bootstrapped confidence intervals for the second element of the normalized cointegration vector based on Wright's (2000) methodology. As for Johansen's tests, following Benati (2020) and Benati *et al.* (2021), I bootstrap them *via* the procedure proposed by Cavaliere *et al.* (2012, henceforth CRT).[45] I select the VAR lag order as the maximum[46] between the lag orders chosen by the Schwartz and the Hannan-Quinn criteria[47] for the VAR in levels, and I estimate the VECM based on Johansen's estimator as detailed in Hamilton (1994). As for Wright's (2000) test, since it has been designed to be equally valid for data-generation processes (DGPs) featuring either exact or near unit roots, following Benati (2020) and Benati *et al.* (2021) I consider two alternative bootstrapping procedures, corresponding to either of the two possible cases. The first procedure involves bootstrapping as in CRT the cointegrated VECM estimated by imposing one cointegration vector. This procedure is the correct one if the data feature *exact*

[44] In particular, the data for individual countries are not *independent* random draws, since all countries experienced common events such as the Great Inflation of the 1970s, the disinflation of the early 1980s, the spread of globalization, and the 2008-2009 financial crisis.

[45] For details see Online Appendix B.2, which also discusses Monte Carlo evidence on the performance of CRT's procedure.

[46] I consider the maximum between the lag orders chosen by the SIC and HQ criteria because the risk associated with selecting a lag order smaller than the true one (model mis-specification) is more serious than the one resulting from choosing a lag order greater than the true one (over-fitting).

[47] On the other hand, I do not consider the Akaike Information Criterion since, as discussed (e.g.) by Luetkepohl (1991), for systems featuring I(1) series the AIC is an inconsistent lag selection criterion, in the sense of not choosing the correct lag order asymptotically.

unit roots. For the alternative possibility in which the two series are *near* unit root processes, I proceed as follows. Based on the just-mentioned cointegrated VECM estimated by imposing one cointegration vector, I compute the implied VAR in levels, which by construction features one, and only one, eigenvalue equal to 1.[48] I then turn such exact unit root VAR into its corresponding near unit root VAR, by shrinking the single unitary eigenvalue to $\lambda = 1 - 0.5\times(1/T)$, where $T$ is the sample length.[49] The bootstrapping procedure I implement for the second possible case, in which the two series are near unit root processes, is based on bootstrapping such near unit root VAR. In practice the two procedures produce near-identical results, and in Table B.1 I therefore uniquely report results based on bootstrapping the VECM estimated by imposing one cointegration vector.

**Table B.1 Bootstrapped $p$-values for Johansen's maximum eigenvalue tests for M1 velocity and a short-term rate, and 90% bootstrapped confidence intervals for the second element of the normalized cointegration vector based on Wright's (2000) tests[a]**

| *Country* | *Period* | $p$-values for maximum eigenvalue tests[b] | Results from Wright's test |
|---|---|---|---|
| Australia | 1969Q3-2019Q4 | 0.0661 | [-0.9255 -0.7013] |
| Canada | 1967Q1-2019Q4 | 0.0279 | [-1.1642 -0.1032] |
| Euro area | 1999Q1-2019Q4 | 0.0896 | [-0.6013 -0.2970] |
| New Zealand | 1990Q1-2019Q4 | 0.1584 | [-0.1643 -0.0642] |
| Norway | 1985Q1-2019Q4 | 0.0000 | NCD[c] |
| Sweden | 1998Q1-2019Q4 | 0.1136 | [-0.3642 -0.3081] |
| United Kingdom | 1955Q1-2019Q4 | 0.0201 | [-0.5323 -0.3441] |
| United States | 1959Q1-2019Q4 | 0.0985 | [-0.5634 -0.3672] |

[a] Based on 10,000 bootstrap replications. [b] Null of 0 *versus* 1 cointegration vectors.
[c] NCD = no cointegration detected.

Based on Wright's (2000) tests, the null hypothesis of cointegration is only rejected for Norway. Likewise, at the 10 per cent level Johansen's tests reject the null of 0 cointegration vectors for all countries except Sweden (marginally), and New Zealand (with a $p$-value of 0.1584). As in Benati (2020), in what follows I will therefore proceed under the assumption that M1 velocity and the short rate are cointegrated in all samples. Online Appendix B.4 reports results from Hansen and Johansen's (1999) Nyblom-type tests for stability in either the cointegration vector, or the vector of loading coefficients, in the estimated VECMs, and discusses Monte Carlo evidence on the performance of the tests. In short, evidence of breaks in either the cointegration

[48] Bootstrapping this VAR would be equivalent to bootstrapping the underlying cointegrated VECM, that is, it would be correct if the data featured exact unit roots.
[49] Once again, for details see Online Appendix B.2.

vector or the loading coefficients is nearly non-existent. In particular, based on either the Selden-Latané specification—which, as discussed, appears to be the one preferred by the data for low-inflation (and therefore low-interest rates) countries—or the log-log, not a single break in either the cointegration vector or the loading coefficients is identified. As for the semi-log, no break in the cointegration vector is identified for any country, whereas only for Norway a break in the loading coefficients is identified, although the $p$-value, at 0.0935, is essentially borderline.

# C Computing Permanent and Transitory GDP by Projecting GDP on Consumption

The left-hand side panel of Figure C.1 in this appendix shows the transitory component of U.S. GDP obtained by projecting log real GDP onto log real consumption, i.e. the residual from the cointegrating regression

$$\ln Y_t = a + b \ln C_t + e_t, \tag{C.1}$$

where $Y_t$ and $C_t$ are real GDP and real consumption, respectively (the two series are described in Online Appendix A.2.9., and their unit root and cointegration properties are discussed in footnote 7 in the main text). I estimate (C.1) *via* a simple OLS regression, but near-identical results are produced by Stock and Watson's (1993) dynamic OLS estimator. Two main findings emerge from the figure. First, the estimated transitory component of GDP captures remarkably well the peaks and troughs of the post-WWII U.S. business cycle as established by the NBER Business-Cycle Dating Committee (i.e., the vertical blue and red bars in the figure). Second, the transitory component interprets a sizeable portion of the fall in output associated with the Great Recession as permanent: this is clearly highlighted, e.g., by the fact that whereas the troughs of annual real GDP growth associated with the Volcker recession and the Great Recession had been equal to -2.6 and -3.9 per cent, respectively, the troughs of the corresponding transitory components of GDP obtained by projecting log real GDP onto log real consumption had been equal to -4.0 and -2.8 per cent, respectively. As a matter of logic, the only possible interpretation of this is that, when viewed though the lenses of consumption, the latter recession had been characterized by a significantly greater decrease in permanent GDP than the former.

The right hand-side panel of Figure C.1 provides simple, but powerful corroborating evidence that this may in fact had been the case. The figure shows log real GDP and rescaled log real consumption,[50] together with $(i)$ up to 2004Q4, the HP-filtered trend of log real GDP,[51] and $(ii)$ starting from 2005Q1, the forecast of the HP-filtered trend, which I computed recursively by exploiting the fact that, in the state-space

[50] I rescaled log real consumption as in Cochrane's (1994*a*) Figure III, i.e. by adding to it the mean log ratio between GDP and consumption.

[51] I set the smoothing parameter to the standard value of 1600 for quarterly data, but qualitatively

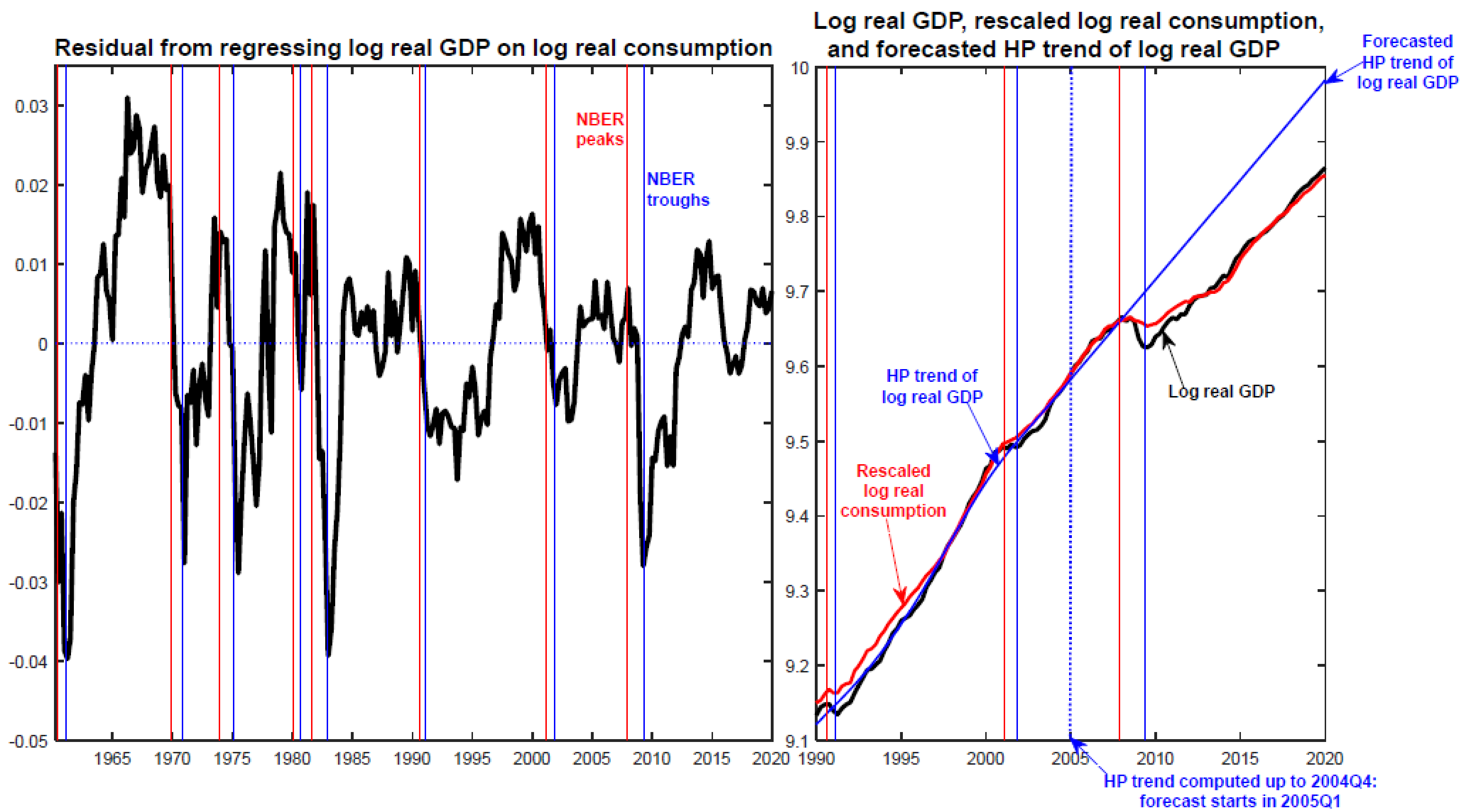


Figure C.1 Computing transitory GDP by projecting GDP on consumption representation of the Hodrick-Prescott filter, the second difference of the HP trend is white noise (see e.g. Harvey and Jaeger, 1993, and King and Rebelo, 1993), so that the trend, $\tau_t$, evolves according to $\tau_t = 2\tau_{t-1} - \tau_{t-2} + \epsilon_t$, with $\epsilon_t$ being a shock. Following the collapse of Lehman Brothers, both GDP and consumption have significantly fallen short compared to the forecast of the HP trend. In particular, at the end of 2019 the shortfall had been for both series equal to about 12 per cent. Crucially, the fact that this has equally held for *both* GDP *and* consumption logically suggests that the shortfalls are permanent: otherwise, by the Permanent Income Hypothesis, consumption would be close to the HP trend.

# D The Monthly Series Used in Section 7.2

As discussed more extensively in Online Appendix A.3, for Canada and the U.K. monthly seasonally adjusted real GDP estimates are available respectively from *Statistics Canada* and from the U.K.'s *National Institute for Economic and Social Research* (NIESR). As for the Euro area I have interpolated seasonally adjusted quarterly real GDP based on Stock and Watson's (2012) methodology, using monthly seasonally adjusted industrial production as the interpolator series. In order to compute nominal GDP, for any of the three countries I have then interpolated to the monthly frequency the quarterly seasonally adjusted GDP deflator based on Stock and Watson's (2012) methodology, using the monthly seasonally adjusted core CPI as the interpolator series.

For the United States, seasonally adjusted monthly series for real and nominal GDP are from Stock and Watson (2012) until 2010, and from *IHS Markit*, a consultancy, after that. Finally, a crucial component of Lucas and Nicolini's (2015) 'New M1' aggregate that is used herein (see the discussion in Section 2), i.e. Money Market Deposit Accounts (MMDAs), is available only at the quarterly frequency. As discussed in Online Appendix A.3.4, I have therefore interpolated MMDAs to the monthly frequency as in Stock and Watson (2012), using as monthly interpolator the seasonally adjusted series for 'Total Checkable Deposits' from the *Federal Reserve Board*. The rationale for using this interpolator series is exactly the same originally advanced by Goldfeld and Sichel (1990, pp. 314-315), and then reiterated by Lucas and Nicolini (2015), for including MMDAs within an expanded, and economically more sensible definition of M1 (see Section 2): MMDAs perform an economic function which is very similar to that of the checkable deposits included in the standard M1 series. Therefore, on the one hand it makes sense to include them within an economically sensible definition of M1; on the other hand, it makes sense to use total checkable deposits as the monthly interpolator for quarterly MMDAs.

similar results are produced by alternative plausible values of the parameter. In order to compute the HP trend I only use data up to 2004Q4 because it could possibly be argued that during the years immediately preceding the financial crisis U.S. GDP had been significantly above trend. Using data up to 2008Q3, however, produces near-identical results.