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Long-Run Evidence on the Quantity Theory of Money

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21-10

June, 2021

DISCUSSION PAPERS

Schanzeneckstrasse 1
CH-3012 Bern, Switzerland
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Abstract

Evidence from low-frequency regressions for 27 countries since the XVIII century suggests that the relationship between broad money growth and inflation has been mostly one-for-one, and largely invariant to changes in the monetary regime. There is little evidence that the relationship had been weaker under commodity standards than it has been under *fiat* standards. Only for the period since the mid-1980s, which has seen the introduction of monetary regimes in which inflation is directly targeted, the relationship appears to have materially weakened. Crucially, however, the slope relationship between the trends of money growth and inflation produced by time-varying parameters VARs has been near-uniformly one-for-one for all countries and sample periods, *including* the one following the end of the Great Inflation. This suggests that, although central banks' targeting of inflation has weakened its relationship with money growth, time-series methods can still recover the one-for-one long-horizon relationship between the series. There is no evidence that, since WWII, inflation's low-frequency relationship with credit growth has been stronger than with money growth. The relationship between money growth and nominal interest rates had been non-existent under commodity standards, and it has only appeared under *fiat* standards.

Keywords: Quantity theory of money; Lucas critique; frequency domain; time-varying parameters VARs.

*This project lasted nearly two decades. Among all the people, in either academia or central banks, who provided comments and suggestions, I wish to especially thank Charlie Bean and Robert Lucas for encouragement in the project's very early stages, and Mark Watson for helpful suggestions on Müller and Watson's (2018) low-frequency regression methodology. I am alone responsible for any mistake or imprecision.

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1 Introduction

In this paper I study several aspects of the quantity theory of money (QTM) based on data for 27 countries, for samples that in many cases extend back in time to the commodity standards era (e.g., for the U.K. since the early XVIII century). My main focus is the long-run relationship between the rate of growth of a broad monetary aggregate (mostly, M2) and inflation, but I also explore, to a more limited extent, the relationship between money growth and either real GDP growth or nominal interest rates.

The theoretical framework motivating the analysis for money growth and inflation is a standard theory of money demand. Consider e.g., without any loss of generality, Cagan's (1956) 'semi-log' specification for the demand for real money balances with unitary income elasticity,[1] i.e. $m_t - p_t = y_t - \delta R_t + v_t$, where m_t, p_t, and y_t are the logarithms of money, prices, and output, R_t is a nominal interest rate, and v_t is a velocity disturbances. Taking first differences we obtain

$$\Delta m_t - \Delta p_t = \Delta y_t - \delta \Delta R_t + \Delta v_t \tag{1}$$

If the right-hand side features no (or a small amount of) spectral power at the very low frequencies, low-frequency money growth and inflation move (essentially) one-for-one. Even ignoring v_t, however, low-frequency fluctuations in either real GDP growth, or the first-difference of the nominal interest rate (due, e.g., to shifts in the natural rate of interest, or the long-horizon component of inflation), would, if sizeable, materially distort the relationship between Δm_t and Δp_t. Accordingly, in line with the analysis of Teles, Uhlig, and Valle e Azevedo (2016) for M1, beyond studying the simple bivariate relationship between broad money growth and inflation I also control for low-frequency shifts in real GDP growth—which I do by focusing on nominal GDP growth, rather than inflation[2]—and the first difference of the interest rate. Likewise, when analyzing the relationship between money growth and interest rates I consider both Δm_t and $\Delta m_t - \Delta y_t$.

The study of the relationship between money growth and real GDP growth is motivated by a key tenet of the QTM, i.e. the Classical Dicothomy.[3] Finally, the relationship between money growth and nominal interest rates was studied by Lucas (1980). The theoretical rationale behind this relationship is the Fisher effect, *via* the impact of money growth on inflation.

[1] Benati, Lucas, Nicolini, and Weber (2021) show that Cagan's (1956) 'semi-log' specification—as well as Meltzer's (1963) 'log-log', and the specification proposed by Selden (1956) and Latané (1960), which is linear in money velocity and the short rate—can be derived within a generalized Baumol-Tobin framework.

[2] As I discuss below, focusing on the relationship between Δp_t and $\Delta m_t - \Delta y_t$ produces qualitatively the same, and numerically very close results. My preference for working with nominal GDP originates from the fact that this does not require to split it into a price and real output indices.

[3] As pointed out by Lucas (1995) in his Nobel lecture, 'The central predictions of the quantity theory are that, in the long run, money growth should be neutral in its effects on the growth rate of production, and should affect the inflation rate on a one-for-one basis.'

I consider two alternative definitions of the long-run, i.e.

(I) the *low-frequency components* of the series, and

(II) the *trends* produced by time-varying parameters (TVP) VARs.

Evidence based on Müller and Watson's (2018) low-frequency regressions for 17 countries, for samples ranging from 84 to 318 years, points in the vast majority of cases to a relationship between money growth and inflation that is either very close to one-for-one, or statistically indistinguishable from it. The fact that confidence intervals for the estimated slope typically contain one, however, partly reflects the sizeable uncertainty that chacterizes the estimates (with the exception of high-inflation countries), which, as discussed by Müller and Watson, is intrinsic to low-frequency regressions.

Evidence for nine countries for which sufficiently long samples are available for both commodity and *fiat* standards suggests that the low-frequency relationship between money growth and inflation had *not* been materially weaker under the former type of monetary regime. This suggests that Rolnick and Weber's (1997) classic finding of a weaker relationship under commodity than under *fiat* standards crucially hinges on their exclusive focus on the raw data.

Only for the period since the mid-1980s, which in many of the countries I analyze has seen the introduction of monetary regimes in which inflation is *directly targeted*, the low-frequency relationship between the two series appears to have materially weakened. A natural explanation for this is the impact of the Lucas (1976) critique analyzed by Sargent and Surico (2011). Crucially, however, the slope relationship between the trends of money growth and inflation produced by the TVP-VARs of Amir-Ahmadi, Matthes, and Wang (2020) has been near-uniformly one-for-one for all countries over the entire sample periods considered herein, *including* the period following the end of the Great Inflation. This shows that although central banks' targeting of the inflation rate has weakened its low-frequency relationship with money growth in recent years, standard time-series methods can still recover the one-for-one long-horizon relationship between the two series. The contrast between these results, and Sargent and Surico's (2011) evidence based on TVP-VARs for the U.S. since the early XX century, originates from the fact that whereas I focus on the VAR-implied trends, they analyzed the frequency-zero components of the series' *deviations* from the trends.

These results, as well as Benati, Lucas, Nicolini, and Weber's (2021) evidence of stability of the long-run demand for M1 in a sample of 38 countries since WWI, provide an important qualification to the dominant narrative of widespread weakness and instability in the relationship between money and prices (and output, and interest rates) that has taken hold since the early 1980s, and especially over the last two decades.[4] Initially, weakness and instability was thought to uniquely pertain to the

[4] Gao, Kulish, and Nicolini (2020) study the relationship between M1 growth, inflation, and nominal interest rates in a sample of 13 OECD countries since 1960. In line with both Benati *et al.* (2021), and the present work, their evidence suggests that at the very low frequencies the three series have maintained a close relationship over the entire sample.

high-to-medium frequencies. In recent years, however, a dominant view has taken hold that, in fact, the relationship between money growth and inflation is weak and unstable *also* at the very low frequencies, to the point that (e.g.) such view features prominently in one of the leading macroeconomics graduate textbooks (see Walsh, 2017, pp. 1-8). My evidence suggests that the long-horizon relationship between money growth (possibly, net of output growth) and inflation has *never* disappeared from the data, and it can still be reliably recovered *via* 'off-the-shelf' time-series methods.[5] The policy implication is that if central banks were to either lose control of, or allow persistent fluctuations in the long-horizon component of broad money growth, corresponding fluctuations in trend inflation would necessarily ensue.[6]

In contrast with the analysis of Teles *et al.* (2016) for M1, for broader aggregates controlling for low-frequency fluctuations in real GDP growth and changes in the interest rate does *not* consistently make the estimated relationship between money growth and inflation closer to one-for-one.

Likewise, in contrast with the evidence produced by Jordà, Schularick and Taylor (2017) based on the raw data, at the very low frequencies there is *no* evidence that, in the post-WWII 'Age of Credit', inflation may have been more strongly correlated with credit growth than with money growth: rather, evidence near-uniformly suggests that the correlation with money growth has been stronger, sometimes markedly so.

In line with the Classical Dicothomy, the slope coefficients in the low-frequency regressions of real GDP growth on money growth are near-uniformly statistically insignificant. The point estimates, however, exhibit a strong *negative* cross-country correlation with those for the corresponding regressions of inflation on money growth. I argue that a plausible explanation for such negative correlation is cross-country variation in the volatility of money growth and inflation.

Finally, the low-frequency relationship between money growth and nominal interest rates had been non-existent under commodity standards, and it has only appeared under *fiat* standards.

The paper is organized as follows. The next section briefly discusses the dataset, which is described in detail in Online Appendix A. Section 3 discusses evidence based on Müller and Watson's (2018) low-frequency regression methodology, whereas Section 4 discusses the corresponding evidence from TVP-VARs. Section 5 concludes.

[5] In fact, as I discuss in Section 4, my application of Amir-Ahmadi *et al.*'s (2020) TVP-VAR methodology is purely 'mechanical': I just run the MATLAB codes found at the website of the *Journal of Business and Economics Statistics*.

[6] In the U.S. the annual growth rate of M2 increased significantly following the outbreak of the COVID pandemic, reaching a peak of 27.1% in February 2021. Although it has markedly decreased since then, it is an open question whether part of the increase will turn out to be permanent. This is the case, in particular, because within an environment characterized by a low natural rate of interest, the Federal Reserve will plausibly be compelled to resort, more and more, to various forms of quantitative easing policies that directly increase monetary aggregates. One way of thinking about this is as a variation on the standard dynamic inconsistency arguments of Kydland and Prescott (1977) and Barro and Gordon (1983).

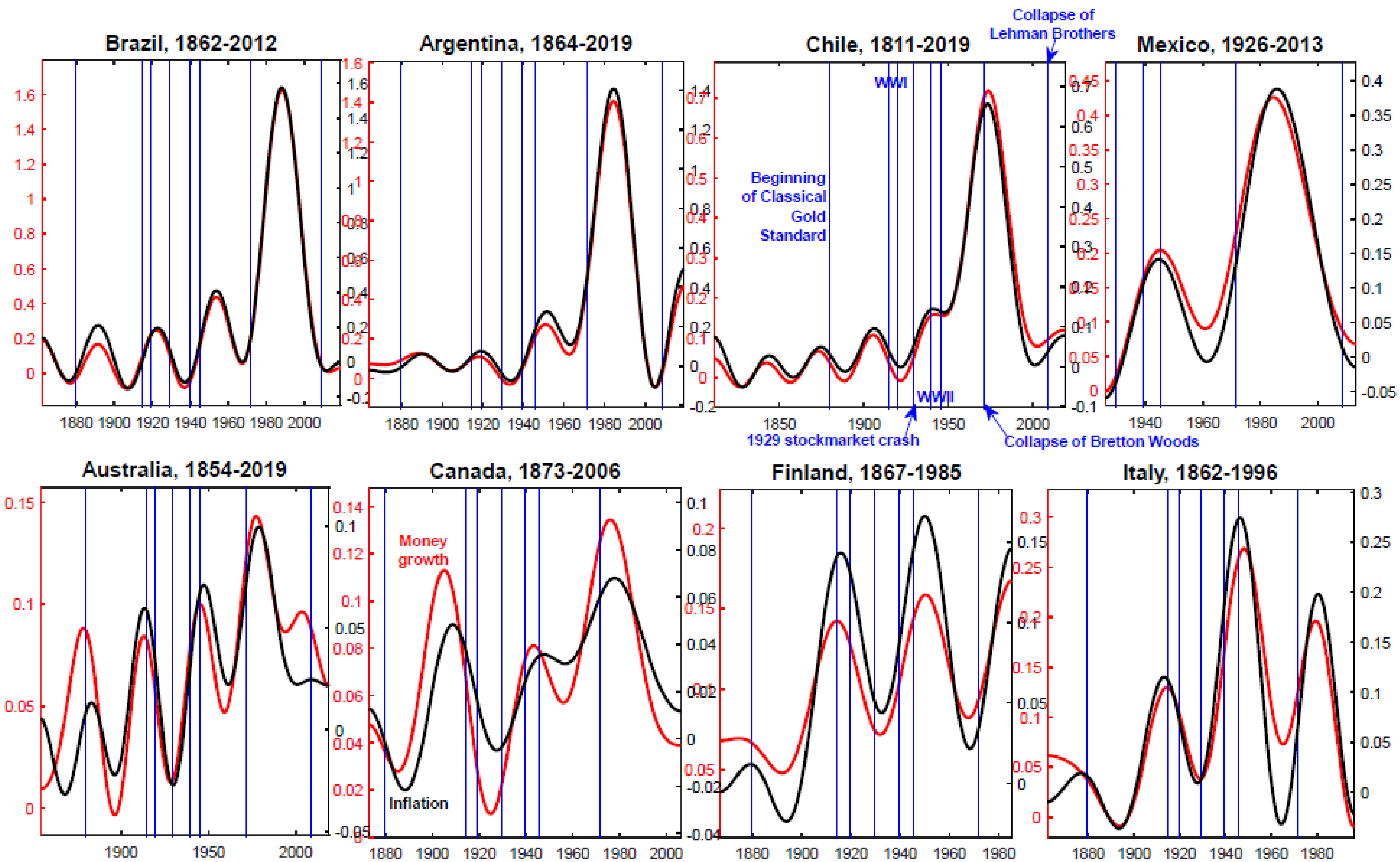

Figure 1a Money growth and inflation: low-frequency components extracted via Müller and Watson's methodology

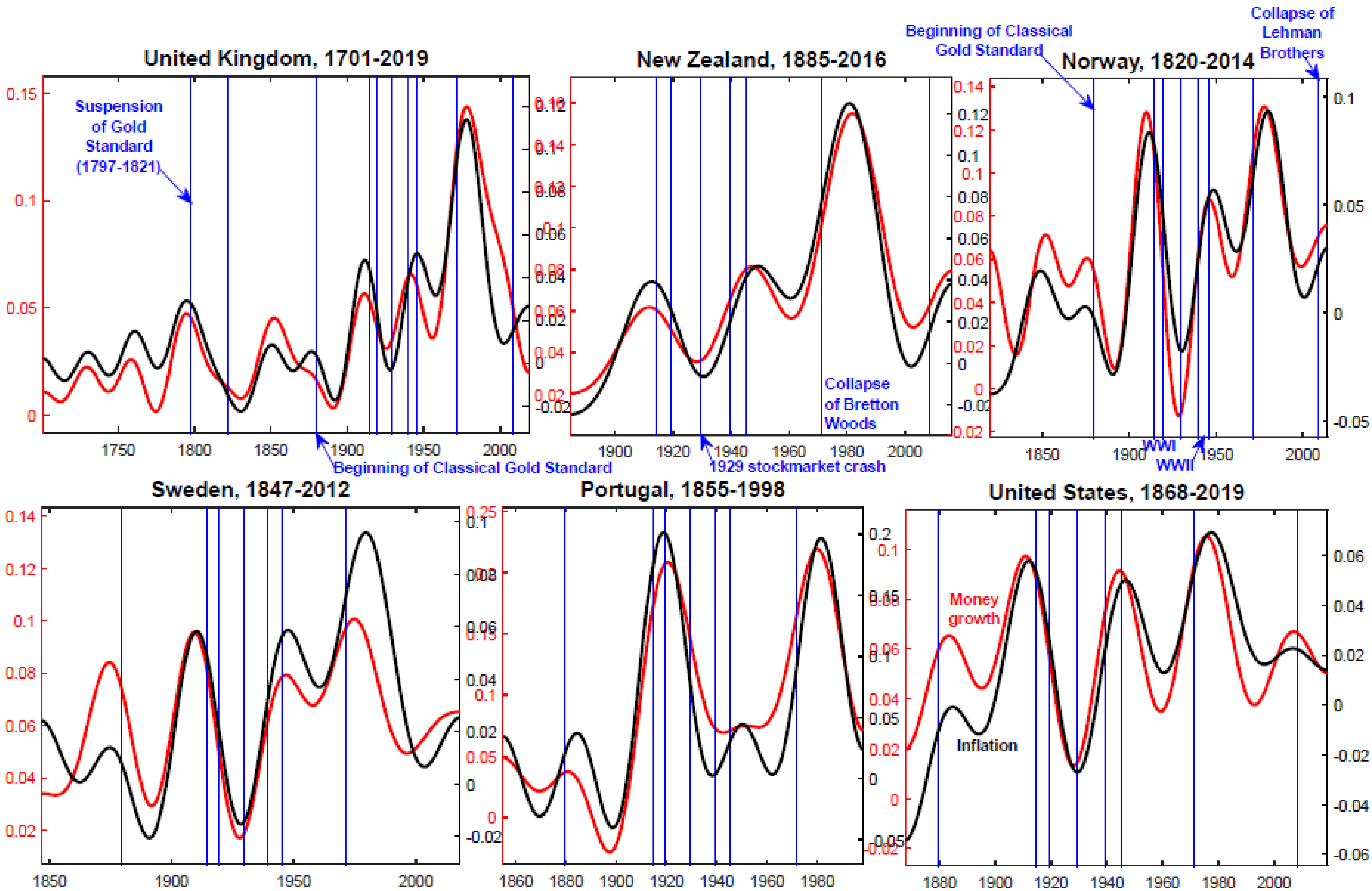

Figure 1b Money growth and inflation: low-frequency components extracted via Müller and Watson's methodology

2 The Data

Throughout the paper I focus on the growth rate of a broad monetary aggregate (mostly, M2), inflation, nominal and real GDP growth, a short-term nominal interest rate, and the growth rate of a credit aggregate (all of the growth rates, and inflation, are computed as log-differences of the relevant series). Online Appendix A describes in detail the data and their sources which, with a few exceptions, are the same as Benati (2020) and Benati, Lucas, Nicolini, and Weber (2021). Almost all of the data are from original sources, i.e. they are from either original hard copy (books, or central banks' and national statistical agencies' publications), or tables available at central banks' or statistical agencies' websites. In the few cases in which I was not able to find data from original sources, I took them from either the *IMF*'s International Financial Statistics or the *OECD*'s Main Economic Indicators database. All of the annual series for total nominal loans, and total nominal loans to real estate, are from Jordà, Schularick and Taylor's dataset, which is available at http://www.macrohistory.net/data/. Throughout the entire paper I end all samples in 2019, in order to avoid that the evidence be distorted by the impact of the COVID pandemic.

Table A.1*a* in the Appendix reports, for the annual data, the chronology of the commodity and *fiat* standards regimes,[7] together with the average and maximum inflation rate for each individual sub-sample. For commodity standards the dataset comprises 11 countries, with samples ranging from 31 to 111 years, and average inflation rates ranging from -0.001 for the U.S. to 0.041 for Finland. For *fiat* standards it features 26 countries, with samples ranging from 52 to 135 years, and average inflation rates ranging from 0.026 for Japan to 0.352 for Argentina.

Turning to the quarterly data, with the exception of the U.K. and the U.S. they uniformly pertain to *fiat* standards. Table A.1*b* in the Appendix reports the full sample periods for the 18 countries in the dataset, together with the average and maximum inflation rates. The samples range from 31 years for New Zealand to 144 years for the U.S., whereas the average inflation rates range from 0.0079 for Taiwan's second sample to 0.0988 for Finland.

Figures 1*a*-1*b* show, for selected countries, the components of money growth and inflation associated with periodicities beyond approximately 30 years,[8] which have been extracted *via* the methodology proposed by Müller and Watson (2018, 2020). The evidence in the top row of Figure 1*a* pertains to countries that, at some point in their history, experienced high, or very high inflation rates, whereas the remaining evidence pertains to low-inflation countries. In each panel the scales on the left- and right-hand side axes are exactly the same,[9] thus allowing for a meaningful comparison

[7] The chronology is based on Tables 1-2 of Rolnick and Weber (1995), Table 1 of Rolnick and Weber (1997), and Table 2.1 of Bernanke and James (1991).

[8] The cutoff points for the low frequencies, which are sample-specific, are reported in Table 1.

[9] E.g., for Sweden the vertical axes range from -0.0314 to 0.1097 for inflation, and from 0.0024 to

between the low-frequency components of the two series.

Two facts emerge from the figures.[10] First, a near-uniformly strong relationship between the low-frequency components of the two series. This is the case in particular for high-inflation countries, but it is quite clearly apparent also for most of the low-inflation countries, such as the U.S.[11] Second, in most cases the relationship does not appear to have been materially different under commodity and *fiat* standards. This is especially clear, e.g., for Chile (1811-1877 *vs.* 1878-2019), Canada (1873-1929 *vs.* 1935-2006), Finland (1868-1913 *vs.* 1915-1985), and Italy (1862-1935 *vs.* 1936-1996).[12]

I now turn to the evidence from Müller and Watson's (2018) low-frequency regressions.

3 Evidence from Low-Frequency Regressions

The methodology proposed by Müller and Watson's (2018, 2020), which is conceptually related to Engle's (1974) classic band spectrum regression estimator, is based on the notion of regressing the series of interest on cosine transforms (essentially, cosine waves) associated with a specific set of low frequencies of interest. A crucial feature of this methodology is that it has been specifically designed to work well with series characterized by a wide array of low-frequency behaviour, from I(0) processes to (near) unit roots, to cointegrated processes. In what follows I work with the (A, B, c, d) model discussed in Section 3.2.1 of Müller and Watson (2018). The parameterization

0.1435 for money growth. In both cases, the range is equal to 0.1411.

[10] Figures A.1*a*-A.1*b* in the Online Appendix report very similar evidence based on Christiano and Fitzgerald's (2003) filter, whereas Figures A.2*a*-A.2*b* report qualitatively similar evidence based on the 30-year differences in the logarithms of money and prices. Additional evidence based on Christiano and Fitzgerald's (2003) filter can be found in Benati (2005*a*, 2009), which also showed evidence for the monetary base and M1.

[11] The evidence in Sargent and Surico's (2011) Figure 3 points towards significant instability in the low-frequency relationship between M2 growth and inflation in the U.S. over the period 1900Q1-2005Q4. A likely explanation for this result, however, is that, as I show in Online Appendix E, the low-frequency filter they used only reliably wipes out the spectral power associated with cycles *faster than 4 years*, whereas it retains sizeable, and sometimes large amount of power associated with comparatively high frequencies (e.g., it retains 41% of the spectral power associated with 8-year cycles). So those results should be interpreted as reflecting the well-known instability that characterizes the relationship at comparatively high frequencies. In fact, Christiano and Fitzgerald's (2003, Section 5, Figures 4 and 5) evidence for the U.S. for frequencies between 20 and 40 years is in line with that in Figure 1.*a* in the present work.

[12] In fact, in several cases the low-frequency component of inflation exhibits *wider* fluctuations than the corresponding component for money growth, thus suggesting a more than one-for-one relationship at the low frequencies. This is the case, e.g., for Finland, and to a slightly lesser extent for Italy and New Zealand. For the U.S. this had been the case for the period until WWI.

characterizing this model produces a local-to-zero spectrum of the form

$$S(\omega) \propto A \begin{bmatrix} (\omega^2 + c_1^2)^{-d_1} & 0 \\ 0 & (\omega^2 + c_2^2)^{-d_2} \end{bmatrix} A' + BB' \quad (2)$$

with A and B being (2×2) matrices, with A unrestricted and B lower triangular. As discussed by Müller and Watson (2018, pp. 785-786), the primary motivation behind the (A, B, c, d) model is that it offers a parsimonious, but flexible way of modelling the local-to-zero spectrum, and it comprises, as special cases, several possibilities of interest. For example, $A = 0$ is associated with the I(0) local-to-zero spectrum, whereas $B = 0$, $c = 0$, and $d_1 = d_2 = 1$ yield the I(1) spectrum.

I perform the estimation based on the MATLAB codes found at Mark Watson's web page. I start by discussing the evidence for money growth and inflation, and I then turn to that for money growth and either real GDP growth or nominal interest rates.

3.1 Money growth and inflation

I start by discussing the evidence obtained without controlling for low-frequency fluctuations in real GDP growth and the first difference of the interest rate. Then, in Section 3.2.6, I discuss the extent to which controlling for such shifts does, or does not make a material difference. To anticipate, it only does in a few instances.

3.1.1 Evidence for the longest available samples

Table 1 reports evidence from low-frequency regressions of either inflation or nominal GDP growth on money growth for the 17 countries with the longest available samples. Although, in all cases, I focus on cycles slower than 30 years, the specific cut-off points for the low frequencies are sample-specific, and range between 30 and 35.2 years. The table reports, for any of the regressions, the posterior median estimate of the slope coefficient together with the confidence interval with 67% coverage probability.[13] For the sake of simplicity, in what follows I will use 'estimate' as a shorthand for 'posterior median estimate of the slope coefficient'.

Focusing on the annual data (the evidence based on quarterly data is qualitatively the same), two main findings emerge from the table:

first, in spite of the length of the samples, ranging from 84 to 318 years, the slopes are quite precisely estimated *only* for the high-inflation countries,[14] for which the width of the confidence interval ranges between 0.035 and 0.182. For all of the remaining countries the estimates are quite imprecise, with the width of the confidence

[13] Table II.1 in the Online Appendix reports the full set of results, including the confidence intervals with 90% coverage probability, and the equal-tail credible intervals with 67 and 90% coverage probability.

[14] Argentina, Brazil, Chile, and Mexico.

Table 1 Full samples: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67 per cent coverage probability in the regression of: inflation on money growth	nominal GDP growth on money growth: *simple estimate*	nominal GDP growth on money growth: *controlling for changes in short rates*
		Based on annual data			
Argentina	1864-2019	31.2	1.041 [1.000 1.068]	1.016 [0.992 1.038]	1.017 [0.992 1.043]
Australia	1854-2019	30.2	0.768 [0.573 0.951]	0.535 [0.069 1.064]	
Brazil	1862-2019	31.6	0.990 [0.964 1.017]	0.990 [0.972 1.007]	
Canada	1873-2006	33.5	0.597 [0.440 0.755]	0.880 [0.707 1.043]	0.601 [0.358 0.843]
Chile	1811-2019	32.2	0.946 [0.857 0.991]	0.932 [0.889 0.964]	0.907 [0.880 0.933]
Denmark	1923-2019	32	0.754 [0.659 0.849]	–[a]	–[a]
Finland	1867-1985	34	1.589 [1.397 1.782]	1.532 [1.299 1.756]	1.753 [1.500 2.005]
France	1910-1994	34	1.047 [0.655 1.443]	0.690 [0.404 0.933]	–[b]
Italy	1862-1996	30	1.091 [0.929 1.263]	1.183 [1.076 1.284]	1.225 [1.134 1.315]
Mexico	1926-2013	35.2	1.104 [1.011 1.193]	1.104 [1.052 1.155]	–[b]
New Zealand	1885-2016	33	1.123 [0.980 1.271]	0.942 [0.622 1.164]	–[b]
Norway	1820-2014	30	0.878 [0.714 1.051]	–[a]	–[a]
Portugal	1855-1998	32	0.974 [0.816 1.143]	0.908 [0.794 1.016]	–[b]
Sweden	1847-2018	31.3	1.086 [0.790 1.384]	1.039 [0.738 1.345]	0.796 [0.357 1.234]
Switzerland	1916-2015	33.3	0.878 [0.646 1.110]	1.096 [0.752 1.438]	0.085 [-0.365 0.534]
United Kingdom	1701-2019	30.4	0.783 [0.678 0.882]	0.881 [0.773 0.982]	0.910 [0.855 0.966]
United States	1869-2019	30.4	1.034 [0.812 1.275]	1.130 [0.931 1.329]	0.995 [0.756 1.234]
		Based on quarterly data			
United Kingdom	1881Q1-2019Q4	30.9	0.899 [0.569 1.252]	–[a]	–[a]
	1895Q2-2019Q4	30.1	0.798 [0.574 1.023]	–[a]	–[a]
United States	1875Q2-2019Q4	32.2	0.965 [0.680 1.268]	1.252 [0.990 1.516]	1.227 [0.945 1.510]

[a] Nominal GDP data are not available. [b] Short rate data are not available.

intervals ranging from 0.19 for Denmark to 0.995 for Australia. The reason for this is that, as discussed by Müller and Watson (2018, 2020), low-frequency regression is intrinsically a small-sample problem, in the sense that the lower the frequencies that are being analyzed, the smaller the number of cosine transforms that are used for the regression.[15] For example, for the U.K., with a sample period of more than three centuries, the estimates in Table 1 have been computed based on 21 cosine transforms, whereas for Norway and the U.S., with samples of 194 and 150 years, the number of cosine transforms is 13 and 10, respectively. Based on such a limited amount of information it is difficult to obtain precise estimates, unless, as in the case of high-inflation countries, the data exhibit wide fluctuations.

Second, in most, although not all cases the estimate in the regression of inflation on money growth is either close to 1, or statistically indistinguishable from it (with the *caveat* that this is partly due to the imprecision of the estimates). Specifically, for five countries[16] it is between 0.95 and 1.05; for two[17] it is between 0.9 and 1.1; and for three additional countries[18] the confidence interval contains 1. Only for the remaining seven countries the estimate is significantly different from one. It is to be noticed, however, that within this group the estimates for Chile and Mexico are equal to 0.946 and 1.104, respectively, so that they are, in fact, quite close to 1. Likewise, confirming the visual impression from Figure 1*a* (see footnote 8), the estimate for Finland is equal to 1.589, thus pointing towards a more than one-for-one low-frequency relationship between money growth and inflation over the entire period 1867-1985. Only for Australia, Canada, Denmark and the U.K. the estimates, ranging between 0.597 and 0.783, are smaller than, and significantly different from 1.

The results for the U.S. contrast with those obtained by Sargent and Surico (2011) for the period 1900Q1-2005Q4. Sargent and Surico estimated the cross-spectrum of M2 growth onto inflation at ω=0 by Fourier-transforming a fixed-coefficients Bayesian VAR estimated in the time domain. They reported a median estimate of 0.55, with a very tight 16-84 credible set (see their Figure 5). Conditional on their methodology, Sargent and Surico's results are extremely robust: e.g., a Classical VAR estimated *via* OLS based on the same data and sample period produces a point estimate of 0.638. These estimates, however, are in contrast with the 1.034 posterior median of the slope for the U.S. over the period 1869-2019 reported in Table 1.[19]

The contrast between the estimates obtained by either (*i*) Fourier-transforming

[15] For cross-spectral analysis, the number of Fourier frequencies that are used in order to compute gains and coherences.

[16] Argentina, Brazil, France, Portugal, and the U.S.

[17] Italy and Sweden.

[18] New Zealand, Norway, and Switzerland.

[19] Even focusing on the data and sample period analyzed by Sargent and Surico (2011), Müller and Watson's (2018) methodology produces an estimate of the slope equal to 0.902, with a 67%-coverage confidence interval stretching from 0.712 to 1.093, whereas cross-spectral methods produce a point estimate of the gain of M2 growth onto inflation at ω=0 equal to 1.043. (The Fast-Fourier-Transform-based methodology I use for cross-spectral analysis is discussed in detail in Online Appendix D.)

VARs estimated in the *time-domain*, or (*ii*) working directly in the *frequency domain* (*via* either Müller and Watson's methodology, or cross-spectral methods) raises the question of which of the two sets of results should be regarded as the more reliable. Online Appendices C and D report Monte Carlo evidence on the performance of Müller and Watson's methodology and, respectively, the cross-spectral methodology I use throughout the paper. In short, both methodologies exhibit an excellent performance. On the other hand, Müller and Watson (2020) warn against computing low-frequency estimates based on models estimated in the time domain, since these models automatically impose restrictions across frequencies, in particular the restriction that the relationship among the series of interest is described by the *same* model at *all* frequencies. As pointed out by Müller and Watson (2020),

> 'While this is sensible if there are tight connections between low-frequency and higher-frequency variability, it can lead to serious misspecification absent these connections. A more robust approach is to conduct inference about the low-frequency characteristics of a time series based solely on the low-frequency characteristics of the sample.'[20]

Within the present context this problem is likely to be especially serious. Whereas the evidence in Figures 1*a*-1*b* (and Figures A.1*a*-A.1*b* and A.2*a*-A.2*b* in the Online Appendix) points towards a strong and apparently stable relationship between money growth and inflation at the very low frequencies, instability and weakness in the relationship at higher frequencies have been extensively documented. Since models estimated in the time domain ought to describe, with the same set of parameters, the relationship at both high and low frequencies, the parameters' estimates are necessarily a 'weighted average' of the two sets of parameters that best characterize the relationship within the two frequency bands. To the extent that the relationship is one-for-one at the low frequencies, but significantly weaker at higher frequencies, this implies that the slope at the low frequencies obtained by Fourier-transforming models estimated in the time-domain will spuriously point towards a relationship that is less than one-for-one. This is a likely explanation for the difference between the results for the U.S. obtained by Fourier-transforming VARs estimated in the time-domain, and those obtained by working directly in the frequency domain.

I now turn to a comparison between commodity and *fiat* standards.

3.1.2 Commodity *versus fiat* standards

In a classic paper, Rolnick and Weber (1997) showed that the relationship between the raw money growth and inflation series had been weaker under commodity standards

[20]This is in line with Engle (1974): 'In the time domain it is very common to exclude some periods such as wars or strikes because they do not conform to the model. [...] However, there is little discussion of whether the same model applies to all frequencies. It may be too much to ask of a model that it explain both slow and rapid shifts in the variables [...].'

Table 2 Commodity *versus fiat* standards: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies

	Commodity standards			*Fiat standards*		
Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67% coverage probability	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67% coverage probability
			Regression of inflation on money growth			
Argentina	1864-1929	33	0.977 [0.459 1.350]	1930-2019	30	1.022 [0.990 1.054]
Canada	1873-1929	38	0.511 [-0.009 1.004]	1935-2006	36	0.529 [0.439 0.612]
Chile	1811-1877	33.5	1.829 [0.693 3.195]	1878-2019	31.6	0.964 [0.877 1.001]
Finland	1868-1930	32	1.983 [1.518 2.476]	1931-1985	36.7	1.200 [0.777 1.627]
Italy	1862-1935	37	1.096 [0.864 1.316]	1936-1996	30.5	1.214 [0.748 1.655]
Norway	1865-1931	33.5	0.915 [0.777 1.051]	1947-2014	34	0.882 [0.573 1.174]
Sweden	1847-1931	34	0.838 [0.422 1.234]	1932-2018	34.8	1.438 [0.868 1.968]
United Kingdom	1701-1796	32	0.489 [0.350 0.743]	1932-2019	35.2	0.662 [0.315 0.997]
	1821-1931	31.7	0.967 [0.760 1.175]			
United States	1869-1932	32.5	1.040 [0.805 1.272]	1933-2019	34.8	0.688 [0.255 1.098]
			Regression of nominal GDP growth on money growth			
Argentina	1864-1929	33	1.013 [0.481 1.400]	1930-2019	30	1.007 [0.979 1.036]
Canada	1873-1929	38	0.759 [0.248 1.265]	1935-2006	36	0.672 [0.501 0.843]
Chile	1811-1877	33.5	1.547 [0.590 2.570]	1878-2019	31.6	0.942 [0.877 0.976]
Finland	1868-1930	31.5	1.739 [1.401 2.043]	1931-1985	36.7	1.173 [0.819 1.587]
Italy	1862-1935	37	1.080 [0.849 1.310]	1936-1996	30.5	1.197 [0.979 1.416]
Norway	1865-1931	33.5	0.921 [0.836 1.012]	1947-2014	34	0.725 [0.367 1.080]
Sweden	1847-1931	34	0.736 [0.318 1.158]	1932-2018	34.8	1.355 [0.869 1.864]
United Kingdom	1701-1796	32	0.946 [0.719 1.293]	1932-2019	35.2	0.712 [0.412 1.006]
	1821-1931	31.7	0.838 [0.608 1.053]			
United States	1869-1932	32.5	1.356 [1.247 1.442]	1933-2019	34.8	1.130 [0.713 1.694]

than it has been under *fiat* standards. Table 2 reports evidence for 8 countries for which at least 50 years of data are available for either type of monetary regime. The main finding is that, when focusing on the very low frequencies, there is little evidence that under commodity standards the relationship may have been weaker than under *fiat* standards. In particular, based on money growth and inflation, for Chile, Finland and the U.S. the posterior median estimate of the slope had been *greater* under commodity standards than it has been under *fiat* standards; for Norway, Argentina, and Canada the two estimates are essentially the same; and for the U.K. evidence is mixed, with the estimate for *fiat* standards being greater than that for commodity standards for the period 1701-1796, but smaller than that for the period 1821-1931. Only for Italy and Sweden evidence clearly suggests that the low-frequency relationship between the two series had been materially weaker under commodity standards. Especially notable are the results for Argentina and Chile. In spite of the fact that under *fiat* standards the two countries have experienced high inflation, and they have even both undergone a short-lived episode of hyperinflation, for Argentina the slopes (0.977 and 1.022) are nearly the same, whereas for Chile the slope for commodity standards (1.829) is almost twice that for *fiat* standards (0.964).

These results suggest that Rolnick and Weber's (1997) finding of a weaker relationship between money growth and inflation under commodity standards hinges on their focus on the raw data and that, when focusing on the very low frequencies at which the QTM should be expected to manifest itself, there is little evidence that the relationship had in fact been weaker.

3.1.3 Additional evidence for *fiat* standards

Table 3 reports additional evidence for *fiat* standards. Based on inflation, out of 20 countries the estimates are between 0.9 and 1.1 for 7 countries, and the confidence intervals contain one in 17 cases. Once again, however, uncertainty is near-uniformly substantial, so that it is often not possible to make strong statements. In fact, for 7 countries the estimated slopes are below 0.9 so that in several cases the fact that the confidence intervals contain one simply reflects the sizeable uncertainty characterizing the estimates.

3.1.4 The period following the end of the Great Inflation

Table 4 reports evidence based on quarterly data for the period since 1985Q1. Since, as discussed in Section 2, I end all samples in 2019, the longest samples in the table are slightly shorter than 35 years. In order to obtain more precise estimates I therefore focus on the frequencies slower than 20 years (the highest frequency I consider in the low-frequency regressions ranges between 21.2 and 23.3 years). Given the short lengths of these samples, however, this evidence should be treated with extreme caution, especially because it pertains to a feature of the data—the long-horizon

Table 3 Additional evidence for *fiat* standard regimes from regressing either inflation or nominal GDP growth on money growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67 per cent coverage probability in the regression of: inflation on money growth	nominal GDP growth on money growth
		Based on annual data		
Australia	1914-2019	30.3	0.868 [0.668 1.057]	0.535 [0.069 1.064]
Brazil	1862-2019	31.6	0.990 [0.964 1.017]	0.990 [0.972 1.007]
Colombia	1956-2019	32	0.986 [0.854 1.115]	1.011 [0.899 1.110]
Finland	1915-1985	36.7	1.200 [0.777 1.627]	1.173 [0.819 1.587]
France	1937-1994	38.7	0.972 [0.368 1.604]	0.690 [0.404 0.933]
Iceland	1961-2019	39.3	1.058 [0.843 1.270]	1.128 [0.847 1.395]
Japan	1956-2018	31.5	0.473 [0.354 0.685]	0.885 [0.746 0.962]
Mexico	1926-2013	35.2	1.104 [1.011 1.193]	1.104 [1.052 1.155]
New Zealand	1914-2016	34.3	1.146 [0.972 1.306]	0.942 [0.622 1.164]
Norway	1947-2014	34	0.882 [0.573 1.174]	0.725 [0.367 1.080]
Paraguay	1963-2015	35.3	1.532 [0.285 2.715]	1.902 [1.095 2.672]
Peru	1960-2018	39.3	1.176 [1.091 1.255]	1.120 [1.048 1.187]
Portugal	1932-1998	33.5	1.098 [0.718 1.473]	1.124 [0.776 1.441]
Saudi Arabia	1964-2019	37.3	0.386 [0.291 0.487]	0.768 [0.622 0.941]
South Korea	1965-2019	36.7	0.480 [0.344 0.628]	0.718 [0.576 0.856]
Spain	1942-1997	37.3	0.415 [-0.413 1.092]	0.725 [0.359 1.049]
South Africa	1966-2019	36	0.917 [0.528 1.288]	0.786 [0.537 1.022]
Sweden	1932-2018	34.8	1.438 [0.868 1.968]	1.355 [0.869 1.864]
Switzerland	1937-2015	31.6	0.691 [0.251 1.093]	0.747 [0.262 1.433]
Venezuela	1951-2017	33.5	0.997 [0.851 1.137]	0.851 [0.730 0.971]
		Based on quarterly data		
Australia	1959Q4-2019Q4	30.3	1.038 [0.762 1.329]	0.972 [0.720 1.272]
Brazil	1975Q2-2019Q4	30.2	2.227 [2.021 2.425]	1.334 [1.265 1.400]
Canada	1968Q1-2019Q4	34.7	0.673 [0.367 0.880]	0.836 [0.284 1.243]
Denmark	1977Q2-2019Q4	43	1.127 [0.902 1.399]	1.143 [1.058 1.210]
Euro area	1970Q3-2019Q4	33.2	0.873 [0.331 1.353]	1.068 [0.380 1.564]
Germany	1960Q2-1998Q4	38.8	1.311 [0.988 1.625]	0.673 [-2.137 3.406]
Japan	1955Q2-2019Q4	32.4	0.468 [0.284 0.775]	0.968 [0.848 1.127]
Netherlands	1957Q2-1997Q4	40.8	0.965 [0.805 1.117]	1.074 [-0.381 2.437][a]
Norway	1978Q1-2019Q4	32	1.053 [0.605 1.614]	0.917 [-0.880 2.765]
South Africa	1966Q1-2019Q4	35.8	0.899 [0.491 1.288]	0.762 [0.498 1.032]
South Korea	1960Q1-2019Q4	30	0.583 [0.288 0.811]	0.842 [0.657 0.975]
Taiwan	1961Q3-2019Q4	39.2	0.342 [0.217 0.524]	0.724 [0.662 0.770]
United Kingdom	1955Q1-2019Q4	32.5	0.782 [0.471 1.091]	0.807 [0.521 1.093]
United States	1933Q2-2019Q4	34.7	0.617 [0.310 0.928]	1.201 [0.882 1.651]

[a]The sample period is 1960Q2-1997Q4.

relationship between two series—that is intrinsically hard to precisely pin down in small samples.[21]

Table 4 Period since 1985: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies[a]

		Posterior median and confidence interval with 67 per cent coverage probability in the regression of:	
Country	Sample period	inflation on money growth	nominal GDP growth on money growth
Australia	1985Q1-2019Q4	0.546 [0.207 0.826]	0.597 [0.348 0.795]
Canada	1985Q1-2019Q4	0.237 [0.061 0.380]	0.078 [-0.240 0.336]
Denmark	1985Q1-2019Q4	0.867 [0.406 1.291]	0.292 [-0.393 0.955]
Euro area	1985Q1-2019Q4	0.799 [-0.295 1.949]	0.810 [-0.740 2.418]
Hong Kong	1985Q1-2019Q4	0.761 [0.446 1.199]	1.059 [0.803 1.383]
Japan	1985Q1-2019Q4	0.325 [0.053 0.671]	0.937 [0.652 1.277]
Norway	1985Q1-2015Q1	0.804 [0.054 1.422]	0.751 [0.638 0.872]
New Zealand	1988Q1-2019Q4	0.687 [-0.082 1.444]	0.130 [-1.275 1.430]
South Korea	1985Q1-2019Q4	0.313 [0.262 0.346]	0.701 [0.546 0.810]
Switzerland	1985Q1-2019Q4	0.586 [0.133 1.043]	0.792 [0.385 1.204]
Taiwan	1985Q1-2019Q4	0.197 [0.032 0.357]	0.547 [0.417 0.678]
United Kingdom	1985Q1-2017Q2	0.321 [0.074 0.559]	0.398 [0.135 0.677]
United States	1985Q1-2019Q4	0.016 [-0.451 0.466]	-0.792 [-1.383 -0.233]

[a] The highest frequency ranges between 21.2 and 23.3 years.

With this *caveat* in mind, the evidence in Table 4 points, overall, towards an often weak low-frequency relationship between money growth and inflation. In particular, for seven countries the estimate of the slope ranges between 0.016 and 0.546, and the confidence interval does not contain one. For the remaining six countries the slope ranges between 0.586 and 0.867, and the confidence interval does contain one, but this simply reflects the large extent of uncertainty characterizing the estimates, as the width of the confidence intervals ranges between 0.753 and 2.244. For the U.S., which has been the focus of most research, the estimated slope is 0.016, with a confidence interval equal to [-0.451 0.466], thus pointing towards the complete disappearance of any relationship between broad money growth and inflation at the very low frequencies.

The natural explanation for this phenomenon is the fact that, in nearly all of the countries in Table 4, the period since the mid-1980s has seen the introduction of monetary regimes in which inflation is *directly targeted*, either *de jure* or *de facto*. As

[21] This is testified by the fact that (e.g.), based on inflation, the width of the confidence intervals with 67% coverage probability is greater than 0.6 and 0.9, respectively, for 9 and 5 countries out of 13.

shows by Sargent and Surico (2011) based on estimated DSGE models, a stronger reaction to (expected) inflation on the part of the monetary authority weakens, and in the limit destroys altogether the relationship between inflation and other macroeconomic variables. A comparison with the corresponding evidence for commodity standards logically suggests that what is crucial is the fact that inflation is being directly targeted, rather than the fact that the monetary regime delivers a strong price stability. For centuries, commodity standards delivered indeed an extraordinary extent of price stability. In spite of this, as shown in Table 2, the low-frequency relationship between money growth and inflation had most of the time been very strong (and, as we will discuss in Section 3.2.6, controlling for low-frequency fluctuations in real GDP growth and changes in nominal interest rates makes this evidence even stronger).

3.1.5 Money growth *versus* credit growth in the 'Age of Credit'

In a series of landmark studies, Jordà, Schularick and Taylor have documented how, over the last century and a half, advanced economies have moved from the 'Age of Money' to the post-WWII 'Age of Credit', characterized by dramatic increases in the ratios between financial aggregates and either GDP or monetary aggregates, and by an increased role played by financial factors in macroeconomic fluctuations. Based on the raw data, in particular, Jordà *et al.* (2017) documented how, in the 'Age of Credit', inflation has become more strongly correlated with credit growth than with broad money growth.

Table 5 explores the low-frequency relationship between inflation (or nominal GDP growth) and either money or credit growth since WWII. I measure credit as either total nominal loans, or nominal loans to the non-real estate sector. Evidence provides *no* support to the notion that, at the very low frequencies, inflation may have been more strongly correlated with credit growth than with money growth. Rather, the slope coefficient in the regression of inflation (or nominal GDP growth) on money growth is near-uniformly greater than the corresponding slope coefficient for either measure of credit growth. This pattern is clearly illustrated by the scatterplots in Figure 2, plotting the posterior median of the slope coefficient in the regression of either inflation or nominal GDP growth on money growth against the posterior medians of the slope coefficient in the corresponding regressions on either measure of credit growth. Nearly all observations lie below the 45^{o} line, showing how, at the very low frequencies, both inflation and nominal GDP growth have been more strongly correlated with money growth than with credit growth.

This evidence suggests that Jordà *et al.*'s (2017) finding of a stronger correlation of inflation with credit growth in the post-WWII 'Age of Credit' crucially hinges on their exclusive focus on the raw data, and that at the very low frequencies money growth is still the variable most strongly correlated with inflation.

I now turn to discussing the extent to which controlling for low-frequency shifts

Table 5 Post-WWII period: evidence from regressing either inflation or nominal GDP growth on the growth rates of money, non real estate loans,[a] or total loans at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67 per cent coverage probability in the regression of either inflation or nominal GDP growth on:		
			money growth	non real estate loans growth	total loans growth
			Regressions for inflation		
Australia	1953-2016	32	0.940 [0.611 1.270]	0.803 [0.556 1.036]	0.787 [0.418 1.140]
Canada	1947-2006	30	0.610 [0.350 0.878]	0.488 [0.218 0.755]	0.532 [0.244 0.820]
Finland	1947-1985	39	1.253 [0.351 2.113]	0.777 [0.629 0.919]	0.670 [0.349 0.959]
France	1947-1994	32	0.873 [-0.178 1.626]	0.822 [0.391 1.304]	0.841 [0.359 1.287]
Italy	1949-1996	32	0.769 [-0.032 1.471]	0.442 [-0.910 1.662]	0.291 [-1.254 1.448]
Japan	1956-2016	30.5	0.472 [0.351 0.676]	0.257 [-0.026 0.433]	0.301 [0.004 0.477]
Norway	1947-2013	33.5	0.742 [0.525 0.967]	0.285 [0.035 0.522]	0.383 [0.060 0.738]
Spain	1947-1997	34	0.441 [-0.375 1.164]	0.440 [-0.179 1.013]	0.574 [-0.225 1.285]
Sweden	1947-2012	33	1.246 [0.562 1.917]	0.386 [0.189 0.569]	0.705 [0.402 0.985]
Switzerland	1947-2015	34.5	0.682 [0.393 0.949]	0.265 [0.028 0.545]	0.741 [0.609 0.939]
United Kingdom	1947-2014	34	0.677 [0.311 1.034]	0.496 [0.280 0.711]	0.641 [0.449 0.844]
United States	1947-2014	34	0.549 [0.103 1.003]	0.425 [-0.011 0.988]	0.367 [-0.153 1.480]
			Regressions for nominal GDP growth		
Australia	1953-2016	37.3	1.141 [0.739 1.575]	0.831 [0.490 1.169]	0.847 [0.281 1.373]
Canada	1947-2006	30	0.662 [0.327 0.984]	0.619 [0.405 0.834]	0.658 [0.440 0.879]
Finland	1947-1985	39	1.030 [-0.342 2.339]	0.740 [0.543 0.922]	0.666 [0.654 0.677]
France	1947-1994	32	1.114 [0.157 1.865]	1.007 [0.731 1.307]	1.027 [0.698 1.303]
Italy	1949-1996	32	0.907 [0.450 1.308]	0.649 [-0.464 1.707]	0.520 [-0.744 1.490]
Japan	1956-2016	30.5	0.879 [0.722 0.958]	0.587 [0.130 0.816]	0.675 [0.238 0.879]
Norway	1947-2013	33.5	0.754 [0.379 1.101]	0.443 [0.244 0.642]	0.559 [0.272 0.883]
Spain	1947-1997	34	0.733 [0.293 1.124]	0.623 [0.329 0.918]	0.819 [0.452 1.170]
Sweden	1947-2012	33	1.091 [0.443 1.694]	0.279 [0.088 0.474]	0.571 [0.270 0.855]
Switzerland	1947-2015	34.5	0.888 [0.452 1.311]	0.404 [0.241 0.670]	0.863 [0.474 1.213]
United Kingdom	1947-2014	34	0.658 [0.333 1.012]	0.519 [0.376 0.650]	0.648 [0.526 0.766]
United States	1947-2014	34	0.467 [-0.012 0.973]	0.405 [-0.025 0.822]	0.441 [-0.097 1.340]

[a] Computed as total nominal loans minus nominal loans to real estate.

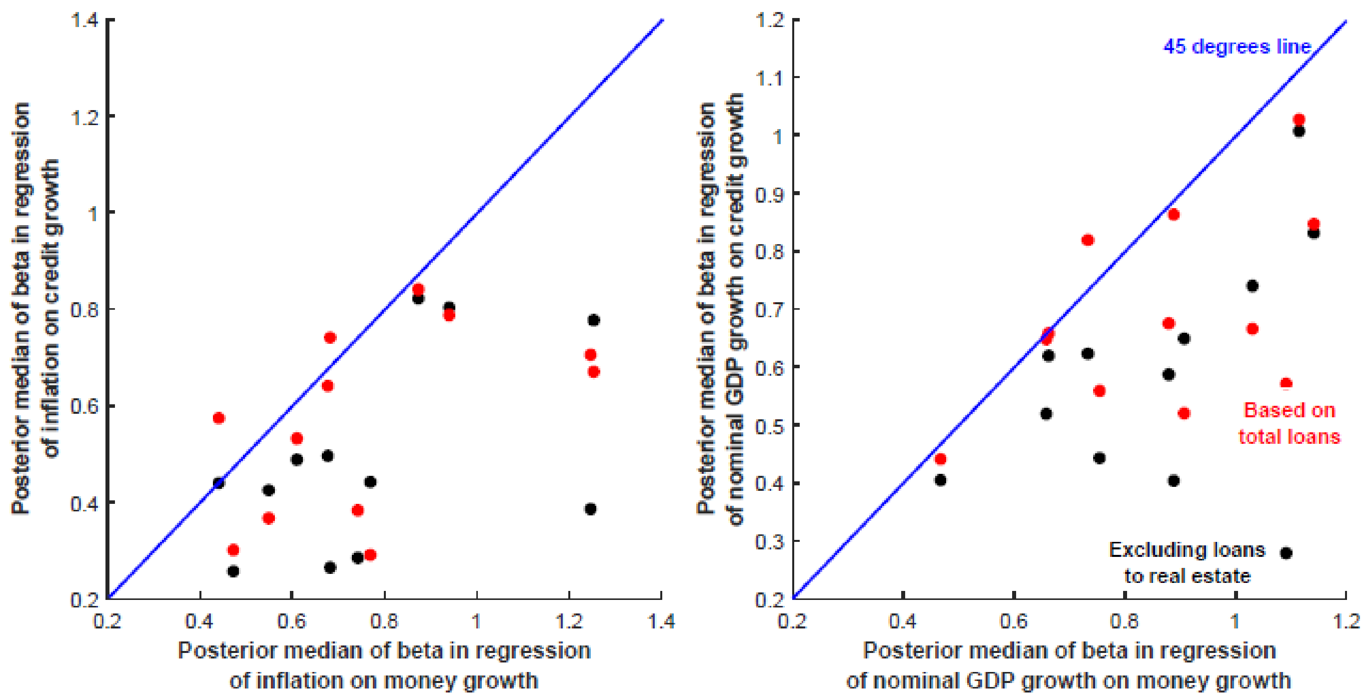

Figure 2 Scatterplots of posterior medians of the slope coefficients in the regression of either inflation or nominal GDP growth on either money growth or credit growth at the very low frequencies

in real GDP growth (i.e., focusing on nominal GDP growth, rather than inflation), and changes in nominal interest rates, does or does not make a difference.

3.1.6 Controlling for low-frequency fluctuations in real GDP growth and changes in nominal interest rates

As shown by Teles, Uhlig, and Valle e Azevedo (2016), for M1 correcting for changes in output growth and the opportunity cost of money (which they do based on the theoretical money-demand elasticities implied by the Baumol-Tobin and Miller-Orr models) makes the relationship between money growth and inflation markedly closer to one-for-one than it is in the raw data.[22] Conceptually in line with Sargent and Surico (2011), a partial exception to this is a sample of inflation-targeting countries, for which the improvement in the fit is comparatively modest.

The evidence in Table 1 shows that, for the longest available samples, controlling for changes in nominal interest rates does not consistently make the estimated relationship closer to one-for-one. Rather, based on annual data, in all but one case the estimated slope is farther away from one than the corresponding slope obtained by simply focusing on nominal GDP growth. By the same token, based on U.S. quarterly data the improvement is marginal, with the slope slightly decreasing from 1.252 to 1.227. The corresponding evidence for Tables 2-5, which is reported in Tables A.5-A.8 in the Appendix, is broadly in line with this, with no consistent improvement compared to simply focusing on nominal GDP growth. Further, controlling for changes in nominal interest rates still suggests (see Table A.8) that, in the post-WWII 'Age of Credit' inflation has been more strongly correlated with money growth than with credit growth.

Turning to the plausibly more important issue of controlling for changes in real GDP growth,[23] evidence is in general ambiguous. For example in Table 1, based on annual data, out of 15 countries the estimates based on inflation and nominal GDP growth are (as expected) near-identical for the 4 high-inflation countries,[24] whereas out of the remaining 11 countries the estimates based on nominal GDP growth are closer to one in only 6 instances. By the same token, for commodity and *fiat* standards the estimated slopes are virtually the same, or very close, for 3 and 4 countries, respectively. For the remaining countries, the estimates based on nominal GDP growth are closer to one for for 4 and 3 countries under commodity

[22] This is compatible with the stability of the long-run demand for M1 documented by Benati, Lucas, Nicolini, and Weber (2021), and with the presence of a non-negligible extent of low-frequency variation in real GDP growth and the first-difference of the nominal interest rate.

[23] Changes in trend output growth have historically been widespread: simple, *prima facie* evidence on this can be obtained by computing average output growth for (e.g.) 10- or 20-year non-overlapping periods. On the other hand, up until the Great Inflation the level of nominal interest rates had historically exhibited very little low-frequency variation. This suggests that, with the exception of the period since the mid-1960s, controlling for changes in the first difference of interest rates should be less important than controlling for changes in output growth.

[24] Argentina, Brazil, Chile, and Mexico.

and *fiat* standards, respectively, whereas the opposite is true for 3 and 2 countries, respectively. The evidence in Tables 3-5 is qualitatively the same. Overall, in contrast with Teles *et al.*'s (2016) evidence for M1, for broader aggregates controlling for low-frequency fluctuations in real GDP growth does not consistently make the estimated relationship between money growth and inflation closer to one-for-one.

I now turn to the long-horizon relationship between money growth and real GDP growth.

3.2 Money growth and real GDP growth

Tables A.2-A.4 in the Appendix report evidence from low-frequency regressions of real GDP growth on money growth for the longest available samples, commodity standards, and *fiat* standards, respectively. The main finding emerging from the three tables is that, in line with the Classical Dicothomy, the confidence interval for the estimated slope coefficient contains zero in 12 cases out of 15 based on the longest available samples; and in 5 cases out of 9, and 20 cases out of 25 for commodity and *fiat* standards, respectively.

Two sets of evidence should be regarded, under this respect, as especially informative.

First, that pertaining to *fiat* standards, under which money growth, and therefore inflation, has consistently exhibited much wider fluctuations than under commodity standards,[25] thus allowing for a comparatively stronger identification of the relationship under study. The fact that, under *fiat* standards, the confidence interval contains zero in 80% of the cases provides therefore strong support for the Classical Dicothomy. In order to fully appreciate this, it is important to keep in mind that, within a Classical context, a perfectly sized test incorrectly rejects the null hypothesis $x\%$ of the time at the $x\%$ level. If we were here performing Classical statistical tests, we should therefore expect to obtain, under the null hypothesis of long-horizon orthogonality between money growth and real GDP growth, 33% of rejections of the null.[26] Given the one-for-one correspondence between performing statistical tests and the construction of confidence intervals, this implies that if the Classical Dicothomy were true, Classical confidence intervals for the estimated slopes should contain zero 33% of the time.[27] Although we are here working within a Bayesian context, the same basic logic applies. This implies that the 20% of cases in which the confidence intervals for the estimated slopes does not contain one is materially lower than what we should expect under the Classical Dicothomy.

[25] As shown in Table A.1a, average inflation rates ranged from -0.001 to 0.041 under commodity standards, and from 0.026 to 0.352 under *fiat* standards.

[26] This corresponds to the fact that, throughout the entire paper, I focus on confidence intervals with 67% coverage probability.

[27] To be precise, this would be the case if the samples were independent (e.g., Monte Carlo) realizations. Although in the real world this is obviously not the case, when performing many tests for many countries and periods, such as here, this logic should approximately hold.

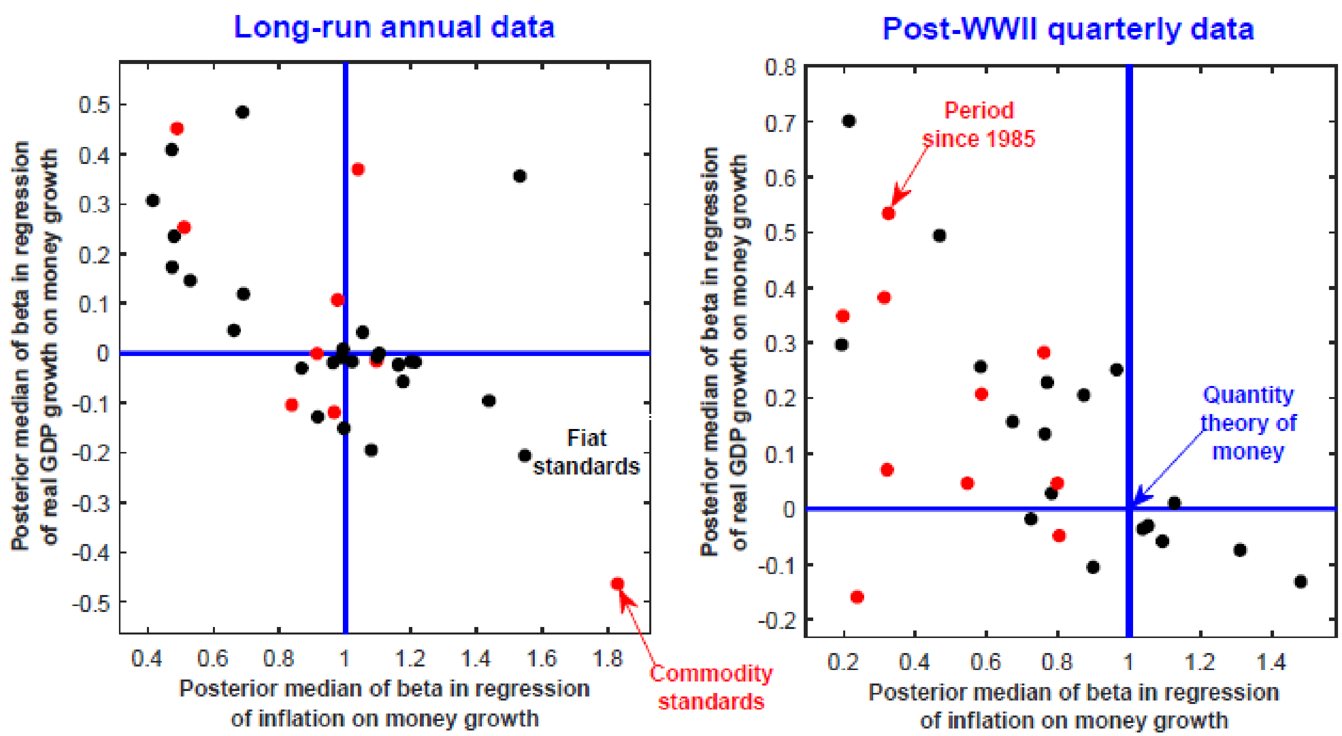

Figure 3 Scatterplots of posterior medians of the slope coefficients in the regression of either inflation or real GDP growth on money growth at the very low frequencies

For exactly the same reason, a second especially informative set of results pertains to the high-inflation countries. Within this group, the confidence interval contains zero for Brazil, Chile, Colombia, and Mexico. For Argentina, Peru, and Venezuela, on the other hand, the confidence interval does not contain zero, and the estimates are consistently negative. In line with the literature on 'inflation crises' (see e.g. Bruno and Easterly, 1998), a natural explanation for this is that the same macroeconomic factors at the root of high, or very high inflation episodes also cause collapses in output,[28] so that the experience of these three countries is in fact compatible with long-horizon orthogonality between money growth (and inflation) and output growth.

Figure 3 shows that, although insignificantly different from zero, the estimated slopes for real GDP growth have consistently exhibited a strong *negative* cross-country correlation with those for the corresponding regressions of inflation on money growth. The fact that the correlation appears to have been the same under commodity and *fiat* standards, and it has not disappeared since the mid-1980s, naturally suggests that it is invariant to changes in monetary policy, and it therefore originates from deep features of the economy.

In order to gauge an idea about what, exactly, may lie at the root of such negative cross-country correlation, consider the following admittedly over-simplified perfect-foresight model. The demand for real money balances is given by (1), and output growth and the velocity disturbance evolve as $\Delta y_t = u_t$ and $\Delta v_t = \eta_t$. Under perfect-foresight the nominal interest rate is described by the Fisher relationship $R_t = R^* + \Delta p_t$, where R^* is the natural rate of interest, which is assumed to be constant. Finally, in line with, e.g., Sargent (1999) the monetary authority is assumed to set inflation equal to its target, which is equal to zero, up to a control error, so that $\Delta p_t = \epsilon_t$. All of the shocks (u_t, η_t, and ϵ_t) are normally distributed, with variances σ_u^2, σ_η^2, and σ_ϵ^2. Under these assumptions, the theoretical slope coefficients in the regressions of Δy_t and Δp_t, respectively, on Δm_t are equal to $\beta_{OLS}^{\Delta y_t \text{ on } \Delta m_t} = \sigma_u^2/\Theta$ and $\beta_{OLS}^{\Delta p_t \text{ on } \Delta m_t} = (1\text{-}\delta)\sigma_\epsilon^2/\Theta$, respectively, with $\Theta = \sigma_u^2 + \sigma_\eta^2 + \sigma_\epsilon^2[\delta^2 + (1\text{-}\delta)^2]$. From this it immediately follows that

$$\frac{\partial \beta_{OLS}^{\Delta y_t \text{ on } \Delta m_t}}{\partial \sigma_\epsilon^2} = -\frac{\sigma_u^2[\delta^2 + (1-\delta)^2]}{\Theta^2} < 0 \text{ and } \frac{\partial \beta_{OLS}^{\Delta p_t \text{ on } \Delta m_t}}{\partial \sigma_\epsilon^2} = \frac{(1-\delta)(\sigma_u^2 + \sigma_\eta^2)}{\Theta^2} > 0 \quad (3)$$

since estimates of δ are consistently around 0.1 (see, e.g., Benati *et al.*, 2021). The implication is that cross-country variation in the volatility of inflation is going to trace out a negative relationship between individual countries' slope coefficients in the regressions of output growth and inflation on money growth. Although the model is extraordinarily simply, and the assumption that the monetary authority directly controls inflation up to a forecast error is extreme, the same basic logic should be expected to hold within more realistic frameworks.

[28] In fact, hyperinflation episodes are typically associated with dramatic output collapses.

Table 6 Full samples: evidence from regressing a short rate on money growth			
Country	Sample period	Highest frequency (in years)	Posterior median and confidence intervals with 67% coverage probability
Argentina	1864-2004	31.3	1.291 [1.161 1.422]
Chile	1866-1995	32.5	1.398 [1.014 1.690]
Finland	1868-1985	33.7	0.151 [-0.009 0.384]
Italy	1862-1996	30	0.066 [-0.177 0.183]
Norway	1823-2013	31.8	0.167 [0.018 0.317]
Sweden	1857-1989	33.3	0.177 [-0.079 0.467]
Switzerland	1916-2015	33.3	0.250 [-0.056 0.538]
United Kingdom	1719-2016	31.4	0.644 [0.344 0.806]
United States	1869-2019	30.2	0.120 [-0.312 0.527]

3.3 Money growth and nominal interest rates

Tables 6 and 7 report evidence from low-frequency regressions of a short rate on money growth for the longest available samples, and for commodity and *fiat* standards, respectively.[29]

The evidence for the longest samples highlights a stark contrast between the high-inflation countries (Argentina and Chile), for which the estimated slopes are around 1.3 and 1.4, respectively, and the remaining countries, for which they range between 0.066 and 0.644. For all countries the confidence intervals do not contain one, thus suggesting that for either group the slopes deviate significantly from the one-for-one relationship implied by the QTM, which was identified by Lucas (1980, Figure 11) for the U.S., based on M1, for the period 1955-1975. At first sight, a natural explanation for the results for Argentina and Chile would seem to be that at high inflation rates money velocity increases, as agents attempt to escape the inflation tax, thus causing inflation to increase more than one-for-one with money growth. By the Fisher effect, this would cause a corresponding more than one-for-one increase in nominal interest rates. A comparison with the evidence in Table 1, however, suggests that this explanation is not quite right, as the slopes in the corresponding regressions of inflation on money growth are 1.041 and 0.946, respectively,[30] so that, as discussed

[29]Tables II.24 and II.25 in the Online Appendix report the corresponding evidence for the regressions of a short rate on money growth minus real GDP growth. Since this evidence is qualitatively the same as that in Tables 6 and 7, in what follows I do not discuss it.

[30]The difference is not due to the different sample periods. Regressions of inflation on money growth for the same sample periods as in Table 6 produce, for Argentina, a slope equal to 1.039, with a confidence interval [1.015 1.062]. The corresponding results for Chile are 0.957 and [0.887 1.014].

in Section 3.1.1, in both countries low-frequency inflation has moved one-for-one with low-frequency money growth.

Table 7 Commodity *versus* *fiat* standards: evidence from regressing a short rate on money growth

Country	Sample period	Highest frequency (in years)	Posterior median and confidence intervals with 67% coverage probability
			Commodity standards
Argentina	1864-1929	33	-0.195 [-0.389 0.056]
Italy	1862-1935	37	-0.010 [-0.056 0.031]
Norway	1865-1931	33.5	0.032 [-0.000 0.064]
Sweden	1857-1931	30	0.057 [-0.039 0.169]
United Kingdom	1719-1796	31.2	0.025 [-0.419 0.537]
	1821-1931	31.7	0.164 [0.073 0.249]
United States	1869-1932	32	0.090 [-0.258 0.437]
			Fiat standards
Argentina	1930-2004	30	1.354 [1.112 1.585]
Canada	1935-2006	36	0.571 [-0.068 1.209]
Chile	1878-1995	33.7	1.404 [0.993 1.702]
Colombia	1956-2018	31.5	1.387 [0.591 2.141]
Finland	1931-1985	36.7	0.061 [-0.288 0.387]
Iceland	1961-2019	39.3	0.567 [0.200 0.900]
Italy	1936-1996	30.5	0.042 [-0.357 0.446]
Japan	1956-2018	31.5	0.339 [0.068 0.476]
New Zealand	1935-2016	32.8	0.939 [0.502 1.442]
Norway	1947-2013	33.5	0.515 [-0.027 1.042]
Portugal	1932-1998	33.5	0.688 [0.190 1.150]
South Korea	1965-2019	36.7	0.610 [0.307 0.956]
South Africa	1966-2019	36	0.971 [0.465 1.455]
Sweden	1932-1989	38.7	0.970 [-1.135 2.281]
Switzerland	1937-2015	31.6	0.169 [-0.229 0.693]
United Kingdom	1932-2016	34	0.985 [0.730 1.313]
United States	1933-2019	34.8	0.477 [-0.498 1.453]

The evidence in Table 7 highlights an equally stark contrast between commodity and *fiat* standards. Whereas for the former monetary regimes the estimated slopes are consistently small, ranging between -0.195 and 0.164, and the confidence intervals near-uniformly contain zero,[31] for the latter regimes evidence is mixed.[32] As expected,

[31] With the single exception of the period 1821-1931 for the4 U.K..

[32] Likewise, the evidence based on quarterly data in Table II.17 in the Online Appendix (mostly, for the post-WWII period) is also mixed. Since this evidence is qualitatively the same as that for *fiat* standards in Table 7 I do not discuss it.

for Argentina, Chile and Colombia the estimates are very high, ranging between 1.354 and 1.404. For the remaining countries, however, evidence is not clear-cut. For four countries[33] the estimates are close to one, ranging between 0.939 and 0.985, and confidence intervals contain one. For Portugal the estimate is lower, but the confidence interval still contains one. For Canada, Norway, and the U.S. the estimates are around 0.5, and confidence intervals contain both zero and one, so that it is not possible to make any strong statement. Finally, for the remaining six countries estimates range between 0.042 and 0.567, and all confidence intervals do not contain one.

This evidence is in line with that in Sargent and Surico (2011, Figure 5):[34] in contrast with the corresponding evidence for money growth and inflation discussed in Section 3.1, the low-frequency relationship between money growth and interest rates appears to have often been less than one-for-one. A further difference is that, as I will discuss in the next section, in this case the series' trends produced by TVP-VARs do not consistently recover a one-for-one long-horizon relationship between the series.

3.4 Comparison with the evidence produced by cross-spectral methods

As discussed by Müller and Watson (2018, 2020), the low-frequency regression methodology the propose should be regarded as superior to standard cross-spectral techniques. If, however, the results it produces turned out to be consistently and materially different from those produced by cross-spectral methods, this could raise doubts about their robustness. In fact, the cross-spectral evidence reported in Tables III.1-III.7 in the Online Appendix is exactly in line with that discussed so far.[35] Because of this, I do not discuss this evidence in detail, and I instead turn to the evidence produced by TVP VARs.

4 Evidence from Time-Varying Parameters VARs

Figures 4*a* and 4*b* show, based on quarterly data, the estimated slope relationships between the trends of money growth and, respectively, inflation and nominal GDP

[33] New Zealand, South Africa, Sweden, and the United Kingdom.

[34] Although, as I discussed in Section 3.1.1, VAR-based evidence should be discounted for the reasons discussed by Müller and Watson (2020). Further, as I will discuss in Section 4, Sargent and Surico's evidence in their Figure 5 pertains to the deviations of the series from the VAR-implied trends.

[35] Online Appendix D discusses the Fast-Fourier Transform (FFT)-based estimator I use to compute cross-spectral objects, and reports Monte Carlo evidence on the excellent performance of Berkowitz and Diebold's (1998) spectral bootstrapping procedure, which I use in order to characterize uncertainty about the estimates.

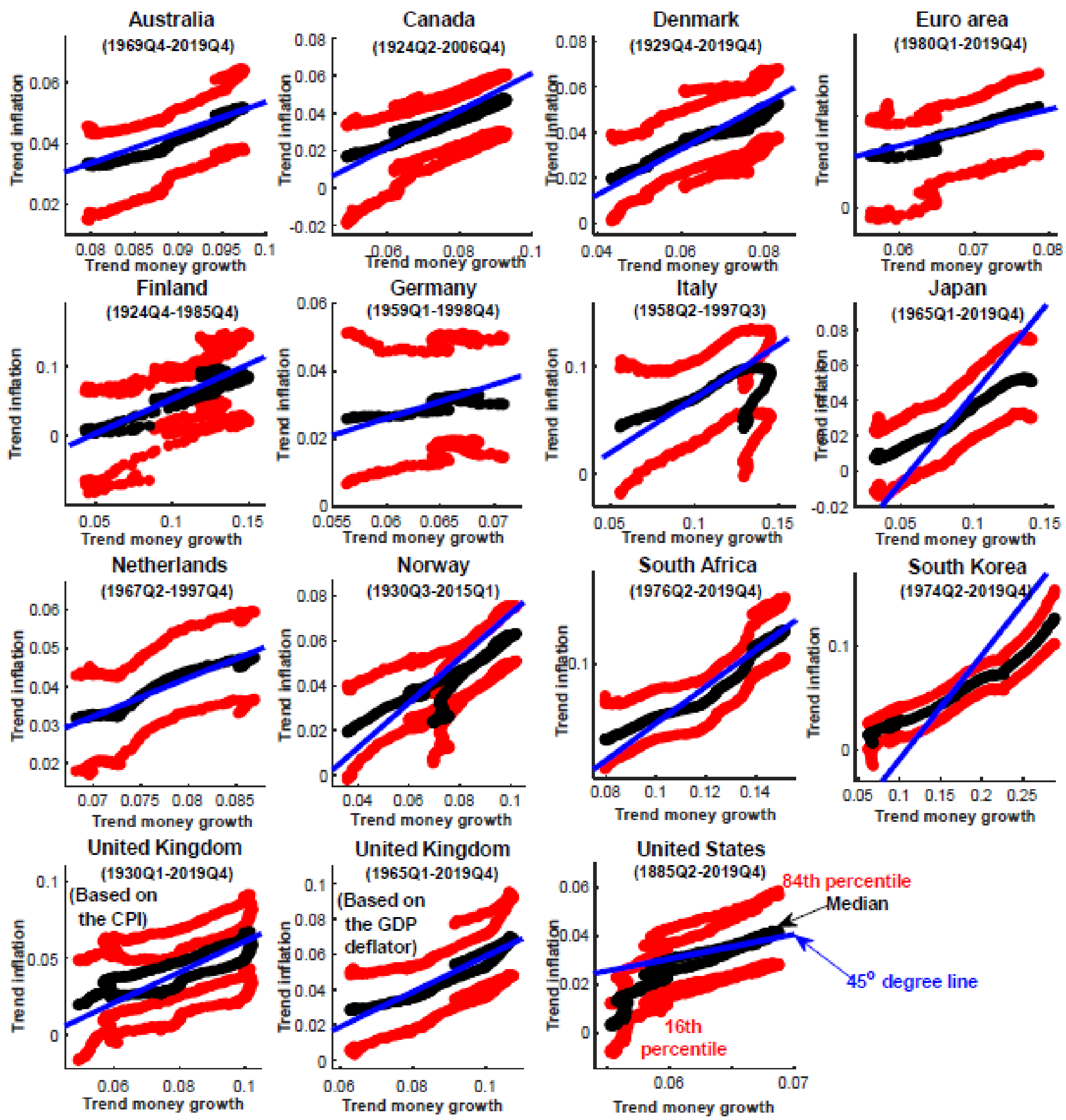

Figure 4a Slope relationship between the trends of money growth and inflation produced by the TVP-VARs of Amir-Ahmadi et al. (2020): median, and 16th and 84th percentiles

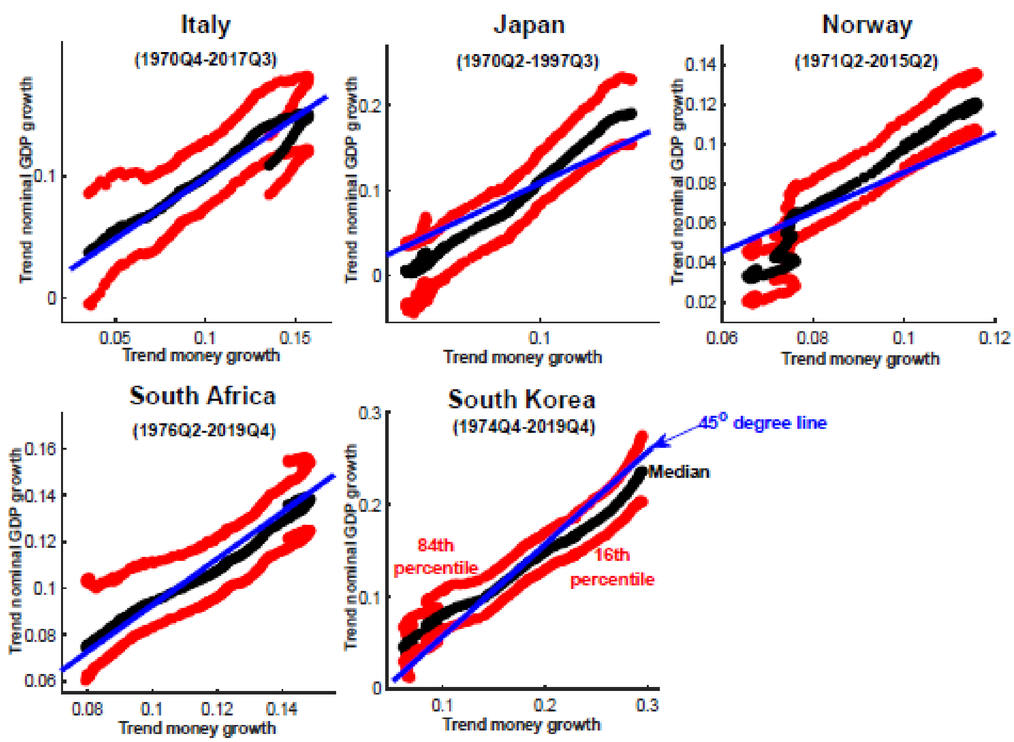

Figure 4b Slope relationship between the trends of money growth and nominal GDP growth produced by the TVP-VARs of Amir-Ahmadi et al. (2020): median, and 16th and 84th percentiles

growth produced by the TVP-VARs of Amir-Ahmadi *et al.* (2020),

$$Y_t = B_{0,t} + B_{1,t}Y_{t-1} + ... + B_{p,t}Y_{t-p} + u_t \tag{4}$$

where $Y_t = [\Delta m_t \ \Delta p_t]'$; the $B_{j,t}$'s, with $j = 0, 1, 2, ..., p$ are time-varying matrices whose elements evolve according to random walks, subject to reflecting barriers in order to keep the VAR stationary on a period-by-period basis;[36] and u_t is a (2×1) vector of reduced-form innovations whose covariance matrix, Ω_t, is factored as

$$\Omega_t = A_t^{-1}\Sigma_t(A_t^{-1})' \tag{5}$$

A_t is a lower triangular matrix with ones on the main diagonal, and the non-zero and non-one elements evolving as random walks. Σ_t is a diagonal matrix whose non-zero elements evolve according to geometric random walks.

Although this setup is the same as Primiceri (2005), the key innovation of Amir-Ahmadi *et al.* (2020) is that, instead of postulating prior distributions for the hyperparameters governing the extent of random-walk time-variation in the elements of the $B_{j,t}$'s, the non-zero and non-one elements of A_t, and the non-zero elements of Σ_t, they treat the hyperparameters as part of a hierarchical model, and estimate them jointly with the all of the other parameters in the model *via* Bayesian methods.

For future reference it is useful to rewrite (4) as

$$Y_t = \underbrace{\overset{\text{VAR-implied trends}}{\overline{[I - B_t(1)]^{-1}B_{0,t}}}}_{\mu_t} + \underbrace{\overset{\text{Deviations from VAR-implied trends}}{\overline{[I - (B_{1,t}L + ... + B_{p,t}L^p)]^{-1}u_t}}}_{A_t(L)u_t} = \mu_t + \Phi_t(L)u_t \tag{6}$$

thus highlighting how TVP-VARs automatically decompose a vector of time series Y_t into two components: the VAR-implied *trends* collected in the vector μ_t, which by construction are convolutions of random walks, and the series' *deviations* from their respective trends, which (as discussed) by construction are instead stationary.

I exactly follow Amir-Ahmadi *et al.* (2020) in terms of both prior distributions, and the algorithm I use to estimate (4) *via* Gibbs-sampling. (In fact, I estimate the model based on Amir-Ahmadi *et al.*'s (2020) MATLAB codes found at the website of the *Journal of Business and Economics Statistics.*) In particular, I use inverse-Gamma priors for the hyperparameters characterizing the hierarchical prior for the extent of random-walk time-variation, and I set $\kappa_{\Omega_b} = \kappa_{\Omega_h} = \kappa_{\Omega_{a,j}} = 0.1$.

Figures 4*a* and 4*b* show scatterplots of the slope relationships between the two elements of the μ_t's together with a 45 degrees line, which allows to properly assess how close the estimated slope relationships are to the one-for-one relationship implied by the QTM. The evidence in Figure 4*a* shows that the slope of the relationship between the trends of money growth and inflation has been statistically indistinguishable from

[36]Following Cogley and Sargent (2001, 2005) this is implemented by rejecting, within the Gibbs-sampling algorithm, all draws associated with non-stationary representations for $A_t(L)u_t$ in expression (12) below.

one-for-one over the *entire* sample period for 9 countries out of 14. The exceptions are Japan and South Korea, for which the slope has clearly been smaller than one; Italy, for which it has been markedly higher than one for the first part of the sample; and Norway and the U.S., which also exhibit instability in the first parts of their respective samples. As for South Africa, although strictly speaking the evidence is compatible with a unitary slope, the overall impression is clearly of some instability along the sample.

Three of these countries (Japan, South Korea, and Italy) have experienced dramatic decelerations of real GDP growth over the sample period, and especially in the first part of their samples.[37] This suggests that controlling for low-frequency shifts in real GDP growth may allow to recover the one-for-one relationship predicted by the QTM. The evidence in Figure 4*b* shows that this is nearly the case. For both South Korea, and especially Japan, the estimated slope is now slightly *greater* than one. For South Africa the evidence is now fully compatible with a unitary slope over the entire sample period. For Italy the relationship is now one-for-one over the entire sample, with the exception of a few observations. And for Norway, although there is still some evidence of instability, the slope is near-uniformly slightly greater than one-for-one.

Overall, the evidence in Figures 4*a* and 4*b* points towards a long-horizon relationship between the trends of money growth and either inflation, or nominal GDP growth, that has been near-uniformly one-for-one for all countries over the entire sample periods, *including* the period following the end of the Great Inflation. This shows that although, as discussed in Section 3.1.4, central banks' targeting of the inflation rate has weakened its low-frequency relationship with money growth in recent years, standard time-series methods can still recover the one-for-one long-horizon relationship predicted by the QTM.

Although, at first sight, these results would appear to be in contrast with the evidence in Sargent and Surico's (2011) Figure 5, in fact this is not the case. This is because whereas I am here focusing on the VAR-implied *trends* (i.e., the μ_t's in expression 12), Sargent and Surico (2011) analyzed instead the frequency-zero components of the series' *deviations* from the trends (i.e., the $\Phi_t(L)u_t$'s). Putting aside Müller and Watson's (2020) warnings about computing low-frequency estimates based on models estimated in the time domain (see the discussion in Section 3.1.1), the evidence in the top panel of Sargent and Surico's (2011) Figure 5 points towards weakness and time-variation in the low-frequency relationship between money growth and inflation *around a time-varying steady-state*. My evidence, on the other hand, pertains to the evolution of the slope relationship between the two series' steady states. The two types of evidence clearly address entirely different issues.

[37]E.g., for Japan average GDP growth decreased from 9.7% in the 1960s to 1.3% in the 2010s.

5 Conclusions

Following the end of the Great Inflation, and especially over the last two decades, a vast literature has documented weakness and instability in the relationship between money growth and inflation. Whereas initially this was thought to uniquely affect the high-to-medium frequencies, in recent years a dominant view has taken hold that, in fact, the relationship between money growth and inflation is weak and unstable *also* at the very low frequencies. In this paper I have reconsidered several aspects of the quantity theory of money based on data for 27 countries, for samples that in many cases extend back in time to the commodity standards era. Evidence from low-frequency regressions suggests that the relationship between broad money growth and inflation has been mostly one-for-one, and largely invariant to changes in the monetary regime. Only for the period since the mid-1980s, which has seen the introduction of monetary regimes in which inflation is directly targeted, the relationship appears to have materially weakened. Crucially, however, the slope relationship between the trends of money growth and inflation produced by time-varying parameters VARs has been near-uniformly one-for-one for all countries and sample periods, including the one following the end of the Great Inflation. This suggests that, although central banks' targeting of inflation has weakened its relationship with money growth, standard time-series methods can still recover the one-for-one long-horizon relationship between the series. The policy implication is that if central banks were to either lose control of, or allow persistent fluctuations in the long-horizon component of broad money growth, corresponding fluctuations in trend inflation would necessarily ensue.

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Tables for the Appendix

Table A.1*a* Annual data: sample periods, and average and maximum inflation[a]

Country	Period	*Average inflation*	*Maximum inflation*
	Commodity standards		
Argentina	1863-1929	0.009	0.220
Canada	1873-1929	0.012	0.218
Chile	1811-1877	0.004	0.169
Italy	1862-1935	0.021	0.355
Norway	1865-1930	0.010	0.362
Sweden	1847-1931	0.010	0.367
United Kingdom	1701-1796	0.004	0.245
	1821-1931	0.004	0.225
United States	1869-1932	-0.001	0.210
	Fiat standards		
Argentina	1930-2004	0.391	3.446
Australia	1914-2019	0.039	0.178
Brazil	1862-2019	0.248	3.417
Canada	1935-2006	0.040	0.142
Chile	1878-2019	0.160	2.072
Colombia	1956-2019	0.131	0.256
Finland	1915-1985	0.094	0.491
France	1937-1994	0.100	0.462
Iceland	1961-2019	0.136	0.573
Italy	1936-1996	0.124	0.886
Japan	1956-2018	0.026	0.188
Mexico	1926-2013	0.130	0.874
New Zealand	1914-2016	0.042	0.210
Norway	1947-2014	0.048	0.236
Paraguay	1963-2015	0.111	0.312
Peru	1960-2018	0.361	4.153
Portugal	1932-1998	0.068	0.235
Saudi Arabia	1964-2019	0.032	0.297
South Africa	1966-2019	0.094	0.222
South Korea	1965-2019	0.073	0.267
Spain	1942-1997	0.088	0.210
Sweden	1932-2018	0.044	0.184
Switzerland	1937-2015	0.031	0.140
United Kingdom	1932-2019	0.047	0.232
United States	1933-2019	0.032	0.119
Venezuela	1951-2014	0.155	0.768

[a] Computed as log-difference of the price level.

Table A.1*b* Quarterly data: sample periods, and average and maximum inflation[a]

Country	*Period*	*Average inflation*	*Maximum inflation*
Australia	1959Q4-2019Q4	0.0470	0.1754
Canada	1914Q2-2006Q4	0.0315	0.1875
	1968Q1-2019Q4	0.0392	0.1513
Denmark	1922Q4-2019Q4	0.0354	0.2668
	1977Q1-2019Q4	0.0322	0.1262
Euro area	1970Q3-2019Q4	0.0436	0.1359
Finland	1914Q4-1985Q4	0.0988	1.4019
Germany	1949Q1-1998Q4	0.0364	0.0967
Hong Kong	1985Q1-2019Q4	0.0280	0.1293
Italy	1948Q2-1997Q3	0.0831	0.2290
Japan	1955Q2-2019Q4	0.0249	0.1991
Netherlands	1957Q2-1997Q4	0.0403	0.1036
New Zealand	1988Q1-2019Q4	0.0860	0.1687
Norway	1920Q3-2019Q4	0.0303	0.2006
	1978Q1-2019Q4	0.0399	0.1233
South Africa	1966Q1-2019Q4	0.0945	0.2497
South Korea	1960Q1-2019Q4	0.0813	0.3267
Switzerland	1985Q1-2019Q4	0.0109	0.0556
Taiwan	1961Q3-2019Q4	0.0343	0.4389
	1982Q1-2019Q4	0.0079	0.0674
United Kingdom	1881Q1-2019Q4	0.0325	0.3459
	1914Q3-2019Q4	0.0380	0.2230
	1955Q1-2019Q4	0.0503	0.2412
United States	1875Q2-2019Q4	0.0217	0.2612

[a] Computed as the 4-quarters rolling average of 4 times the log-difference of the price level.

Table A.2 Full samples: evidence from regressing real GDP growth on money growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67% coverage probability:
Argentina	1864-2019	31.2	-0.025 [-0.044 -0.005]
Australia	1854-2019	38	-0.206 [-0.538 0.134]
Brazil	1862-2019	31.6	-0.009 [-0.028 0.009]
Canada	1873-2006	33.5	0.274 [0.155 0.385]
Chile	1811-2019	32.2	-0.007 [-0.033 0.028]
Finland	1867-1985	33.7	-0.089 [-0.179 0.083]
France	1910-1994	30	0.173 [-0.063 0.398]
Italy	1862-1996	30	0.079 [-0.013 0.171]
Mexico	1926-2013	35.2	0.000 [-0.111 0.105]
New Zealand	1885-2016	32.8	-0.195 [-0.416 0.025]
Portugal	1855-1998	32	0.016 [-0.107 0.132]
Sweden	1847-2018	31.3	-0.013 [-0.128 0.102]
Switzerland	1916-2015	33.3	0.205 [-0.071 0.479]
United Kingdom	1701-2019	30.4	0.089 [0.023 0.230]
United States	1869-2019	30.4	0.094 [-0.071 0.258]

For details, see Section 3.2.

Table A.3 Commodity standards: evidence from regressing real GDP growth on money growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67% coverage probability:
Argentina	1864-1929	33	0.107 [-0.190 0.436]
Canada	1873-1929	38	0.253 [0.041 0.495]
Chile	1811-1877	33.5	-0.464 [-1.513 0.429]
Italy	1862-1935	37	-0.016 [-0.046 0.017]
Norway	1865-1931	33.5	-0.001 [-0.061 0.063]
Sweden	1847-1931	34	-0.104 [-0.192 -0.016]
United Kingdom	1701-1796	32	0.452 [0.338 0.565]
	1821-1931	31.7	-0.119 [-0.289 0.040]
United States	1869-1932	32.5	0.370 [0.096 0.609]

For details, see Section 3.2.

Table A.4 *Fiat* standards: evidence from regressing real GDP growth on money growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67% coverage probability:
Argentina	1930-2019	30	-0.017 [-0.030 -0.005]
Australia	1914-2019	38	-0.206 [-0.538 0.134]
Brazil	1862-2019	31.6	-0.009 [-0.028 0.009]
Canada	1935-2006	36	0.146 [-0.105 0.397]
Chile	1878-2019	31.6	-0.010 [-0.040 0.018]
Colombia	1956-2019	31.5	0.008 [-0.073 0.087]
Finland	1915-1985	36.7	-0.017 [-0.268 0.200]
France	1937-1994	30	0.173 [-0.063 0.398]
Iceland	1961-2019	39.3	0.000 [-0.091 0.086]
Italy	1936-1996	30.5	-0.018 [-0.288 0.237]
Japan	1956-2018	31.5	0.409 [0.116 0.543]
Mexico	1926-2013	35.2	0.000 [-0.111 0.105]
New Zealand	1914-2016	32.8	-0.195 [-0.416 0.025]
Norway	1947-2014	33.5	-0.030 [-0.267 0.212]
Paraguay	1963-2015	35.3	0.356 [-0.094 0.774]
Peru	1960-2018	39.3	-0.057 [-0.078 -0.037]
Portugal	1932-1998	33.5	-0.007 [-0.241 0.216]
South Korea	1965-2019	36.7	0.235 [0.094 0.362]
Spain	1942-1997	37.3	0.307 [-0.067 0.751]
South Africa	1966-2019	36	-0.128 [-0.477 0.183]
Sweden	1932-2018	34.8	-0.096 [-0.439 0.260]
Switzerland	1937-2015	31.6	0.119 [-0.249 0.547]
United Kingdom	1932-2019	35.2	0.046 [-0.048 0.139]
United States	1933-2019	34.8	0.485 [-0.015 1.015]
Venezuela	1951-2017	33.5	-0.151 [-0.198 -0.104]

For details, see Section 3.2.

Table A.5 Commodity *versus fiat* standards: evidence from regressing nominal GDP growth on money growth at the very low frequencies controlling for changes in short rates

Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67% coverage probability
		Commodity standards	
Argentina	1864-1929	32.5	0.790 [0.331 1.249]
Canada	1873-1929	37.3	0.075 [-0.739 0.889][a]
Italy	1862-1935	36.5	1.284 [0.960 1.608]
Norway	1865-1931	33	1.064 [0.846 1.282]
Sweden	1857-1931	37	0.784 [0.357 1.210]
United Kingdom	1719-1796	30.8	0.762 [0.474 1.050]
	1821-1931	31.4	0.853 [0.688 1.018]
United States	1869-1932	31.5	1.333 [1.212 1.454]
		Fiat standards	
Argentina	1930-2004	37	0.986 [0.964 1.009]
Canada	1935-2006	35.5	0.601 [0.358 0.843]
Italy	1936-1996	30	1.279 [1.081 1.478]
Norway	1947-2014	33	0.256 [0.125 0.387]
Sweden	1932-2018[c]	38	1.127 [0.097 2.156]
United Kingdom	1932-2016	33.6	0.735 [0.602 0.867]
United States	1933-2019	34.4	1.083 [0.725 1.441]

[a] Based on the long rate.

Table A.6 Additional evidence for *fiat* standard regimes from regressing nominal GDP growth on money growth at the very low frequencies controlling for changes in short rates

Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67 per cent coverage probability
		Based on annual data	
Colombia	1956-2018	31	0.989 [0.815 1.163]
Finland	1931-1985	36	1.321 [0.636 2.006]
Japan	1956-2018	31	0.948 [0.902 0.994]
New Zealand	1935-2016	32.4	0.776 [0.508 1.043]
Portugal	1932-1998	33	1.105 [0.692 1.518]
South Korea	1965-2019	36	0.750 [0.732 0.769]
Spain	1942-1989	31.3	0.222 [0.189 0.255]
South Africa	1966-2019	35.3	0.723 [0.569 0.878]
Switzerland	1937-2015	31.2	0.313 [-0.360 0.985]
		Based on quarterly data	
Australia	1968Q2-2019Q4	34.3	1.077 [0.816 1.339]
Canada	1968Q1-2019Q4	34.3	1.501 [1.107 1.895]
Euro area	1970Q3-2019Q4	32.8	1.424 [0.443 2.405]
Japan	1955Q2-2019Q4	32.1	0.959 [0.880 1.038]
South Africa	1966Q1-2019Q4	35.7	0.707 [0.530 0.884]
South Korea	1960Q1-2019Q4	37	0.749 [0.731 0.767]
Taiwan	1961Q4-2019Q4	38.7	0.730 [0.669 0.791]
United Kingdom	1955Q2-2019Q4	32.3	0.672 [0.566 0.778]
United States	1933Q2-2019Q4	34.6	0.997 [0.695 1.299]

[a] The sample period is 1960Q2-1997Q4.

Table A.7 Period since 1985: evidence from regressing nominal GDP growth on money growth at the very low frequencies controlling for changes in short rates

Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67 per cent coverage probability
Australia	1985Q1-2019Q4	23.2	0.679 [0.647 0.711]
Canada	1985Q1-2019Q4	23.2	0.105 [0.064 0.146]
Denmark	1985Q1-2019Q4	23.2	0.486 [-0.323 1.294]
Euro area	1985Q1-2019Q4	23.2	1.135 [-0.518 2.788]
Hong Kong	1985Q1-2019Q4	23.2	1.061 [0.814 1.308]
Japan	1985Q1-2019Q4	23.2	0.848 [0.664 1.032]
Norway	1985Q1-2015Q1	23	0.706 [0.653 0.760]
New Zealand	1988Q1-2019Q4	21	-0.308 [-0.801 0.185]
South Korea	1985Q1-2019Q4	23.2	0.707 [0.617 0.797]
Switzerland	1985Q1-2019Q4	23.2	2.449 [1.590 3.308]
Taiwan	1985Q1-2019Q4	23.2	0.600 [0.537 0.663]
United Kingdom	1985Q1-2017Q2	23.2	2.319 [2.214 2.424]
United States	1985Q1-2019Q4	23.2	-1.139 [-1.165 -1.114]

Table A.8 Post-WWII period: evidence from regressing nominal GDP growth on the growth rates of money, non real estate loans,[a] or total loans at the very low frequencies controlling for changes in short rates

Country	Sample period	Highest frequency (in years)	Posterior median and confidence interval with 67 per cent coverage probability in the regression of either inflation or nominal GDP growth on:		
			money growth	non real estate loans growth	total loans growth
Canada	1947-2006	39.3	0.357 [0.129 0.585]	0.721 [0.228 1.214]	0.720 [0.452 0.989]
Italy	1949-1996	31.3	1.433 [0.541 2.325]	0.343 [-0.525 1.211]	0.495 [-0.345 1.334]
Japan	1956-2016	30.5	0.949 [0.899 0.998]	0.723 [0.577 0.870]	0.785 [0.660 0.909]
Norway	1947-2013	33.5	0.256 [0.125 0.387]	0.052 [-0.161 0.265]	0.069 [-0.180 0.317]
Switzerland	1947-2015	34.5	0.747 [0.106 1.388]	0.367 [0.114 0.621]	1.345 [0.332 2.357]
United Kingdom	1947-2014	34	0.889 [0.522 1.256]	0.483 [0.351 0.616]	0.632 [0.510 0.754]
United States	1947-2014	34	0.373 [-0.054 0.801]	0.080 [-1.879 2.039]	0.220 [-0.677 1.117]

[a] Computed as total nominal loans minus nominal loans to real estate.

Online appendix for: Long-Run Evidence on the Quantity Theory of Money

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University of Bern*

A The Data

Almost all of the data used in this paper are from original sources. Specifically, they are from either (*i*) original hard copy (books, or central banks' and national statistical agencies' hard-copy publications), in which case I have entered the data manually into Excel; or (*ii*) tables in either Excel or simple text format available at central banks' or national statistical agencies' websites. In the few cases (discussed below) in which I was not able to find the data in original documents, I took them from either the *International Monetary Fund*'s International Financial Statistics (henceforth, IMF and IFS, respectively) or the *Organisation for Economic Co-operation and Development*'s (OECD) Main Economic Indicators database. All of the annual series for total nominal loans and total nominal loans to real estate are from the Jordà, Schularick and Taylor dataset, available at http://www.macrohistory.net/data/. In the light of this paper's exclusive focus on broad monetary aggregates I do not consider the Netherlands, since the aggregate labelled as M2 in the Rolnick and Weber (1997) dataset is, in fact, M1: this can be trivially checked by comparing the series labelled as M2 in the sheets 'AllData' and 'Money' with the series labelled as M1 (in column L) in the sheet 'Netherlands' (on the other hand, the series there labelled as M2 is only available for 36 years).

A.1 Annual data

A.1.1 Argentina

An annual series for M3, available for the period 1863-2004, is from Table 7.1.4 ("Agregados Monetarios") of the *Banco Central de la República Argentina* (Ar-

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gentina's central bank, henceforth, *Banco Central*). The series has been extended to 2019 based on the monthly M3 data found at:

http://www.bcra.gov.ar/PublicacionesEstadisticas/Cuadros_estandarizados_series_estadisticas.as
in the Excel file Series Mensuales, sheet M3_total ('M3 - Total, saldos a fin de mes'). The series has been converted to the annual frequency by taking annual averages. Annual series for real GDP ('PBI a precios de mercado, mill. $ de 1993') and the GDP deflator ('Precios Implícitos - Demanda Agregada, PBI a precios de mercado'), available for the period 1821-2004, are from From Table 3 ('Cuentas Nacionales') of Ferreres (2005). Both series have been updated to 2019 based on the quarterly series from the *Instituto Nacional de Estadística y Censos de la República Argentina* (INDEC), which I have converted to the annual frequency by taking annual averages. Nominal GDP has been computed as the product of real GDP and the GDP deflator.

A.1.2 Australia

A series for M3, available for the period 1841-2019, is from Pope (1986) until 1959, and from the *Reserve Bank of Australia* (Australia's central bank, henceforth, *RBA*) after that (the acronym of this series is DMAM3N). A series for real GDP ('Gross domestic product: Chain volume measures'; series' code is A2304755F), available for the period 1960-2019, is from the *Australian Bureau of Statistics* (henceforth, *ABS*). A series for the CPI, available for the period 1850-2019, is from the *ABS*. A series for nominal GDP, available since 1960, is from the *ABS* ("Gross domestic product: Current prices; A2304617J; $ Millions"). For the period before 1960, this series has been linked to series from Mitchell (2007) and from Dincecco and Prado (2013), which are both available from the website of the *Global Price and Income History Group* at the University of California at Davis, at: http://gpih.ucdavis.edu/.

A.1.3 Brazil

A series for M2, available for the period 1839-2014, is from the Rolnick and Weber (1997) dataset (available at the website of the Federal Reserve Bank of Minneapolis) until 1987, and from the IMF's IFS since then. A series for real GDP, available for the period 1861-2014, is from the Rolnick and Weber (1997) dataset until 1992, and from the IMF's IFS since then. A series for the CPI, available for the period 1861-2019, is from the Rolnick and Weber (1997) dataset until 1991,[1] and from the IMF's IFS since then. A series for nominal GDP, available for the period 1861-2012, is from Mitchell (1998) until 1992, and from IBGE (the Brazilian Institute of Geography and Statistics) since then.

[1]Specifically, I constructed the series by linking the two series labelled as 'Onody' and 'Haddad/Contador' in the sheet 'Brazil'.

A.1.4 Canada

An annual series for nominal GDP, available since 1870, has been constructed by linking the Urquhart (1986) series available from *Statistics Canada* (Canada's national statistical agency, henceforth, *SC*), which is available for the period 1870-1924; series 0380-0515, v96392559 (1.1) from *SC*, for the period 1925-1980; and series 0384-0038, v62787311 (1.2.38) from *SC*, for the period 1981-2013. Likewise, a series for the GDP deflator has been constructed by linking the corresponding GDP deflator series from Urquhart (1986) and *SC*. A monthly series for M2, available for the period July 1871-December 1967, is from Metcalf, Redish, and Shearer (1996). After that, we link it to the series labelled as "M2 (net) (currency outside banks, chartered bank demand and notice deposits, chartered bank personal term deposits, adjustments to M2 (net) (continuity adjustments and inter-bank demand and notice deposits))" from *SC*, which is available until December 2006. For the period since December 2006 we were not able to find an M2 series that could be reliably linked to the previous ones (over the last several decades, Canada's monetary aggregates have undergone a number of redefinitions, which complicates the task of constructing consistent long-run series for either of them). As a result, we end the sample in December 2006. The annual M2 series has been computed by taking annual averages of the monthly series.

A.1.5 Chile

Annual series for the GDP deflator, real GDP and M2 are from Braun-Llona *et al.* (1998) for the period 1810-1995. As for the period 1996-2012, they are from the *Banco Central do Chile (* Chile's central bank). Specifically, the GDP deflator and real GDP are from *Banco Central*'s 'Anuarios de Cuentas Nacionales', whereas M2 is from *Banco Central*'s 'Base Monetaria y Agregados Monetarios Privados'. Nominal GDP has been computed as the product of real GDP and the GDP deflator.

A.1.6 Colombia

An annual series for M2 for the period 1955-2019 is from the *Banco de la Republica* (Colombia's central bank) since 1982, and from the IMF's IFS before that. Since 1982 the IMF series is near-identical to the one from the *Banco de la Republica*, which justifies the linking. Data for real and nominal GDP and the GDP deflator have been kindly provided by David Perez Reyna. The original sources are the *Base de Cuentas Nacionales* of DANE (Colombia's statistical agency) and the *Banco de la Republica*.

A.1.7 Finland

Long-run monthly data for M2 for the period January 1866-December 1985 have been generously provided by Tarmo Haavisto. The data come from his Ph.D. dissertation, Haavisto (1992), and have been converted to the annual frequency by taking simple

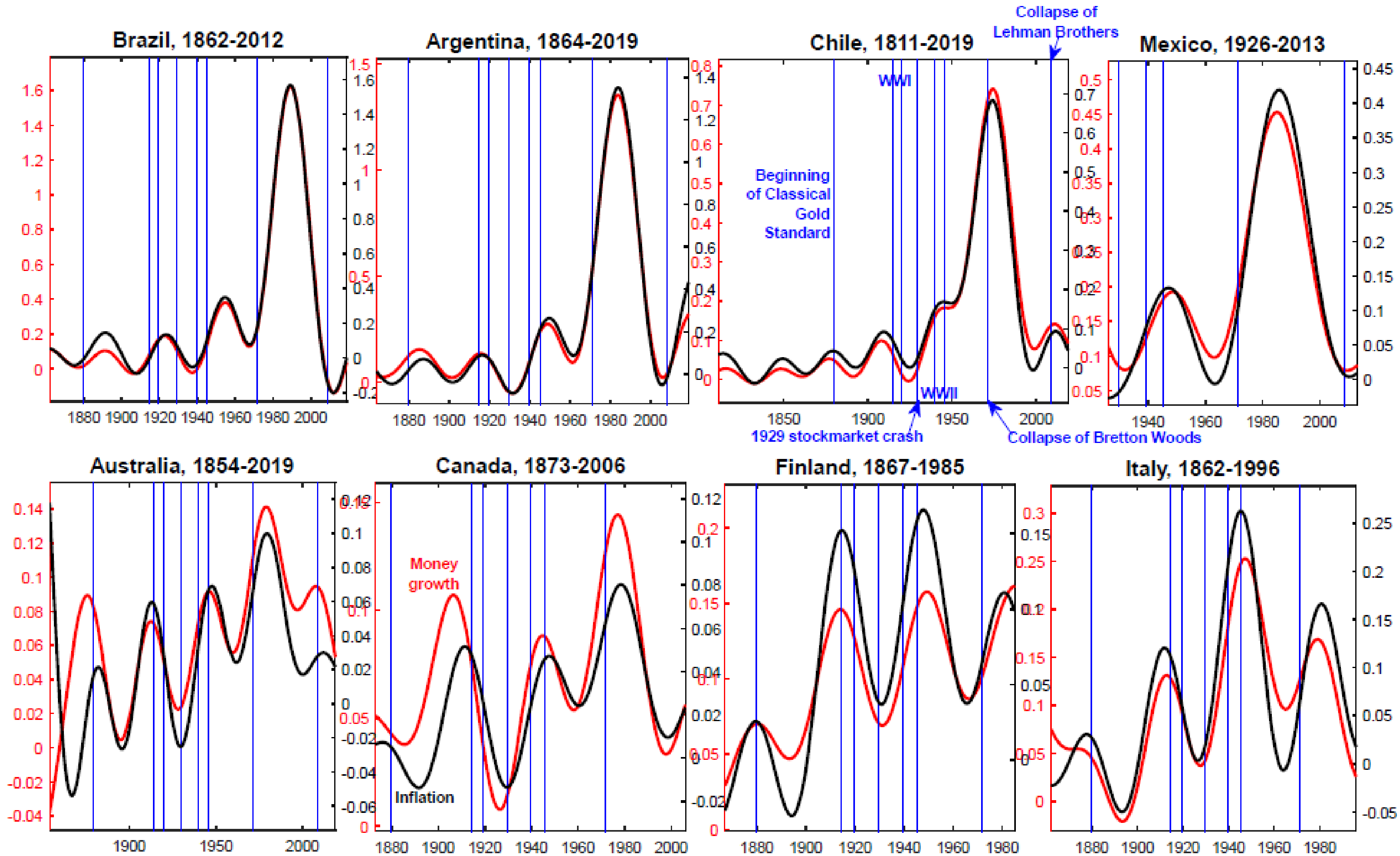

Figure A.1a Money growth and inflation: components with periodicities beyond 30 years, extracted via Christiano and Fitzgerald's (2003) filter

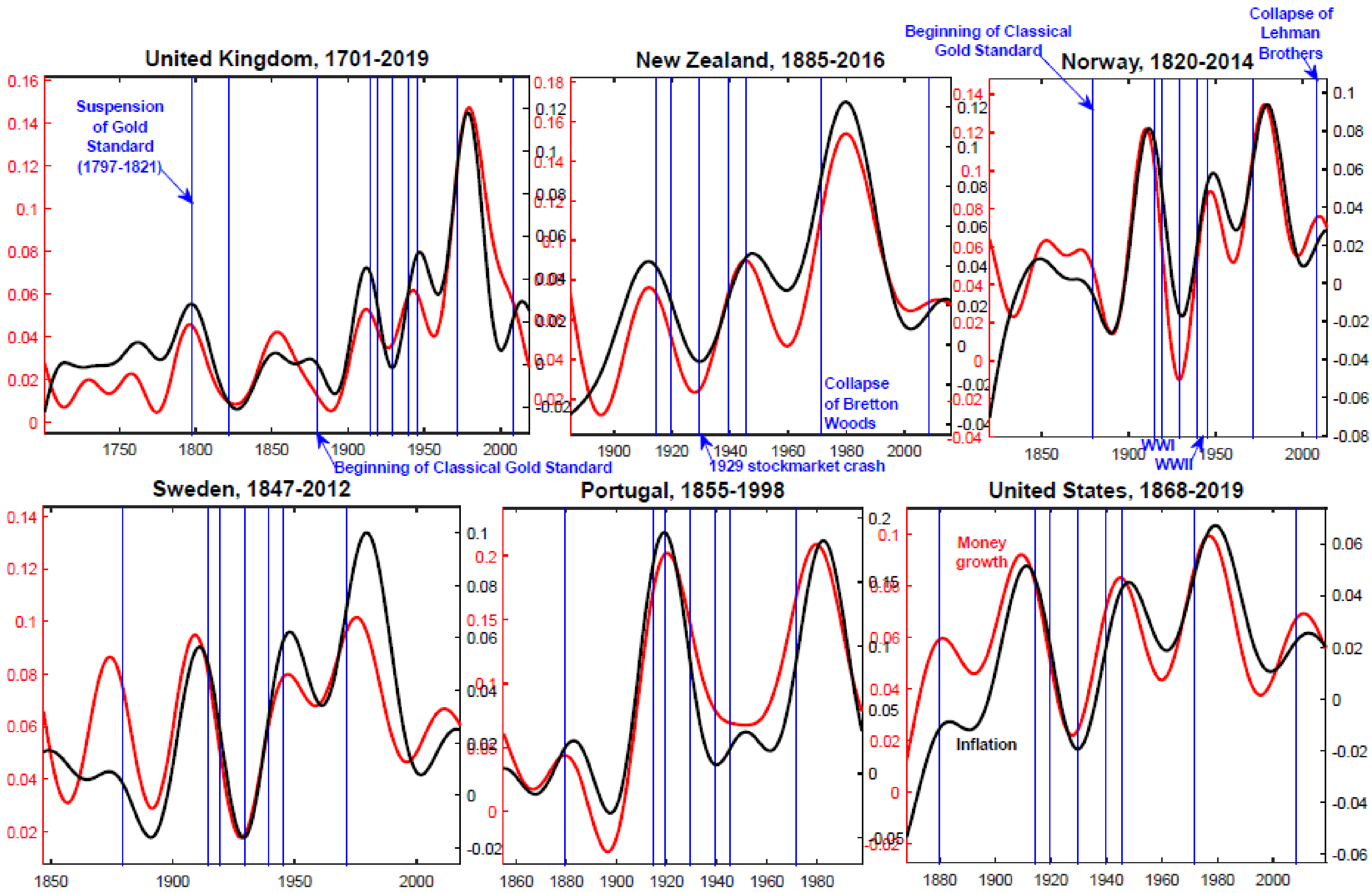

Figure A.1b Money growth and inflation: components with periodicities beyond 30 years, extracted via Christiano and Fitzgerald's (2003) filter

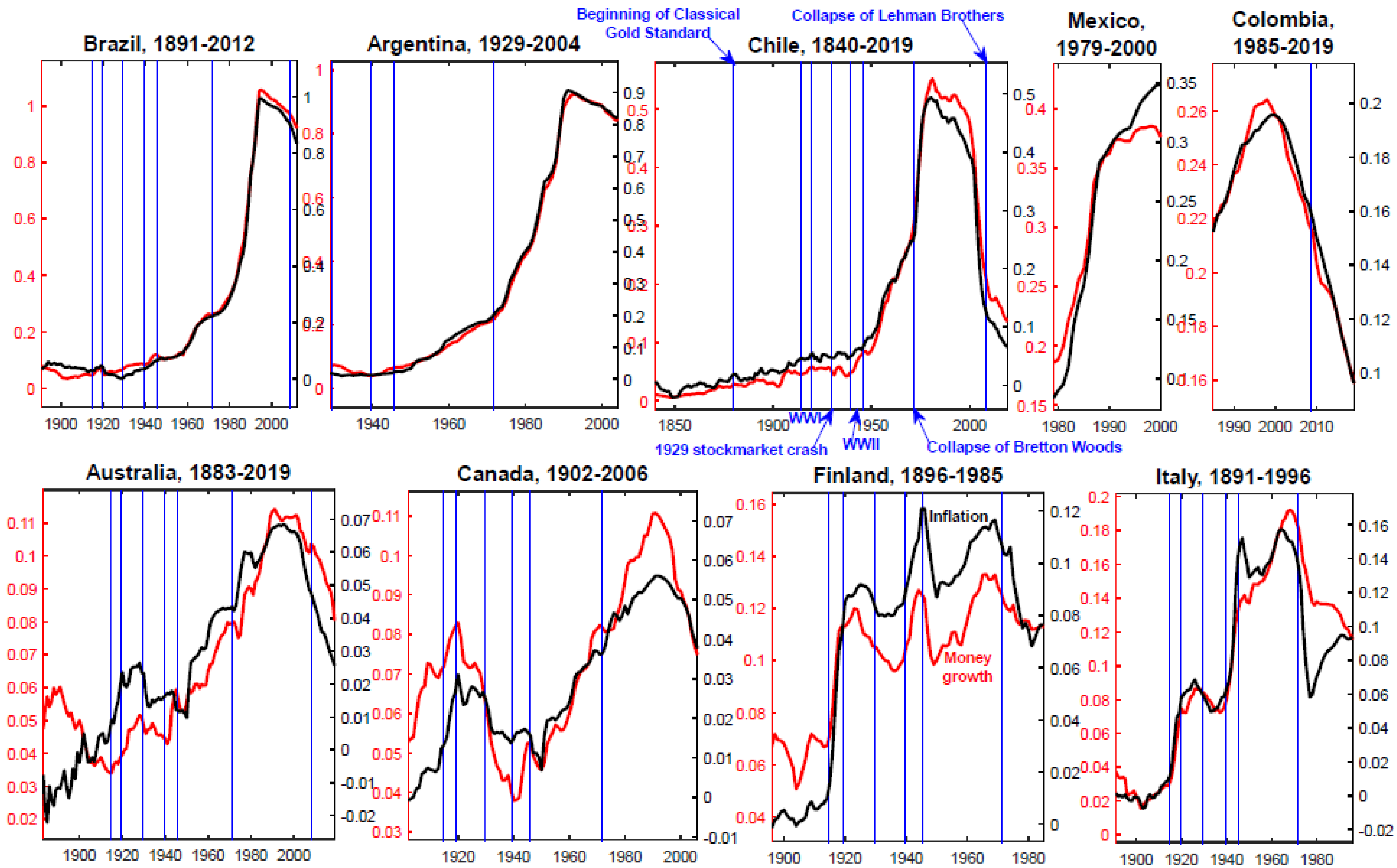

Figure A.2a Money growth and inflation: 30-year log-differences of broad money and prices

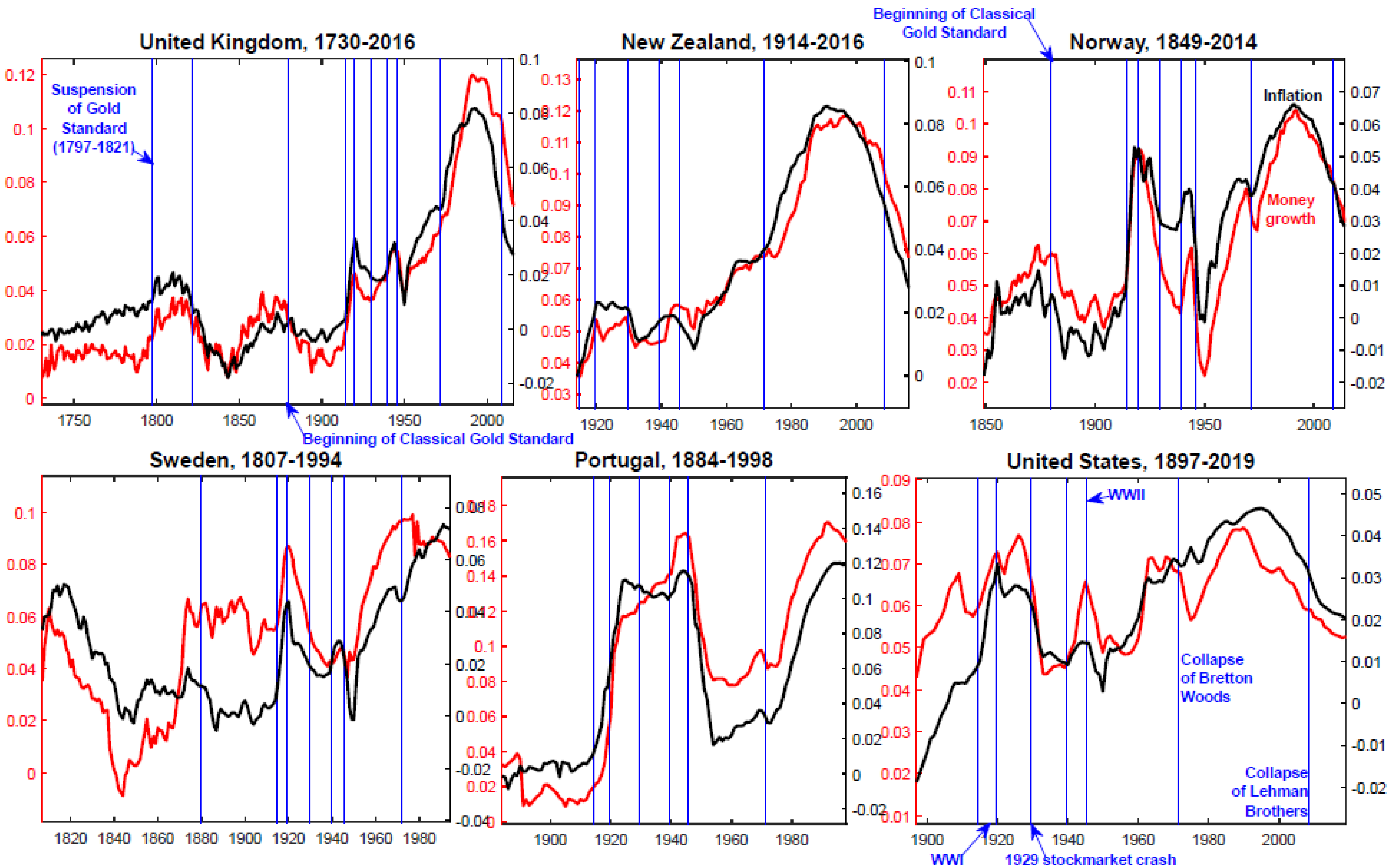

Figure A.2b Money growth and inflation: 30-year log-differences of broad money and prices

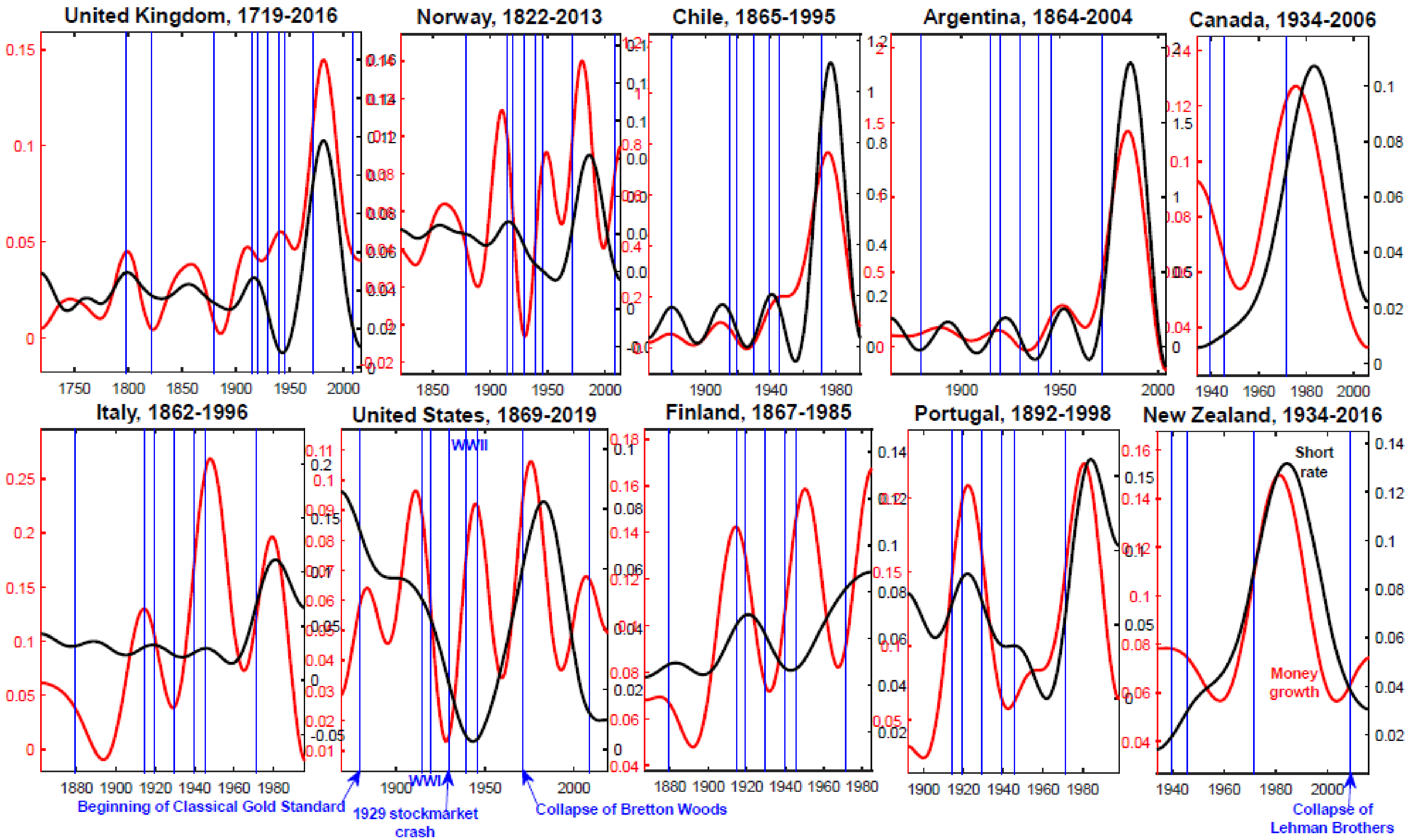

Figure A.3 Money growth and a short-term nominal interest rate: low-frequency components extracted via Müller and Watson's methodology

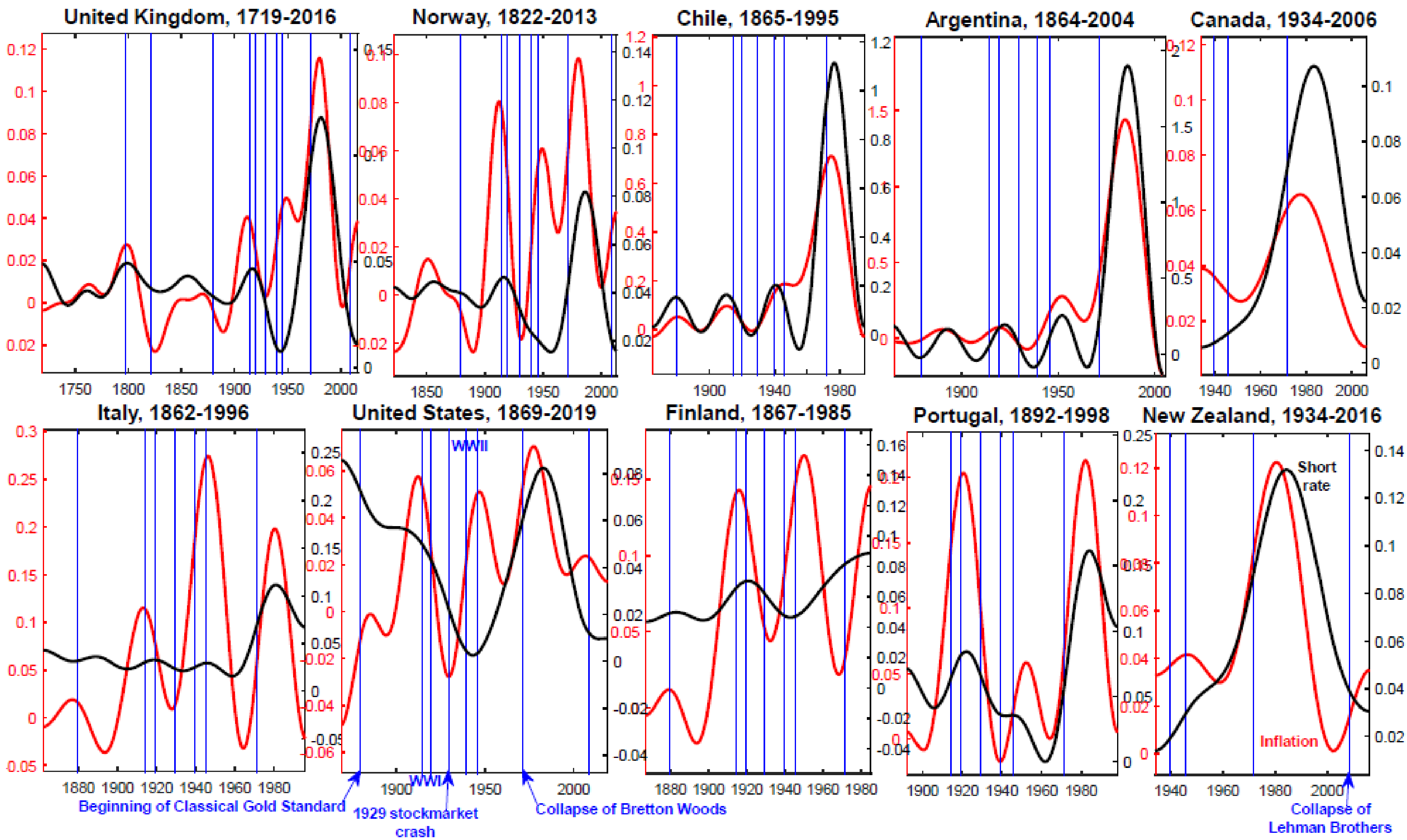

Figure A.4 Inflation and a short-term nominal interest rate: low-frequency components extracted via Müller and Watson's methodology annual averages. Series for nominal and real GDP and the GDP deflator, available since 1860, are from Finland's Historical Statistics, which are available from the web page of *Statistics Finland* (Finland's national statistical agency). (To be precise, from the homepage of Statistics Finland, look at Home > Statistics > National Accounts > Annual national accounts > Tables.)

A.1.8 France

Series for M2, real GDP, and prices are from the Rolnick and Weber (1997) dataset.

A.1.9 Germany

A series for nominal GDP, available for the periods 1851-1913 and 1949-1992, is from the Rolnick and Weber (1997) dataset. A series for the price level, available for the period 1820-1991, is from the Rolnick and Weber (1997) dataset. A series for M2, available for the period 1838-1923 (with a discontinuity in 1850) is from the Rolnick and Weber (1997) dataset.

A.1.10 Iceland

Series for nominal and real GDP, and the GDP deflator, all available since 1948, are from the IMF's IFS. Two series for M1 and M3 are from Iceland's central bank since 1994, and from the OECD's Main Economic Indicators before that. Over the period of overlapping, the series from the OECD are near-identical to those from the central bank, which justifies their linking.

A.1.11 Italy

Series for nominal GDP at current market prices, real GDP in chained 2005 euros, and the implied GDP deflator, all available for the period 1861-2010, are from the sheet "Tab_03' in the Excel spreadsheet 'Data_Na150-1.1.xls", which is available at the *Banca d'Italia*'s website at http://www.bancaditalia.it/statistiche/tematiche/stat-storiche/index.html. The spreadsheet contains the estimates of the Italian National Accounts' aggregates, which are extensively discussed in Baffigi (2011). A series for M2, available for the period 1861-1996, is from Muscatelli and Spinelli (2000).

A.1.12 Japan

A series for M2, available for the period 1868-1979, is from the Rolnick and Weber (1997) dataset until 1979, and from the *Bank of Japan* since then (over the period of overlapping the two series are identical, which justifies their linking). A series for the GDP deflator, available for the periods 1885-1940 and 1955-2018, is from the Rolnick and Weber (1997) dataset for the former period (it has been computed as the ratio between nominal and real GDP). As for the latter, it has been computed again as the ratio between nominal and real GDP based on data from the *Economic and Social Research Institute*, Cabinet Office, Government of Japan.

A.1.13 Mexico

A series for M2 for the period 1925-2000 is from the *Instituto Nacional de Estadistica y Geografia* (Mexico's national statistical agency, henceforth *INEGI*), "Estadisticas Historicas de Mexico, 2014". Annual series for nominal GDP and the GDP deflator are from *INEGI*, "Estadisticas Historicas de Mexico 2014", for the period 1925-1970; from the IMF's IFS for the period 1970-1988; from *Banco Central do Mexico* (Mexico's central bank) for the period 1988-2004; and from *INEGI* for the period since 2004. The four series have been linked *via* splicing.

A.1.14 New Zealand

A series for M3, available for the period 1884-2016, has been constructed by linking the series from Table 24.21 of Sheppard, Guerin, and Lee (1990), which is available for the period 1884-1989, and the series from *Reserve Bank of New Zealand*'s (henceforth, *RBNZ*) spreadsheet hc1.xls, which is available at the *RBNZ*'s website. Series for nominal and real GDP, and the GDP deflator, are from *Statistics New Zealand* (New Zealand's statistical agency), and they are available since 1860.

A.1.15 Norway

A series for M2, available since 1919, is from the Historical Statistics of *Norges Bank* (Norway's central bank), which are available at its website. All historical statistics for Norway's monetary aggregates are from Klovland (2004). Series for nominal GDP and the GDP deflator, all available since 1830, are from *Norges Bank*'s Historical Statistics (for all series, the period 1940-1945 is missing).

A.1.16 Paraguay

A series for nominal M2 in thousands of *guaranies,* available for the period 1962-2015, is from the website of *Banco Central del Paraguay* (Paraguay's central bank, henceforth *BCP*). Series for nominal and real GDP*,* available for the period 1950-2018, are from the website of *BCP*.

A.1.17 Peru

All of the data for Peru are from the *Banco Central de Reserva del Peru* (i.e. Peru's central bank). Specifically, series for nominal GDP in millions of nuevos soles, and real GDP in millions of nuevos soles at 2007 prices, are both available since 1950. By the same token, a series for CPI inflation, computed as the annual percentage change in the series 'Índice de precios (índice 2009 = 100) - Índice de Precios al Consumidor

(IPC)', is available since 1950. Two series for M1 ('Liquidez del sistema bancario (fin de periodo) - Dinero (millones S/)') and M2 ('Liquidez del sistema bancario (fin de periodo) - Cuasidinero (millones S/)')—the series' codes are PD09863MA and PD09866MA respectively—are available since 1959.

A.1.18 Portugal

A series for M2 for the period 1854-1998 is from Table 5 of Mata and Valerio (2011). A series for real GDP in 1914 prices for the period 1854-1998 has been constructed by linking the series from Nunes, Mata, and Valerio (1989) and that from Mata and Valerio (2011). A series for nominal GDP for the period 1854-1998 is from the Rolnick and Weber (1997) dataset.

A.1.19 Saudi Arabia

All of the data for Saudi Arabia are from the *Saudi Arabian Monetary Authority* (i.e. Saudi Arabia's central bank). Specifically, series for M1, M2, M3 and the CPI are available since 1963, whereas a series for the non-oil portion of nominal GDP is available since 1966.

A.1.20 South Africa

Series for nominal and real GDP, and M2 (the acronyms are KBP6006J, KBP6006Y, and KBP1373J respectively) are all from the website of the *South African Reserve Bank* (South Africa's central bank), at: https://www.resbank.co.za.

A.1.21 South Korea

All of the data for South Korea are from the website of the *Central Bank of Korea* (henceforth, *BOK*), at: http://ecos.bok.or.kr. Specifically, a series for M2 is from Table 1.1. ('Money & Banking (Monetary Aggregates, Deposits, Loans & Discounts etc.'); series for nominal and real GDP, and the GDP deflator, are from Tables 10.2.1.1 ('GDP and GNI by Economic Activities'), 10.2.2.2 ('Expenditures on GDP'), and 10.2.3.1. ('GDP Deflator by Economic Activities') respectively.

A.1.22 Spain

A series for M2 for the periods 1874-1935 and 1941-1997 is from Cuadro 9.16 "Agregados Monetarios, 1865-1998" of Barciela-López, Carreras, and Tafunell (2005), pp. 697-699 (the series is labeled as "M2, datos a fin de ano, en millones de pesetas"). A series for nominal GDP for the period 1850-2000 is from Cuadro 17.7 of Barciela-López, Carreras, and Tafunell (2005), pp. 1338-1340 (the series is labeled as "El PIB a precios corrientes, 1850-2000, millones de pesetas"; PIB is the Spanish acronym of

GDP). A series for the GDP deflator for the period 1850-2000 is from Cuadro 17.169 of Barciela-López, Carreras, and Tafunell (2005), pp. 1359-1361.

A.1.23 Sweden

An M2 series available for the period 1732-1994 is from the Rolnick and Weber (1997) dataset. An M3 series available for the period 1846-2012 is from the spreadsheet VolumeIIChapter7MoneySupply.xls, which is available from the website of the *Riksbank* (Sweden's central bank). A series for real GDP ('GDP volume by expenditure') available for the period 1732-2014 is from the spreadsheet VolumeIICh4GDP.xls at the *Riksbank*'s website (the series has been computed as the product of GDP *per capita* and population). A series for nominal GDP ('Nominal GDP Current prices, mn daler kmt 1620-1776, mn SEK 1777- (1 SEK = 18 kmt)') available for the period 1777-2014 is from the spreadsheet VolumeIICh4GDP.xls at the *Riksbank*'s website.

A.1.24 Switzerland

A series for M3 (based on the 1995 definition), available for the period 1907-2015, is from the website of the *Swiss National Bank* (Switzerland's central bank, henceforth *SNB*). A series for nominal GDP, available for the period 1851-1910, is from Switzerland's historical statistics database, at: http://www.fsw.uzh.ch/histstat/main.php. A series for nominal GDP, available for the period 1914-2015, has been constructed by linking a series from Switzerland's historical statistics and the nominal GDP series from SECO (Switzerland's State Secretariat for Economic Affairs). The GDP deflator series has been constructed as the ratio between nominal and real GDP.

A.1.25 United Kingdom

All U.K. data are from version 3.1 of the dataset 'A millennium of macroeconomic data', which is available from the *Bank of England*'s website.[2] The first version of the dataset (which was called 'Three centuries of macroeconomic data') was discussed in detail in Hills and Dimsdale (2010). Specifically, a series for broad money ('Composite broad money measure based on M3/M4, Year end data, break-adjusted stock') is available for the period 1270-2016; a series for real GDP ('Real UK GDP at market prices, geographically-consistent estimate based on post-1922 borders, £mn, Chained Volume measure, 2013 prices'), is available for the period 1700-2016; a series for nominal GDP ('Nominal UK GDP at market prices, £mn, Constant border UK (GB+NI) definition'), is available for the period 1700-2016. The GDP deflator series has been constructed as the ratio between nominal and real GDP.

[2] At: https://www.bankofengland.co.uk/statistics/research-datasets.

A.1.26 United States

A series for M2 has been constructed by linking the series from Balke and Gordon (1986), which is available for the period 1869-1983, to the M2 series from the *Board of Governors of the Federal Reserve System* (from release H.6, Money Stock Measures). The linked series is from Balke and Gordon (1986) until 1958, and from the Federal Reserve after that. A series for the GDP deflator has been constructed by linking the series from Officer and Williamson (2015)—see at http://www.measuringworth.com—and the GDP deflator series from Table 1.1.4 of the *National Income and Product Accounts* (NIPA). Likewise, a series for nominal GDP has been constructed by linking the series from Officer and Williamson (2015) to the nominal GDP series from Table 1.1.5 of the NIPA.

A.1.27 Venezuela

Series for nominal GDP ("Producto Interno Bruto, Millones de Bolívares a Precios Corrientes"), real GDP ("Producto Interno Bruto"), and M2 ("Liquidez Monetaria, (M2) Saldos al final de cada período en millones de bolívares"), are from the *Banco Central de Venezuela* (Venezuela's central bank). Nominal and real GDP are available for the period 1950-2017, whereas M2 is available since 1940.

A.2 Quarterly data

A.2.1 Australia

Seasonally adjusted series for the GDP deflator, real GDP (chain volume measure) and nominal GDP are all from the *Australian Bureau of Statistics*. The series' codes are A2303730T, A2304402X, and A2304418T respectively, and they are all available since 1959Q3. A seasonally adjusted M3 series from the *Reserve Bank of Australia* (the series' code is DMAM3S) is available since 1965Q2, and it has been extended back to 1959Q3 based on its seasonally unadjusted counterpart (the series' code is DMAM3N), which has been seasonally adjusted *via* ARIMA X-12. A short-term and a long-term interest rate ('3-Month or 90-day Rates and Yields: Interbank Rates for Australia, Percent', and 'Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Australia, Percent', respectively), both from the OECD, are available since 1968Q1 and 1969Q3, respectively.

A.2.2 Brazil

Seasonally adjusted monthly series for the monetary base, M1 and M2 available since January 1975 are from the *Banco Central do Brasil*. They have been converted to the quarterly frequency by taking averages within the quarter. Quarterly seasonally unadjusted series for nominal GDP and CPI inflation, available since 1975Q1, are from *IBGE*, and they have been seasonally adjusted *via* ARIMA X-12. A quarterly seasonally unadjusted series for the Treasury bill rate, available since 1975Q1, is from FRED II (the acronym is INTGSTBRM193N).

A.2.3 Canada

A monthly seasonally unadjusted CPI series available since January 1914 ('CPI, Canada, 2016A000011124, All-items, v41690973') is from *Statistics Canada*. The series has been converted to the quarterly frequency by taking averages within the quarter, and it has been seasonally adjusted *via* ARIMA X-12. A quarterly seasonally adjusted series for real GDP since 1947Q3 has been constructed by linking a series for real GDP in 1986 constant prices (available until 1997Q2) to a series in chained 2012 dollars (available since 1961Q1). Quarterly data for real GDP for the period before 1947Q3—which are used uniquely in order to calibrate the Bayesian priors when working with time-varying parameters VARs—have been computed by interpolating the real GDP annual series described in Appendix A.1.4. A quarterly seasonally adjusted series for the GDP deflator, available since 1961Q1, is from *Statistics Canada*. A seasonally adjusted quarterly M2 series available for the period 1871Q3-2006Q4 has been constructed by first linking the monthly seasonally unadjusted 'M2 net' series from the *Bank of Canada* ('M2 net: currency outside banks, chartered bank demand and notice deposits, chartered bank personal term deposits, adjustments to M2 net; continuity adjustments and inter-bank demand and notice deposits; the series' code is v37198'), which is available since January 1947, and the monthly seasonally unadjusted M2 series from Metcalf, Redish, and Shearer (1996), which is available for the period July 1871-December 1967. The resulting linked series has then been converted to the quarterly frequency by taking averages within the quarter, and it has been seasonally adjusted *via* ARIMA X-12. A seasonally unadjusted monthly 'M2 gross' series available since January 1968 ('M2 gross: currency outside banks, chartered bank demand and notice deposits, chartered bank personal term deposits, adjustments to M2 gross; continuity adjustments and inter-bank demand and notice deposits; the series' code is v41552786') has been converted to the quarterly frequency by taking averages within the quarter, and it has been seasonally adjusted *via* ARIMA X-12. In order to calibrate the Bayesian priors when working with time-varying parameters VARs, this series has been extended back in time based on the rate of growth of the previously mentioned 'M2 net' aggregate. Monthly short-term and long-term interest rates ('Treasury bill auction, average yields, 3 month', and 'Government of Canada marketable bonds, average yields over 10 years', respectively), both from the *Bank of Canada*, are available since October 1935 and January 1919, respectively, and they have been converted to the quarterly frequency by taking averages within the quarter.

A.2.4 Denmark

Seasonally adjusted series for M2, the CPI, and a short-term and a long-term nominal interest rate, all available for the period 1922Q3-2011Q4, are from the database of long-term Danish historical statistics collected by Kim Abildgren, which is available at: https://sites.google.com/view/kim-abildgren/home. The four series have been updated to 2019Q4 based on data from either *Statistics Denmark* or *Danmarks Nationalbank* (Denmark's national statistical agency and central bank, respectively). Quarterly seasonally adjusted series for real GDP and the GDP deflator, both available since 1977Q1, are from the IMF's IFS.

A.2.5 Euro area

All of the data for the Euro area are from the *European Central Bank* (ECB), and they are available since 1970Q1. Seasonally adjusted series for the GDP deflator and real GDP have been constructed by linking the series from the Area Wide Model (AWM) database, which are available up to 2017Q4, to the real GDP and GDP deflator series from the ECB's Statistical Data Warehouse (SDW), which are available since 1995Q1. A series for the short rate has been constructed by linking the short rate series from the AWM database ('STN') to the series for the 3-month Euribor rate from the ECB's SDW. A series for the long rate has been constructed by linking the long rate series from the AWM database ('LTR') to the series for the Euro area average 10-year government bond yield from the ECB's SDW ('10y government bond yield, FM.M.U2.EUR.4F.BB.U2_10Y.YLD, Euro area (changing composition) - Benchmark bond - Euro area 10-year Government Benchmark bond yield'). A quarterly series for M2 has been constructed by first linking the monthly seasonally adjusted historical M2 series from

https://www.ecb.europa.eu/stats/pdf/money/aggregates/historical_1970s_sa.pdf and the M2 series from the ECB's SDW ('BSI.M.U2.Y.V.M20.X.1.U2.2300.Z01.E: Monetary aggregate M2, Outstanding amounts at the end of the period (stocks), Euro'), and then converting the resulting linked series to the quarterly frequency by taking averages within the quarter.

A.2.6 Finland

A monthly seasonally adjusted M2 series for the period January 1866-December 1985 has been generously provided by Tarmo Haavisto. The series come from his Ph.D. dissertation, Haavisto (1992). A monthly seasonally adjusted cost-of-living index, available since August 1939, is from *Statistics Finland*. Both series have been converted to the quarterly frequency by taking averages within the quarter.

A.2.7 Germany

A quarterly seasonally unadjusted M2 series available for the period 1948Q4-1998Q4 has been kindly provided by the Bundesbank, and it has been seasonally adjusted *via* ARIMA X-12. A monthly seasonally adjusted CPI series available since June 1948 is from the Bundesbank's website ('Consumer price index / seasonally adjusted / 1

/ 2 / 3, USFB99,USFB99_FLAGS, Unit,2000=100'), and it has been converted to the quarterly frequency by taking averages within the quarter. Seasonally adjusted series for real and nominal GDP, and the GDP deflator, all available since 1960Q1, are from the IMF's IFS. A monthly series for a short-term nominal interest rate ('BBK01.SU0101, Geldmarktsätze am Frankfurter Bankplatz / Tagesgeld / Monatsdurchschnitt, % p.a., Eins') available since January 1960 is from the Bundesbank's website, and it has been converted to the quarterly frequency by taking averages within the quarter. A monthly series for a long-term nominal interest rate ('Germany, Interest Rates, Government Securities, Government Bonds, Percent per Annum') available since January 1957 is from the IMF's IFS, and it has been converted to the quarterly frequency by taking averages within the quarter.

A.2.8 Hong Kong

Seasonally adjusted series for real and nominal GDP, and the GDP deflator, all available since 1973Q1, are from the website of Hong Kong's Census and Statistics Department. A seasonally adjusted monthly M2 series available since January 1985 is from Table 2.2 ('Money supply') from the website of the *Hong Kong Monetary Authority* (HKMA), and it has been converted to the quarterly frequency by taking averages within the quarter. A seasonally unadjusted monthly series for the interbank rate (HIBOR) available since January 1985 is from Table 6.3.1 from the HKMA's website, and it has been converted to the quarterly frequency by taking averages within the quarter. A seasonally unadjusted monthly series for a long rate ('Interest Rates, Government Securities, Government Bonds for Republic of Korea, Percent per Annum, Monthly, Not Seasonally Adjusted') available since July 1973, is from the IMF's IFS, and it has been converted to the quarterly frequency by taking averages within the quarter.

A.2.9 Italy

A monthly seasonally unadjusted series for the CPI net of tobacco ('VIGR: Costo della vita - Indice generale al netto dei tabacchi'), available since January 1947, is from Italy's national statistical agency, *ISTAT*. A monthly seasonally unadjusted M2 series, available since January 1948, has been reconstructed by Eugenio Gaiotti of *Banca d'Italia*. Both series have been seasonally adjusted *via* ARIMA X-12 and converted to the quarterly frequency by taking averages within the quarter. A quarterly seasonally adjusted series for real GDP, available for the period 1960Q1-2017Q3, is from the Ohanian and Raffo (2011) database. A monthly seasonally unadjusted series for the *Banca d'Italia*'s discount rate, available for the period January 1964-December 1998, is from the IMF's IFS, and it has been converted to the quarterly frequency by taking averages within the quarter.

A.2.10 Japan

A monthly seasonally unadjusted series for the CPI ('CPI: All items less imputed rent'), available since August 1946, is from Japan's national statistical agency, *Statistics Japan*. A monthly seasonally unadjusted series for M2 including certificates of deposits is from the *Bank of Japan*. Both series have been seasonally adjusted *via* ARIMA X-12 and converted to the quarterly frequency by taking averages within the quarter. A monthly seasonally unadjusted series for the *Bank of Japan*'s discount rate, available since January 1883, is from the *Bank of Japan*'s website, and it has been converted to the quarterly frequency by taking averages within the quarter. Quarterly seasonally adjusted series for real and nominal GDP and the GDP deflator, available since 1955Q2, are from the *Economic and Social Research Institute*, Cabinet Office, Government of Japan.

A.2.11 Mexico

A quarterly seasonally adjusted series for the GDP deflator available since 1985Q4 is from INEGI. A quarterly seasonally unadjusted series for nominal GDP available since 1985Q4 is from INEGI, and it has been seasonally adjusted *via* ARIMA X-12. A series for the Treasury bill rate is from FRED II (the acronym is INTGSTMXM193N). A quarterly seasonally adjusted series for M2 available since 1985Q4 is from the *Banco de México*'s 'Agregados monetarios y activos financieros internos'.

A.2.12 Netherlands

Monthly seasonally unadjusted series for the CPI ('CPI: wage earners, median inc.') and M2 ('M2: National definition'), available since January 1957, and for the period January 1957-December 1997, respectively, are from the IMF's IFS. Both series have been seasonally adjusted *via* ARIMA X-12 and converted to the quarterly frequency by taking averages within the quarter. Quarterly, seasonally adjusted series for real GDP and the GDP deflator, available since 1960Q1, are from the OECD. A quarterly, seasonally adjusted series for nominal GDP has been computed as the product of the series for real GDP and the GDP deflator. A series for the 10-year government bond yields, available since 1960Q1, is from the IMF's IFS. A daily series for the fixed advance rate (a short-term monetary policy rate) has been constructed based on data from the *Nederlandsche Bank* (Netherlands' central bank), and it has then been converted to the quarterly frequency by taking averages within the quarter.[3]

[3] To be precise, *Nederlandsche Bank* does not provide the actual daily time series for the fixed advance rate, but rather the dates at which the rate had been changed (starting from August 3, 1898), together with the new value of the fixed advance rate prevailing starting from that date. Based on this information, I constructed a daily series for the fixed advance rate starting on August 3, 1898, *via* a straightforward MATLAB program, and I then converted the series to the quarterly frequency by taking averages within the quarter.

A.2.13 New Zealand

A quarterly seasonally adjusted CPI series available since 1925Q3 has been constructed by linking the CPI series from the spreadsheet ha3discontinued.xls from the website of the *Reserve Bank of New Zealand* (RBNZ),[4] which is available until 2013Q1, and the CPI series from the spreadsheet hm1.xls, again from the RBNZ's website, which is available since 1988Q1 ('CPI.Q.C.ia'). Both series are originally from New Zealand's statistical agency, *Statistics New Zealand*. A quarterly seasonally unadjusted M3 series available since 1986Q2 is from the RBNZ's website, and it has been seasonally adjusted *via* ARIMA X-12. Quarterly series for real GDP ('Expenditure-based gross domestic product, Real $m, NZDm(r), GDE.Q.EY.RA'), and nominal GDP ('Expenditure-based gross domestic product, Nominal $m, NZDm, GDE.Q.EY.NA'), both available since 1987Q2, are from the RBNZ's website. Monthly seasonally unadjusted series for a short rate ('Bank bill yields, 30 days') and a long rate ('Secondary market government bond yields, 10 year') are both from *Statistics New Zealand*, and have been converted to the quarterly frequency by taking averages within the quarter. A monthly seasonally adjusted series for 'broad money', available since January 1988, is from the RBNZ's website, and it has been converted to the quarterly frequency by taking averages within the quarter (the series' acronym is MCA.MDB.BM).

A.2.14 Norway

A quarterly seasonally adjusted series for real GDP, available for the period 1960Q1-2017Q3, is from the Ohanian and Raffo (2011) database. A monthly seasonally adjusted M2 series ('M2 broad money, (old definition)') available for the period January 1919-March 2015, is from Klovland (2004), and it is available at the website of *Norges Bank* (Norway's central bank).[5] A monthly seasonally adjusted CPI series available since January 1920 is from the collection of Norway's historical statistics at *Norges Bank*'s website. A monthly series for a short-term rate available since January 1961 has been constructed by linking the series for the Eurokrone 3 month rate from *Norges Bank*, which is available until December 1986, and the series for the 3-month Norway interbank overnight rate (NIBOR), which is available up until the present (over the period of overlapping, the two series are very close, which justifies their linking). A monthly series for a long rate ('Norway: Interest Rates, Government Securities, Government Bonds, Percent per Annum') available since January 1961, is from the IMF's IFS. All monthly series have then been converted to the quarterly frequency by taking averages within the quarter. Quarterly seasonally unadjusted series for nominal GDP and the GDP deflator, available since 1978Q1, are from *Statistics Norway* (Norway's statistical agency), and they have been seasonally adjusted *via* ARIMA X-12.

[4] At: https://www.rbnz.govt.nz/statistics/discontinued-statistics.

[5] A second M2 series from *Statistics Norway* (Norway's statistical agency) is available since January 2008, but the two series are defined in a different way, and they cannot be linked.

A.2.15 South Africa

A monthly seasonally unadjusted M2 series ('Monetary aggregates / Money supply: M2, KBP1373M, Rand millions'), available since January 1966, is from the South African Reserve Bank (SARB), and it has been converted to the quarterly frequency by taking averages within the quarter, and then seasonally adjusted *via* ARIMA X-12. A monthly seasonally unadjusted series for a short rate ('KBP1401M: Bankrate (lowest rediscount rate at SARB)'), available since January 1960, is from the SARB, and it has been converted to the quarterly frequency by taking averages within the quarter. Quarterly seasonally adjusted series for nominal and real GDP ('KBP6006K: Gross domestic product at market prices, Current prices, Seasonally adjusted' and 'KBP6006C: Gross domestic product at market prices, Constant 2010 prices, Seasonally adjusted', respectively), both available since 1960Q1, are from the SARB. The GDP deflator has been computed as the ratio between nominal and real GDP. A monthly seasonally unadjusted series for a long rate ('Long-Term Government Bond Yields: 10-year for South Africa, Percent, Quarterly, Not Seasonally Adjusted'), available since January 1960, is from the IMF's IFS, and it has been converted to the quarterly frequency by taking averages within the quarter.

A.2.16 South Korea

All of the data for South Korea are from the *Bank of Korea* (South Korea's central bank). Quarterly seasonally adjusted series for nominal and real GDP and the GDP deflator ('Expenditures on GDP, seasonally adjusted, current prices, quarterly', 'Expenditures on GDP, seasonally adjusted, chained 2010 year prices, quarterly' and 'GDP deflator, seasonally adjusted, quarterly, 2015=100', respectively) are all available since 1960Q1. A monthly seasonally adjusted M2 series ('M2: Broad Money, End Of Period, Billion Won'), available since January 1960, has been converted to the quarterly frequency by taking averages within the quarter. The series is from the *Bank of Korea*'s Table 1.1 ('Money & Banking (Monetary Aggregates, Deposits, Loans & Discounts etc.)'). A monthly seasonally unadjusted series for the *Bank of Korea*'s discount rate, available since January 1964, has been converted to the quarterly frequency by taking averages within the quarter.

A.2.17 Switzerland

Quarterly seasonally adjusted series for nominal and real GDP and the GDP deflator ('Gross domestic product, ESA 2010, quarterly aggregates of Gross Domestic Product, expenditure approach, seasonally and calendar adjusted data, in million Swiss Francs, at current prices', 'Gross domestic product, ESA 2010, quarterly aggregates of Gross Domestic Product, expenditure approach, seasonally and calendar adjusted data, in million Swiss Francs, at prices of the preceding year, chained values, reference year 2010' and 'Gross domestic product, ESA 2010, quarterly aggregates of Gross Domestic Product, expenditure approach, seasonally and calendar adjusted data, implicit chain price indexes', respectively), all available since 1980Q1, are from the website of Switzerland's *State Secretariat for Economic Affairs* (SECO). A monthly seasonally adjusted M2 series ('M2, level, seasonally adjusted, in million Swiss Francs'), available since January 1985, has been converted to the quarterly frequency by taking averages within the quarter. Monthly seasonally unadjusted series for the interbank rate and the 10-year government bond yield ('Interbank Rates for Switzerland, 3-Month or 90-day Rates and Yields' and 'Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Switzerland, Percent, Monthly, Not Seasonally Adjusted', respectively), available since January 1974 and January 1960, respectively, have been converted to the quarterly frequency by taking averages within the quarter. Both series are from the OECD's Main Economic Indicators database.

A.2.18 Taiwan

Quarterly seasonally adjusted series for nominal and real GDP, available since 1961Q1 and 1982Q1, respectively, are from the website of Taiwan's Directorate-General of Budget, Accounting, and Statistics (DGBAS). The GDP deflator has been computed as the ratio between nominal and real GDP. A monthly seasonally adjusted M2 series ('M2, end of period, 100 millions of N.T. dollars'), available since October 1961, has been converted to the quarterly frequency by taking averages within the quarter. A monthly seasonally unadjusted series for the discount rate of the central bank of the Republic of China Taiwan (Taiwan's central bank) is from from the central bank's website, is available since October 1961, and it has been converted to the quarterly frequency by taking averages within the quarter. A monthly seasonally adjusted CPI series, available since January 1959, is from DGBAS, and it has been converted to the quarterly frequency by taking averages within the quarter.

A.2.19 United Kingdom

Quarterly seasonally adjusted series for nominal and real GDP ('YBHA: Gross domestic product at market prices: current price, seasonally adjusted £m' and 'ABMI: Domestic expenditure at market prices, gross domestic product, £ million at chained volume measures', respectively), available since 1955Q1, are from the *Office for National Statistics* (ONS). The GDP deflator has been computed as the ratio between nominal and real GDP. Series for monetary aggregates, interest rates, the CPI and the wholesale price index (WPI) are all from 'A millennium of macroeconomic data for the U.K.' (Version 3), the *Bank of England*'s collection of historical macroeconomic and financial statistics available from its website. Monthly seasonally unadjusted series for a short- and long-term nominal interest rate ('Spliced series for Treasury bill allotment rate/discount rate at the weekly tender, % pa' and 'Medium-term/10 year bond yield, % pa', respectively), available for the periods January 1923-January 2017 and January 1935-January 2017, have been converted to the quarterly frequency by taking averages within the quarter. A quarterly seasonally adjusted break-adjusted M4 series is available for the period 1880Q4-2016Q4. The series has been updated to 2019Q4 by linking it to the series LPMVUBR ('Monthly break adjusted level of monetary financial institutions' sterling M4 liabilities to private sector (in sterling millions) seasonally adjusted') from the *Bank of England*'s database, which I have converted to the quarterly frequency by taking averages within the quarter. Monthly seasonally adjusted CPI and WPI series ('Spliced monthly consumer price index' and 'Spliced wholesale/producer price index'), available since July 1914 and January 1790, respectively, have been updated to the present based on the corresponding data from the ONS, and have then been converted to the quarterly frequency by taking averages within the quarter.

A.2.20 United States

Quarterly seasonally adjusted series for real GNP and the GNP deflator since 1875Q1 are from Balke and Gordon (1986) up until 1946Q4, and from the U.S. Department of Commerce: Bureau of Economic Analysis (BEA) after that (the real GNP and the GNP deflator series' acronyms in FRED II, the Federal Reserve Bank of St. Louis' internet data portal, are GNPC96 and GNPDEF respectively). By the same token, a quarterly seasonally adjusted M2 series since 1875Q1 is from Balke and Gordon (1986) up until 1958Q4, and from the Board of Governors of the Federal Reserve System after that (the FRED II acronym is M2SL). A monthly seasonally unadjusted series for the commercial paper rate, available since January 1857, is from the NBER Historical database (NBER series is 13002, 'Commercial Paper Rates for New York') until March 1971; between April 1971 and December 1996, it is the series CP3M ('3-Month Commercial Paper Rate') from the G.13 release ('Selected Interest Rates') from the Board of Governors of the Federal Reserve System; after that, it is CPF3M ('3-Month AA Financial Commercial Paper Rate '), from the Board of Governors of the Federal Reserve System. Over the period of overlapping the three series are near-identical, which justifies their linking. Finally, the resulting monthly linked series has been converted to the quarterly frequency by taking averages within the quarter. A quarterly series for a long-term nominal interest rate available since 1875Q1 has been constructed by linking the corporate bond yield series from Balke and Gordon (1986) and the series for the BAA corporate bond yield from the Board of Governors of the Federal Reserve System. Specifically, the linked series is Balke and Gordon's up until 1918Q4, and the Federal Reserve's BAA corporate bond yield after that. Finally, for the post-WWII period we also consider quarterly seasonally adjusted series for real GDP and the GDP deflator ('Real gross domestic product, billions of chained 2012 dollars, quarterly, seasonally adjusted annual rate' and 'Gross domestic product: chain-type price index, index 2012=100, quarterly, seasonally adjusted', respectively), both available since 1947Q1 from the BEA (the series' acronyms in FRED II are are GDPC1 and GDPCTPI, respectively); and monthly seasonally unadjusted series for the 3-month Treasury bill rate and the 10-year Treasury constant maturity rate (the series' acronyms in FRED II are TB3MS and GS10, respectively), which are both from the Board of Governors of the Federal Reserve System, and are available since January 1934 and April 1953, respectively (we convert both series to the quarterly frequency by taking averages within the quarter).

B Monte Carlo Evidence on the Performance of the Bootstrapping Procedures for Elliot *et al.*'s (1996) and Kwiatkowski *et al.*'s (1992) Tests

Table B.1 in this appendix reports Monte Carlo evidence on the performance of the bootstrapping procedures we use for Elliot *et al.*'s (1996) unit root tests and Kwiatkowski *et al.*'s (1992) stationarity tests, respectively. The table reports, for different Data-Generation Processes (DGPs), and four different sample lengths (in years, 25, 50, 75, and 100, respectively), the fractions of Monte Carlo replications for which the null hypothesis—i.e., that the series is I(1) and, respectively, I(0)—is rejected at the 10 per cent level. In order to properly assess the Monte Carlo experiments, it is important to keep in mind that a perfectly sized test incorrectly rejectes the null hypothesis x per cent of the time at the x per cent level, so that, ideally, all of the entries in the table should be equal to 0.1.

For each DGP, and any of the four lengths for the artificial samples, we perform 5,000 Monte Carlo simulations, and for each simulation we bootstrap the test statistics based on 10,000 bootstrap replications. In order to use plausible, realistic DGPs, we simulate, *via* bootstrapping, estimated AR(p) representations for inflation as follows.

As for Kwiatkowski *et al.*'s (1992) tests, for which the null hypothesis is that the DGP is I(0), we need I(0) DGPs, we estimate AR(p) representations for the *level* of inflation based on U.S. samples for either the Gold Standard or the post-WWII period. In both instances we consider either annual or quarterly data. Based on annual data the samples are 1870-1913 and 1946-2019, respectively, whereas based on quarterly data they are 1875Q2-1914Q2 and 1947Q2-2019Q4. We select the lag order, p, as the maximum between the lag orders selected by the Schwartz and Hannan-Quinn criteria. In all cases the estimated DGPs are I(0). In line with the evidence in (e.g.) Benati (2008), however, the DGPs estimated based on post-WWII data exhibit a significantly greater extent of persistence than those estimated based on data from the Gold Standard era: e.g., based on quarterly data the sum of the AR coefficients is equal to -0.0076 for the Gold Standard, and to 0.7774 for the post-WWII pewriod. For each Monte Carlo replication we simulate the estimated AR(p) by bootstrapping it as in Diebold and Chen (1996). For each simulation we use a pre-sample of 100 observations which we then discard in order to eliminate dependence on initial conditions, thus making the artificial samples effectively independent of one another. Based on each artificial sample we then perform the Kwiatkowski *et al.*'s

(1992) stationarity tests exactly as we do based on the actual data, i.e. bootstrapping the test statistic as in Diebold and Chen (1996), thus obtaining a bootstrapped p-value for each artificial sample. In this way we build up the Monte Carlo distribution of the bootstrapped p-values of the test. For each estimated DGP, and any sample length, the table reports the fraction of p-values smaller than 10 per cent out of the 5,000 Monte Carlo simulations.

Turning to Elliot *et al.*'s (1996) unit root tests, here we rather need, as the null hypothesis to be simulated, an I(1) DGP. In the light of the well-known and widespread evidence of stationarity for inflation under metallic standards (see e.g. Benati (2008)), here we exclusively focus upon DGPs estimated based on post-WWII samples. Specifically, based on the previously mentioned samples (either annual or quarterly), we estimate, exactly as before, AR(p) models for the *first-difference* of inflation, rather than for its level. Then, for each Monte Carlo simulation we bootstrap the estimated DGP exactly as for Kwiatkowski *et al.*'s (1992) tests, and we *cumulate* the bootstrapped realization, thus obtaining an artificial sample for inflation upon which we have imposed a unit root. Based on each artificial sample we then perform Elliot *et al.*'s (1996) tests exactly as we do based on the actual data, thus building up the Monte Carlo distribution of the bootstrapped p-values of the test. For each estimated DGP, and any sample length, the table reports the fraction of p-values smaller than 10 per cent out of the 5,000 Monte Carlo simulations.

Table B.1 Monte Carlo evidence on the performance of the bootstrapping procedures for Elliot *et al.*'s (1996) and Kwiatkowski *et al.*'s (1992) tests: fractions of replications for which the null hypothesis is rejected at the 10 per cent level

Sample length (in years)	I: Elliot *et al.*'s (1996) tests based on post-WWII data: Annual data	Quarterly data	II: Kwiatkowski *et al.*'s (1992) tests — *Annual data*: Gold Standard	*Annual data*: Post-WWII	*Quarterly data*: Gold Standard	*Quarterly data*: Post-WWII
25	0.0726	0.0998	0.1046	0.1312	0.0998	0.1180
50	0.0754	0.1198	0.1090	0.1208	0.1042	0.1038
75	0.0718	0.1156	0.1108	0.1052	0.1092	0.1012
100	0.0734	0.1134	0.1036	0.0990	0.1006	0.1056

The results in the table are uniformly excellent for either of the two tests based on *quarterly* data, with the fractions of Monte Carlo simulations for which the null hypothesis is rejected at the 10 per cent level ranging from 0.0998 to 0.1198 for Elliot *et al.*'s (1996) tests, and from 0.0998 to 0.1180 based on Kwiatkowski *et al.*'s (1992) tests, in both instances close, or even very close to the ideal of 0.1. As for the DGPs estimated based on annual data, results are good for Kwiatkowski *et al.*'s (1992) tests, with the fractions of rejections of the null ranging from 0.0990 to 0.1312. For Elliot *et al.*'s (1996) tests results are less satisfactory, with the fractions ranging from

0.0718 to 0.0754, so that here the tests tend to reject the null hypothesis slightly less frequently than they ideally should.

Table C.1 Monte Carlo evidence on the performance of Müller and Watson's (2018) low-frequency regression methodology: median, and 16-84 percentiles of the Monte Carlo distribution of the median estimate

Sample length (in years)	Gold Standard	Post-WWII
	Annual data	
50	0.945 [0.334 1.526]	0.930 [0.369 1.478]
75	0.952 [0.471 1.446]	0.957 [0.469 1.423]
100	0.977 [0.548 1.400]	0.953 [0.579 1.345]
200	0.985 [0.703 1.271]	0.974 [0.716 1.222]
	Quarterly data	
50	0.990 [0.608 1.375]	0.923 [0.298 1.553]
75	0.993 [0.658 1.314]	0.947 [0.412 1.488]
100	0.995 [0.733 1.262]	0.965 [0.523 1.399]
200	1.006 [0.823 1.176]	0.976 [0.673 1.278]

C Monte Carlo Evidence on the Performance of Müller and Watson's (2018) Low-Frequency Regression Methodology

Tables C.1 and C.2 in this appendix reports Monte Carlo evidence on the performance of Müller and Watson's (2018) low-frequency regression methodology along two dimensions, (*i*) the methodology's ability to correctly recover the point estimate of the slope parameter, and (*ii*) its ability to reliably characterize the authentic extent of uncertainty pertaining to the point estimates.

Based on either annual or quarterly data, and for either the Gold Standard or post-WWII samples discussed in the previous Section of this Online Appendix (i.e., based on annual data, 1870-1913 and 1946-2019, and based on quarterly data 1875Q2-1914Q2 and 1947Q2-2019Q4, respectively), we estimate bivariate VARs for M2 growth and inflation (computed as the log-differences of the M2 and GNP deflator series), setting the lag order to 2 based on annual data, and to 4 based on quarterly data. All of the estimated VARs turn out to be stationary. In doing so, we rescale the inflation series in such a way that the resulting VAR-implied average gain of M2 growth onto inflation at frequencies equal to or beyond 30 years is equal to 1. Then, for each estimated DGP, and for any of four lengths for the artificial samples (in years, 50, 75, 100, or 200), we generate artificial samples for M2 growth and inflation by bootstrapping the estimated VARs. Exactly as in the previous section, for each simulation we use a pre-sample of 100 observations which we then discard in order to eliminate dependence on initial conditions, thus making the artificial samples effectively independent of one another. Based on each artificial sample we then regress inflation upon M2 growth based on Müller and Watson's (2018) low-frequency regression methodology exactly as we do based on the actual data, taking 30 years as the 'cutoff point' for the set of low frequencies. In this way, for each Monte Carlo simulation we obtain (*i*) a median estimate of the slope coefficient in the regression, which, exactly as we do in the main text of this paper, we consider as a *point estimate* of the slope, and (*ii*) *confidence intervals*. As for (*i*), we compare the Monte Carlo distribution of the median (i.e., point) estimate to the value we impose upon the VAR, i.e. 1. As for (*ii*), we compare the Monte Carlo distribution of the estimated width of the confidence interval with 67 per cent coverage probability to the true width as defined by the Monte Carlo distribution computed in (*i*).

Starting from the evidence in Table C.1, the performance of Müller and Watson's methodology is near-perfect conditional on the Gold Standard DGP estimated based on quarterly data. Conditional on the other three DGPs, on the other hand, the performance is slightly worse. In particular, whereas for sample lenghts of 200 years (based on either annual or quarterly data) the median of the distribution of the point estimate ranges from 0.974 to 0.985, shorter sample lengths produce materially lower values: e.g., with 50-years samples the median of the distribution of the point estimate ranges from 0.923 to 0.945. As we will see in the next section of this Online Appendix, cross-spectral methods suffer from exactly the same small-sample problem, with both the gain and the coherence between M2 growth and inflation exhibiting some downward bias (see Figure D.2 in Online Appendix D).

Turning to the coverage properties of Müller and Watson's methodology, the evidence in Table C.2 is near-uniformly excellent across the board. In particular, first, for 200-years samples the median of the Monte Carlo distribution of the estimated width is near-identical to the true width in three cases out of four, and in the fourth case the difference (0.504 versus 0.526) is quite modest. Second, as the sample size decreases, the methodology's ability to reliably characterize the authentic extent of uncertainty pertaining to the point estimates deteriorates (as expected), but, quite remarkably, such deterioration is comparatively minor. For example, even based on annual data, the true width and the median of the Monte Carlo distribution are equal to 1.169 and 1.177 based on the Gold Standard DGP, and to 1.122 and 1.159 based on the post-WWII one. The implication is that, essentially irrespective of the sample length, Müller and Watson's methodology is reliably characterizes the authentic extent of uncertainty pertaining to the point estimates.

Table C.2 Monte Carlo evidence on the performance of Müller and Watson's (2018) low-frequency regression methodology: true width of the confidence interval with 67 per cent coverage probability, and median of the Monte Carlo distribution of the estimated width

Sample length (in years)	Gold Standard		Post-WWII	
	True	Median	True	Median
	Annual data			
50	1.169	1.177	1.122	1.159
75	1.001	0.999	0.926	0.966
100	0.827	0.822	0.770	0.793
200	0.564	0.566	0.504	0.526
	Quarterly data			
50	0.757	0.779	1.272	1.323
75	0.654	0.640	1.093	1.113
100	0.525	0.532	0.893	0.899
200	0.360	0.359	0.606	0.600

D Monte Carlo Evidence on the Performance of Berkowitz and Diebold's (1998) Spectral Bootstrapping Procedure

Let x_t and y_t be two jointly covariance-stationary series, with x_t being, in the language of 'transfer function models',[6] the 'input' series, and y_t being the 'output' series (in our case, x_t and y_t are money growth and, respectively, either inflation or nominal GDP growth); let $F_x(\omega_j)$ and $F_y(\omega_j)$ be the smoothed spectra of the two series at the Fourier frequency ω_j; and let $C_{x,y}(\omega_j)$ and $Q_{x,y}(\omega_j)$ be the smoothed co-spectrum and, respectively, quadrature spectrum between x_t and y_t at the Fourier frequency ω_j.

I estimate the spectral densities of x_t and y_t, the co-spectrum, and the quadrature spectrum, based on the Fast Fourier Transform (FFT) estimator of the spectral density matrix, by smoothing the periodograms and, respectively, the cross-periodogram in the frequency domain by means of a Bartlett spectral window.

I select the spectral bandwidth automatically *via* the following multivariate version of the procedure proposed by Beltrao and Bloomfield (1987). As found, e.g.,

[6] See e.g. Wei (2005).

in Hansen and Sargent (1980), the log-likelihood of the data can approximately be decomposed, up to a constant, as

$$\log L \simeq -\frac{1}{2}\sum_{\omega_j\in\Omega}\log\left\{\det\left[S\left(\omega_j\right)\right]\right\}-\frac{1}{2}\sum_{\omega_j\in\Omega}\text{trace}\left[S\left(\omega_j\right)^{-1}I\left(\omega_j\right)\right] \tag{D.1}$$

where Ω is the set of the Fourier frequencies, and $S\left(\omega_j\right)$ and $I\left(\omega_j\right)$ are the spectral density matrix and the unsmoothed sample spectral density matrix (i.e., the matrix of the periodograms and cross-periodograms) at the Fourier frequency ω_j, respectively. For each value of the spectral bandwidth $M = 1, 2, ..., [T^{1/2}]$—with $[x]$ meaning 'the largest integer of x'—I compute, based on (D.1), the approximated cross-validated log-likelihood based on the 'leave-one-out' estimate of the spectral density matrix,[7] $\hat{S}_{CV}\left(\omega_j, M\right)$.[8] Finally, I select the optimal value of M as the one associated with the largest value of $\hat{S}_{CV}\left(\omega_j, M\right)$.

For a specific Fourier frequency ω_j, the estimated smoothed gain and coherence are defined as[9]

$$\Gamma(\omega_j) = \frac{[C_{x,y}(\omega_j)^2 + Q_{x,y}(\omega_j)^2]^{\frac{1}{2}}}{F_x(\omega_j)} \tag{D.2}$$

$$K(\omega_j) = \left\{\frac{C_{x,y}(\omega_j)^2 + Q_{x,y}(\omega_j)^2}{F_x\left(\omega_j\right)\cdot F_y\left(\omega_j\right)}\right\}^{\frac{1}{2}} \tag{D.3}$$

As it is well known, the gain is the absolute value of the slope coefficient in the regression of y_t on x_t at the frequency ω_j, whereas the coherence is the R-squared of such regression.

I compute confidence bands for the estimated cross-spectral objects based on the first of the two non-parametric multivariate spectral bootstrapping procedures proposed by Berkowitz and Diebold (1998) hencefort BD. As BD show *via* Monte Carlo, such procedure generates confidence intervals with superior coverage properties compared to those based on the approximated asymptotic formulas found, e.g., in Chapter 8 of Koopmans (1974). BD's spectral bootstrap—a multivariate generalisation of the Franke and Hardle (1992) univariate bootstrap—can be briefly described as follows. Let Z_t=$[x_t,\, y_t]'$, and let $\hat{S}(\omega_j)$ be the smoothed sample spectral density matrix (i.e., the consistent estimator of $S(\omega_j)$), for the random vector Z_t, all corresponding to the Fourier frequency ω_j. As it is well known—see e.g. Brillinger (1981)—$I(\omega_j)$ converges

[7]See Beltrao and Bloomfield (1987, pp. 23-24). For an extended discussion of cross-validation as a general solution to the bandwidth problem in kernel density estimation, see, e.g., Marron (1985).

[8]Specifically, I estimate $\hat{S}_{CV}\left(\omega_j, M\right)$ by smoothing $I\left(\omega_k\right)$ over all the Fourier frequencies $\omega_k \in \Omega$, $\omega_k \neq \omega_j$, with a Bartlett spectral window of width $2M$+1.

[9]It is to be noticed that the literature presents alternative, slightly different definitions of the gain and the coherence—on this, see (e.g.) Hamilton (1994), page 275. The gain, for example, is sometimes defined as the numerator of (D.2), whereas the coherence is defined as the square of (D.3).

in distribution to a N-dimensional complex Wishart distribution with one degree of freedom and scale matrix equal to $S(\omega_j)$, i.e.

$$I(\omega_j) \xrightarrow{d} W_{N,C}\left(1, S\left(\omega_j\right)\right) \tag{D.4}$$

where $W_{s,C}\left(h, H\right)$ is a s-dimensional complex Wishart distribution with h degrees of freedom and scale matrix H. BD propose to draw from

$$I^k(\omega_j) = S\left(\omega_j\right)^{\frac{1}{2}} W^k_{2,C}\left(1, I_N\right) S\left(\omega_j\right)^{\frac{1}{2}} \tag{D.5}$$

for all the Fourier frequencies ω_j=$2\pi j/T$, j=1,2, ..., $[T/2]$, with T being the sample length. Confidence bands are computed by first getting a smoothed estimate of the spectral density matrix, $S\left(\omega_j\right)$. Then, for each ω_j=$2\pi j/T$, j=1,2, ..., $[T/2]$, I generate 10,000 random draws from (D.5), thus getting bootstrapped, artificial (unsmoothed) periodograms, I smooth them exactly as I previously did with $I(\omega_j)$, and based on the bootstrapped, smoothed spectral density matrices I compute gains and coherences, thus building up their empirical distributions. Finally, I compute the confidence bands based on the percentiles of the distribution.[10]

Figures D.1-D.2 in this Online Appendix report Monte Carlo evidence on the performance of the FFT-based estimator of the spectral density matrix, whereas Figures D.3-D.4 show Monte Carlo evidence on the coverage properties of BD's spectral bootstrap procedure. For both sets of Figures I consider, as Data-Generation Processes (DGPs), fixed-coefficients VARs estimated *via* OLS based on annual U.S. data for M2 growth and GNP deflator inflation for the periods 1870-1913 and 1946-2013, respectively, setting the lag order to p=2. For any of four sample lengths (either 25, 50, 75, or 100 years) I simulate (i.e., bootstrap) the estimated DGPs 5,000 times. Based on each artificial sample I then compute either (*i*) the FFT-based estimate of the spectral density matrix, or (*ii*) the 16th, 84th, 5th, and 95th percentiles of the bootstrapped distribution of the cross-spectral gain and coherence between the two artificial series based on BD's procedure (i.e., the 1- and 2-standard deviations confidence bands).

Figures D.1-D.2 show, in blue, the true values (as implied by the point estimates of the VARs) of the log spectral densities of the two series, and of the gain and the

[10]Since, in general, the medians of the bootstrapped distributions of the gain and the coherence at each frequency ω_j are not numerically identical to the simple estimates of the two objects based on (D.2) and (D.3), I rescale the two distributions so that their medians are indeed equal to such estimates. Given that the gain is, by construction, greater than or equal to zero, whereas the coherence is between 0 and 1, I perform such rescaling based on the log and, respectively, the logit transformations. To be clear, this implies that (e.g.) for the gain, for each frequency ω_j I subtract from the log of the bootstrapped distribution of the gain at ω_j its median, I add to it the log of the simple estimate of the gain at ω_j, and I then take the exponential of the resulting distribution, thus obtaining a botstrapped distribution which, by construction, is exactly centered around the simple estimate. For the coherence I follow the same procedure, with the only difference that I use the logit, instead of the log transformation.

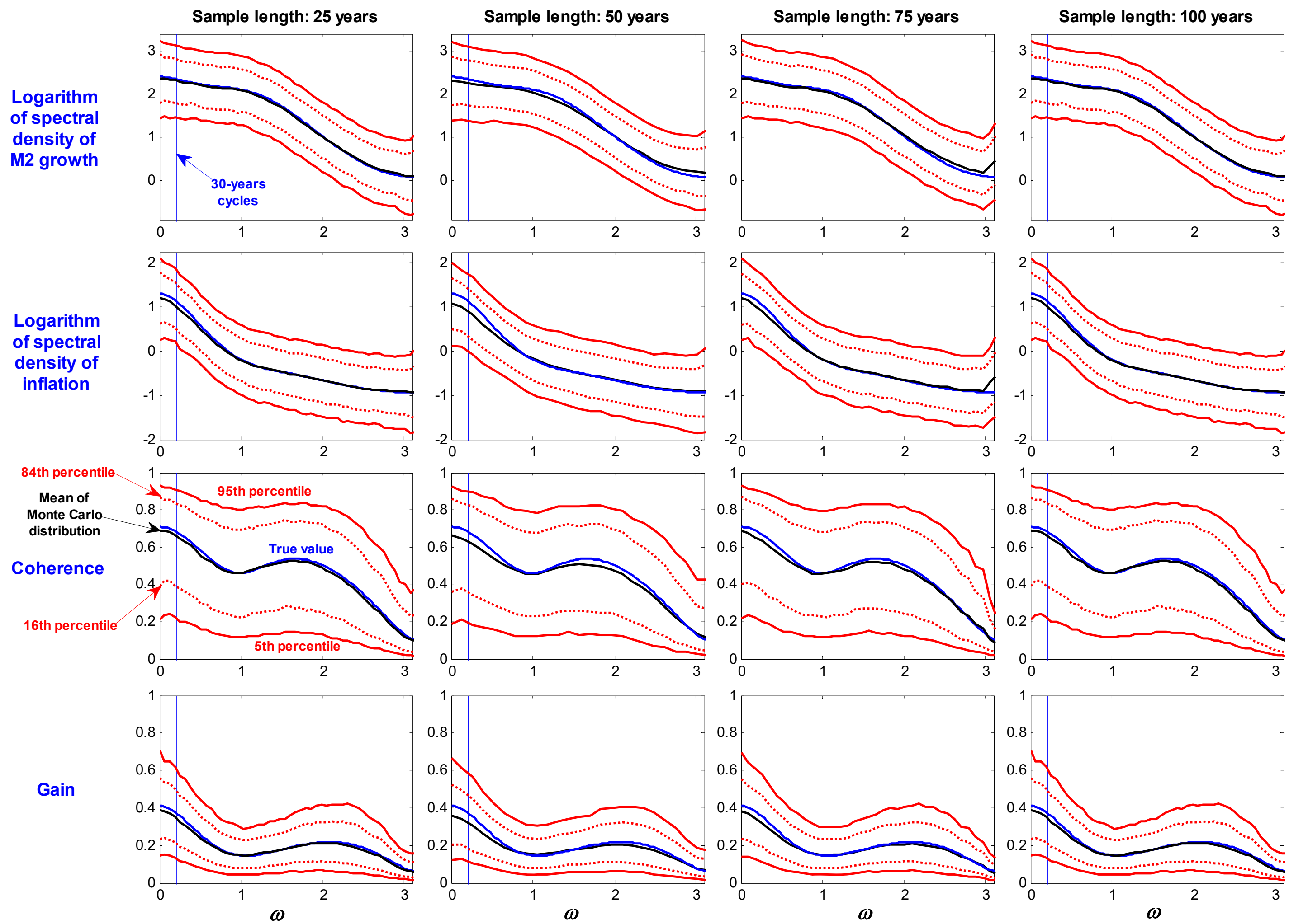

Figure D.1 Monte Carlo evidence on the performance of the FFT-based estimator of the spectral density matrix conditional on the Gold Standard DGP: true cross-spectral objects, and means and 16-84 and 5-95 percentiles of the Monte Carlo distributions

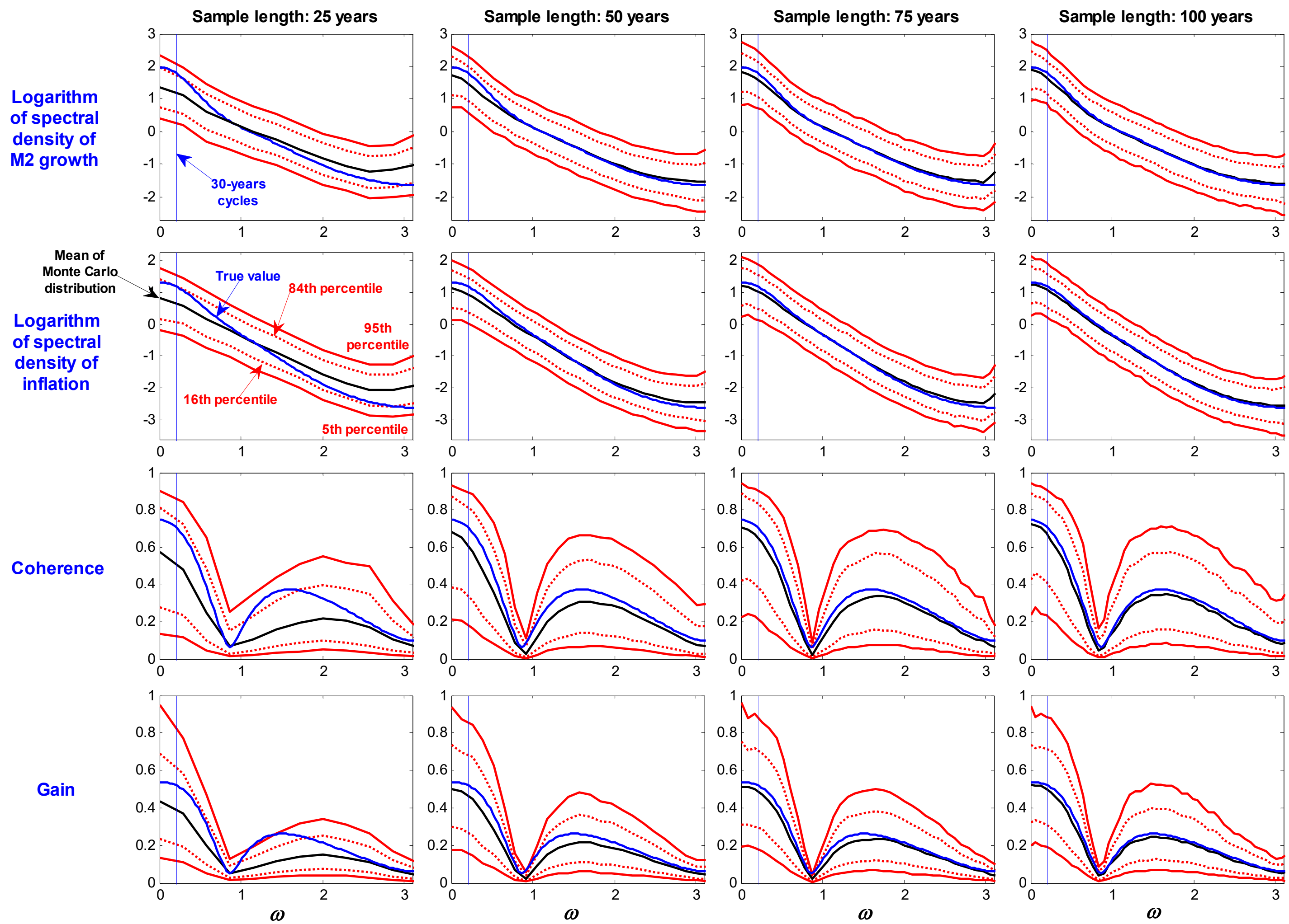

Figure D.2 Monte Carlo evidence on the performance of the FFT-based estimator of the spectral density matrix conditional on the post-WWII DGP: true cross-spectral objects, and means and 16-84 and 5-95 percentiles of the Monte Carlo distributions

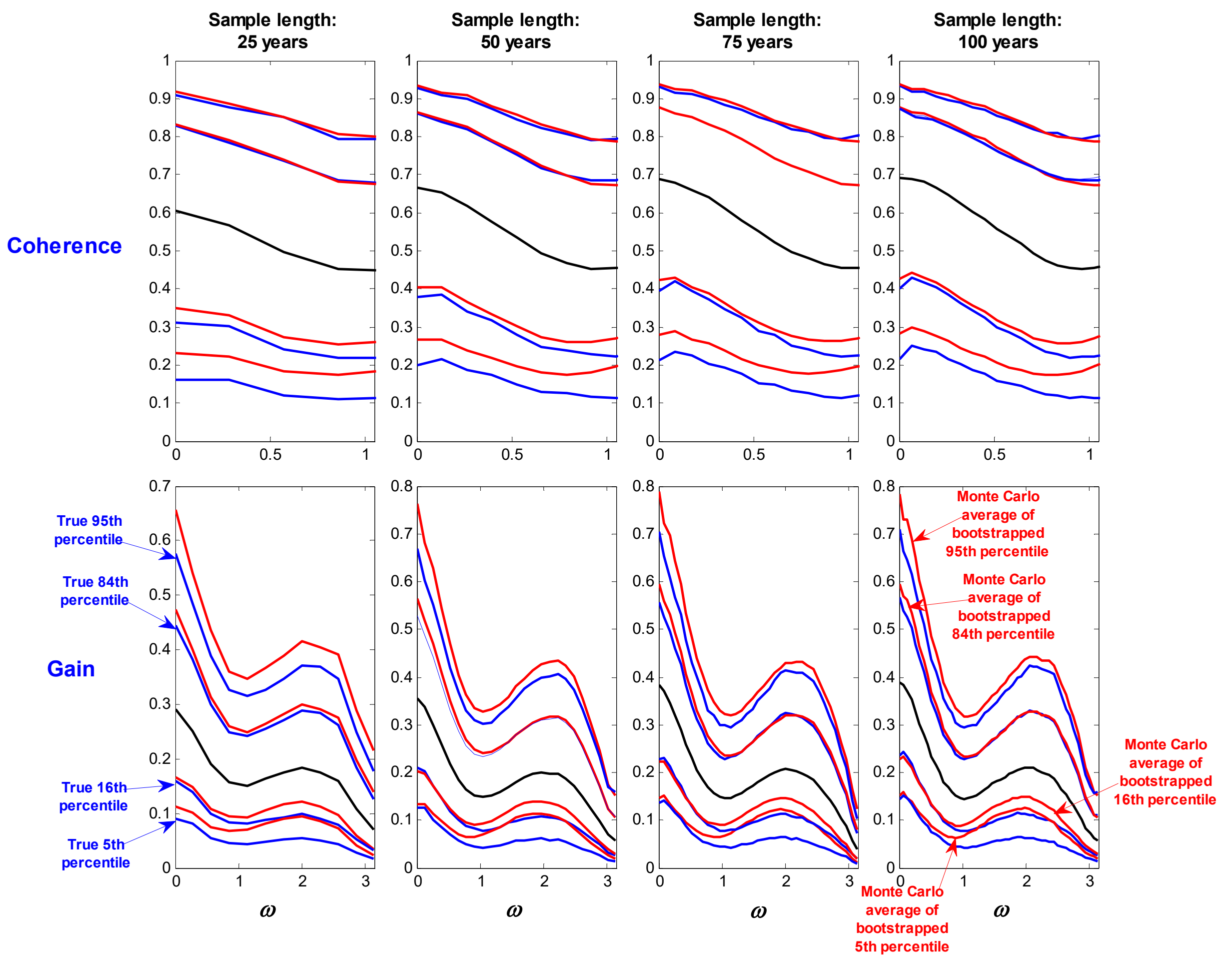

Figure D.3 Monte Carlo evidence on the coverage properties of Berkowitz and Diebold's (1998) spectral bootstrap procedure conditional on the Gold Standard DGP: percentiles of the Monte Carlo distributions of the gain and the coherence, and averages, across the Monte Carlo replications, of the corresponding confidence bands

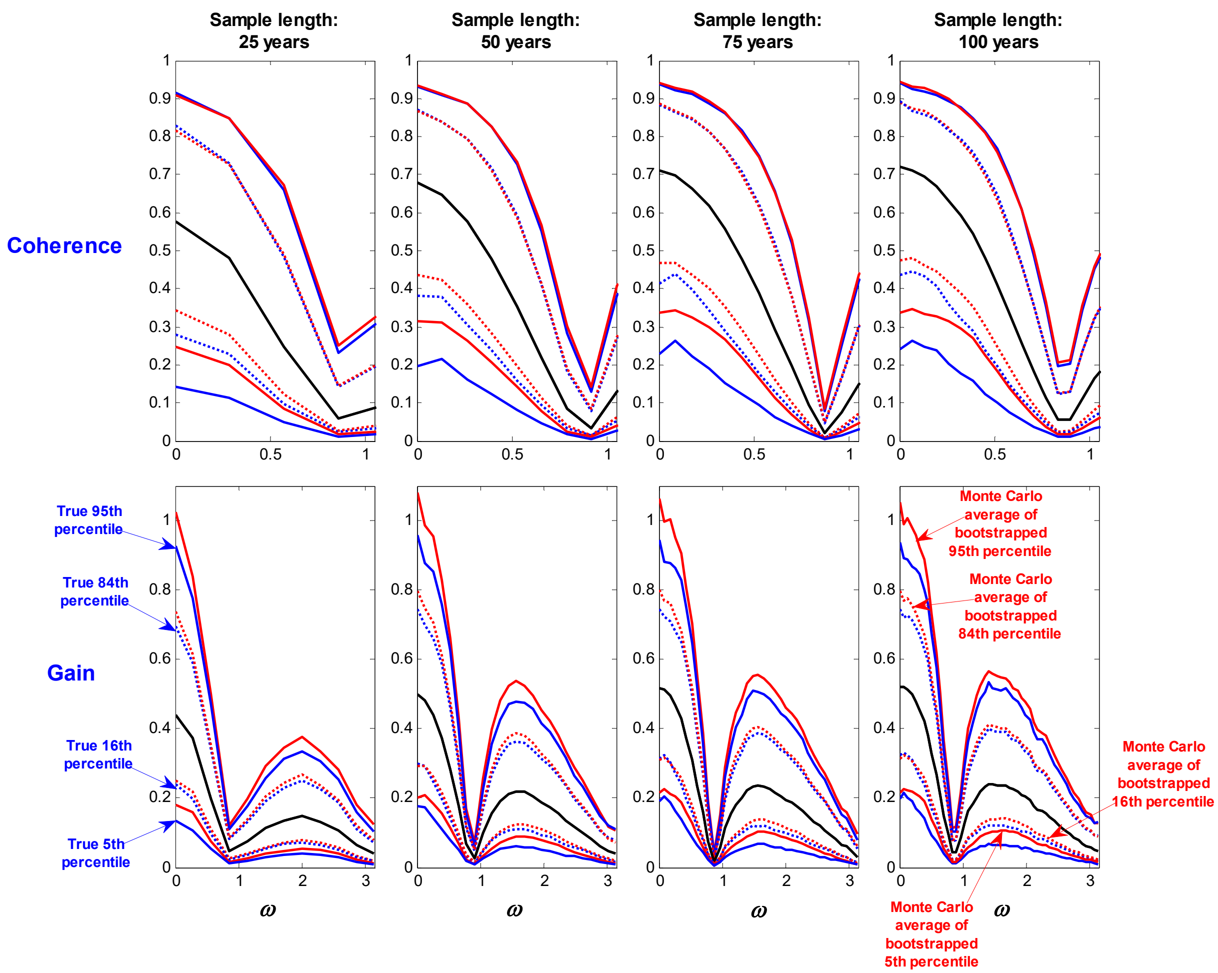

Figure D.4 Monte Carlo evidence on the coverage properties of Berkowitz and Diebold's (1998) spectral bootstrap procedure conditional on the post-WWII DGP: percentiles of the Monte Carlo distributions of the gain and the coherence, and averages, across the Monte Carlo replications, of the corresponding confidence bands coherence between them; and in black and red, respectively, the mean, and the 16th, 84th, 5th, and 95th percentiles of the corresponding Monte Carlo distributions of the spectral objects. The performance of the FFT-based estimator of the spectral density matrix emerges as uniformly excellent, with the means of the Monte Carlo distributions of the estimates of the log spectral densities, and of the gain and the coherence, being near-uniformly close, or even very close to the true values. This is especially clear conditional on the Gold Standard DGP, whereas conditional on the post-WWII DGP there is some difference for short sample lengths (in particular, for $T = 25$ years).

Turning to the coverage properties of BD's procedure, the blue lines in Figures D.3-D.4 show the percentiles of the Monte Carlo distributions of the FFT-based estimates of the gain and coherence (i.e., the very same percentiles reported in Figures D.1-D.2)—which characterize the *true* extent of uncertainty pertaining to the estimates of the two objects—whereas the red lines report the averages, across all of the Monte Carlo simulations, of the corresponding confidence bands produced by BD's procedure. Intuitively, if BD's procedure worked perfectly, the red lines in Figures D.3-D.4 would be identical to the corresponding blue lines. A comparison between the two sets of lines provides therefore evidence on how effective BD's procedure is at capturing the true extent of estimation uncertainty pertaining to the two cross-spectral objects. Overall, the evidence in the two figures suggests that BD's procedure is quite reliable. Starting from the coherence, based on either DGP, and for any sample size, the upper percentiles (84th and 95th) are captured remarkably well, with the red lines being near-uniformly indistinguishable from the corresponding blue lines. For the lower percentiles (5th and 16th), on the other hand, the performance is less impressive, with BD's procedure consistently pointing towards a *smaller* extent of uncertainty than in reality. For example, with $T = 50$, the true 5th percentile of the Monte Carlo distribution of the coherence at ω=0 is equal to 0.2 conditional on either DGP, whereas the corresponding average values, across all of the the Monte Carlo simulations, are equal to 0.27 conditional on the Gold Standard DGP, and 0.31 for the post-WWII DGP. As for the gain, the overall pattern appears to be exactly the opposite. Focusing, in particular, on the low frequencies, the 5th and 16th percentiles are captured very well (with the partial exception of the 5th percentile with $T = 25$ years), whereas for the 84th and 95th percentiles BD's procedure consistently points towards a *greater* extent of uncertainty than in reality. For example, with $T = 50$, the true 95th percentile of the Monte Carlo distribution of the gain at ω=0 is equal to 0.66 conditional on the Gold Standard DGP, and to 0.95 for the post-WWII DGP, whereas the corresponding average values, across all of the the Monte Carlo simulations, are equal to 0.76 and 1.09, respectively.

Summing up, based on either DGP, BD's procedure captures very well the true extent of uncertainty pertaining to estimates of the gain at the low frequencies for the lower percentiles of the distribution (i.e., 5th and 16th), whereas it points towards a greater extent of uncertainty than in reality for the higher percentiles (84th and

95th). For the coherence, on the other hand, it is exactly the opposite.

E Assessing the Reliability of the Low-Pass Filter Used by Sargent and Surico (2011)

Figure 3 of Sargent and Surico (2011) shows, for the United States, scatterplots of low-frequency inflation against low-frequency M2 growth for several sub-samples since the beginning of the XX century, where the low-frequency components of the two series have been extracted *via* a version of Lucas' (1980) low-pass filter. The frequency-domain properties of the filter—in particular, its ability to reliably approximate the ideal low-pass filter—crucially depend on a 'bandwidth' parameter, k, which Sargent and Surico (2011), as detailed in the replication code found at the *American Economic Review* website, set to $k = 8$. Overall, the evidence in Sargent and Surico's (2011) Figure 3 clearly points towards instability in the low-frequency relationship between the two series, with the slope being most of the time smaller than one, thus suggesting that at the low frequencies money growth and inflation move less than one-for-one.

The contrast between these results and those reported in Figure 2 in the main text of the present work raises the obvious question of which, among the two sets of results, should be regarded as the more reliable.

The methodology I used in order to produce the evidence reported in my Figure 1 in the main text, pioneered by Müller and Watson's (2017, 2018, 2019), is based on the notion of regressing the series of interest on cosine transforms (i.e., essentially cosine waves) associated with specific (low) Fourier frequencies of interest. The low-frequency component of the series of interest is then simply the fitted value from such low-frequency regression. By the same token, the band-pass filter I used in order to produce the qualitatively similar evidence in Figures A.1*a*-A.1*b* in the Online Appendix was derived by Christiano and Fitzgerald (2003) as an optimal approximation to the ideal band-pass filter, with optimality being defined in terms of a loss function defined in the frequency domain, based on the integral over the interval $[0\ \pi]$ of the squared difference between the filter's frequency-response function,[11] and the corresponding frequency-response function of the ideal band-pass filter.

Lucas' (1980) filter, on the other hand, was *ad hoc*, in the sense on not having been derived as an optimal approximation to the ideal band-pass filter. As a consequence, no matter what low-frequency band a researcher is interested in, the filter provides a poor approximation to the ideal low-pass filter. In particular, Lucas' filter suffers from a significant extent of 'high-frequency leakage': for example, with $k = 8$ (the value used by Sargent and Surico, 2011) it only effectively kills off the spectral power associated with cycles *faster than 4 years*, retaining instead non-negligible,

[11] The frequency-response function of a filter goes under a number of different labels, being often also referred to as the 'squared gain of the filter'.

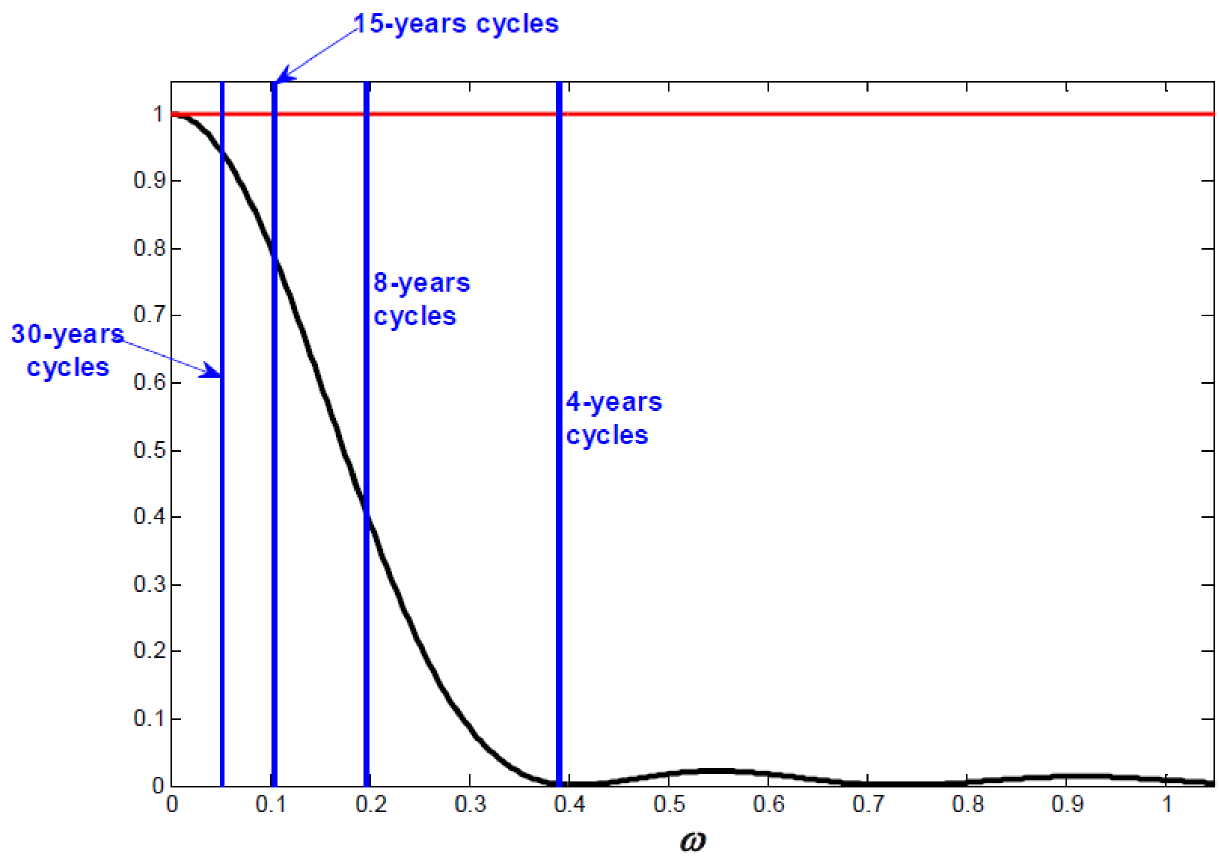

Figure E.1 Frequency-response function of the filter used by Sargent and Surico (2011)

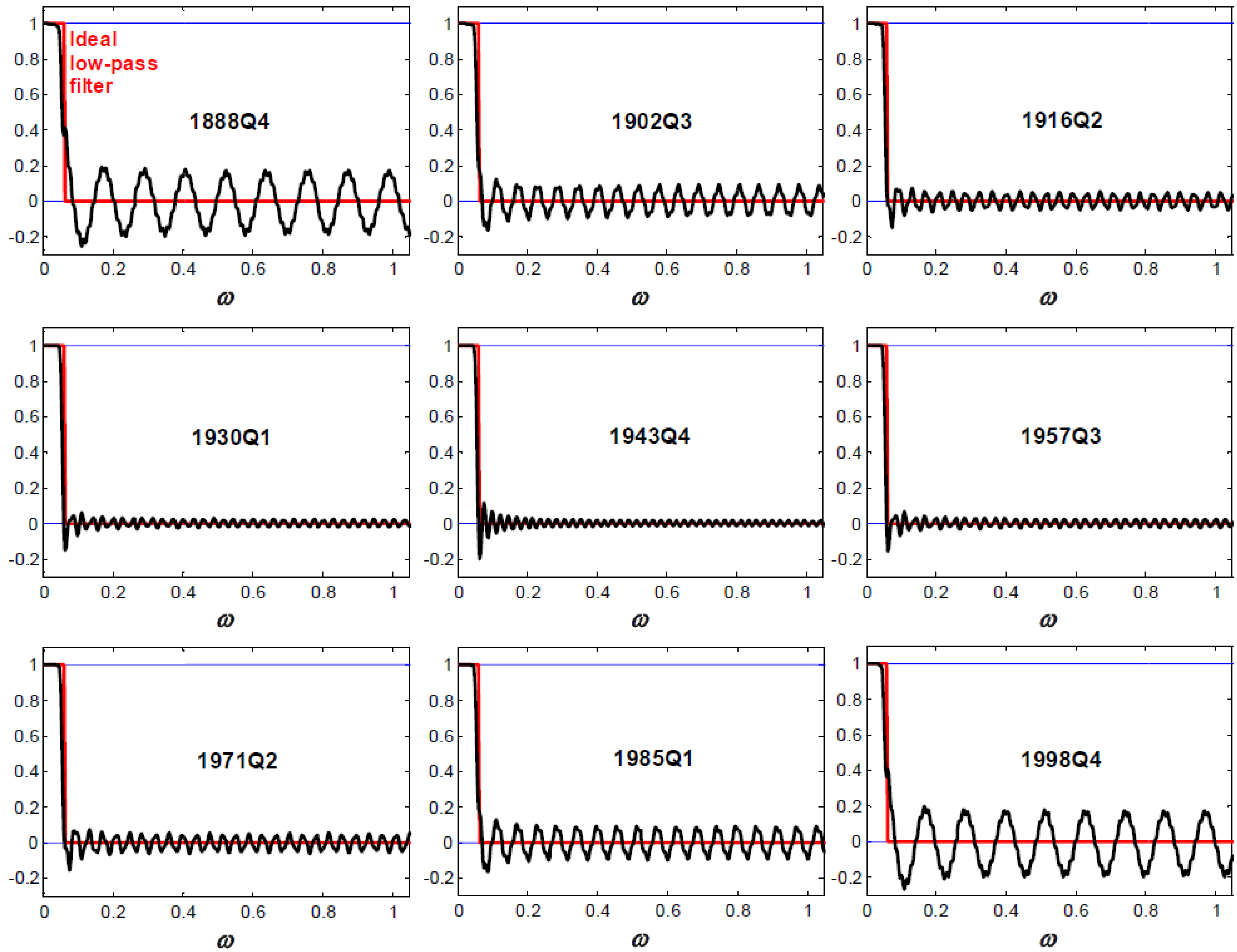

Figure E.2 Frequency-response function of the low-pass Christiano and Fitzgerald (2003) filter for the frequency band beyond 30 years (selected quarters) and sometimes sizeable fractions of spectral power for cycles which are not typically associated with the notion of 'long run'.

Figure E.1 in this Online Appendix shows the frequency-response function of Lucas' filter. Several facts about the filter's ability to reliably extract components associated with a specific range of low frequencies are clearly apparent from the figure. In particular,

(I) no matter what range of low frequencies a researcher is interested in, the filter does a poor job at separating it from adjacent frequencies. If, for example, a researcher is interested in extracting all frequencies beyond 8 years, the filter in fact destroys a non-negligible portion of the spectral power which should instead be preserved (e.g., for cycles of exactly 8 years, it only keeps 41% of the spectral power), whereas it retains non-negligible portions of the spectral power it should discard (e.g., in the range between 4 and 8 years, the filter retains fractions of spectral power ranging between 0 and 41%). In fact, the filter is only effective at wiping out the spectral power associated with cycles faster than 4 years, whereas for all frequencies lower than 4 years it is clearly sub-optimal—often starkly so—no matter what specific range of frequencies a researcher is interested in.

(II) If we take as a working definition of the 'long run' the set of frequencies beyond 30 years, Figure E.1 shows how the filter does a poor job at separating them from higher frequencies (the same holds for alternative plausible working definitions of the long run). Specifically, for the frequency band between 8 and 15 years the filter retains fractions of spectral power ranging between 41 and 79%, whereas for the frequency band between 15 and 30 years it preserves fractions between 79 and 94.5%.

These results show that exploring the low-frequency relationship between money growth and inflation based on Lucas' filter is fraught with dangers, because the filter retains, to a non-negligible and often sizeable extent, components of the data which many researchers would not associate with any meaningful notion of 'long run'. This implies that, in general, the filter extracts a set of quite heterogeneous frequencies, and the results it produces are therefore sub-optimal compared to those produced by filters such as Christiano and Fitzgerald's, or approaches such as that of Müller and Watson's (2017, 2018, 2019).

For example, Figure E.2 in this Online Appendix shows the frequency-response function of the Christiano and Fitzgerald (2003) low-pass filter I used to produce Figures A.1*a*-A.1*b*. Since the structure of the filter is observation-specific (i.e., each observation in the sample has its own set of filter coefficients, and therefore its own filter), the figure shows the frequency-response function for the quarters corresponding to 10, 20, 30, ..., 90 per cent of the sample, for a sample of length equal to that used by Sargent and Surico (2011). In each panel, the red line represents the ideal low-pass filter that the Christiano-Fitzgerald filter aims at replicating. The figure illustrates the well-known point[12] that the approximation the Christiano-Fitzgerald filter provides to an ideal filter is remarkably good around the middle of the sample, and it instead

[12] See the extended discussion in Christiano and Fitzgerald (2003).

systematically deteriorates for observations that are closer and closer to either the beginning or the end of the sample. Further, the figure shows how the goodness of the approximation is symmetric with respect to the middle of the sample: it is still very good around either 20 or 80% of the sample length, whereas it starts to significantly deteriorate around either the first or the last 10% of the sample.

A comparison between Figures E.1 and E.2 therefore suggests that, between the evidence reported in Figure 3 of Sargent and Surico (2011), and that in Figures A.1*a*-A.1*b* (as well as the very similar evidence in Figure 1 in the main text of the present work) the latter evidence should be regarded as the more reliable. The implication is that evidence from low-frequency filtering, when reliably computed, provides little support to the notion that the relationship between money growth and inflation at the very low frequencies may have been unstable since the metallic standard era, and may be weaker under regimes oriented to price stability, such as e.g. metallic standards.

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Tables for the Online Appendix

I: Evidence from Elliot *et al.*'s (1996) unit root tests and Kwiatkowski *et al.* s (1992) stationarity tests

Table I.1 Bootstrapped *p*-values for Elliot *et al.*'s (1996) unit root tests for the longest available samples

		Money growth		Inflation		Nominal GDP growth	
Country	*Period*	p=1	p=2	p=1	p=2	p=1	p=2
Argentina	1864-2019	0.005	0.007	0.007	0.007	0.006	0.005
Australia	1854-2019	0.000	0.000	0.000	0.000	0.000	0.000
Brazil	1862-2014	0.026	0.037	0.020	0.039	0.016	0.045
Canada	1873-2006	0.000	0.000	0.000	0.000	0.000	0.000
Chile	1811-2019	0.017	0.027	0.002	0.000	0.000	0.000
Finland	1867-1985	0.000	0.000	0.002	0.010	0.001	0.018
France	1910-1994	0.015	0.037	0.005	0.033	–	–
Italy	1862-1996	0.010	0.022	0.005	0.004	0.002	0.006
Mexico	1926-2013	0.001	0.008	0.070	0.184	0.056	0.133
New Zealand	1885-2016	0.000	0.004	0.004	0.038	0.000	0.000
Norway	1820-2014	0.000	0.000	0.000	0.000	0.000	0.000
Portugal	1855-1998	0.000	0.003	0.001	0.022	0.004	0.035
Sweden	1847-2018	0.000	0.000	0.000	0.000	0.000	0.000
Switzerland	1916-2015	0.002	0.001	0.000	0.000	0.000	0.000
United Kingdom	1701-2019	0.000	0.000	0.000	0.000	0.000	0.000
United States	1868-2019	0.000	0.000	0.000	0.000	0.000	0.000

For details, see Section 2.

Table I.2 Bootstrapped *p*-values for Elliot *et al.*'s (1996) unit root tests by monetary standard

Country	*Period*	Money growth		Inflation		Nominal GDP growth	
		p=1	p=2	p=1	p=2	p=1	p=2
		Commodity standards					
Argentina	1863-1929	0.0007	0.0022	0.0005	0.0041	0.0001	0.0011
Canada	1873-1929	0.0033	0.0051	0.0026	0.0092	0.0007	0.0054
Chile	1811-1877	0.0002	0.0018	0.0009	0.0058	0.0004	0.0067
Italy	1862-1935	0.0037	0.0145	0.0050	0.0202	0.0026	0.0232
Norway	1865-1931	0.0419	0.0856	0.0052	0.0191	0.0018	0.0186
Sweden	1847-1931	0.0042	0.0087	0.0000	0.0000	0.0000	0.0000
United Kingdom	1701-1796	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1821-1931	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
United States	1870-1932	0.0317	0.0721	0.0230	0.0921	0.0202	0.0882
		Fiat standards					
Argentina	1930-2019	0.0354	0.0507	0.0274	0.0362	0.0329	0.0392
Australia	1914-2019	0.0003	0.0032	0.0011	0.0051	0.0000	0.0000
Brazil	1864-2012	0.0282	0.0457	0.0173	0.0523	0.0163	0.0518
Canada	1935-2006	0.0454	0.1256	0.0079	0.0922	0.0004	0.0044
Chile	1878-2019	0.0487	0.0839	0.0121	0.0033	0.0000	0.0000
Colombia	1956-2019	0.2073	0.5045	0.5598	0.8642	0.5558	0.8534
Finland	1915-1985	0.0215	0.0310	0.0156	0.1323	0.0038	0.0801
France	1937-1994	0.2156	0.4302	0.1569	0.3193	0.0128	0.0000
Italy	1936-1996	0.4291	0.2923	0.1849	0.0913	0.0961	0.0909
Japan	1956-2018	0.0022	0.2183	0.1945	0.3994	0.5012	0.6971
Mexico	1926-2013	0.001	0.008	0.0699	0.1839	0.056	0.133
New Zealand	1914-2016	0.0000	0.0087	0.0042	0.0470	0.0000	0.0029
Norway	1947-2014	0.0008	0.0046	0.0004	0.0027	0.0007	0.0198
Paraguay	1963-2015	0.0471	0.1377	0.2223	0.3202	0.0962	0.2123
Portugal	1932-1998	0.0282	0.0422	0.1489	0.3077	0.0570	0.3134
South Korea	1965-2019	0.4669	0.2302	0.5108	0.6096	0.6746	0.8134
Spain	1942-1997	0.1216	0.2562	0.0217	0.1395	0.0051	0.0352
South Africa	1966-2019	0.0066	0.0224	0.2049	0.4277	0.0731	0.1826
Sweden	1932-2018	0.0001	0.0006	0.0016	0.0241	0.0000	0.0077
Switzerland	1937-2012	0.0106	0.0159	0.0026	0.0041	0.0003	0.0304
United Kingdom	1932-2019	0.0355	0.0576	0.0279	0.1142	0.0089	0.0356
United States	1933-2019	0.0004	0.0186	0.0007	0.0250	0.0000	0.0000
Venezuela	1951-2017	0.9497	0.9569	0.9788	0.9408	0.9702	0.9325

Table I.3 Bootstrapped *p*-values for Kwiatkowski *et al.*'s (1992) stationarity tests tests by monetary standard

		Money growth		Inflation		Nominal GDP growth	
Country	*Period*	p=1	p=2	p=1	p=2	p=1	p=2
		Commodity standards					
Argentina	1863-1929	0.8009	0.7754	0.5387	0.5793	0.6631	0.6768
Canada	1873-1929	0.5042	0.3907	0.2503	0.2576	0.1511	0.1605
Chile	1811-1877	0.1389	0.0720	0.7086	0.7134	0.3739	0.3734
Italy	1862-1935	0.5868	0.5677	0.4815	0.4496	0.3216	0.3507
Norway	1865-1931	0.8680	0.8837	0.8661	0.7879	0.7679	0.6798
Sweden	1847-1931	0.9303	0.9526	0.9740	0.9188	0.9180	0.8311
United Kingdom	1701-1796	0.3825	0.1588	0.1153	0.0258	0.0441	0.0060
	1821-1931	0.7378	0.7624	0.2990	0.3134	0.7357	0.7770
United States	1870-1932	0.6715	0.5481	0.2250	0.2904	0.5705	0.6139
		Fiat standards					
Argentina	1930-2019	0.4124	0.2876	0.4311	0.2389	0.4519	0.2425
Australia	1914-2019	0.1139	0.1057	0.4283	0.4192	0.2767	0.3859
Brazil	1864-2012	0.1914	0.1684	0.1859	0.1792	0.1463	0.1478
Canada	1935-2006	0.6331	0.6453	0.6121	0.6267	0.2223	0.1485
Chile	1878-2019	0.2575	0.2659	0.3459	0.2294	0.2474	0.2102
Colombia	1956-2019	0.2503	0.3498	0.3500	0.4856	0.2851	0.4199
Finland	1915-1985	0.5780	0.5783	0.7362	0.7550	0.7090	0.6939
France	1937-1994	0.0236	0.0580	0.1972	0.2214	0.2276	0.1085
Italy	1936-1996	0.2991	0.2782	0.5198	0.3638	0.3989	0.3297
Japan	1956-2018	0.0000	0.0059	0.0216	0.0357	0.0229	0.0330
Mexico	1926-2013	0.0793	0.1191	0.2963	0.3251	0.2812	0.3283
New Zealand	1914-2016	0.1350	0.1354	0.4727	0.4330	0.3329	0.2345
Norway	1947-2014	0.2937	0.3401	0.4563	0.4439	0.0803	0.0827
Paraguay	1963-2015	0.4443	0.5889	0.5302	0.5144	0.4753	0.4508
Portugal	1932-1998	0.2315	0.2246	0.1729	0.1808	0.0777	0.0728
South Korea	1965-2019	0.0284	0.0134	0.0495	0.1062	0.0308	0.0882
Spain	1942-1997	0.2567	0.2872	0.5780	0.6424	0.2132	0.2585
South Africa	1966-2019	0.1205	0.0820	0.1843	0.2455	0.0423	0.0570
Sweden	1932-2018	0.3995	0.3989	0.2973	0.2323	0.0520	0.0349
Switzerland	1937-2012	0.2201	0.2183	0.0243	0.0426	0.0049	0.0026
United Kingdom	1932-2019	0.5353	0.5179	0.5283	0.4506	0.3852	0.3449
United States	1933-2019	0.7478	0.8183	0.4649	0.4300	0.0541	0.0597
Venezuela	1951-2017	0.7629	0.6688	0.6028	0.6210	0.7632	0.7779

Table I.4 Bootstrapped *p*-values for Elliot *et al.*'s (1996) unit root tests (quarterly data, full samples)

Country	Period	Money growth		Inflation		Nominal GDP growth	
		p=4	p=8	p=4	p=8	p=4	p=8
Australia	1959Q4-2019Q4	0.0002	0.0012	0.0270	0.1850	0.0009	0.0873
Brazil	1975Q2-2019Q4	0.0008	0.0229	0.0002	0.0036	0.0005	0.0060
Canada	1914Q2-2006Q4	0.0002	0.0114	0.0001	0.0002	–	–
	1968Q1-2019Q4	0.0509	0.2435	0.0672	0.2395	0.0219	0.2535
Denmark	1922Q4-2019Q4	0.0000	0.0006	0.0000	0.0000	–	–
	1977Q1-2019Q4	0.0062	0.0574	0.0437	0.2532	0.0096	0.1971
Euro area	1970Q3-2019Q4	0.0531	0.0827	0.6845	0.8186	0.4545	0.7060
Finland	1914Q4-1985Q4	0.0010	0.0256	0.0000	0.0017	–	–
Germany	1949Q1-1998Q4	0.0005	0.0703	0.0001	0.0026	–	–
Hong Kong	1985Q1-2019Q4	0.0114	0.1790	0.3337	0.3831	0.0495	0.0821
Italy	1948Q2-1997Q3	0.2364	0.6367	0.0991	0.3138	–	–
Japan	1955Q2-2019Q4	0.3260	0.5314	0.0166	0.0592	0.1966	0.3315
Mexico	1986Q1-2017Q3	0.0151	0.0005	0.0351	0.0000	0.0235	0.0000
Netherlands	1957Q2-1997Q4	0.0003	0.0337	0.0492	0.4458	–	–
New Zealand	1960Q1-1986Q4	0.0022	0.0590	0.3344	0.5570	–	–
	1988Q1-2019Q4	0.0218	0.0778	0.0001	0.0040	0.0002	0.0015
Norway	1920Q3-2019Q4	0.0001	0.0156	0.0020	0.0042	–	–
	1978Q1-2019Q4	0.0002	0.0166	0.0008	0.1951	0.0456	0.3836
South Africa	1966Q1-2019Q4	0.0017	0.0065	0.0053	0.1289	0.0020	0.0564
South Korea	1960Q1-2019Q4	0.0219	0.2082	0.0197	0.2162	0.0574	0.4635
Switzerland	1985Q1-2019Q4	0.0151	0.0082	0.0746	0.4574	0.1088	0.3704
Taiwan	1961Q3-2019Q4	0.0000	0.0005	0.0000	0.0041	0.0037	0.3303
	1982Q1-2019Q4	0.2728	0.2323	0.0041	0.0457	0.0008	0.1333
United Kingdom	1881Q1-2019Q4	0.0001	0.0040	0.0000	0.0000	–	–
	1895Q2-2019Q4	0.0040	0.0132	0.0000	0.0000	–	–
	1955Q1-2019Q4	0.0406	0.0737	0.0893	0.1634	0.0452	0.4925
United States	1875Q2-2019Q4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table I.5 Bootstrapped *p*-values for Kwiatkowski *et al.*'s (1992) stationarity tests (quarterly data, full samples)

		Money growth		Inflation		Nominal GDP growth	
Country	*Period*	p=4	p=8	p=4	p=8	p=4	p=8
Australia	1959Q4-2019Q4	0.1520	0.1250	0.0985	0.1977	0.0067	0.0542
Brazil	1975Q2-2019Q4	0.1792	0.2842	0.2999	0.3328	0.1816	0.2546
Canada	1914Q2-2006Q4	0.1628	0.3365	0.1975	0.2623	–	–
	1968Q1-2019Q4	0.0294	0.0643	0.0037	0.0330	0.0003	0.0069
Denmark	1922Q4-2019Q4	0.1501	0.1501	0.1714	0.1617	–	–
	1977Q1-2019Q4	0.1139	0.1903	0.0028	0.0345	0.0004	0.0019
Euro area	1970Q3-2019Q4	0.0129	0.0209	0.0669	0.0825	0.0086	0.0265
Finland	1914Q4-1985Q4	0.7595	0.8287	0.3627	0.5836	–	–
Germany	1949Q1-1998Q4	0.0357	0.1154	0.0620	0.1143	–	–
Hong Kong	1985Q1-2019Q4	0.0167	0.0968	0.2531	0.2570	0.0770	0.0701
Italy	1948Q2-1997Q3	0.0805	0.2580	0.1712	0.3575	–	–
Japan	1955Q2-2019Q4	0.0029	0.0134	0.0015	0.0087	0.0003	0.0031
Mexico	1986Q1-2017Q3	0.0099	0.0001	0.0114	0.0001	0.0093	0.0000
Netherlands	1957Q2-1997Q4	0.1257	0.2297	0.2526	0.4742	–	–
New Zealand	1960Q1-1986Q4	0.4650	0.2594	0.0211	0.0767	–	–
	1988Q1-2019Q4	0.8129	0.8505	0.8538	0.5614	0.8741	0.8705
Norway	1920Q3-2019Q4	0.0317	0.1175	0.1194	0.0722	–	–
	1978Q1-2019Q4	0.0173	0.0302	0.0015	0.0957	0.0135	0.1039
South Africa	1966Q1-2019Q4	0.0674	0.0519	0.0126	0.0733	0.0029	0.0200
South Korea	1960Q1-2019Q4	0.0012	0.0103	0.0001	0.0045	0.0000	0.0099
Switzerland	1985Q1-2019Q4	0.9909	0.9688	0.0393	0.1487	0.0648	0.0912
Taiwan	1961Q3-2019Q4	0.7002	0.6517	0.0345	0.0764	0.0000	0.0033
	1982Q1-2019Q4	0.0067	0.0072	0.0285	0.0650	0.0003	0.0044
United Kingdom	1881Q1-2019Q4	0.0577	0.0914	0.1329	0.1733	–	–
	1895Q2-2019Q4	0.3790	0.4062	0.3540	0.5303	–	–
	1955Q1-2019Q4	0.2415	0.2349	0.1680	0.1773	0.0331	0.1550
United States	1875Q2-2019Q4	0.9444	0.9537	0.0469	0.0617	0.3331	0.3637

Table I.6 Bootstrapped *p*-values for Elliot *et al.*'s (1996) unit root tests and Kwiatkowski *et al.*'s (1992) stationarity tests, for the period since 1985Q1

Country	*Period*	Money growth		Inflation		Nominal GDP growth	
		p=4	p=8	p=4	p=8	p=4	p=8
	Elliot et al.'s (1996) unit root tests						
Australia	1985Q1-2019Q4	0.050	0.062	0.008	0.069	0.035	0.042
Brazil	1975Q2-2019Q4	0.0043	0.0530	0.0016	0.0085	0.0021	0.0152
Canada	1985Q1-2019Q4	0.391	0.293	0.076	0.326	0.023	0.117
Denmark	1985Q1-2019Q4	0.000	0.013	0.001	0.041	0.000	0.021
Euro area	1985Q1-2019Q4	0.017	0.173	0.039	0.423	0.014	0.119
Hong Kong	1985Q1-2019Q4	0.012	0.176	0.338	0.382	0.047	0.082
Japan	1985Q1-2017Q3	0.262	0.045	0.061	0.281	0.021	0.237
Mexico	1986Q1-2017Q3	0.0181	0.0006	0.0319	0.0000	0.0265	0.0001
Norway	1985Q1-2015Q1	0.000	0.013	0.000	0.007	0.002	0.010
New Zealand	1987Q4-2016Q4	0.007	0.029	0.000	0.001	0.012	0.116
South Korea	1985Q1-2019Q4	0.387	0.339	0.027	0.603	0.107	0.468
Switzerland	1985Q1-2019Q4	0.015	0.008	0.122	0.484	0.005	0.077
Taiwan	1985Q1-2017Q4	0.198	0.093	0.022	0.144	0.001	0.091
United Kingdom	1985Q1-2017Q2	0.205	0.176	0.047	0.403	0.048	0.209
United States	1985Q1-2019Q4	0.008	0.082	0.1379	0.044	0.132	0.223
	Kwiatkowski et al.'s (1992) stationarity tests						
Australia	1985Q1-2019Q4	0.288	0.232	0.120	0.242	0.130	0.157
Brazil	1975Q2-2019Q4	0.0212	0.0602	0.0642	0.0754	0.0260	0.0425
Canada	1985Q1-2019Q4	0.570	0.552	0.064	0.106	0.094	0.080
Denmark	1985Q1-2019Q4	0.612	0.678	0.018	0.062	0.005	0.002
Euro area	1985Q1-2019Q4	0.372	0.457	0.012	0.042	0.008	0.013
Hong Kong	1985Q1-2019Q4	0.018	0.095	0.258	0.257	0.075	0.071
Japan	1985Q1-2017Q3	0.163	0.114	0.174	0.266	0.031	0.112
Mexico	1986Q1-2017Q3	0.0118	0.0005	0.0096	0.0001	0.0082	0.0002
Norway	1985Q1-2015Q1	0.228	0.264	0.011	0.056	0.140	0.177
New Zealand	1987Q4-2016Q4	0.485	0.448	0.465	0.238	0.786	0.822
South Korea	1985Q1-2019Q4	0.027	0.020	0.002	0.069	0.002	0.020
Switzerland	1985Q1-2019Q4	0.990	0.968	0.049	0.155	0.082	0.114
Taiwan	1985Q1-2017Q4	0.005	0.007	0.058	0.082	0.001	0.007
United Kingdom	1985Q1-2017Q2	0.131	0.078	0.036	0.170	0.015	0.043
United States	1985Q1-2019Q4	0.224	0.262	0.190	0.089	0.066	0.022

Table I.4 Empirical rejection frequencies for bootstrapped Elliot *et al.*'s (1996) unit root tests under the assumption of stationarity, by monetary standard

Country	*Period*	Money growth	Inflation	Nominal GDP growth
		Commodity standards		
Argentina	1863-1929	1.0000	0.9860	0.0000
Canada	1873-1929	0.9900	0.9968	0.9998
Chile	1811-1877	0.9988	1.0000	1.0000
Italy	1862-1935	0.9968	0.9928	0.9998
Norway	1865-1931	0.8962	0.9810	0.9966
Sweden	1847-1931	0.9996	0.9990	0.9984
United Kingdom	1701-1796	1.000	1.000	1.000
	1821-1931	1.000	1.000	1.000
United States	1870-1932	0.9856	0.9958	1.0000
		Fiat standards		
Argentina	1930-2019	0.9610	0.9928	0.9932
Australia	1914-2019	1.0000	0.9986	0.9890
Brazil	1864-2012	0.9322	0.9588	0.7890
Canada	1935-2006	0.8848	0.9906	0.9988
Chile	1878-2019	0.8378	0.9962	0.9944
Colombia	1956-2019	0.7388	0.1204	0.2922
Finland	1915-1985	0.9846	0.9674	0.9894
France	1937-1994	0.6080	0.4598	0.7392
Italy	1936-1996	0.2530	0.7188	0.5476
Japan	1956-2018	0.7486	0.5738	0.2564
Mexico	1926-2013	0.5876	0.2716	0.2648
New Zealand	1914-2016	1.000	0.9936	0.9998
Norway	1947-2014	0.9998	0.9374	0.9998
Paraguay	1963-2015	0.9980	0.4106	0.6726
Portugal	1932-1998	0.9170	0.5366	0.6016
South Korea	1965-2019	0.2040	0.1300	0.0984
Spain	1942-1997	0.8166	0.9786	0.9998
South Africa	1966-2019	0.9806	0.6542	0.8670
Sweden	1932-2018	1.0000	0.9962	0.9996
Switzerland	1937-2012	0.9900	0.9990	0.9972
United Kingdom	1932-2019	0.8772	0.8370	0.9394
United States	1933-2019	0.9906	0.9958	0.9984
Venezuela	1951-2017			

Table I.5 Empirical rejection frequencies for bootstrapped Kwiatkowski *et al.*'s (1992) stationarity tests under the assumption of stationarity, by monetary standard

Country	*Period*	Money growth	Inflation	Nominal GDP growth
		Commodity standards		
Argentina	1863-1929	0.1018	0.1012	0.0984
Canada	1873-1929	0.1024	0.1058	0.0922
Chile	1811-1877	0.0932	0.1016	0.0968
Italy	1862-1935	0.1092	0.1064	0.1076
Norway	1865-1931	0.1084	0.1056	0.1010
Sweden	1847-1931	0.1060	0.1022	0.1052
United Kingdom	1701-1796	0.1064	0.0888	0.1126
	1821-1931	0.1010	0.0980	0.1002
United States	1870-1932	0.1078	0.1026	0.0946
		Fiat standards		
Argentina	1930-2019	0.1112	0.1106	0.1060
Australia	1914-2019	0.1030	0.1004	0.1110
Brazil	1864-2012	0.1064	0.1086	0.1184
Canada	1935-2006	0.0576	0.0580	0.0566
Chile	1878-2019	0.1174	0.1100	0.1096
Colombia	1956-2019	0.1188	0.1984	0.1314
Finland	1915-1985	0.1144	0.1072	0.1032
France	1937-1994	0.1122	0.1200	0.0984
Italy	1936-1996	0.1386	0.1214	0.1178
Japan	1956-2018	0.1170	0.1182	0.1358
Mexico	1926-2013	0.1148	0.1296	0.1308
New Zealand	1914-2016	0.1068	0.1092	0.1022
Norway	1947-2014	0.1010	0.1186	0.1012
Paraguay	1963-2015	0.0998	0.1244	0.1160
Portugal	1932-1998	0.1158	0.1168	0.1176
South Korea	1965-2019	0.1434	0.1896	0.2534
Spain	1942-1997	0.1042	0.1012	0.0952
South Africa	1966-2019	0.1008	0.1184	0.1020
Sweden	1932-2018	0.1168	0.1146	0.1090
Switzerland	1937-2012	0.1074	0.1024	0.1104
United Kingdom	1932-2019	0.1168	0.1146	0.1090
United States	1933-2019	0.1026	0.1164	0.1052
Venezuela	1951-2017			

II: Full set of results based on Müller and Watson's (2018) low-frequency regressions

Table II.1 Full samples: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			I: Based on money growth and inflation		
Argentina	1864-2019	31.2	1.041	[1.000 1.068] [0.958 1.091]	[1.013 1.068] [0.991 1.091]
Australia	1854-2019	30.2	0.768	[0.573 0.951] [0.433 1.099]	[0.573 0.951] [0.433 1.099]
Brazil	1862-2019	31.6	0.990	[0.964 1.017] [0.945 1.040]	[0.964 1.017] [0.945 1.040]
Canada	1873-2006	33.5	0.597	[0.440 0.755] [0.325 0.925]	[0.440 0.755] [0.325 0.890]
Chile	1811-2019	32.2	0.946	[0.857 0.991] [0.804 1.023]	[0.902 0.991] [0.868 1.023]
Finland	1867-1985	34	1.589	[1.397 1.782] [1.068 2.118]	[1.397 1.782] [1.235 1.946]
France	1910-1994	34	1.047	[0.655 1.443] [0.179 1.813]	[0.655 1.443] [0.293 1.813]
Italy	1862-1996	30	1.091	[0.929 1.263] [0.637 1.402]	[0.929 1.263] [0.798 1.402]
Mexico	1926-2013	35.2	1.104	[1.011 1.193] [0.894 1.295]	[1.011 1.193] [0.932 1.273]
New Zealand	1885-2016	33	1.123	[0.980 1.271] [0.766 1.386]	[0.980 1.271] [0.873 1.386]
Norway	1820-2014	30	0.878	[0.714 1.051] [0.592 1.363]	[0.714 1.051] [0.592 1.193]
Portugal	1855-1998	32	0.974	[0.816 1.143] [0.625 1.271]	[0.816 1.143] [0.699 1.271]
Sweden	1847-2018	31.3	1.086	[0.790 1.384] [0.566 1.649]	[0.790 1.384] [0.566 1.649]
Switzerland	1916-2015	33.3	0.878	[0.646 1.110] [0.435 1.321]	[0.646 1.110] [0.435 1.321]
United Kingdom	1701-2019	30.4	0.783	[0.678 0.882] [0.598 0.964]	[0.678 0.882] [0.598 0.964]
United States	1869-2019	30.4	1.034	[0.812 1.275] [0.634 1.674]	[0.812 1.275] [0.634 1.495]

For details, see Section 3.

Table II.1 (continued) Full samples: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			II: Based on money growth and nominal GDP growth		
Argentina	1864-2019	31.2	1.016	[0.992 1.038] [0.961 1.056]	[0.992 1.038] [0.975 1.056]
Australia	1854-2019	34	0.535	[0.069 1.064] [-0.556 1.664]	[0.069 0.998] [-0.441 1.509]
Brazil	1862-2019	31.6	0.990	[0.972 1.007] [0.934 1.032]	[0.972 1.007] [0.958 1.021]
Canada	1873-2006	33.5	0.880	[0.707 1.043] [0.563 1.243]	[0.707 1.043] [0.572 1.192]
Chile	1811-2019	32.2	0.932	[0.889 0.964] [0.850 0.990]	[0.900 0.964] [0.876 0.990]
Finland	1867-1985	33.7	1.532	[1.299 1.756] [1.140 2.127]	[1.299 1.756] [1.140 1.940]
France	1910-1994	30.0	0.690	[0.404 0.933] [0.018 1.316]	[0.438 0.933] [0.138 1.265]
Italy	1862-1996	30.0	1.183	[1.076 1.284] [0.875 1.373]	[1.076 1.284] [0.996 1.373]
Mexico	1926-2013	35.2	1.104	[1.052 1.155] [0.991 1.204]	[1.052 1.155] [1.009 1.200]
New Zealand	1885-2016	32.8	0.942	[0.622 1.164] [0.346 1.457]	[0.731 1.147] [0.500 1.350]
Portugal	1855-1998	32	0.908	[0.794 1.016] [0.636 1.106]	[0.794 1.016] [0.711 1.106]
Sweden	1847-2018	31.3	1.039	[0.738 1.345] [0.522 1.930]	[0.738 1.345] [0.522 1.627]
Switzerland	1916-2015	33.3	1.096	[0.752 1.438] [0.420 1.835]	[0.752 1.438] [0.442 1.743]
United Kingdom	1701-2019	30.4	0.881	[0.773 0.982] [0.673 1.130]	[0.773 0.982] [0.673 1.062]
United States	1869-2019	30.4	1.130	[0.931 1.329] [0.783 1.624]	[0.931 1.329] [0.783 1.512]

For details, see Section 3.

Table II.2 Commodity standards: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			I: Based on money growth and inflation		
Argentina	1864-1929	33	0.977	[0.459 1.350] [-0.114 1.791]	[0.509 1.350] [0.045 1.731]
Canada	1873-1929	38	0.511	[-0.009 1.004] [-0.658 1.653]	[-0.009 1.004] [-0.658 1.653]
Chile	1811-1877	33.5	1.829	[0.693 3.195] [-0.561 4.850]	[0.693 3.124] [-0.415 4.330]
Finland	1868-1914	31.3	0.425	[0.177 0.763] [-0.153 1.163]	[0.177 0.675] [-0.153 1.015]
Italy	1862-1935	37	1.096	[0.864 1.316] [0.585 1.586]	[0.864 1.316] [0.645 1.535]
Norway	1865-1931	33.5	0.915	[0.777 1.051] [0.574 1.266]	[0.777 1.051] [0.630 1.222]
Sweden	1847-1931	34	0.838	[0.422 1.234] [-0.045 1.630]	[0.422 1.234] [0.063 1.617]
United Kingdom	1701-1796	32	0.489	[0.350 0.743] [0.212 0.944]	[0.350 0.647] [0.234 0.800]
	1821-1931	31.7	0.967	[0.760 1.175] [0.428 1.506]	[0.760 1.175] [0.581 1.354]
United States	1869-1932	32.5	1.040	[0.805 1.272] [0.456 1.632]	[0.805 1.272] [0.538 1.558]
			II: Based on money growth and nominal GDP growth		
Argentina	1864-1929	33	1.013	[0.481 1.400] [-0.035 1.942]	[0.570 1.400] [0.123 1.836]
Canada	1873-1929	38	0.759	[0.248 1.265] [-0.392 1.923]	[0.248 1.265] [-0.392 1.923]
Chile	1811-1877	33.5	1.547	[0.590 2.570] [-0.425 3.873]	[0.590 2.570] [-0.425 3.654]
Finland	1868-1914	31.3	0.692	[0.438 1.016] [0.112 1.400]	[0.438 0.931] [0.112 1.267]
Italy	1862-1935	37	1.080	[0.849 1.310] [0.539 1.620]	[0.849 1.310] [0.602 1.557]
Norway	1865-1931	33.5	0.921	[0.836 1.012] [0.715 1.143]	[0.836 1.012] [0.744 1.111]
Sweden	1847-1931	34	0.736	[0.318 1.158] [-0.223 1.611]	[0.318 1.158] [-0.079 1.551]
United Kingdom	1701-1796	32	0.946	[0.719 1.293] [0.454 1.619]	[0.719 1.190] [0.532 1.407]
	1821-1931	31.7	0.838	[0.608 1.053] [0.429 1.248]	[0.608 1.053] [0.429 1.248]
United States	1869-1932	32.5	1.356	[1.247 1.442] [1.156 1.536]	[1.267 1.442] [1.180 1.529]

For details, see Section 3.

Table II.3 ***Fiat* standards: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology**

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			I: Based on money growth and inflation		
Argentina	1930-2019	30	1.022	[0.990 1.054] [0.962 1.088]	[0.990 1.054] [0.962 1.088]
Australia	1914-2019	30.3	0.868	[0.668 1.057] [0.509 1.226]	[0.668 1.057] [0.509 1.226]
Brazil	1862-2019	31.6	0.990	[0.964 1.017] [0.945 1.040]	[0.964 1.017] [0.945 1.040]
Canada	1935-2006	36	0.529	[0.439 0.612] [0.347 0.720]	[0.439 0.612] [0.347 0.703]
Chile	1878-2019	31.6	0.964	[0.877 1.001] [0.836 1.055]	[0.927 1.001] [0.897 1.029]
Colombia	1956-2019	32	0.986	[0.854 1.115] [0.710 1.262]	[0.854 1.115] [0.710 1.262]
Finland	1915-1985	36.7	1.200	[0.777 1.627] [0.227 2.482]	[0.777 1.594] [0.227 2.157]
France	1937-1994	38.7	0.972	[0.368 1.604] [-0.529 2.500]	[0.368 1.519] [-0.359 2.275]
Iceland	1961-2019	39.3	1.058	[0.843 1.270] [0.562 1.551]	[0.843 1.270] [0.562 1.551]
Italy	1936-1996	30.5	1.214	[0.748 1.655] [0.187 2.234]	[0.748 1.655] [0.253 2.150]
Japan	1956-2018	31.5	0.473	[0.354 0.685] [0.192 0.837]	[0.354 0.602] [0.223 0.742]
Mexico	1926-2013	35.2	1.104	[1.011 1.193] [0.894 1.295]	[1.011 1.193] [0.932 1.273]
New Zealand	1914-2016	34.3	1.146	[0.972 1.306] [0.704 1.457]	[0.972 1.306] [0.827 1.457]
Norway	1947-2014	34	0.882	[0.573 1.174] [0.224 1.546]	[0.573 1.174] [0.245 1.483]
Paraguay	1963-2015	35.3	1.532	[0.285 2.715] [-1.172 4.171]	[0.285 2.715] [-1.172 4.171]
Peru	1960-2018	39.3	1.176	[1.091 1.255] [0.988 1.367]	[1.091 1.255] [0.988 1.363]
Portugal	1932-1998	33.5	1.098	[0.718 1.473] [0.315 1.886]	[0.718 1.473] [0.315 1.886]
Saudi Arabia	1964-2019	37.3	0.386	[0.291 0.487] [0.171 0.652]	[0.291 0.479] [0.171 0.608]
South Korea	1965-2019	36.7	0.480	[0.344 0.628] [0.078 0.896]	[0.365 0.590] [0.198 0.744]
Spain	1942-1997	37.3	0.415	[-0.413 1.092] [-1.344 2.054]	[-0.293 1.092] [-1.142 1.976]
South Africa	1966-2019	36	0.917	[0.528 1.288] [0.021 1.817]	[0.528 1.288] [0.021 1.817]
Sweden	1932-2018	34.8	1.438	[0.868 1.968] [0.227 2.631]	[0.868 1.968] [0.369 2.487]
Switzerland	1937-2015	31.6	0.691	[0.251 1.093] [-0.216 1.616]	[0.251 1.093] [-0.156 1.484]
United Kingdom	1932-2019	35.2	0.662	[0.315 0.997] [0.015 1.327]	[0.315 0.997] [0.015 1.306]
United States	1933-2019	34.8	0.688	[0.255 1.098] [-0.118 1.521]	[0.255 1.098] [-0.118 1.471]
Venezuela	1951-2017	33.5	0.997	[0.851 1.137] [0.685 1.322]	[0.851 1.137] [0.715 1.283]

For details, see Section 3.

Table II.3 (continued) *Fiat* standards: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			II: Based on money growth and nominal GDP growth		
Argentina	1930-2019	30	1.007	[0.979 1.036] [0.954 1.063]	[0.979 1.036] [0.954 1.063]
Australia	1914-2019	34	0.535	[0.069 1.064] [-0.556 1.664]	[0.069 0.998] [-0.441 1.509]
Brazil	1862-2019	31.6	0.990	[0.972 1.007] [0.934 1.032]	[0.972 1.007] [0.958 1.021]
Canada	1935-2006	36	0.672	[0.501 0.843] [0.273 1.070]	[0.501 0.843] [0.318 1.022]
Chile	1878-2019	31.6	0.942	[0.877 0.976] [0.836 1.026]	[0.908 0.976] [0.880 1.002]
Colombia	1956-2019	31.5	1.011	[0.899 1.110] [0.775 1.236]	[0.899 1.110] [0.791 1.218]
Finland	1915-1985	36.7	1.173	[0.819 1.587] [0.333 2.325]	[0.819 1.527] [0.333 2.014]
France	1937-1994	30	0.690	[0.404 0.933] [0.018 1.316]	[0.438 0.933] [0.138 1.265]
Iceland	1961-2019	39.3	1.128	[0.847 1.395] [0.491 1.751]	[0.847 1.395] [0.491 1.751]
Italy	1936-1996	30.5	1.197	[0.979 1.416] [0.691 1.687]	[0.979 1.416] [0.751 1.631]
Japan	1956-2018	31.5	0.885	[0.746 0.962] [0.604 1.048]	[0.791 0.962] [0.684 1.048]
Mexico	1926-2013	35.2	1.104	[1.052 1.155] [0.991 1.204]	[1.052 1.155] [1.009 1.200]
New Zealand	1914-2016	32.8	0.942	[0.622 1.164] [0.346 1.457]	[0.731 1.147] [0.500 1.350]
Norway	1947-2014	34	0.725	[0.367 1.080] [-0.097 1.547]	[0.367 1.080] [-0.013 1.467]
Paraguay	1963-2015	35.3	1.902	[1.095 2.672] [0.118 3.651]	[1.095 2.672] [0.118 3.651]
Peru	1960-2018	39.3	1.120	[1.048 1.187] [0.956 1.278]	[1.048 1.187] [0.956 1.278]
Portugal	1932-1998	33.5	1.124	[0.776 1.441] [0.437 1.797]	[0.776 1.441] [0.437 1.778]
Saudi Arabia	1964-2019	35.3	0.768	[0.622 0.941] [0.428 1.132]	[0.622 0.907] [0.437 1.083]
South Korea	1965-2019	36.7	0.718	[0.576 0.856] [0.333 1.092]	[0.605 0.818] [0.448 0.966]
Spain	1942-1997	37.3	0.725	[0.359 1.049] [-0.117 1.510]	[0.386 1.049] [-0.020 1.473]
South Africa	1966-2019	36	0.786	[0.537 1.022] [0.228 1.339]	[0.537 1.022] [0.228 1.339]
Sweden	1932-2018	34.8	1.355	[0.869 1.864] [0.329 2.488]	[0.869 1.813] [0.486 2.223]
Switzerland	1937-2015	31.6	0.747	[0.262 1.433] [-0.235 1.929]	[0.293 1.209] [-0.112 1.639]
United Kingdom	1932-2019	35.2	0.712	[0.412 1.006] [0.085 1.376]	[0.412 1.006] [0.144 1.270]
United States	1933-2019	34.8	1.130	[0.713 1.694] [0.219 2.267]	[0.713 1.571] [0.324 1.984]
Venezuela	1951-2017	33.5	0.851	[0.730 0.971] [0.583 1.126]	[0.730 0.971] [0.612 1.093]

For details, see Section 3.

Table II.4 Period since 1985: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			I: Based on money growth and inflation		
Australia	1985Q1-2019Q4	23.3	0.546	[0.207 0.826] [-0.161 1.194]	[0.334 to 0.740] [0.063 to 1.012]
Canada	1985Q1-2019Q4	23.3	0.237	[0.061 to 0.380] [-0.218 to 0.589]	[0.073 to 0.380] [-0.140 to 0.589]
Denmark	1985Q1-2019Q4	23.3	0.867	[0.406 to 1.291] [-0.185 to 1.873]	[0.553 to 1.170] [0.143 to 1.591]
Euro area	1985Q1-2019Q4	23.3	0.799	[-0.295 to 1.949] [-1.901 to 3.725]	[-0.125 to 1.751] [-1.373 to 2.999]
Hong Kong	1985Q1-2019Q4	23.3	0.761	[0.446 to 1.199] [0.030 to 1.651]	[0.446 to 1.063] [0.072 to 1.459]
Japan	1985Q1-2019Q4	23.3	0.325	[0.053 to 0.671] [-0.292 to 1.019]	[0.053 to 0.565] [-0.277 to 0.899]
Norway	1985Q1-2015Q1	23.2	0.804	[0.054 to 1.422] [-0.759 to 2.236]	[0.336 to 1.262] [-0.241 to 1.834]
New Zealand	1988Q1-2019Q4	21.2	0.687	[-0.082 to 1.444] [-1.083 to 2.447]	[-0.082 to 1.444] [-1.083 to 2.447]
South Korea	1985Q1-2019Q4	23.3	0.313	[0.262 to 0.346] [0.210 to 0.399]	[0.283 to 0.341] [0.243 to 0.378]
Switzerland	1985Q1-2019Q4	23.3	0.586	[0.133 to 1.043] [-0.815 to 1.675]	[0.133 to 1.043] [-0.514 to 1.675]
Taiwan	1985Q1-2019Q4	23.3	0.197	[0.032 to 0.357] [-0.230 to 0.600]	[0.032 to 0.346] [-0.185 to 0.546]
United Kingdom	1985Q1-2017Q2	23.3	0.321	[0.074 to 0.559] [-0.245 to 0.897]	[0.093 to 0.542] [-0.181 to 0.823]
United States	1985Q1-2019Q4	23.3	0.016	[-0.451 to 0.466] [-1.022 to 1.018]	[-0.451 to 0.466] [-1.022 to 1.018]
			II: Based on money growth and nominal GDP growth		
Australia	1985Q1-2019Q4	23.3	0.597	[0.348 to 0.795] [-0.017 to 1.175]	[0.405 to 0.783] [0.163 to 1.040]
Canada	1985Q1-2019Q4	23.3	0.078	[-0.240 to 0.336] [-0.691 to 0.680]	[-0.198 to 0.336] [-0.559 to 0.680]
Denmark	1985Q1-2019Q4	23.3	0.292	[-0.393 to 0.955] [-1.608 to 2.352]	[-0.333 to 0.955] [-1.145 to 1.872]
Euro area	1985Q1-2019Q4	23.3	0.810	[-0.740 to 2.418] [-2.997 to 5.246]	[-0.601 to 2.251] [-2.403 to 4.197]
Hong Kong	1985Q1-2019Q4	23.3	1.059	[0.803 to 1.383] [0.445 to 1.805]	[0.803 to 1.325] [0.490 to 1.659]
Japan	1985Q1-2019Q4	23.3	0.937	[0.652 to 1.277] [0.293 to 1.654]	[0.652 to 1.194] [0.327 to 1.520]
Norway	1985Q1-2015Q1	23.3	0.751	[0.638 to 0.872] [0.415 to 1.103]	[0.638 to 0.872] [0.490 to 1.031]
New Zealand	1988Q1-2019Q4	21.2	0.130	[-1.275 to 1.430] [-2.991 to 3.211]	[-1.275 to 1.430] [-2.991 to 3.211]
South Korea	1985Q1-2019Q4	23.3	0.701	[0.546 to 0.810] [0.327 to 1.019]	[0.590 to 0.810] [0.432 to 0.959]
Switzerland	1985Q1-2019Q4	23.3	0.792	[0.385 to 1.204] [-0.468 to 1.772]	[0.385 to 1.204] [-0.181 to 1.772]
Taiwan	1985Q1-2019Q4	23.3	0.547	[0.417 to 0.678] [0.167 to 0.853]	[0.417 to 0.663] [0.243 to 0.819]
United Kingdom	1985Q1-2017Q2	23.3	0.398	[0.135 to 0.677] [-0.229 to 1.038]	[0.135 to 0.631] [-0.149 to 0.927]
United States	1985Q1-2019Q4	23.3	-0.792	[-1.383 to -0.233] [-2.103 to 0.426]	[-1.383 to -0.233] [-2.103 to 0.426]

Table II.5 Full samples: evidence from regressing real GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Argentina	1864-2019	31.2	-0.025	[-0.044 -0.005] [-0.059 0.009]	[-0.036 -0.014] [-0.045 -0.006]
Australia	1854-2019	38	-0.206	[-0.538 0.134] [-0.931 0.550]	[-0.538 0.134] [-0.931 0.550]
Brazil	1862-2019	31.6	-0.009	[-0.028 0.009] [-0.043 0.050]	[-0.028 0.009] [-0.043 0.024]
Canada	1873-2006	33.5	0.274	[0.155 0.385] [0.013 0.479]	[0.155 0.385] [0.066 0.479]
Chile	1811-2019	32.2	-0.007	[-0.033 0.028] [-0.057 0.067]	[-0.033 0.018] [-0.057 0.036]
Finland	1867-1985	33.7	-0.089	[-0.179 0.083] [-0.245 0.212]	[-0.179 0.016] [-0.245 0.096]
France	1910-1994	30	0.173	[-0.063 0.398] [-0.403 0.783]	[-0.063 0.398] [-0.342 0.690]
Italy	1862-1996	30	0.079	[-0.013 0.171] [-0.085 0.326]	[-0.013 0.171] [-0.085 0.243]
Mexico	1926-2013	35.2	0.000	[-0.111 0.105] [-0.244 0.250]	[-0.111 0.105] [-0.208 0.203]
New Zealand	1885-2016	32.8	-0.195	[-0.416 0.025] [-0.631 0.247]	[-0.416 0.025] [-0.631 0.247]
Portugal	1855-1998	32	0.016	[-0.107 0.132] [-0.212 0.269]	[-0.107 0.132] [-0.212 0.229]
Sweden	1847-2018	31.3	-0.013	[-0.128 0.102] [-0.213 0.201]	[-0.128 0.102] [-0.213 0.201]
Switzerland	1916-2015	33.3	0.205	[-0.071 0.479] [-0.380 0.884]	[-0.071 0.479] [-0.307 0.706]
United Kingdom	1701-2019	30.4	0.089	[0.023 0.230] [-0.033 0.297]	[0.023 0.154] [-0.033 0.203]
United States	1869-2019	30.4	0.094	[-0.071 0.258] [-0.333 0.391]	[-0.071 0.258] [-0.211 0.391]

For details, see Section 3.

Table II.6 Commodity standards: evidence from regressing real GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Argentina	1864-1929	33	0.107	[-0.190 0.436] [-0.480 0.785]	[-0.190 0.398] [-0.480 0.706]
Canada	1873-1929	38	0.253	[0.041 0.495] [-0.214 0.741]	[0.041 0.454] [-0.214 0.724]
Chile	1811-1877	33.5	-0.464	[-1.513 0.429] [-2.274 1.379]	[-1.221 0.328] [-2.001 1.137]
Italy	1862-1935	37	-0.016	[-0.046 0.017] [-0.083 0.062]	[-0.046 0.017] [-0.078 0.051]
Norway	1865-1931	33.5	-0.001	[-0.061 0.063] [-0.141 0.153]	[-0.061 0.063] [-0.121 0.133]
Sweden	1847-1931	34	-0.104	[-0.192 -0.016] [-0.270 0.068]	[-0.192 -0.016] [-0.270 0.068]
United Kingdom	1701-1796	32	0.452	[0.338 0.565] [0.216 0.697]	[0.338 0.565] [0.242 0.669]
	1821-1931	31.7	-0.119	[-0.289 0.040] [-0.562 0.326]	[-0.289 0.040] [-0.428 0.174]
United States	1869-1932	32.5	0.370	[0.096 0.609] [-0.297 0.979]	[0.096 0.609] [-0.211 0.903]

For details, see Section 3.

Table II.7 *Fiat* standards: evidence from regressing real GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Argentina	1930-2019	30	-0.017	[-0.030 -0.005] [-0.042 0.013]	[-0.030 -0.005] [-0.042 0.006]
Australia	1914-2019	38	-0.206	[-0.538 0.134] [-0.931 0.550]	[-0.538 0.134] [-0.931 0.550]
Brazil	1862-2019	31.6	-0.009	[-0.028 0.009] [-0.043 0.050]	[-0.028 0.009] [-0.043 0.024]
Canada	1935-2006	36	0.146	[-0.105 0.397] [-0.441 0.717]	[-0.105 0.397] [-0.380 0.662]
Chile	1878-2019	31.6	-0.010	[-0.040 0.018] [-0.065 0.062]	[-0.040 0.018] [-0.065 0.041]
Colombia	1956-2019	31.5	0.008	[-0.073 0.087] [-0.172 0.187]	[-0.073 0.087] [-0.160 0.174]
Finland	1915-1985	36.7	-0.017	[-0.268 0.200] [-0.600 0.540]	[-0.213 0.173] [-0.462 0.416]
France	1937-1994	30	0.173	[-0.063 0.398] [-0.403 0.783]	[-0.063 0.398] [-0.342 0.690]
Iceland	1961-2019	39.3	0.000	[-0.091 0.086] [-0.192 0.193]	[-0.091 0.086] [-0.192 0.193]
Italy	1936-1996	30.5	-0.018	[-0.288 0.237] [-0.585 0.545]	[-0.288 0.237] [-0.547 0.506]
Japan	1956-2018	31.5	0.409	[0.116 0.543] [-0.085 0.695]	[0.250 0.543] [0.055 0.690]
Mexico	1926-2013	35.2	0.000	[-0.111 0.105] [-0.244 0.250]	[-0.111 0.105] [-0.208 0.203]
New Zealand	1914-2016	32.8	-0.195	[-0.416 0.025] [-0.631 0.247]	[-0.416 0.025] [-0.631 0.247]
Norway	1947-2014	33.5	-0.030	[-0.267 0.212] [-0.577 0.547]	[-0.267 0.198] [-0.528 0.473]
Paraguay	1963-2015	35.3	0.356	[-0.094 0.774] [-0.635 1.346]	[-0.094 0.774] [-0.635 1.346]
Peru	1960-2018	39.3	-0.057	[-0.078 -0.037] [-0.104 -0.010]	[-0.078 -0.037] [-0.103 -0.012]
Portugal	1932-1998	33.5	-0.007	[-0.241 0.216] [-0.460 0.476]	[-0.241 0.216] [-0.460 0.438]
South Korea	1965-2019	36.7	0.235	[0.094 0.362] [-0.058 0.518]	[0.153 0.314] [0.045 0.417]
Spain	1942-1997	37.3	0.307	[-0.067 0.751] [-0.542 1.239]	[-0.067 0.662] [-0.521 1.115]
South Africa	1966-2019	36	-0.128	[-0.477 0.183] [-0.883 0.598]	[-0.477 0.183] [-0.883 0.598]
Sweden	1932-2018	34.8	-0.096	[-0.439 0.260] [-0.771 0.711]	[-0.439 0.227] [-0.708 0.533]
Switzerland	1937-2015	31.6	0.119	[-0.249 0.547] [-0.668 1.002]	[-0.249 0.493] [-0.588 0.828]
United Kingdom	1932-2019	35.2	0.046	[-0.048 0.139] [-0.137 0.237]	[-0.048 0.139] [-0.137 0.237]
United States	1933-2019	34.8	0.485	[-0.015 1.015] [-0.568 1.730]	[-0.015 0.961] [-0.465 1.439]
Venezuela	1951-2017	33.5	-0.151	[-0.198 -0.104] [-0.259 -0.044]	[-0.198 -0.104] [-0.246 -0.054]

For details, see Section 3.

Table II.8 Post-WWII period: evidence from regressing inflation on either money growth or credit[a] growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
		I: Based on money growth			
Australia	1953-2016	32	0.940	[0.611 1.270] [0.228 1.691]	[0.611 1.270] [0.291 1.609]
Canada	1947-2006	30	0.610	[0.350 0.878] [0.068 1.169]	[0.350 0.878] [0.068 1.169]
Finland	1947-1985	39	1.253	[0.351 2.113] [-1.988 4.452]	[0.351 2.113] [-1.988 4.363]
France	1947-1994	32	0.873	[-0.178 1.626] [-1.255 2.730]	[0.131 1.563] [-0.778 2.505]
Italy	1949-1996	32	0.769	[-0.032 1.471] [-1.177 2.642]	[-0.032 1.471] [-1.004 2.408]
Japan	1956-2016	30.5	0.472	[0.351 0.676] [0.188 0.840]	[0.351 0.601] [0.219 0.733]
Norway	1947-2013	33.5	0.742	[0.525 0.967] [0.282 1.211]	[0.525 0.967] [0.288 1.211]
Spain	1947-1997	34	0.441	[-0.375 1.164] [-1.390 2.149]	[-0.316 1.164] [-1.260 2.149]
Sweden	1947-2012	33	1.246	[0.562 1.917] [-0.152 2.738]	[0.562 1.917] [-0.152 2.628]
Switzerland	1947-2015	34.5	0.682	[0.393 0.949] [0.095 1.311]	[0.393 0.949] [0.123 1.229]
United Kingdom	1947-2014	34	0.677	[0.311 1.034] [-0.078 1.494]	[0.311 1.034] [-0.044 1.422]
United States	1947-2014	34	0.549	[0.103 1.003] [-0.343 1.483]	[0.103 0.972] [-0.343 1.433]
		II: Based on credit growth			
Australia	1953-2016	32	0.803	[0.556 1.036] [0.263 1.358]	[0.556 1.036] [0.309 1.298]
Canada	1947-2006	30	0.488	[0.218 0.755] [-0.079 1.050]	[0.218 0.755] [-0.064 1.050]
Finland	1947-1985	39	0.777	[0.629 0.919] [0.259 1.304]	[0.629 0.919] [0.259 1.289]
France	1947-1994	32	0.822	[0.391 1.304] [-0.288 1.762]	[0.435 1.169] [-0.012 1.602]
Italy	1949-1996	32	0.442	[-0.910 1.662] [-3.133 3.592]	[-0.682 1.466] [-2.112 2.799]
Japan	1956-2016	30.5	0.257	[-0.026 0.433] [-0.289 0.618]	[0.049 0.433] [-0.180 0.605]
Norway	1947-2013	33.5	0.285	[0.035 0.522] [-0.274 0.807]	[0.035 0.522] [-0.222 0.786]
Spain	1947-1997	34	0.440	[-0.179 1.013] [-0.941 1.790]	[-0.140 1.013] [-0.799 1.714]
Sweden	1947-2012	33	0.386	[0.189 0.569] [-0.018 0.807]	[0.189 0.569] [-0.005 0.764]
Switzerland	1947-2015	34.5	0.265	[0.028 0.545] [-0.199 0.765]	[0.103 0.428] [-0.072 0.602]
United Kingdom	1947-2014	34	0.496	[0.280 0.711] [0.066 0.966]	[0.280 0.711] [0.066 0.942]
United States	1947-2014	34	0.425	[-0.011 0.988] [-0.442 1.491]	[-0.011 0.893] [-0.442 1.333]

[a] Computed as total nominal loans minus nominal loans to real estate.

Table II.9 Post-WWII period: evidence from regressing nominal GDP growth on either money growth or credit[a] growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
		I: Based on money growth			
Australia	1953-2016	37.3	1.141	[0.739 1.575] [0.248 2.120]	[0.739 1.575] [0.248 2.120]
Canada	1947-2006	30	0.662	[0.327 0.984] [-0.060 1.371]	[0.327 0.984] [0.000 1.343]
Finland	1947-1985	39	1.030	[-0.342 2.339] [-3.901 5.898]	[-0.342 2.339] [-3.901 5.763]
France	1947-1994	32	1.114	[0.157 1.865] [-0.864 2.912]	[0.402 1.806] [-0.442 2.663]
Italy	1949-1996	32	0.907	[0.450 1.308] [-0.205 1.978]	[0.450 1.275] [-0.105 1.844]
Japan	1956-2016	30.5	0.879	[0.722 0.958] [0.598 1.042]	[0.785 0.958] [0.679 1.042]
Norway	1947-2013	33.5	0.754	[0.379 1.101] [-0.097 1.560]	[0.379 1.101] [0.000 1.484]
Spain	1947-1997	34	0.733	[0.293 1.124] [-0.255 1.655]	[0.325 1.124] [-0.185 1.655]
Sweden	1947-2012	33	1.091	[0.443 1.694] [-0.145 2.521]	[0.443 1.694] [-0.145 2.315]
Switzerland	1947-2015	34.5	0.888	[0.452 1.311] [0.043 1.839]	[0.452 1.311] [0.043 1.723]
United Kingdom	1947-2014	34	0.658	[0.333 1.012] [-0.067 1.406]	[0.333 0.960] [0.001 1.300]
United States	1947-2014	34	0.467	[-0.012 0.973] [-0.567 1.564]	[-0.012 0.942] [-0.478 1.426]
		II: Based on credit growth			
Australia	1953-2016	37.3	0.831	[0.490 1.169] [0.048 1.634]	[0.490 1.169] [0.048 1.634]
Canada	1947-2006	30	0.619	[0.405 0.834] [0.203 1.057]	[0.405 0.834] [0.203 1.057]
Finland	1947-1985	39	0.740	[0.543 0.922] [0.061 1.420]	[0.543 0.922] [0.061 1.402]
France	1947-1994	32	1.007	[0.731 1.307] [0.296 1.609]	[0.759 1.229] [0.473 1.506]
Italy	1949-1996	32	0.649	[-0.464 1.707] [-2.070 3.224]	[-0.198 1.471] [-1.320 2.474]
Japan	1956-2016	30.5	0.587	[0.130 0.816] [-0.168 1.038]	[0.299 0.816] [0.013 1.033]
Norway	1947-2013	33.5	0.443	[0.244 0.642] [0.040 0.868]	[0.244 0.629] [0.046 0.833]
Spain	1947-1997	34	0.623	[0.329 0.918] [-0.094 1.340]	[0.329 0.918] [-0.018 1.263]
Sweden	1947-2012	33	0.279	[0.088 0.474] [-0.159 0.745]	[0.088 0.474] [-0.118 0.689]
Switzerland	1947-2015	34.5	0.404	[0.241 0.670] [-0.053 0.933]	[0.241 0.571] [0.047 0.763]
United Kingdom	1947-2014	34	0.519	[0.376 0.650] [0.229 0.821]	[0.376 0.650] [0.229 0.806]
United States	1947-2014	34	0.405	[-0.025 0.822] [-0.463 1.328]	[-0.025 0.822] [-0.447 1.248]

[a] Computed as total nominal loans minus nominal loans to real estate.

Table II.10 Post-WWII period: evidence from regressing either inflation or nominal GDP growth on total loans growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
		I: Based on inflation			
Australia	1953-2016	32	0.787	[0.418 1.140] [0.009 1.608]	[0.418 1.140] [0.049 1.525]
Canada	1947-2006	30	0.532	[0.244 0.820] [-0.038 1.108]	[0.244 0.820] [-0.017 1.108]
Finland	1947-1985	39	0.670	[0.349 0.959] [-0.461 1.770]	[0.349 0.959] [-0.461 1.770]
France	1947-1994	32	0.841	[0.359 1.287] [-0.313 1.789]	[0.456 1.209] [-0.014 1.653]
Italy	1949-1996	32	0.291	[-1.254 1.448] [-3.338 3.244]	[-0.840 1.257] [-2.275 2.653]
Japan	1956-2016	30.5	0.301	[0.004 0.477] [-0.306 0.705]	[0.077 0.477] [-0.167 0.671]
Norway	1947-2013	33.5	0.383	[0.060 0.738] [-0.336 1.096]	[0.060 0.689] [-0.264 1.032]
Spain	1947-1997	34	0.574	[-0.225 1.285] [-1.277 2.327]	[-0.172 1.285] [-1.078 2.234]
Sweden	1947-2012	33	0.705	[0.402 0.985] [0.092 1.302]	[0.402 0.985] [0.092 1.302]
Switzerland	1947-2015	34.5	0.741	[0.609 0.939] [0.432 1.109]	[0.609 0.886] [0.468 1.026]
United Kingdom	1947-2014	34	0.641	[0.449 0.844] [0.237 1.082]	[0.449 0.844] [0.237 1.060]
United States	1947-2014	34	0.367	[-0.153 1.480] [-0.817 2.227]	[-0.153 0.927] [-0.669 1.542]
		II: Based on nominal GDP growth			
Australia	1953-2016	37.3	0.847	[0.281 1.373] [-0.400 2.048]	[0.281 1.373] [-0.400 2.048]
Canada	1947-2006	30	0.658	[0.440 0.879] [0.223 1.111]	[0.440 0.879] [0.223 1.111]
Finland	1947-1985	39	0.666	[0.654 0.677] [0.623 0.708]	[0.654 0.677] [0.623 0.708]
France	1947-1994	32	1.027	[0.698 1.303] [0.276 1.659]	[0.768 1.255] [0.457 1.570]
Italy	1949-1996	32	0.520	[-0.744 1.490] [-2.394 2.985]	[-0.378 1.339] [-1.533 2.446]
Japan	1956-2016	30.5	0.675	[0.238 0.879] [-0.110 1.100]	[0.408 0.879] [0.116 1.100]
Norway	1947-2013	33.5	0.559	[0.272 0.883] [-0.021 1.221]	[0.272 0.844] [-0.011 1.128]
Spain	1947-1997	34	0.819	[0.452 1.170] [-0.069 1.701]	[0.452 1.170] [0.016 1.632]
Sweden	1947-2012	33	0.571	[0.270 0.855] [-0.054 1.259]	[0.270 0.855] [-0.021 1.171]
Switzerland	1947-2015	34.5	0.863	[0.474 1.213] [-0.116 1.607]	[0.474 1.213] [0.024 1.607]
United Kingdom	1947-2014	34	0.648	[0.526 0.766] [0.395 0.908]	[0.526 0.766] [0.395 0.902]
United States	1947-2014	34	0.441	[-0.097 1.340] [-0.764 1.896]	[-0.009 0.928] [-0.493 1.434]

Table II.11 Quarterly data: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			I: Based on money growth and inflation		
Australia	1959Q4-2019Q4	30.3	1.038	[0.762 1.329] [0.482 1.674]	[0.762 1.311] [0.507 1.584]
Brazil	1975Q2-2019Q4	30.2	2.227	[2.021 2.425] [1.771 2.678]	[2.021 2.425] [1.771 2.678]
Canada	1914Q2-2006Q4	30.9	0.725	[0.559 0.891] [0.411 1.042]	[0.559 0.891] [0.411 1.042]
	1968Q1-2019Q4	34.7	0.673	[0.367 0.880] [0.097 1.139]	[0.450 0.880] [0.197 1.139]
Denmark	1922Q4-2019Q4	32.5	0.708	[0.611 0.805] [0.459 0.922]	[0.611 0.805] [0.526 0.895]
	1977Q2-2019Q4	43	1.127	[0.902 1.399] [0.298 2.031]	[0.902 1.339] [0.353 1.901]
Euro area	1970Q3-2019Q4	33.2	0.873	[0.331 1.353] [-0.374 2.038]	[0.403 1.306] [-0.137 1.844]
Finland	1914Q4-1985Q4	35.6	1.483	[1.338 1.661] [1.127 1.862]	[1.338 1.633] [1.176 1.809]
Germany	1949Q1-1998Q4	38.8	1.311	[0.988 1.625] [0.103 2.460]	[0.988 1.625] [0.103 2.460]
Hong Kong	1985Q1-2019Q4	35	0.770	[0.157 1.481] [-1.480 3.185]	[0.157 1.333] [-1.432 2.913]
Italy	1948Q2-1997Q3	37.5	0.763	[-0.518 2.043] [-3.872 5.527]	[-0.518 2.043] [-3.872 5.154]
Japan	1955Q2-2019Q4	32.4	0.468	[0.284 0.775] [-0.024 1.116]	[0.284 0.655] [0.068 0.871]
Mexico	1986Q1-2017Q3	31.3	1.093	[0.984 1.208] [0.693 1.517]	[0.984 1.195] [0.720 1.466]
Netherlands	1957Q2-1997Q4	40.8	0.965	[0.805 1.117] [0.373 1.534]	[0.805 1.117] [0.373 1.498]
New Zealand	1988Q1-2019Q4	32	0.214	[0.133 0.289] [-0.085 0.504]	[0.133 0.289] [-0.071 0.482]
Norway	1920Q3-2019Q4	33.3	0.766	[0.661 0.870] [0.569 0.963]	[0.661 0.870] [0.569 0.963]
	1978Q1-2019Q4	42	1.053	[0.605 1.614] [-0.844 3.064]	[0.605 1.493] [-0.618 2.711]
South Africa	1966Q1-2019Q4	35.8	0.899	[0.491 1.288] [-0.062 1.818]	[0.491 1.288] [-0.062 1.818]
South Korea	1960Q1-2019Q4	30	0.583	[0.288 0.811] [0.043 1.070]	[0.404 0.749] [0.203 0.942]
Switzerland	1985Q1-2019Q4	35	1.481	[-0.440 3.252] [-5.510 8.015]	[-0.440 3.252] [-5.510 8.015]
Taiwan	1961Q3-2019Q4	39.2	0.342	[0.217 0.524] [0.060 0.769]	[0.217 0.477] [0.060 0.648]
	1982Q1-2019Q4	38.3	0.193	[0.134 0.265] [-0.030 0.438]	[0.134 0.249] [-0.015 0.397]
United Kingdom	1881Q1-2019Q4	30.9	0.899	[0.569 1.252] [0.192 1.548]	[0.569 1.252] [0.302 1.548]
	1895Q2-2019Q4	30.1	0.798	[0.574 1.023] [0.280 1.217]	[0.574 1.023] [0.395 1.217]
	1955Q1-2019Q4	32.5	0.782	[0.471 1.091] [0.108 1.500]	[0.471 1.091] [0.143 1.428]
United States	1875Q2-2019Q4	32.2	0.965	[0.680 1.268] [0.469 1.715]	[0.680 1.268] [0.469 1.521]

Table II.11 (continued) Quarterly data: evidence from regressing either inflation or nominal GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			II: Based on money growth and nominal GDP growth		
Australia	1959Q4-2019Q4	30.3	0.972	[0.720 1.272] [0.418 1.659]	[0.720 1.256] [0.466 1.555]
Brazil	1975Q2-2019Q4	30.2	1.334	[1.265 1.400] [1.179 1.486]	[1.265 1.400] [1.179 1.486]
Canada	1968Q1-2019Q4	34.7	0.836	[0.284 1.243] [-0.366 1.791]	[0.389 1.243] [-0.172 1.791]
Denmark	1977Q2-2019Q4	43	1.143	[1.058 1.210] [0.857 1.407]	[1.068 1.210] [0.893 1.385]
Euro area	1970Q3-2019Q4	33.2	1.068	[0.380 1.564] [-0.313 2.360]	[0.535 1.564] [-0.059 2.168]
Germany	1960Q2-1998Q4	38.8	0.673	[-2.137 3.406] [-9.847 10.681]	[-2.137 3.406] [-9.847 10.681]
Hong Kong	1985Q1-2019Q4	35	1.009	[0.761 1.296] [0.099 1.986]	[0.761 1.237] [0.118 1.876]
Japan	1955Q2-2019Q4	32.4	0.968	[0.848 1.127] [0.688 1.283]	[0.869 1.067] [0.750 1.176]
Mexico	1986Q1-2017Q3	31.3	1.034	[0.919 1.157] [0.610 1.484]	[0.919 1.143] [0.639 1.430]
New Zealand	1988Q1-2019Q4	32	0.917	[-0.880 2.765] [-5.540 7.656]	[-0.880 2.567] [-5.407 7.198]
Norway	1978Q1-2019Q4	32	0.917	[-0.880 2.765] [-5.540 7.656]	[-0.880 2.567] [-5.407 7.198]
South Africa	1966Q1-2019Q4	35.8	0.762	[0.498 1.032] [0.161 1.379]	[0.498 1.032] [0.161 1.379]
South Korea	1960Q1-2019Q4	30	0.842	[0.657 0.975] [0.387 1.223]	[0.703 0.975] [0.529 1.120]
Switzerland	1985Q1-2019Q4	35	-1.408	[-4.881 1.730] [-13.659 10.508]	[-4.881 1.730] [-13.659 10.508]
Taiwan	1961Q3-2019Q4	39.2	0.724	[0.662 0.770] [0.581 0.851]	[0.672 0.770] [0.609 0.835]
	1982Q1-2019Q4	38.3	0.492	[0.288 0.654] [-0.216 1.146]	[0.313 0.654] [-0.111 1.088]
United Kingdom	1955Q1-2019Q4	32.5	0.807	[0.521 1.093] [0.200 1.459]	[0.521 1.093] [0.238 1.391]
United States	1875Q2-2019Q4	32.2	1.252	[0.990 1.516] [0.796 1.778]	[0.990 1.516] [0.796 1.738]

Table II.12 Quarterly data: evidence from regressing real GDP growth on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Australia	1959Q4-2019Q4	30.3	-0.036	[-0.197 0.168] [-0.401 0.362]	[-0.197 0.134] [-0.372 0.301]
Canada	1947Q2-2006Q4	39.5	-0.018	[-0.269 0.230] [-0.600 0.560]	[-0.269 0.230] [-0.600 0.560]
	1968Q1-2019Q4	34.7	0.158	[-0.110 0.391] [-0.530 0.724]	[-0.110 0.391] [-0.440 0.724]
Denmark	1977Q2-2019Q4	43	0.011	[-0.341 0.289] [-1.174 1.109]	[-0.298 0.289] [-1.027 1.014]
Euro area	1970Q3-2019Q4	33.2	0.206	[0.053 0.351] [-0.150 0.551]	[0.067 0.343] [-0.106 0.516]
Germany	1949Q1-1998Q4	38.8	-0.074	[-2.029 1.827] [-7.392 6.887]	[-2.029 1.827] [-7.392 6.887]
Hong Kong	1985Q1-2019Q4	35	0.229	[-0.196 0.582] [-1.238 1.613]	[-0.147 0.582] [-1.102 1.556]
Italy	1948Q2-1997Q3	37.5	0.136	[-0.335 0.543] [-1.454 1.704]	[-0.335 0.543] [-1.419 1.610]
Mexico	1986Q1-2017Q3	31.3	-0.058	[-0.065 -0.051] [-0.083 -0.033]	[-0.065 -0.052] [-0.081 -0.036]
Japan	1955Q2-2019Q4	32.4	0.494	[0.318 0.587] [0.161 0.754]	[0.395 0.587] [0.277 0.691]
New Zealand	1988Q1-2019Q4	32	0.701	[-1.176 2.632] [-6.044 7.741]	[-1.176 2.426] [-5.904 7.263]
Norway	1978Q1-2019Q4	42	-0.030	[-0.600 0.402] [-2.098 1.941]	[-0.531 0.402] [-1.778 1.668]
South Africa	1966Q1-2019Q4	35.8	-0.105	[-0.486 0.217] [-0.918 0.673]	[-0.486 0.217] [-0.918 0.673]
South Korea	1960Q1-2019Q4	30	0.257	[0.128 0.387] [0.029 0.476]	[0.183 0.331] [0.102 0.408]
Switzerland	1985Q1-2019Q4	35	-0.131	[-0.702 0.396] [-2.211 1.814]	[-0.702 0.396] [-2.211 1.814]
Taiwan	1982Q1-2019Q4	38.3	0.297	[0.023 0.515] [-0.654 1.176]	[0.056 0.515] [-0.513 1.098]
United Kingdom	1955Q1-2019Q4	32.5	0.029	[-0.072 0.133] [-0.185 0.252]	[-0.072 0.122] [-0.168 0.223]
United States	1875Q2-2019Q4	32.2	0.252	[0.054 0.437] [-0.094 0.596]	[0.054 0.437] [-0.094 0.596]

Table II.13 Evidence from regressing a short rate on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Argentina	1864-2004	31.3	1.291	[1.161 1.422] [1.007 1.528]	[1.161 1.422] [1.057 1.528]
Chile	1866-1995	32.5	1.398	[1.014 1.690] [0.666 1.923]	[1.134 1.690] [0.939 1.923]
Finland	1868-1985	33.7	0.151	[-0.009 0.384] [-0.131 0.614]	[-0.009 0.308] [-0.131 0.442]
Italy	1862-1996	30	0.066	[-0.177 0.183] [-0.344 0.397]	[-0.053 0.183] [-0.149 0.283]
Norway	1823-2013	31.8	0.167	[0.018 0.317] [-0.095 0.437]	[0.018 0.317] [-0.095 0.437]
Sweden	1857-1989	33.3	0.177	[-0.079 0.467] [-0.278 0.874]	[-0.079 0.434] [-0.278 0.638]
Switzerland	1916-2015	33.3	0.250	[-0.056 0.538] [-0.471 1.048]	[-0.056 0.538] [-0.313 0.805]
United Kingdom	1719-2016	31.4	0.644	[0.344 0.806] [0.173 1.078]	[0.490 0.806] [0.371 0.946]
United States	1869-2019	30.2	0.120	[-0.312 0.527] [-0.616 1.244]	[-0.312 0.527] [-0.616 0.873]

For details, see Section 3.

Table II.14 Evidence from regressing a short rate on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			Commodity standards		
Argentina	1864-1929	33	-0.195	[-0.389 0.056] [-0.626 0.306]	[-0.389 0.015] [-0.578 0.231]
Italy	1862-1935	37	-0.010	[-0.056 0.031] [-0.124 0.091]	[-0.056 0.031] [-0.106 0.079]
Norway	1865-1931	33.5	0.032	[-0.000 0.064] [-0.044 0.108]	[-0.000 0.064] [-0.034 0.098]
Sweden	1857-1931	30	0.057	[-0.039 0.169] [-0.151 0.262]	[-0.039 0.154] [-0.130 0.244]
United Kingdom	1719-1796	31.2	0.025	[-0.419 0.537] [-0.629 0.704]	[-0.299 0.329] [-0.575 0.625]
	1821-1931	31.7	0.164	[0.073 0.249] [0.006 0.384]	[0.073 0.249] [0.006 0.321]
United States	1869-1932	32	0.090	[-0.258 0.437] [-0.661 0.847]	[-0.211 0.399] [-0.552 0.731]
			Fiat standards		
Argentina	1930-2004	30	1.354	[1.112 1.585] [0.858 1.807]	[1.112 1.585] [0.903 1.807]
Canada	1935-2006	36	0.571	[-0.068 1.209] [-0.810 2.056]	[-0.068 1.209] [-0.692 1.883]
Chile	1878-1995	33.7	1.404	[0.993 1.702] [0.631 1.940]	[1.119 1.702] [0.903 1.940]
Colombia	1956-2018	31.5	1.387	[0.591 2.141] [-0.384 3.109]	[0.591 2.141] [-0.253 2.986]
Finland	1931-1985	36.7	0.061	[-0.288 0.387] [-0.987 0.847]	[-0.261 0.387] [-0.721 0.847]
Iceland	1961-2019	39.3	0.567	[0.200 0.900] [-0.243 1.348]	[0.200 0.900] [-0.243 1.348]
Italy	1936-1996	30.5	0.042	[-0.357 0.446] [-0.761 0.694]	[-0.316 0.348] [-0.648 0.613]
Japan	1956-2018	31.5	0.339	[0.068 0.476] [-0.101 0.638]	[0.175 0.476] [0.002 0.623]
New Zealand	1935-2016	32.8	0.939	[0.502 1.442] [0.058 1.988]	[0.502 1.352] [0.128 1.736]
Norway	1947-2013	33.5	0.515	[-0.027 1.042] [-0.585 1.713]	[-0.027 1.042] [-0.564 1.598]
Portugal	1932-1998	33.5	0.688	[0.190 1.150] [-0.278 1.784]	[0.190 1.150] [-0.261 1.646]
South Korea	1965-2019	36.7	0.610	[0.307 0.956] [-0.164 1.407]	[0.387 0.829] [0.113 1.123]
South Africa	1966-2019	36	0.971	[0.465 1.455] [-0.177 2.062]	[0.465 1.455] [-0.177 2.062]
Sweden	1932-1989	38.7	0.970	[-1.135 2.281] [-3.139 4.250]	[-0.382 2.217] [-2.025 3.854]
Switzerland	1937-2015	31.6	0.169	[-0.229 0.693] [-0.686 1.126]	[-0.229 0.557] [-0.585 0.907]
United Kingdom	1932-2016	34	0.985	[0.730 1.313] [0.439 1.599]	[0.740 1.235] [0.517 1.449]
United States	1933-2019	34.8	0.477	[-0.498 1.453] [-1.986 2.955]	[-0.498 1.453] [-1.388 2.404]

For details, see Section 3.

Table II.15 Evidence from regressing a short rate on inflation at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Argentina	1864-2004	31.3	1.241	[1.107 1.376] [0.864 1.483]	[1.107 1.376] [1.005 1.483]
Chile	1866-1995	32.5	1.435	[0.988 1.721] [0.634 1.954]	[1.148 1.721] [0.939 1.954]
Finland	1868-1985	33.7	0.071	[-0.029 0.216] [-0.157 0.360]	[-0.029 0.169] [-0.102 0.251]
Italy	1862-1996	30	0.078	[-0.083 0.171] [-0.251 0.359]	[-0.022 0.171] [-0.099 0.252]
Norway	1823-2013	31.8	0.135	[-0.012 0.284] [-0.224 0.487]	[-0.012 0.284] [-0.127 0.398]
Sweden	1857-1989	33.3	0.179	[-0.013 0.506] [-0.238 0.769]	[-0.013 0.361] [-0.163 0.516]
Switzerland	1916-2015	33.3	0.356	[0.069 0.625] [-0.170 1.068]	[0.069 0.625] [-0.153 0.855]
United Kingdom	1719-2016	31.4	0.548	[0.367 0.744] [0.046 1.239]	[0.367 0.744] [0.226 0.911]
United States	1869-2019	30.2	-0.018	[-0.524 0.297] [-1.084 0.988]	[-0.338 0.297] [-0.579 0.541]

For details, see Section 3.

Table II.16 Evidence from regressing a short rate on inflation at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			Commodity standards		
Argentina	1864-1929	33	-0.277	[-0.450 -0.151] [-0.567 -0.019]	[-0.398 -0.160] [-0.519 -0.047]
Italy	1862-1935	37	-0.019	[-0.067 0.015] [-0.121 0.061]	[-0.062 0.015] [-0.104 0.053]
Norway	1865-1931	33.5	0.042	[0.012 0.070] [-0.026 0.110]	[0.012 0.070] [-0.018 0.101]
Sweden	1857-1931	30	0.100	[0.028 0.168] [-0.076 0.267]	[0.028 0.168] [-0.033 0.231]
United Kingdom	1719-1796	31.2	0.111	[-0.397 0.618] [-1.024 1.245]	[-0.397 0.618] [-0.877 1.099]
	1821-1931	31.7	0.101	[0.012 0.191] [-0.122 0.366]	[0.012 0.191] [-0.059 0.264]
United States	1869-1932	32	-0.067	[-0.407 0.175] [-0.746 0.515]	[-0.356 0.175] [-0.640 0.459]
			Fiat standards		
Argentina	1930-2004	30	1.323	[1.117 1.531] [0.841 1.768]	[1.117 1.531] [0.920 1.726]
Canada	1935-2006	36	1.395	[0.386 2.345] [-0.719 3.639]	[0.386 2.345] [-0.575 3.420]
Chile	1878-1995	33.7	1.434	[1.014 1.749] [0.582 2.009]	[1.134 1.749] [0.893 1.996]
Colombia	1956-2018	31.5	1.476	[0.821 2.128] [-0.012 2.909]	[0.821 2.128] [0.143 2.793]
Finland	1931-1985	36.7	-0.053	[-0.326 0.185] [-0.710 0.510]	[-0.326 0.185] [-0.666 0.510]
Iceland	1949-2019	35.5	0.502	[0.151 0.833] [-0.252 1.203]	[0.151 0.833] [-0.197 1.169]
Italy	1936-1996	30.5	0.076	[-0.183 0.353] [-0.487 0.545]	[-0.163 0.277] [-0.402 0.498]
Japan	1956-2018	31.5	0.660	[0.166 0.962] [-0.155 1.254]	[0.323 0.962] [-0.048 1.248]
New Zealand	1935-2016	32.8	0.648	[0.219 1.222] [-0.307 1.780]	[0.219 1.092] [-0.167 1.495]
Norway	1947-2013	33.5	0.568	[0.011 1.085] [-0.556 1.722]	[0.011 1.085] [-0.504 1.639]
Portugal	1932-1998	33.5	0.674	[0.407 0.978] [0.138 1.338]	[0.407 0.950] [0.138 1.229]
South Korea	1965-2019	36.7	1.134	[0.350 1.648] [-0.912 2.703]	[0.563 1.648] [-0.241 2.430]
South Africa	1966-2019	36	0.745	[-0.005 1.443] [-0.907 2.337]	[-0.005 1.443] [-0.907 2.337]
Sweden	1932-1989	38.7	1.206	[0.846 1.559] [0.328 2.065]	[0.943 1.460] [0.596 1.793]
Switzerland	1937-2015	31.6	0.315	[-0.037 0.725] [-0.449 1.129]	[-0.037 0.644] [-0.337 0.967]
United Kingdom	1932-2016	34	0.824	[0.392 1.411] [-0.036 1.958]	[0.392 1.267] [-0.001 1.677]
United States	1933-2019	34.8	1.347	[0.613 2.363] [-0.190 3.223]	[0.613 2.062] [0.018 2.732]

For details, see Section 3.

Table II.17 Quarterly data: evidence from regressing a short rate on money growth at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Australia	1968Q2-2019Q4	34.7	1.105	[-1.360 3.018] [-4.664 6.182]	[-0.534 2.580] [-2.738 4.607]
Brazil	1975Q2-2019Q4	30.2	1.522	[1.474 1.569] [1.411 1.632]	[1.474 1.569] [1.411 1.632]
Canada	1935Q3-2006Q4	35.6	0.481	[-0.162 1.123] [-0.931 1.975]	[-0.162 1.123] [-0.825 1.817]
	1968Q1-2019Q4	34.7	0.370	[-0.648 1.328] [-2.182 2.701]	[-0.648 1.328] [-1.962 2.580]
Denmark	1922Q4-2019Q4	32.5	0.245	[-0.025 0.620] [-0.305 0.860]	[-0.025 0.514] [-0.240 0.737]
	1977Q2-2019Q4	43	1.271	[-0.163 2.404] [-3.558 5.747]	[0.013 2.404] [-2.960 5.362]
Euro area	1970Q3-2019Q4	33.2	0.442	[-0.993 1.429] [-2.330 2.994]	[-0.615 1.429] [-1.777 2.649]
Germany	1959Q4-1998Q4	39.3	1.068	[-0.971 2.910] [-6.124 8.063]	[-0.971 2.910] [-6.124 8.063]
Hong Kong	1985Q2-2019Q4	35	0.419	[-0.145 0.887] [-1.527 2.254]	[-0.080 0.887] [-1.346 2.178]
Italy	1964Q2-1997Q3	33.5	0.159	[-1.583 1.949] [-5.968 6.533]	[-1.583 1.758] [-5.968 5.980]
Japan	1958Q3-2019Q4	32.4	0.399	[0.159 0.567] [-0.022 0.773]	[0.258 0.525] [0.105 0.671]
Mexico	1986Q1-2017Q3	31.3	1.378	[1.378 1.379] [1.376 1.381]	[1.378 1.379] [1.376 1.381]
South Africa	1966Q1-2019Q4	35.8	0.984	[0.451 1.504] [-0.202 2.122]	[0.451 1.459] [-0.202 2.122]
South Korea	1964Q2-2019Q4	37.3	0.599	[0.300 0.906] [-0.127 1.366]	[0.397 0.811] [0.121 1.078]
Taiwan	1982Q2-2019Q4	38.3	0.286	[0.044 0.477] [-0.553 1.060]	[0.073 0.477] [-0.428 0.992]
United Kingdom	1923Q2-2017Q2[a]	31.4	0.654	[0.327 0.938] [0.062 1.243]	] [0.327 0.938] [0.062 1.173]
	1923Q2-2017Q2[b]	31.4	0.654	[0.327 0.938] [0.062 1.243]	[0.327 0.938] [0.062 1.173]
	1955Q2-2017Q2[c]	31.1	0.729	[0.444 1.013] [0.134 1.318]	[0.444 0.998] [0.148 1.267]
United States	1875Q2-2019Q4	32.2	0.198	[-0.308 0.707] [-0.703 1.490]	[-0.308 0.707] [-0.703 1.134]

[a] Based on the wholesale price index. [b] Based on the consumer price index. [a] Based on the GDP deflator.

Table II.18 Quarterly data: evidence from regressing a short rate on inflation at the very low frequencies based on Müller and Watson's (2018) methodology

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Australia	1968Q2-2019Q4	34.7	0.703	[-0.804 2.063] [-3.160 4.288]	[-0.538 1.882] [-2.234 3.400]
Brazil	1975Q2-2019Q4	30.2	0.678	[0.638 0.718] [0.587 0.768]	[0.638 0.718] [0.587 0.768]
Canada	1935Q3-2006Q4	35.6	1.344	[0.546 2.145] [-0.320 3.164]	[0.546 2.145] [-0.195 2.956]
	1968Q1-2019Q4	34.7	0.790	[-0.983 2.146] [-3.382 4.491]	[-0.541 1.989] [-2.255 3.671]
Denmark	1922Q4-2019Q4	32.5	0.464	[0.110 0.897] [-0.147 1.213]	[0.110 0.792] [-0.147 1.051]
	1977Q2-2019Q4	43	1.056	[-0.330 2.205] [-3.848 5.662]	[-0.170 2.205] [-3.383 5.377]
Euro area	1970Q3-2019Q4	33.2	0.821	[-0.468 1.707] [-1.996 3.126]	[-0.021 1.607] [-1.105 2.604]
Germany	1959Q4-1998Q4	39.3	0.717	[-0.732 2.026] [-4.393 5.688]	[-0.732 2.026] [-4.393 5.688]
Hong Kong	1985Q2-2019Q4	35	0.313	[-0.641 1.194] [-2.961 3.562]	[-0.641 1.194] [-2.961 3.562]
Italy	1964Q2-1997Q3	33.5	0.673	[-0.412 1.709] [-3.122 4.418]	[-0.412 1.709] [-3.122 4.418]
Japan	1958Q3-2019Q4	32.4	0.659	[0.149 0.971] [-0.212 1.294]	[0.300 0.971] [-0.054 1.276]
Mexico	1986Q1-2019Q4	34.3	1.259	[1.203 1.306] [1.059 1.447]	[1.209 1.306] [1.078 1.436]
South Africa	1966Q1-2019Q4	35.8	0.734	[-0.022 1.451] [-1.029 2.394]	[-0.022 1.451] [-1.029 2.394]
South Korea	1964Q2-2019Q4	37.3	1.113	[0.604 1.481] [-0.212 2.295]	[0.716 1.481] [0.147 2.023]
Taiwan	1982Q2-2019Q4	38.3	1.349	[-0.270 2.691] [-4.233 6.613]	[-0.083 2.691] [-3.715 6.121]
United Kingdom	1923Q2-2017Q2[a]	31.4	0.146	[-0.216 0.622] [-0.632 1.101]	[-0.216 0.497] [-0.492 0.838]
	1923Q2-2017Q2[b]	31.4	0.553	[0.094 1.083] [-0.352 1.658]	[0.094 0.995] [-0.288 1.373]
	1955Q2-2017Q2[c]	31.1	0.612	[0.169 1.032] [-0.284 1.551]	[0.169 1.032] [-0.245 1.451]
United States	1875Q2-2019Q4	32.2	0.115	[-0.288 0.496] [-1.025 1.296]	[-0.288 0.496] [-0.593 0.829]

[a] Based on the wholesale price index. [b] Based on the consumer price index. [a] Based on the GDP deflator.

Table II.19 Full samples: evidence from regressing inflation on money growth minus real GDP growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Argentina	1864-2019	31.2	1.016	[0.993 1.037] [0.963 1.055]	[0.993 1.037] [0.977 1.055]
Brazil	1862-2019	31.6	0.986	[0.962 1.009] [0.916 1.028]	[0.962 1.009] [0.945 1.028]
Canada	1873-2006	33.5	0.760	[0.555 0.967] [0.305 1.235]	[0.555 0.967] [0.384 1.141]
Chile	1811-2019	32.2	0.933	[0.890 0.965] [0.847 0.989]	[0.902 0.965] [0.878 0.989]
Finland	1867-1985	33.7	1.445	[1.221 1.688] [0.995 2.047]	[1.221 1.688] [1.067 1.894]
France	1910-1994	30.0	0.695	[0.274 1.088] [-0.412 1.683]	[0.274 1.055] [-0.232 1.575]
Italy	1862-1996	30.0	1.221	[1.116 1.319] [0.914 1.469]	[1.116 1.319] [1.035 1.404]
Mexico	1926-2013	35.2	1.092	[1.033 1.147] [0.966 1.205]	[1.033 1.147] [0.986 1.200]
New Zealand	1885-2016	32.8	0.888	[0.601 1.054] [0.390 1.273]	[0.705 1.054] [0.521 1.213]
Portugal	1855-1998	32	0.982	[0.886 1.078] [0.811 1.162]	[0.886 1.078] [0.811 1.162]
Sweden	1847-2018	31.3	0.941	[0.666 1.220] [0.180 1.710]	[0.666 1.220] [0.457 1.467]
Switzerland	1916-2015	33.3	0.789	[0.433 1.145] [0.089 1.459]	[0.433 1.145] [0.119 1.459]
United Kingdom	1701-2019	30.4	0.860	[0.752 0.965] [0.663 1.097]	[0.752 0.965] [0.663 1.045]
United States	1869-2019	30.4	1.107	[0.897 1.307] [0.748 1.666]	[0.897 1.307] [0.748 1.486]

For details, see Section 3.

Table II.20 Commodity standards: evidence from regressing inflation on money growth minus real GDP growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Argentina	1864-1929	33	0.940	[0.230 1.438] [-0.216 2.019]	[0.469 1.380] [-0.030 1.850]
Canada	1873-1929	38	0.640	[-0.014 1.258] [-0.858 2.212]	[-0.014 1.258] [-0.858 2.128]
Chile	1811-1877	33.5	1.092	[0.508 1.662] [-0.123 2.342]	[0.508 1.662] [-0.061 2.286]
Finland	1868-1914	31.3	0.576	[0.238 0.996] [-0.184 1.490]	[0.238 0.914] [-0.184 1.360]
Italy	1862-1935	37	1.073	[0.837 1.295] [0.529 1.604]	[0.837 1.295] [0.597 1.548]
Norway	1865-1931	33.5	0.891	[0.702 1.071] [0.425 1.347]	[0.702 1.071] [0.494 1.297]
Sweden	1847-1931	34	0.731	[0.341 1.078] [-0.113 1.473]	[0.341 1.078] [-0.007 1.437]
United Kingdom	1701-1796	32	0.644	[0.291 1.143] [-0.169 1.637]	[0.291 0.967] [-0.008 1.266]
	1821-1931	31.7	0.826	[0.648 1.003] [0.468 1.158]	[0.648 1.003] [0.494 1.158]
United States	1869-1932	32.5	1.421	[1.170 1.682] [0.896 1.965]	[1.181 1.682] [0.952 1.923]

For details, see Section 3.

Table II.21 *Fiat* standards: evidence from regressing inflation on money growth minus real GDP growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Argentina	1930-2019	30	1.007	[0.980 1.035] [0.954 1.062]	[0.980 1.035] [0.956 1.062]
Australia	1961-2019	38.7	1.128	[0.815 1.466] [0.394 1.931]	[0.815 1.466] [0.394 1.910]
Brazil	1862-2019	31.6	0.986	[0.962 1.009] [0.916 1.028]	[0.962 1.009] [0.945 1.028]
Canada	1935-2006	36	0.573	[0.491 0.645] [0.380 0.737]	[0.495 0.645] [0.408 0.725]
Chile	1878-2019	31.6	0.941	[0.879 0.975] [0.836 1.022]	[0.909 0.975] [0.881 0.999]
Colombia	1956-2019	31.5	1.009	[0.910 1.105] [0.806 1.220]	[0.910 1.105] [0.814 1.203]
Finland	1915-1985	36.7	1.212	[1.051 1.414] [0.857 1.594]	[1.061 1.365] [0.911 1.526]
France	1937-1994	32	1.136	[-0.324 2.258] [-1.751 3.794]	[0.209 2.019] [-0.921 3.149]
Iceland	1961-2019	39.3	1.042	[0.764 1.300] [0.430 1.654]	[0.764 1.300] [0.430 1.654]
Italy	1936-1996	30.5	1.260	[1.098 1.422] [0.896 1.617]	[1.098 1.411] [0.936 1.583]
Japan	1956-2018	31.5	0.798	[0.682 0.905] [0.551 1.011]	[0.682 0.905] [0.561 1.011]
Mexico	1926-2013	35.2	1.092	[1.033 1.147] [0.966 1.205]	[1.033 1.147] [0.986 1.200]
New Zealand	1914-2016	32.8	0.888	[0.601 1.054] [0.390 1.273]	[0.705 1.054] [0.521 1.213]
Norway	1947-2013	34	0.619	[0.184 1.056] [-0.351 1.560]	[0.184 1.023] [-0.242 1.450]
Paraguay	1963-2015	35.3	2.511	[2.126 2.899] [1.600 3.398]	[2.126 2.899] [1.627 3.398]
Peru	1960-2018	39.3	1.114	[1.046 1.177] [0.966 1.262]	[1.046 1.177] [0.966 1.262]
Portugal	1932-1998	33.5	1.110	[0.832 1.385] [0.531 1.712]	[0.832 1.385] [0.531 1.686]
South Korea	1965-2019	36.7	0.629	[0.487 0.788] [0.157 1.088]	[0.493 0.751] [0.299 0.933]
Spain	1942-1997	37.3	0.904	[0.225 1.477] [-0.561 2.315]	[0.397 1.393] [-0.259 2.061]
South Africa	1914-2019	30.3	1.008	[0.983 1.034] [0.963 1.056]	[0.983 1.034] [0.963 1.056]
Sweden	1932-2018	34.8	1.239	[0.835 1.619] [0.298 2.187]	[0.860 1.619] [0.513 1.982]
Switzerland	1937-2015	31.6	0.638	[0.222 1.054] [-0.349 1.650]	[0.222 1.054] [-0.136 1.428]
United Kingdom	1932-2019	35.2	0.739	[0.419 1.059] [0.069 1.409]	[0.419 1.059] [0.137 1.351]
United States	1933-2019	34.8	0.659	[0.167 1.019] [-0.339 1.518]	[0.263 1.019] [-0.088 1.382]
Venezuela	1951-2017	33.5	0.873	[0.767 0.979] [0.638 1.116]	[0.767 0.979] [0.663 1.087]

For details, see Section 3.

Table II.22 Period since 1985: evidence from regressing inflation on money growth minus real GDP growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Australia	1985Q1-2019Q4	23.3	0.572	[0.274 0.790] [-0.070 1.145]	[0.374 0.764] [0.114 1.023]
Canada	1985Q1-2019Q4	23.3	0.183	[0.019 0.326] [-0.223 0.528]	[0.019 0.326] [-0.177 0.528]
Denmark	1985Q1-2019Q4	23.3	0.460	[0.107 0.739] [-0.448 1.122]	[0.159 0.739] [-0.225 1.122]
Euro area	1985Q1-2019Q4	23.3	0.111	[-1.282 1.155] [-2.919 2.518]	[-1.009 1.155] [-2.408 2.518]
Hong Kong	1985Q1-2019Q4	23.3	0.985	[0.642 1.296] [0.225 1.731]	[0.642 1.296] [0.225 1.731]
Japan	1985Q1-2019Q4	23.3	0.714	[0.143 1.422] [-0.634 2.225]	[0.143 1.246] [-0.566 1.940]
Norway	1985Q1-2015Q1	23.3	0.806	[0.640 0.973] [0.376 1.237]	[0.667 0.945] [0.502 1.122]
New Zealand	1988Q1-2019Q4	21.2	0.389	[0.350 0.429] [0.297 0.480]	[0.350 0.429] [0.297 0.480]
South Korea	1985Q1-2019Q4	23.3	0.491	[0.323 0.589] [0.160 0.734]	[0.384 0.589] [0.241 0.714]
Switzerland	1985Q1-2019Q4	23.3	0.773	[0.237 1.311] [-0.794 2.059]	[0.237 1.311] [-0.453 2.042]
Taiwan	1985Q1-2019Q4	23.3	0.670	[0.092 0.542] [-0.211 0.847]	[0.092 0.527] [-0.194 0.794]
United Kingdom	1985Q1-2017Q2	23.3	0.350	[0.080 0.645] [-0.296 1.021]	[0.101 0.588] [-0.195 0.878]
United States	1985Q1-2019Q4	23.3	-0.005	[-0.258 0.247] [-0.558 0.547]	[-0.258 0.247] [-0.558 0.547]

Table II.23 Quarterly data: evidence from regressing inflation on money growth minus real GDP growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Australia	1959Q4-2019Q4	30.3	0.933	[0.699 1.187] [0.372 1.553]	[0.699 1.187] [0.447 1.478]
Canada	1968Q1-2019Q4	34.7	0.664	[0.205 1.067] [-0.260 1.570]	[0.247 1.067] [-0.236 1.570]
Denmark	1977Q2-2019Q4	43	1.135	[1.025 1.239] [0.715 1.533]	[1.025 1.239] [0.744 1.504]
Euro area	1970Q3-2019Q4	33.2	1.019	[0.258 1.659] [-0.598 2.656]	[0.388 1.659] [-0.380 2.444]
Germany	1960Q2-1998Q4	38.8	0.346	[-0.817 1.395] [-3.753 4.332]	[-0.817 1.395] [-3.753 4.220]
Hong Kong	1985Q1-2019Q4	35	1.056	[0.769 1.313] [0.018 2.039]	[0.769 1.313] [0.018 2.039]
Japan	1955Q2-2019Q4	32.4	0.992	[0.814 1.302] [0.627 1.574]	[0.814 1.190] [0.627 1.396]
Mexico	1986Q1-2017Q3	31.3	1.032	[0.923 1.148] [0.631 1.470]	[0.923 1.135] [0.658 1.407]
New Zealand	1988Q1-2019Q4	32	0.085	[-0.266 0.375] [-1.155 1.249]	[-0.225 0.375] [-1.038 1.177]
Norway	1978Q1-2019Q4	32	1.028	[0.996 1.054] [0.919 1.133]	[1.000 1.054] [0.927 1.126]
South Africa	1966Q1-2019Q4	35.8	0.815	[0.584 1.042] [0.280 1.343]	[0.584 1.042] [0.280 1.343]
South Korea	1960Q1-2019Q4	30	0.798	[0.531 0.983] [0.236 1.336]	[0.604 0.983] [0.381 1.191]
Switzerland	1985Q1-2019Q4	35	0.946	[-1.246 3.078] [-7.261 8.753]	[-1.246 3.078] [-7.261 8.753]
Taiwan	1982Q1-2019Q4	38.3	0.276	[0.270 0.283] [0.251 0.301]	[0.270 0.283] [0.253 0.299]
United Kingdom	1955Q1-2019Q4	32.5	0.816	[0.522 1.108] [0.170 1.489]	[0.522 1.108] [0.228 1.420]
United States	1875Q2-2019Q4	32.2	1.197	[0.898 1.514] [0.659 1.856]	[0.898 1.514] [0.659 1.773]

Table II.24 Evidence from regressing a short rate on money growth minus real GDP growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Argentina	1864-2004	31.3	1.266	[1.137 1.389] [0.994 1.499]	[1.137 1.389] [1.042 1.499]
Chile	1866-1995	32.5	1.367	[1.012 1.630] [0.676 1.831]	[1.114 1.630] [0.929 1.831]
Finland	1868-1985	33.7	0.119	[-0.028 0.315] [-0.151 0.554]	[-0.028 0.279] [-0.151 0.411]
Italy	1862-1996	30	0.079	[-0.155 0.200] [-0.344 0.420]	[-0.047 0.200] [-0.147 0.300]
Sweden	1857-1989	33.3	0.302	[0.055 0.566] [-0.128 0.885]	[0.055 0.533] [-0.128 0.731]
Switzerland	1916-2015	33.3	0.420	[0.139 0.685] [-0.091 1.059]	[0.139 0.685] [-0.091 0.918]
United Kingdom	1719-2016	31.4	0.681	[0.366 0.848] [0.215 1.184]	[0.523 0.848] [0.405 0.965]
United States	1869-2019	30.2	0.055	[-0.358 0.466] [-1.129 1.253]	[-0.358 0.466] [-0.663 0.794]

For details, see Section 3.

Table II.25 Evidence from regressing a short rate on money growth minus real GDP growth at the very low frequencies

Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
			Commodity standards		
Argentina	1864-1929	33	-0.152	[-0.401 0.221] [-0.700 0.451]	[-0.386 0.084] [-0.623 0.333]
Italy	1862-1935	37	-0.007	[-0.054 0.035] [-0.119 0.095]	[-0.054 0.035] [-0.103 0.081]
Norway	1865-1931	33.5	0.029	[-0.005 0.063] [-0.050 0.109]	[-0.005 0.061] [-0.039 0.097]
Sweden	1857-1931	30	0.065	[-0.025 0.162] [-0.137 0.256]	[-0.025 0.149] [-0.109 0.237]
United Kingdom	1719-1796	31.2	0.131	[-0.393 0.752] [-0.900 1.278]	[-0.393 0.649] [-0.866 1.128]
	1821-1931	31.7	0.127	[0.051 0.198] [-0.009 0.332]	[0.051 0.198] [-0.009 0.260]
United States	1869-1932	32	-0.154	[-0.600 0.276] [-1.091 0.698]	[-0.560 0.237] [-0.947 0.616]
			Fiat standards		
Argentina	1930-2004	30	1.344	[1.115 1.563] [0.875 1.773]	[1.115 1.563] [0.917 1.773]
Canada	1935-2006	36	0.973	[0.524 1.420] [0.025 1.998]	[0.524 1.420] [0.059 1.897]
Chile	1878-1995	33.7	1.361	[0.983 1.645] [0.634 1.877]	[1.091 1.645] [0.884 1.877]
Colombia	1956-2018	31.5	1.495	[0.769 2.185] [-0.052 2.986]	[0.769 2.185] [0.064 2.886]
Finland	1931-1985	36.7	0.046	[-0.283 0.354] [-0.970 0.796]	[-0.283 0.354] [-0.723 0.796]
Iceland	1961-2019	39.3	0.600	[0.284 0.905] [-0.099 1.275]	[0.284 0.905] [-0.099 1.275]
Italy	1936-1996	30.5	0.068	[-0.281 0.454] [-0.661 0.690]	[-0.243 0.352] [-0.555 0.605]
Japan	1956-2018	31.5	0.475	[0.015 0.771] [-0.308 1.086]	[0.132 0.771] [-0.179 1.051]
New Zealand	1935-2016	32.8	0.834	[0.545 1.132] [0.285 1.497]	[0.545 1.105] [0.285 1.353]
Norway	1947-2013	33.5	0.466	[-0.059 0.962] [-0.535 1.552]	[-0.059 0.962] [-0.535 1.431]
Portugal	1932-1998	33.5	0.751	[0.372 1.160] [-0.019 1.677]	[0.372 1.140] [-0.019 1.521]
South Korea	1965-2019	36.7	0.811	[0.539 1.125] [0.068 1.619]	[0.596 1.038] [0.300 1.329]
South Africa	1966-2019	36	0.798	[0.354 1.233] [-0.194 1.776]	[0.354 1.233] [-0.194 1.776]
Sweden	1932-1989	38.7	0.649	[-0.344 1.694] [-1.598 3.460]	[-0.249 1.538] [-1.243 2.673]
Switzerland	1937-2015	31.6	0.292	[-0.062 0.700] [-0.422 1.157]	[-0.062 0.627] [-0.365 0.944]
United Kingdom	1932-2016	34	0.982	[0.711 1.313] [0.419 1.644]	[0.711 1.246] [0.461 1.487]
United States	1933-2019	34.8	1.062	[0.182 2.104] [-0.826 3.173]	[0.182 1.945] [-0.557 2.740]

For details, see Section 3.

Table II.26 Quarterly data: evidence from regressing a short rate on money growth minus real GDP growth at the very low frequencies based on Müller and Watson's (2018) methodology					
Country	Sample period	Highest frequency (in years)	Posterior median	Confidence intervals with 67 and 90% coverage probability	Equal-tail credible intervals with 67 and 90% coverage probability
Australia	1968Q2-2019Q4	34.7	1.930	[0.474 4.323] [-2.260 7.142]	[0.474 3.460] [-1.580 5.422]
Canada	1935Q3-2006Q4	35.6	0.791	[0.330 1.234] [-0.265 1.871]	[0.330 1.234] [-0.265 1.849]
	1968Q1-2019Q4	34.7	0.440	[-0.649 1.537] [-2.061 2.970]	[-0.649 1.537] [-2.061 2.970]
Denmark	1977Q2-2019Q4	43	1.158	[-0.381 2.600] [-4.690 6.695]	[-0.381 2.600] [-4.282 6.285]
Euro area	1970Q3-2019Q4	33.2	0.338	[-1.357 1.679] [-2.911 3.651]	[-1.049 1.679] [-2.514 3.316]
Germany	1959Q4-1998Q4	39.3	0.832	[0.646 1.013] [0.179 1.495]	[0.646 1.013] [0.179 1.495]
Hong Kong	1985Q2-2019Q4	35	0.405	[-0.439 1.226] [-2.725 3.373]	[-0.439 1.226] [-2.725 3.373]
Italy	1964Q2-1997Q3	33.5	0.402	[-1.424 2.040] [-6.206 6.661]	[-1.424 2.040] [-6.206 6.661]
Japan	1958Q3-2019Q4	32.4	0.734	[0.273 0.994] [-0.045 1.311]	[0.432 0.994] [0.109 1.273]
Mexico	1986Q1-2017Q3	31.3	1.302	[1.294 1.312] [1.270 1.338]	[1.294 1.311] [1.272 1.332]
South Africa	1966Q1-2019Q4	35.8	0.800	[0.342 1.248] [-0.223 1.807]	[0.342 1.248] [-0.223 1.807]
South Korea	1964Q2-2019Q4	37.3	0.805	[0.538 1.075] [0.063 1.579]	[0.595 1.024] [0.315 1.305]
Taiwan	1982Q2-2019Q4	38.3	0.370	[-0.037 0.751] [-1.177 1.866]	[-0.037 0.751] [-1.069 1.726]
United Kingdom	1955Q2-2017Q2	31.1	0.704	[0.377 1.050] [0.052 1.380]	[0.377 0.996] [0.080 1.305]
United States	1875Q2-2019Q4	32.2	0.115	[-0.463 0.696] [-1.391 1.648]	[-0.463 0.696] [-0.901 1.168]

III: Full set of results based on cross-spectral methods

Table III.1 Cross-spectral gain and coherence of money growth onto either inflation or nominal GDP growth at ω=0: point estimates, bootstrapped confidence intervals,[a] p-values for testing the null hypothesis that the gain is equal to 1, and fraction of the mass of the bootstrapped distribution of the coherence beyond 0.9

Country	*Period*	Gain at ω=0, and 16-84 and 5-95 confidence intervals	p-value	Coherence at ω=0, and 16-84 and 5-95 confidence intervals	Fraction beyond 0.9
		I: Based on money growth and inflation			
Argentina	1863-2019	1.045 [0.991 1.099] [0.940 1.150]	0.199	0.998 [0.991 0.999] [0.971 1.000]	0.991
Australia	1854-2019	0.960 [0.545 1.371] [0.223 1.735]	0.458	0.906 [0.587 0.976] [0.229 0.989]	0.517
Brazil	1862-2019	0.960 [0.918 1.006] [0.873 1.055]	0.189	0.999 [0.995 1.000] [0.986 1.000]	0.996
Canada	1873-2006	0.714 [0.427 1.008] [0.198 1.268]	0.166	0.889 [0.608 0.961] [0.249 0.983]	0.464
Chile	1811-2019	0.884 [0.818 0.954] [0.751 1.037]	0.083	0.995 [0.984 0.999] [0.950 1.000]	0.978
Finland	1867-1985	1.655 [1.210 2.100] [0.831 2.468]	0.082	0.937 [0.778 0.982] [0.532 0.992]	0.637
France	1910-1994	0.809 [0.375 1.279] [0.131 1.720]	0.336	0.676 [0.237 0.911] [0.068 0.963]	0.182
Italy	1862-1996	0.937 [0.701 1.198] [0.479 1.438]	0.396	0.959 [0.844 0.987] [0.625 0.994]	0.753
Mexico	1926-2013	1.082 [0.933 1.206] [0.764 1.337]	0.267	0.988 [0.947 0.996] [0.841 0.998]	0.917
New Zealand	1885-2016	1.031 [0.771 1.272] [0.509 1.483]	0.446	0.956 [0.851 0.987] [0.616 0.994]	0.760
Norway	1820-2014	1.052 [0.588 1.530] [0.243 2.022]	0.452	0.837 [0.428 0.963] [0.139 0.984]	0.372
Portugal	1855-1998	0.831 [0.590 1.092] [0.383 1.344]	0.243	0.931 [0.713 0.983] [0.399 0.992]	0.594
Sweden	1847-2018	1.516 [0.752 2.278] [0.288 2.914]	0.244	0.825 [0.427 0.950] [0.141 0.976]	0.333
Switzerland	1916-2015	0.760 [0.417 1.179] [0.172 1.665]	0.260	0.883 [0.572 0.974] [0.237 0.990]	0.457
United Kingdom	1701-2019	0.801 [0.586 1.008] [0.379 1.204]	0.168	0.955 [0.817 0.985] [0.527 0.994]	0.723
United States	1868-2019	1.668 [0.908 2.471] [0.390 3.104]	0.193	0.822 [0.492 0.936] [0.199 0.963]	0.293

[a] Spectral bootstrapping has been implemented as in Berkowitz and Diebold (1998). For details, see Section 3.1.

Table III.1 (continued) Cross-spectral gain and coherence of money growth onto either inflation or nominal GDP growth at ω=0: point estimates, bootstrapped confidence intervals,[a] p-values for testing the null hypothesis that the gain is equal to 1, and fraction of the mass of the bootstrapped distribution of the coherence beyond 0.9

Country	*Period*	Gain at ω=0, and 16-84 and 5-95 confidence intervals	p-value	Coherence at ω=0, and 16-84 and 5-95 confidence intervals	Fraction beyond 0.9
		II: Based on money growth and nominal GDP growth			
Argentina	1863-2019	1.017 [0.980 1.057] [0.941 1.098]	0.309	0.999 [0.996 1.000] [0.986 1.000]	0.997
Australia	1854-2019	0.960 [0.543 1.369] [0.232 1.765]	0.459	0.906 [0.591 0.976] [0.225 0.989]	0.517
Brazil	1862-2019	0.985 [0.946 1.023] [0.907 1.062]	0.338	0.999 [0.995 1.000] [0.986 1.000]	0.996
Canada	1873-2006	0.714 [0.432 1.009] [0.212 1.265]	0.167	0.889 [0.606 0.961] [0.269 0.984]	0.464
Chile	1811-2019	0.884 [0.817 0.952] [0.746 1.034]	0.081	0.995 [0.984 0.999] [0.954 1.000]	0.982
Finland	1867-1985	1.655 [1.214 2.100] [0.830 2.472]	0.083	0.937 [0.773 0.982] [0.511 0.992]	0.639
Italy	1862-1996	0.937 [0.695 1.202] [0.487 1.431]	0.395	0.959 [0.843 0.988] [0.635 0.995]	0.750
Mexico	1926-2013	1.073 [0.961 1.173] [0.832 1.267]	0.243	0.994 [0.973 0.999] [0.921 1.000]	0.963
New Zealand	1885-2016	1.031 [0.766 1.272] [0.515 1.474]	0.448	0.956 [0.853 0.986] [0.614 0.994]	0.762
Portugal	1855-1998	0.831 [0.596 1.075] [0.368 1.329]	0.236	0.931 [0.708 0.983] [0.385 0.992]	0.600
Sweden	1847-2018	1.642 [0.721 2.646] [0.258 3.492]	0.243	0.793 [0.368 0.952] [0.112 0.983]	0.308
Switzerland	1916-2015	0.760 [0.415 1.181] [0.161 1.716]	0.267	0.883 [0.562 0.973] [0.228 0.990]	0.456
United Kingdom	1701-2019	0.972 [0.819 1.111] [0.647 1.248]	0.411	0.981 [0.936 0.993] [0.817 0.997]	0.902
United States	1868-2019	1.668 [0.923 2.523] [0.422 3.193]	0.188	0.822 [0.502 0.934] [0.209 0.963]	0.292

[a] Spectral bootstrapping has been implemented as in Berkowitz and Diebold (1998). For details, see Section 3.1.

Table III.2 Commodity standards: cross-spectral gain and coherence of money growth onto either inflation or nominal GDP growth at ω=0: point estimates, bootstrapped confidence intervals,[a] p-values for testing the null hypothesis that the gain is equal to 1, and fraction of the mass of the bootstrapped distribution of the coherence beyond 0.9

Country	*Period*	Gain at ω=0, and 16-84 and 5-95 confidence intervals	p-value	Coherence at ω=0, and 16-84 and 5-95 confidence intervals	Fraction beyond 0.9
		I: Based on money growth and inflation			
Argentina	1863-1929	0.696 [0.243 1.279] [0.076 1.801]	0.286	0.742 [0.330 0.928] [0.116 0.970]	0.228
Canada	1873-1929	0.506 [0.185 0.870] [0.062 1.201]	0.098	0.599[0.208 0.854] [0.062 0.944]	0.105
Chile	1811-1877	1.289 [0.415 2.493] [0.123 3.579]	0.385	0.713 [0.299 0.916] [0.095 0.963]	0.197
Italy	1862-1935	1.127 [0.852 1.383] [0.566 1.663]	0.220	0.952 [0.763 0.991] [0.454 0.997]	0.670
Norway	1865-1930	0.879 [0.796 0.956] [0.717 1.027]	0.076	0.992 [0.972 0.998] [0.929 0.999]	0.969
Sweden	1847-1931	0.723 [0.357 1.148] [0.132 1.588]	0.245	0.787 [0.362 0.954] [0.114 0.984]	0.301
United Kingdom	1701-1796	0.436 [0.130 0.955] [0.039 1.439]	0.142	0.293 [0.080 0.612] [0.023 0.781]	0.001
	1821-1931	1.054 [0.750 1.384] [0.460 1.679]	0.419	0.934 [0.731 0.985] [0.412 0.994]	0.609
United States	1868-1932	1.157 [0.741 1.556] [0.381 1.947]	0.338	0.934 [0.700 0.985] [0.350 0.993]	0.607
		II: Based on money growth and nominal GDP growth			
Argentina	1863-1929	0.815 [0.376 1.278] [0.149 1.705]	0.333	0.836 [0.484 0.958] [0.182 0.983]	0.356
Canada	1873-1929	0.818 [0.436 1.215] [0.179 1.562]	0.314	0.807 [0.411 0.940] [0.144 0.975]	0.287
Chile	1811-1877	1.389 [0.502 2.545] [0.151 3.678]	0.338	0.819 [0.405 0.960] [0.143 0.985]	0.346
Italy	1862-1935	1.130 [0.825 1.423] [0.547 1.704]	0.316	0.944 [0.734 0.988] [0.413 0.996]	0.643
Norway	1865-1930	0.903 [0.746 1.048] [0.603 1.181]	0.244	0.968 [0.874 0.992] [0.704 0.996]	0.797
Sweden	1847-1931	0.642 [0.282 1.038] [0.098 1.388]	0.179	0.696 [0.243 0.927] [0.069 0.969]	0.218
United Kingdom	1701-1796	0.932 [0.460 1.471] [0.180 1.974]	0.441	0.623 [0.210 0.906] [0.066 0.969]	0.169
	1821-1931	0.837 [0.574 1.102] [0.348 1.333]	0.261	0.889 [0.605 0.965] [0.291 0.981]	0.467
United States	1868-1932	1.319 [1.112 1.507] [0.904 1.679]	0.087	0.986 [0.944 0.996] [0.853 0.998]	0.917

[a] Spectral bootstrapping has been implemented as in Berkowitz and Diebold (1998). For details, see Section 3.1.

Table III.3 *Fiat* standards: cross-spectral gain and coherence of money growth onto either inflation or nominal GDP growth at ω=0: point estimates, bootstrapped confidence intervals,[a] *p*-values for testing the null hypothesis that the gain is equal to 1, and fraction of the mass of the bootstrapped distribution of the coherence beyond 0.9

Country	*Period*	Gain at ω=0, and 16-84 and 5-95 confidence intervals	p-value	Coherence at ω=0, and 16-84 5-95 confidence intervals	Fraction beyond 0.9
		I: Based on money growth and inflation			
Argentina	1930-2019	0.998 [0.962 1.043] [0.926 1.091]	0.473	0.999 [0.998 1.000] [0.994 1.000]	0.999
Australia	1914-2019	0.778 [0.439 1.134] [0.190 1.484]	0.249	0.898 [0.565 0.978] [0.228 0.992]	0.494
Brazil	1862-2019	0.985 [0.946 1.023] [0.907 1.062]	0.338	0.999 [0.995 1.000] [0.986 1.000]	0.996
Canada	1935-2006	0.603 [0.431 0.752] [0.250 0.905]	0.026	0.977 [0.902 0.991] [0.678 0.996]	0.843
Chile	1878-2019	0.901 [0.804 0.998] [0.708 1.099]	0.156	0.991 [0.963 0.997] [0.887 0.999]	0.943
Colombia	1956-2019	0.966 [0.698 1.192] [0.420 1.418]	0.435	0.975 [0.878 0.994] [0.613 0.997]	0.812
Finland	1915-1985	1.641 [1.078 2.094] [0.546 2.492]	0.135	0.947 [0.764 0.987] [0.427 0.995]	0.669
France	1937-1994	1.194 [0.641 1.769] [0.248 2.411]	0.355	0.845 [0.427 0.968] [0.136 0.990]	0.382
Italy	1936-1996	1.215 [0.831 1.607] [0.477 2.034]	0.275	0.901 [0.595 0.971] [0.244 0.988]	0.502
Japan	1956-2018	0.452 [0.288 0.612] [0.141 0.790]	0.018	0.955 [0.771 0.988] [0.429 0.995]	0.696
Mexico	1926-2013	1.082 [0.933 1.206] [0.764 1.337]	0.267	0.988 [0.947 0.996] [0.841 0.998]	0.917
New Zealand	1914-2016	0.946 [0.647 1.240] [0.366 1.531]	0.424	0.935 [0.710 0.982] [0.365 0.992]	0.616
Norway	1947-2014	0.747 [0.486 1.029] [0.250 1.340]	0.181	0.940 [0.720 0.986] [0.348 0.994]	0.632
Paraguay	1963-2015	1.286 [0.454 2.407] [0.150 3.496]	0.378	0.695 [0.266 0.925] [0.082 0.972]	0.213
Portugal	1932-1998	1.088 [0.616 1.501] [0.264 1.831]	0.419	0.911 [0.658 0.972] [0.306 0.988]	0.543
South Korea	1965-2019	0.530 [0.264 0.780] [0.097 1.030]	0.057	0.913 [0.607 0.979] [0.241 0.992]	0.538
Spain	1942-1997	0.372 [0.119 0.785] [0.039 1.188]	0.084	0.456 [0.116 0.817] [0.035 0.928]	0.074
South Africa	1966-2019	0.871 [0.493 1.228] [0.201 1.562]	0.352	0.929 [0.659 0.985] [0.296 0.994	0.587
Sweden	1932-2018	1.539 [0.689 2.362] [0.227 3.011]	0.261	0.847 [0.461 0.953] [0.152 0.979]	0.372
Switzerland	1937-2012	0.750 [0.382 1.104] [0.142 1.435]	0.234	0.908 [0.613 0.972] [0.262 0.988]	0.521
United Kingdom	1932-2019	0.748 [0.388 1.096] [0.144 1.431]	0.224	0.860 [0.473 0.961] [0.157 0.982]	0.404
United States	1933-2019	0.641 [0.282 0.987] [0.094 1.301]	0.151	0.700 [0.296 0.877] [0.087 0.931]	0.106
Venezuela	1951-2017	1.005 [0.741 1.283] [0.499 1.557]	0.493	0.945 [0.738 0.989] [0.430 0.996]	0.640

[a] Spectral bootstrapping has been implemented as in Berkowitz and Diebold 1998. For details, see Section 3.1.

Table III.3 (continued) *Fiat* standards: cross-spectral gain and coherence of money growth onto either inflation or nominal GDP growth at ω=0: point estimates, bootstrapped confidence intervals,[a] *p*-values for testing the null hypothesis that the gain is equal to 1, and fraction of the mass of the bootstrapped distribution of the coherence beyond 0.9

Country	*Period*	Gain at ω=0, and 16-84 and 5-95 confidence intervals	p-value	Coherence at ω=0, and 16-84 and 5-95 confidence intervals	Fraction beyond 0.9
		II: Based on money growth and nominal GDP growth			
Argentina	1930-2019	0.988 [0.951 1.031] [0.913 1.080]	0.366	0.999 [0.997 1.000] [0.990 1.000]	0.998
Australia	1914-2019	0.628 [0.247 1.152] [0.087 1.733]	0.225	0.715 [0.254 0.944] [0.071 0.982]	0.255
Brazil	1862-2019	0.985 [0.946 1.023] [0.907 1.062]	0.338	0.999 [0.995 1.000] [0.986 1.000]	0.996
Canada	1935-2006	0.674 [0.412 0.924] [0.191 1.177]	0.108	0.913 [0.635 0.976] [0.259 0.989]	0.540
Chile	1878-2019	1.008 [0.370 1.735] [0.119 2.385]	0.496	0.759 [0.347 0.930] [0.109 0.972]	0.236
Colombia	1956-2019	0.999 [0.759 1.220] [0.510 1.450]	0.497	0.979 [0.886 0.996] [0.660 0.999]	0.823
Finland	1915-1985	1.583 [0.994 2.031] [0.466 2.421]	0.162	0.951 [0.758 0.988] [0.412 0.995]	0.676
France	1937-1994	0.547 [0.339 0.761] [0.172 0.976]	0.044	0.905 [0.577 0.983] [0.224 0.995]	0.514
Italy	1936-1996	1.180 [0.953 1.423] [0.727 1.683]	0.207	0.966 [0.844 0.992] [0.604 0.997]	0.757
Japan	1956-2018	0.898 [0.779 1.008] [0.643 1.140]	0.174	0.994 [0.969 0.999] [0.892 1.000]	0.947
Mexico	1926-2013	1.073 [0.961 1.173] [0.832 1.267]	0.243	0.994 [0.973 0.999] [0.921 1.000]	0.963
New Zealand	1914-2016	0.790 [0.407 1.163] [0.161 1.495]	0.275	0.820 [0.420 0.941] [0.134 0.973]	0.313
Norway	1947-2014	0.791 [0.417 1.194] [0.156 1.666]	0.287	0.879 [0.504 0.974] [0.183 0.990]	0.453
Paraguay	1963-2015	1.546 [0.676 2.443] [0.219 3.286]	0.268	0.830 [0.428 0.959] [0.133 0.984]	0.352
Portugal	1932-1998	1.132 [0.589 1.628] [0.234 2.036]	0.396	0.924 [0.661 0.980] [0.308 0.991]	0.573
South Korea	1965-2019	0.769 [0.479 1.012] [0.227 1.228]	0.171	0.950 [0.769 0.986] [0.421 0.995]	0.686
Spain	1942-1997	0.683 [0.362 1.022] [0.141 1.317]	0.173	0.869 [0.514 0.961] [0.186 0.984]	0.420
South Africa	1966-2019	0.812 [0.553 1.051] [0.300 1.266]	0.208	0.949 [0.766 0.985] [0.414 0.993]	0.676
Sweden	1932-2018	1.646 [0.766 2.508] [0.283 3.244]	0.234	0.904 [0.566 0.979] [0.235 0.991]	0.510
Switzerland	1937-2012	1.009 [0.464 1.591] [0.164 2.201]	0.493	0.867 [0.470 0.970] [0.155 0.987]	0.427
United Kingdom	1932-2019	0.767 [0.400 1.139] [0.151 1.484]	0.253	0.840 [0.436 0.952] [0.142 0.980]	0.354
United States	1933-2019	1.108 [0.594 1.597] [0.249 1.992]	0.412	0.847 [0.496 0.950] [0.187 0.972]	0.364
Venezuela	1951-2017	0.886 [0.657 1.125] [0.456 1.355]	0.301	0.949 [0.755 0.991] [0.441 0.996]	0.654

[a] Spectral bootstrapping has been implemented as in Berkowitz and Diebold 1998. For details, see Section 3.1.

Table III.4 Quarterly data: cross-spectral gain and coherence of money growth onto either inflation or nominal GDP growth at ω=0: point estimates, bootstrapped confidence intervals,[a] p-values for testing the null hypothesis that the gain is equal to 1, and fraction of the mass of the bootstrapped distribution of the coherence beyond 0.9

Country	*Period*	Gain at ω=0, and 16-84 5-95 confidence intervals	p-value	Coherence at ω=0, and 16-84 5-95 confidence intervals	Fraction beyond 0.9
		I: Based on money growth and inflation			
Australia	1959Q4-2019Q4	1.018 [0.655 1.384] [0.314 1.747]	0.476	0.921 [0.647 0.981] [0.278 0.991]	0.560
Canada	1914Q2-2006Q4	0.576 [0.360 0.773] [0.180 0.958]	0.038	0.883 [0.619 0.954] [0.275 0.979]	0.445
	1968Q1-2019Q4	0.749 [0.470 1.037] [0.219 1.342]	0.188	0.935 [0.669 0.988] [0.283 0.996]	0.598
Denmark	1922Q4-2019Q4	0.725 [0.513 0.924] [0.309 1.135]	0.102	0.963 [0.817 0.993] [0.530 0.998]	0.730
Euro area	1970Q3-2019Q4	1.182 [0.727 1.630] [0.343 2.030]	0.334	0.926 [0.703 0.976] [0.324 0.991]	0.592
Finland	1914Q4-1985Q4	1.411 [1.197 1.617] [0.968 1.833]	0.060	0.987 [0.946 0.996] [0.849 0.998]	0.916
Germany	1949Q1-1998Q4	0.756 [0.225 1.689] [0.067 2.561]	0.376	0.596 [0.225 0.870] [0.070 0.945]	0.116
Hong Kong	1985Q1-2019Q4	0.782 [0.516 1.020] [0.249 1.240]	0.179	0.939 [0.705 0.985] [0.326 0.993]	0.631
Italy	1948Q2-1997Q3	0.751 [0.265 1.375] [0.082 2.041]	0.335	0.544 [0.153 0.832] [0.046 0.928]	0.081
Japan	1955Q2-2019Q4	0.501 [0.324 0.670] [0.165 0.852]	0.024	0.957 [0.782 0.988] [0.428 0.996]	0.704
Netherlands	1957Q2-1997Q4	0.766 [0.370 1.163] [0.137 1.490]	0.272	0.892 [0.564 0.970] [0.218 0.988]	0.479
New Zealand	1988Q1-2019Q4	1.039 [0.319 2.137] [0.106 3.158]	0.482	0.607 [0.204 0.890] [0.066 0.953]	0.144
Norway	1920Q3-2019Q4	0.791 [0.557 1.017] [0.336 1.227]	0.175	0.951 [0.788 0.986] [0.475 0.993]	0.691
	1978Q1-2019Q4	0.865 [0.580 1.122] [0.295 1.360]	0.287	0.939 [0.734 0.982] [0.371 0.991]	0.635
South Africa	1966Q1-2019Q4	0.883 [0.518 1.239] [0.218 1.638]	0.352	0.932 [0.638 0.989] [0.247 0.997]	0.587
South Korea	1960Q1-2019Q4	0.549 [0.329 0.750] [0.150 0.984]	0.045	0.936 [0.690 0.984] [0.310 0.993]	0.616
Switzerland	1985Q1-2019Q4	0.173 [0.050 0.371] [0.015 0.576]	0.004	0.325 [0.094 0.638] [0.028 0.809]	0.014
Taiwan	1961Q3-2019Q4	0.070 [0.023 0.145] [0.007 0.216]	0.000	0.476 [0.156 0.765] [0.051 0.881]	0.035
	1982Q1-2019Q4	0.199 [0.074 0.376] [0.023 0.577]	0.009	0.767 [0.309 0.956] [0.095 0.985]	0.306
United Kingdom	1881Q1-2019Q4	0.687 [0.300 1.133] [0.105 1.533]	0.234	0.790 [0.376 0.935] [0.126 0.974]	0.254
	1895Q2-2019Q4	0.624 [0.286 1.007] [0.104 1.353]	0.163	0.831 [0.421 0.954] [0.144 0.979]	0.349
	1955Q1-2019Q4	0.785 [0.373 1.261] [0.139 1.771]	0.311	0.853 [0.449 0.971] [0.152 0.990]	0.405
United States	1875Q2-2019Q4	1.255 [0.579 2.036] [0.216 2.714]	0.351	0.699 [0.292 0.893] [0.098 0.952]	0.144

Table III.4 (continued) Quarterly data: cross-spectral gain and coherence of money growth onto either inflation or nominal GDP growth at ω=0: point estimates, bootstrapped confidence intervals,[a] p-values for testing the null hypothesis that the gain is equal to 1, and fraction of the mass of the bootstrapped distribution of the coherence beyond 0.9

Country	*Period*	Gain at ω=0, and 16-84 5-95 confidence intervals	p-value	Coherence at ω=0, and 16-84 5-95 confidence intervals	Fraction beyond 0.9
		II: Based on money growth and nominal GDP growth			
Australia	1959Q4-2019Q4	0.960 [0.549 1.375] [0.237 1.786]	0.457	0.860 [0.511 0.959] [0.176 0.985]	0.374
Canada	1968Q1-2019Q4	0.854 [0.424 1.324] [0.149 1.823]	0.365	0.880 [0.485 0.976] [0.167 0.991]	0.457
Euro area	1970Q3-2019Q4	1.350 [0.833 1.891] [0.378 2.413]	0.237	0.917 [0.646 0.976] [0.249 0.990]	0.553
Hong Kong	1985Q1-2019Q4	0.972 [0.748 1.185] [0.521 1.387]	0.445	0.967 [0.882 0.989] [0.666 0.995]	0.810
Japan	1955Q2-2019Q4	1.004 [0.902 1.094] [0.779 1.194]	0.480	0.996 [0.984 0.999] [0.947 0.999]	0.975
New Zealand	1988Q1-2019Q4	0.003 [0.001 0.006] [0.000 0.009]	0.000	0.002 [0.000 0.012] [0.000 0.029]	0.000
Norway	1978Q1-2019Q4	0.887 [0.642 1.113] [0.381 1.317]	0.296	0.964 [0.851 0.987] [0.555 0.993]	0.773
South Africa	1966Q1-2019Q4	0.810 [0.519 1.067] [0.253 1.295]	0.222	0.937 [0.725 0.981] [0.336 0.992]	0.631
South Korea	1960Q1-2019Q4	0.793 [0.593 0.964] [0.365 1.130]	0.122	0.977 [0.887 0.994] [0.642 0.998]	0.822
Switzerland	1985Q1-2019Q4	0.296 [0.103 0.558] [0.032 0.775]	0.018	0.523 [0.171 0.790] [0.051 0.898]	0.049
Taiwan	1961Q3-2019Q4	0.130 [0.039 0.266] [0.012 0.396]	0.001	0.489 [0.137 0.809] [0.039 0.904]	0.055
	1982Q1-2019Q4	0.527 [0.352 0.701] [0.187 0.869]	0.023	0.940 [0.766 0.984] [0.420 0.994]	0.665
United Kingdom	1955Q1-2019Q4	0.823 [0.427 1.246] [0.161 1.654]	0.325	0.879 [0.499 0.969] [0.178 0.989]	0.448
United States	1875Q2-2019Q4	1.534 [0.892 2.278] [0.414 3.006]	0.202	0.825 [0.465 0.961] [0.177 0.987]	0.342

Table III.5 Period since 1985: cross-spectral gain and coherence of money growth onto either inflation or nominal GDP growth at ω=0: point estimates, bootstrapped confidence intervals,[a] *p*-values for testing the null hypothesis that the gain is equal to 1, and fraction of the mass of the bootstrapped distribution of the coherence beyond 0.9

Country	*Period*	Gain at ω=0, and 16-84 5-95 confidence intervals	*p*-value	Coherence at ω=0, and 16-84 5-95 confidence intervals	Fraction beyond 0.9
		I: Based on money growth and inflation			
Australia	1985Q1-2019Q4	0.581 [0.296 0.831] [0.107 1.054]	0.064	0.775 [0.385 0.905] [0.109 0.957]	0.174
Canada	1985Q1-2019Q4	0.251 [0.109 0.414] [0.038 0.595]	0.009	0.712 [0.250 0.928] [0.069 0.969]	0.222
Denmark	1985Q1-2019Q4	0.320 [0.105 0.619] [0.034 0.898]	0.034	0.727 [0.312 0.915] [0.105 0.960]	0.196
Euro area	1985Q1-2019Q4	0.555 [0.199 1.045] [0.065 1.586]	0.178	0.584 [0.169 0.889] [0.047 0.958]	0.145
Hong Kong	1985Q1-2019Q4	0.782 [0.516 1.020] [0.249 1.240]	0.179	0.939 [0.705 0.985] [0.326 0.993]	0.631
Japan	1985Q1-2019Q4	0.438 [0.163 0.756] [0.053 1.062]	0.064	0.856 [0.486 0.967] [0.181 0.987]	0.407
Norway	1985Q1-2019Q4	0.544 [0.226 0.908] [0.076 1.254]	0.113	0.681 [0.234 0.902] [0.065 0.950]	0.162
New Zealand	1988Q1-2019Q4	0.602 [0.177 1.366] [0.053 2.055]	0.283	0.578 [0.210 0.855] [0.067 0.933]	0.097
South Korea	1985Q1-2019Q4	0.347 [0.220 0.464] [0.102 0.589]	0.003	0.957 [0.798 0.988] [0.448 0.995]	0.713
Switzerland	1985Q1-2019Q4	0.173 [0.053 0.378] [0.017 0.575]	0.007	0.325 [0.093 0.631] [0.027 0.808]	0.016
Taiwan	1985Q1-2019Q4	0.256 [0.102 0.448] [0.035 0.643]	0.010	0.820 [0.382 0.960] [0.130 0.984]	0.351
United Kingdom	1985Q1-2019Q4	0.597 [0.295 0.941] [0.111 1.285]	0.129	0.808 [0.367 0.950] [0.111 0.976]	0.321
United States	1985Q1-2019Q4	0.183 [0.055 0.399] [0.017 0.617]	0.010	0.412 [0.127 0.733] [0.037 0.884]	0.041
		II: Based on money growth and nominal GDP growth			
Australia	1985Q1-2019Q4	0.564 [0.299 0.783] [0.122 0.998]	0.050	0.752 [0.396 0.888] [0.140 0.949]	0.138
Canada	1985Q1-2019Q4	0.014 [0.004 0.033] [0.001 0.055]	0.000	0.032 [0.006 0.112] [0.002 0.217]	0.000
Denmark	1985Q1-2019Q4	0.258 [0.086 0.508] [0.028 0.768]	0.021	0.530 [0.156 0.855] [0.042 0.940]	0.102
Euro area	1985Q1-2019Q4	0.829 [0.300 1.585] [0.092 2.350]	0.399	0.606 [0.178 0.899] [0.050 0.959]	0.157
Hong Kong	1985Q1-2019Q4	0.972 [0.748 1.185] [0.521 1.387]	0.445	0.967 [0.882 0.989] [0.666 0.995]	0.810
Japan	1985Q1-2019Q4	1.034 [0.627 1.388] [0.285 1.686]	0.460	0.958 [0.815 0.986] [0.495 0.994]	0.735
Norway	1985Q1-2019Q4	0.555 [0.282 0.824] [0.114 1.055]	0.067	0.799 [0.441 0.926] [0.168 0.957]	0.253
New Zealand	1988Q1-2019Q4	0.003 [0.001 0.006] [0.000 0.009]	0.000	0.002 [0.000 0.011] [0.000 0.026]	0.000
South Korea	1985Q1-2019Q4	0.681 [0.494 0.844] [0.290 1.016]	0.056	0.980 [0.901 0.995] [0.667 0.997]	0.842
Switzerland	1985Q1-2019Q4	0.340 [0.121 0.628] [0.042 0.860]	0.024	0.579 [0.201 0.822] [0.066 0.919]	0.069
Taiwan	1985Q1-2019Q4	0.613 [0.482 0.745] [0.349 0.892]	0.024	0.975 [0.904 0.993] [0.728 0.997]	0.846
United Kingdom	1985Q1-2019Q4	0.731 [0.440 1.078] [0.211 1.447]	0.208	0.868 [0.503 0.966] [0.184 0.986]	0.425
United States	1985Q1-2019Q4	0.359 [0.102 0.866] [0.032 1.434]	0.118	0.420 [0.113 0.783] [0.035 0.908]	0.057

[a] Spectral bootstrapping has been implemented as in Berkowitz and Diebold (1998). For details, see Section 3.1.

Table III.6 Post-WWII period: cross-spectral gain of either money or credit[a] growth onto either inflation or nominal GDP growth: point estimates, and *p*-values for testing the null hypothesis that the gain is equal to 1

		Based on money growth				*Based on credit growth*			
		Frequency zero		Frequencies beyond 30 years		Frequency zero		Frequencies beyond 30 years	
Country	*Period*	Gain	*p*-value	Average gain	*p*-value	Gain	*p*-value	Average gain	*p*-value
	I: Based on inflation								
Australia	1953-2016	0.934	0.434	0.824	0.286	0.696	0.193	0.545	0.076
Canada	1947-2006	0.558	0.091	0.608	0.106	0.391	0.027	0.448	0.024
Finland	1947-1985	1.367	0.276	1.448	0.260	0.663	0.058	0.620	0.021
France	1947-1994	0.301	0.028	0.573	0.156	0.565	0.142	0.652	0.194
Italy	1949-1996	0.826	0.383	0.837	0.380	0.823	0.420	0.654	0.307
Japan	1956-2016	0.451	0.020	0.428	0.011	0.350	0.024	0.313	0.009
Norway	1947-2013	0.750	0.195	0.812	0.250	0.397	0.049	0.399	0.019
Spain	1947-1997	0.372	0.076	0.465	0.124	0.307	0.030	0.410	0.067
Sweden	1947-2012	1.220	0.369	1.099	0.423	0.358	0.020	0.302	0.002
Switzerland	1947-2015	0.814	0.226	0.710	0.088	0.204	0.008	0.142	0.000
United Kingdom	1947-2014	0.614	0.179	0.587	0.150	0.508	0.069	0.485	0.039
United States	1947-2014	0.732	0.250	0.604	0.130	0.237	0.018	0.329	0.033
	II: Based on nominal GDP growth								
Australia	1953-2016	1.072	0.443	0.952	0.449	0.787	0.307	0.664	0.177
Canada	1947-2006	0.595	0.132	0.653	0.166	0.533	0.038	0.581	0.038
Finland	1947-1985	1.312	0.293	1.430	0.242	0.649	0.012	0.640	0.003
France	1947-1994	0.553	0.136	0.813	0.342	0.839	0.329	0.892	0.379
Italy	1949-1996	0.925	0.415	0.923	0.394	0.999	0.500	0.848	0.413
Japan	1956-2016	0.897	0.171	0.884	0.144	0.726	0.151	0.688	0.104
Norway	1947-2013	0.788	0.293	0.800	0.299	0.529	0.082	0.493	0.029
Spain	1947-1997	0.704	0.206	0.740	0.227	0.585	0.075	0.623	0.065
Sweden	1947-2012	1.151	0.409	0.916	0.436	0.287	0.015	0.223	0.001
Switzerland	1947-2015	1.100	0.403	0.979	0.477	0.359	0.015	0.304	0.000
United Kingdom	1947-2014	0.586	0.167	0.545	0.110	0.514	0.025	0.493	0.010
United States	1947-2014	0.723	0.309	0.494	0.115	0.335	0.041	0.418	0.023

Table III.7 Post-WWII period: cross-spectral gain of either money or total loans growth onto either inflation or nominal GDP growth: point estimates, and *p*-values for testing the null hypothesis that the gain is equal to 1

		Based on money growth				*Based on credit growth*			
		Frequency zero		Frequencies beyond 30 years		Frequency zero		Frequencies beyond 30 years	
Country	*Period*	Gain	*p*-value	Average gain	*p*-value	Gain	*p*-value	Average gain	*p*-value
	I: Based on inflation								
Australia	1953-2016	0.934	0.433	0.824	0.296	0.744	0.277	0.641	0.174
Canada	1947-2006	0.558	0.091	0.608	0.106	0.420	0.041	0.477	0.043
Finland	1947-1985	1.367	0.276	1.448	0.260	0.725	0.162	0.728	0.124
France	1947-1994	0.301	0.024	0.573	0.143	0.644	0.193	0.756	0.261
Italy	1949-1996	0.826	0.383	0.837	0.380	0.320	0.083	0.430	0.145
Japan	1956-2016	0.451	0.020	0.428	0.011	0.386	0.024	0.353	0.010
Norway	1947-2013	0.750	0.195	0.812	0.247	0.635	0.149	0.604	0.070
Spain	1947-1997	0.372	0.076	0.465	0.124	0.373	0.065	0.515	0.128
Sweden	1947-2012	1.220	0.369	1.099	0.423	0.731	0.215	0.501	0.062
Switzerland	1947-2015	0.814	0.227	0.710	0.094	0.557	0.137	0.388	0.029
United Kingdom	1947-2014	0.614	0.179	0.587	0.150	0.632	0.102	0.617	0.083
United States	1947-2014	0.732	0.250	0.604	0.130	0.227	0.025	0.263	0.023
	II: Based on nominal GDP growth								
Australia	1953-2016	1.072	0.436	0.952	0.448	0.835	0.361	0.763	0.295
Canada	1947-2006	0.595	0.131	0.653	0.160	0.562	0.054	0.613	0.060
Finland	1947-1985	1.312	0.292	1.430	0.244	0.736	0.091	0.764	0.077
France	1947-1994	0.553	0.132	0.813	0.339	0.921	0.402	1.001	0.499
Italy	1949-1996	0.925	0.421	0.923	0.401	0.714	0.345	0.747	0.362
Japan	1956-2016	0.897	0.172	0.884	0.145	0.793	0.177	0.765	0.133
Norway	1947-2013	0.788	0.294	0.800	0.294	0.735	0.237	0.694	0.123
Spain	1947-1997	0.704	0.215	0.740	0.235	0.717	0.209	0.760	0.226
Sweden	1947-2012	1.151	0.411	0.916	0.443	0.655	0.159	0.434	0.037
Switzerland	1947-2015	1.100	0.399	0.979	0.476	0.811	0.330	0.656	0.145
United Kingdom	1947-2014	0.586	0.171	0.545	0.111	0.639	0.054	0.623	0.036
United States	1947-2014	0.723	0.296	0.494	0.114	0.390	0.082	0.401	0.031