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A New Approach to Estimating the Natural Rate of Interest

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A New Approach to Estimating the Natural Rate of Interest*

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Abstract

Building upon the insight that M1 velocity is the permanent component of nominal interest rates—see Benati (2020)—I propose a novel, and straightforward approach to estimating the natural rate of interest, which is conceptually related to Cochrane's (1994*a*) proposal to estimate the permanent component of GNP by exploiting the informational content of consumption. Under monetary regimes (such as inflation-targeting) making inflation I(0), the easiest way to implement the proposed approach is to (*i*) project the monetary policy rate onto M1 velocity—thus obtaining an estimate of the *nominal* natural rate—and then (*ii*) subtract from this inflation's sample average (or target), thus obtaining the *real* natural rate. More complex implementations based on structural VARs produce very similar estimates. Compared to existing approaches, the one proposed herein presents two key advantages: (1) under regimes making inflation I(0), M1 velocity is equal, up to a linear transformation, to the real natural rate, so that the natural rate is, in fact, *observed*; and (2) based on a high-frequency estimate of nominal GDP, the natural rate can be computed at the monthy or even weekly frequency. In the U.S., Euro area, and Canada the natural rate dropped sharply in the months following the collapse of Lehman Brothers. Likewise, the 1929 stock market crash was followed in the U.S. by a dramatic decrease in the natural rate.

Keywords: Natural rate of interest; money velocity; structural VARs; unit roots; cointegration.

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1 Introduction

Since the outbreak of the financial crisis, the natural rate of interest has been one of the most intensely discussed issues in both policymaking circles and academia. Currently, there are two approaches to estimating the natural rate. In the first, which was originally proposed by Laubach and Williams (2003), the natural rate is modelled as an I(1) process, usually a pure unit root;[1] it is embedded within a semi-structural framework also featuring a Phillips curve; and it is extracted from the data *via* the Kalman filter. In the second approach the natural rate is instead estimated based on a fully-specified DSGE model.[2]

In this paper I illustrate a novel, and straightforward method to estimate the natural rate of interest, which in line with the recent non-DSGE literature I define as a pure unit root process, specifically as the permanent component of the *ex post* real short-term (monetary policy) rate. The approach is conceptually related to Cochrane's (1994*a*) proposal to estimate the permanent component of GNP by exploiting the informational content of consumption, and builds upon the insight that M1 velocity[3] is, to a close approximation, the permanent component of nominal interest rates (see Benati, 2020). This suggests that in the same way as, as argued by Cochrane (1994*a*), consumption can be treated as a good estimate of permanent GNP, M1 velocity can be regarded as a reliable estimate of the permanent component of the nominal short-term rate, R_t^P, i.e. of the *nominal natural rate*.

Further, basic economic logic suggests that R_t^P is driven by (*i*) permanent inflation shocks (*via* the Fisher effect), and (*ii*) permanent shocks to the *real natural rate* of interest, i.e., $R_t^P = \pi_t^P + r_t^N$, where π_t^P is the permanent component of inflation, and r_t^N is the real natural rate. This implies that under monetary regimes, such as inflation-targeting, that cause inflation to be I(0)[4]—so that π_t^P=0—permanent shifts in M1 velocity, V_t, *uniquely* reflect permanent fluctuations in the natural rate of interest, so that, e.g., $V_t = \alpha + \beta r_t^N + \eta_t$, where η_t is a 'small'[5] I(0) component, and the rest of the notation is obvious. Under these regimes, the easiest way to implement the proposed approach is to

(1) project the monetary policy rate onto M1 velocity—thus obtaining an estimate of the nominal natural rate—and then

(2) subtract from this inflation's sample average (or target), thus obtaining the real natural rate.

[1] See e.g. Holston, Laubach and Williams (2017), and Fiorentini, Galesi, Pérez-Quirós and Sentana (2018). To be precise, in these papers the natural rate is modelled as the sum of two pure random walks (see Holston *et al.*'s equations 6, 8 and 9, and Fiorentini *et al.*'s equations 3, 5 and 6), one being trend GDP growth, and the other an additional 'catch-all' factor.

[2] See e.g. Del Negro, Giannone, Giannoni, and Tambalotti (2017).

[3] Defined as the ratio between nominal GDP and nominal M1, i.e. as the inverse of M1 balances expressed as a fraction of GDP.

[4] See Benati (2008).

[5] In the sense of explaining close to *nil* of fluctuations in velocity.

More complex implementations based on structural VARs produce very similar estimates. If, on the other hand, over the sample period inflation had been I(1), so that $\pi_t^P \neq 0$, in order to compute the real natural rate it is necessary to purge the nominal natural rate of permanent inflations shocks. This can be accomplished, e.g., based on a cointegrated SVAR for M1 velocity, the short rate and inflation (and possible other series).

Compared to existing approaches, the one proposed herein presents two advantages. First, since under regimes making inflation I(0) M1 velocity is equal, up to a linear transformation, to the real natural rate, this implies that, under such regimes, *the real natural rate of interest is observed*. This is of obvious interest to policy-makers, and (e.g.) it implies that a consistent decrease in M1 velocity under such a regime—such as the progressive fall that has been going on in several inflation-targeting countries since the early 1990s—provides *direct evidence* of a fall in the natural rate. Second, since M1 is observed (at least) at the weekly frequency, and interest rates are observed on a continuous basis, based on a high-frequency estimate of nominal GDP the natural rate can in principle be computed at the monthy, or even weekly frequency. In an application based on monthly data I show that in the U.S., Euro area, and Canada the natural rate fell sharply in the months following the collapse of Lehman Brothers. More generally, my evidence suggests that in all of the countries I analyze herein the real natural rate has been declinining at least since the early 1990s, and that at the end of the sample, in 2019, it had highly likely been negative in several of them.

The paper is organized as follows. The next section discusses the data, whereas Section 3 discusses the close conceptual similarity between the present work and Cochrane (1994*a*). Section 4 estimates the nominal natural rate of interest, whereas Section 5 explores the integration properties of inflation by monetary regime. Section 6 estimates the real natural rate, whereas Section 7 discusses the advantages of the proposed approach compared to existing alternatives, and computes monthly natural rate estimates for the U.S., the Euro area, the U.K., and Canada. Section 8 discusses two applications of the proposed methodology, pertaining to the evolution of the natural rate during the Great Depression, and to the impact of the COVID pandemic. Section 9 concludes.

2 The Data

Online Appendix A describes the data and their sources in detail. In brief, nearly all of data are from the datasets assembled by Benati (2020) and Benati, Lucas, Nicolini and Weber (2021), which for the post-WWII period I have updated to 2019Q4.[6] All of the series are standard, with the single exception that, following Lucas and

[6] With the exception of the exercise in Section 8.2 I exclude the year 2020 from all samples, in order to avoid that my results could be distorted by the impact of the COVID pandemic.

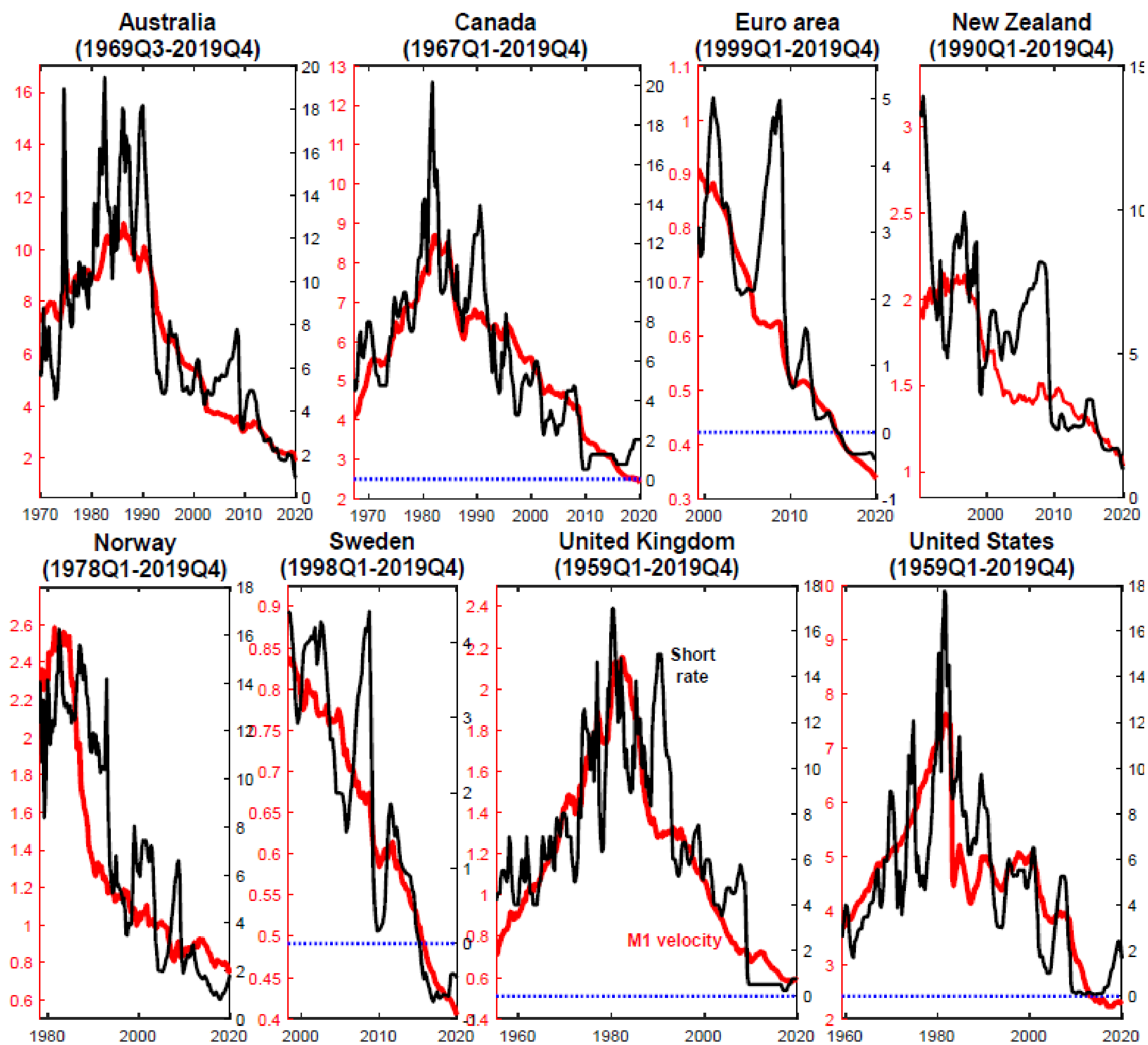

Figure 1a M1 velocity and a nominal short-term interest rate over the post-WWII period

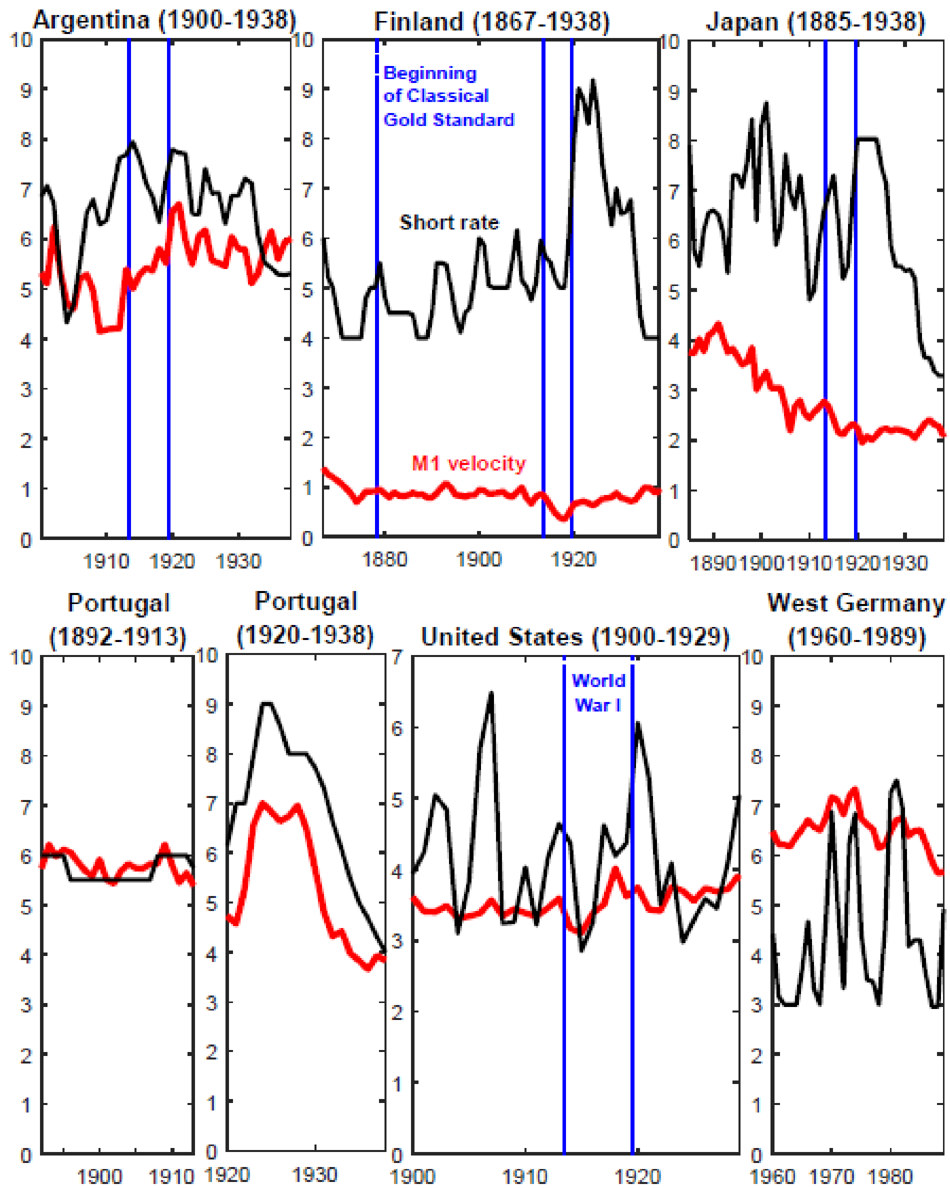

Figure 1b M1 velocity and a nominal short-term interest rate for West Germany and selected pre-World War II samples

Nicolini (2015), Benati (2020), and Benati *et al.* (2021), for the United States I consider, instead of the standard M1 aggregate produced by the Federal Reserve, one of the modifications that had originally been suggested by Goldfeld and Sichel (1990, pp. 314-315) in order to restore the stability of the long-run demand for M1. This alternative M1 series—which Lucas and Nicolini (2015) label as 'New M1'—is obtained by adding to the standard M1 aggregate Money Market Deposits Accounts (MMDAs). The rationale for doing so is that MMDAs perform an economic function very similar to that of the 'checkable deposits' included in the standard M1 series (on this see the discussion in Lucas and Nicolini, 2015).

Figure 1*a* shows, for the eight countries analyzed herein, M1 velocity and a short-term nominal interest rate over the post-WWII period. Visual impression suggests the following three facts, which as shown by Benati (2020) are indeed confirmed by a proper econometric analysis: (*i*) M1 velocity and the short rate are both I(1); (*ii*) the two series are cointegrated; and, crucially, (*iii*) up to a linear transformation, M1 velocity is, essentially, the stochastic trend of the short rate. Figure 1*b* shows the same type of evidence for selected pre-World War II samples and for West Germany. The evidence for Portugal during the interwar period, with the hump-shaped fluctuation in the short rate being mirrored by a corresponding fluctuation in velocity, is qualitatively in line with the post-WWII evidence in Figure 1*a*. This is also the case, although to a lesser extent, for interwar Japan, with both series exhibiting an overall downward trend. For all other countries, however, the lack of any discernible trend in the short rate is mirrored by the broad flatness of M1 velocity. This is especially the case for Finland, Portugal (1892-1913), the United States, and to a lesser extent for Argentina and West Germany.

This evidence naturally suggests that the large fluctuations in M1 velocity that have characterized the post-WWII period have been caused, under a stable demand for M1 balances as a fraction of GDP, by permanent fluctuations in both inflation and the real natural rate of interest injecting a unit root in nominal short-term interest rates. On the other hand, as the evidence in Figure 1*b* shows, when nominal interest rates do not exhibit any trend, M1 velocity is likewise essentially flat. In turn, this suggests that, to the extent that (*i*) inflation will remain under the control of the monetary authority, and therefore I(0), and (*ii*) the decline in the real natural rate of interest will ultimately stop, the fall in velocity that has been going on since the early 1980s (see Figure 1*a*) will also cease.

I now turn to discussing the conceptual similarity between the present work and Cochrane's (1994*a*) analysis for consumption and output.

3 Conceptual Similarity With Cochrane (1994*a*)

The best way to illustrate the approach I am advocating herein is to highlight its close conceptual similarity with Cochrane's (1994*a*) proposal to estimate the permanent component of GNP by exploiting the informational content of consumption.

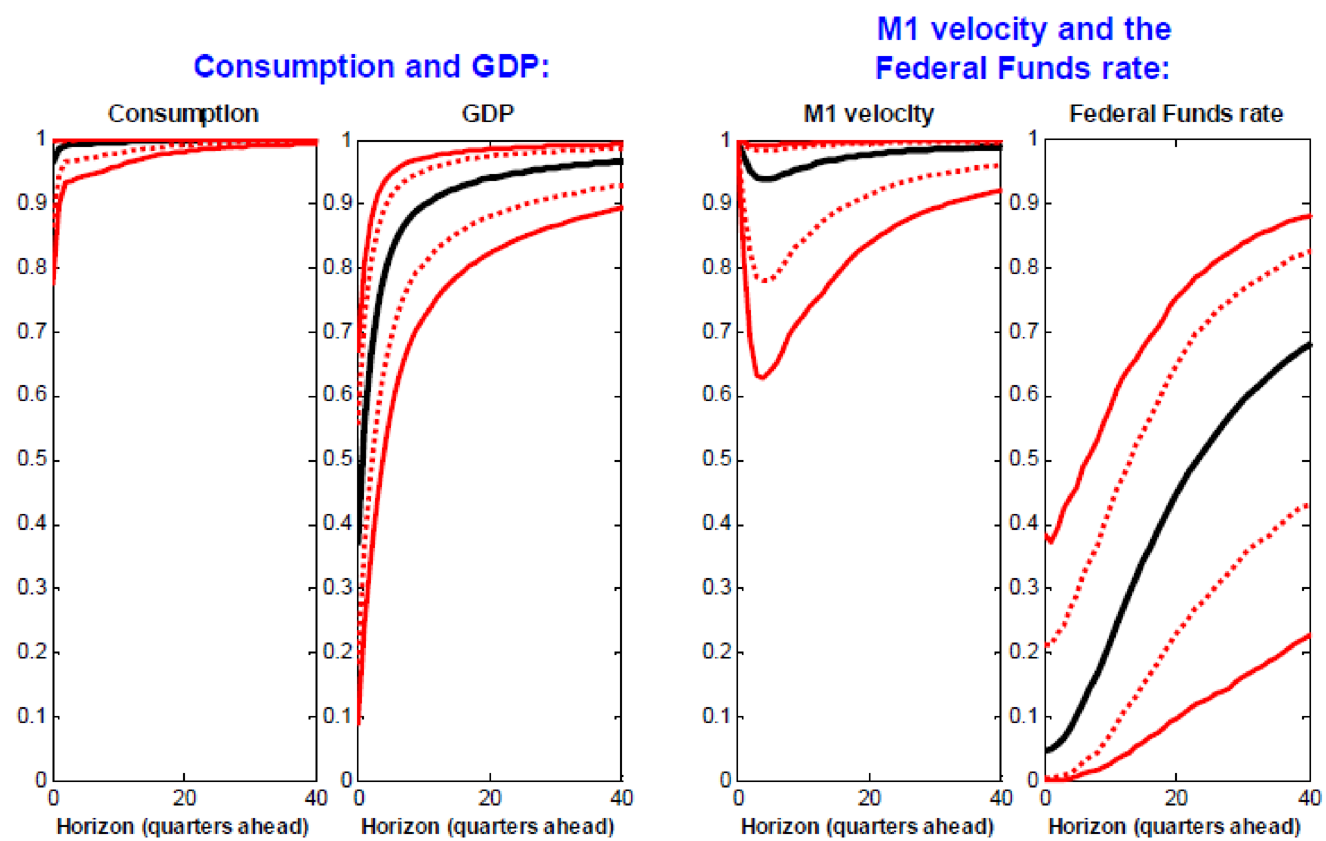

Figure 2 United States: fractions of forecast error variance explained by the permanent shock, based on cointegrated VARs featuring either (i) consumption and GDP or (ii) M1 velocity and the Federal Funds rate (with 1- and 2-standard deviations bootstrapped confidence bands)

3.1 Cochrane (1994*a*): consumption is the permanent component of GNP

In his investigation of the permanent income hypothesis (PIH) based on cointegrated structural VARs (SVARs) for consumption and GNP, Cochrane (1994*a*) documented how consumption is, to a close approximation, the permanent component of GNP. The first two panels of Figure 2 report evidence in line with Cochrane's for the United States for the period 1947Q1-2019Q4, based on a cointegrated SVAR for the logarithms of real consumption and real GDP identified *via* long-run restrictions.[7] The two panels report the fractions of forecast error variance (FEV) of consumption and GDP, respectively, explained by the common permanent shock, which is identified as the only shock having a permanent impact on the two series. In line with Cochrane's (1994*a*) Table I.2, the shock explains (nearly) 100 per cent of the FEV of consumption at all horizons up to 10 years ahead, whereas GDP contains, at the short horizons, a sizeable transitory component.[8] As pointed out by Cochrane (1994*a*),

> 'It is natural to interpret these features of the data via the simple permanent income model. The model predicts that consumption is a random walk and that consumption and total income are cointegrated. If consumption does not change, consumers must think any fluctuation in GNP is transitory. [...] by observing consumption, we separate GNP into permanent and transitory components, as viewed by consumers. [...] Thus, consumption provides a good measure of the trend in GNP, since it measures consumers' expectations of long-run GNP.'

As stressed by Cochrane (1994*b*), this is because

> 'Each person has information about his own prospects, most of which is idiosyncratic. Total consumption aggregates all this information about aggregate activity.'

3.2 M1 velocity is the permanent component of the short rate

As documented by Benati (2020), since World War I M1 velocity has been, to a close approximation, the permanent component of the short-term nominal interest

[7] The data are described in Online Appendix A.2.9. Elliot, Rothenberg, and Stock (1996) unit root tests with an intercept and a time trend strongly suggest that both series are I(1), with p-values bootstrapped as in Diebold and Chen (1996) ranging between 0.4693 and 0.6659 for GDP, and between 0.9514 and 0.9912 for consumption. By the same token, Johansen's maximum eigenvalue test, bootstrapped as in Cavaliere *et al.* (2012), provides clear evidence of cointegration, with a p-value equal to 2.0e-4.

[8] In Cochrane's (1994*a*) Table I.2 the permanent ('consumption') shock explains 97 per cent of the variance of $\Delta \ln C_t$, whereas it explains only 30 per cent of the variance of $\Delta \ln Y_t$, where C_t and Y_t are consumption and GDP, respectively.

rate, so that the time-series relationship between the two series has been *exactly* the same as that between consumption and GDP.[9] The last two panels of Figure 2 report, for the United States for the period 1959Q1-2019Q4, the fractions of FEV explained by the common permanent shock based on a cointegrated SVAR for M1 velocity and the Federal Funds rate identified *via* long-run restrictions. Once again, the permanent shock is identified as the only shock having a permanent impact on the two series. In line with the previously discussed evidence for consumption and GDP, the permanent shock explains (close to) 100 per cent of the FEV of M1 velocity at all horizons, whereas the Federal Funds rate contains a sizeable transitory component, which is in fact dominant at all horizons up to about five years ahead. Benati's (2020) Figure 1*b* reports the corresponding evidence for the post-WWII United Kingdom, whereas Figures 3*a* and 4*a* report evidence for eight additional countries for the post-WWII period, and for ten countries since World War I, respectively. With the single exception of Taiwan, and to a lesser extent of Japan, the evidence there is in line with that reported in Figure 2 in the present work, with the permanent shock explaining (close to) 100 per cent of the FEV of M1 velocity at all horizons. Short-term nominal interest rates, on the other hand, consistently feature a sizeable, and often dominant transitory component.

3.2.1 Interpretation

This evidence suggests that the bivariate relationship between M1 velocity and the short rate is well captured, to a first approximation, by a simple model in which (1) the short rate, R_t, is the sum of two components, a random walk, R_t^P, and a stationary AR(p) process, R_t^T,

$$R_t = R_t^P + R_t^T \tag{1}$$

$$R_t^P = R_{t-1}^P + u_t \tag{2}$$

$$R_t^T = \phi_1 R_{t-1}^T + ...\phi_p R_{t-p}^T + v_t \tag{3}$$

—with, just for the sake of simplicity, u_t and v_t being orthogonal white noise processes—and (2) M1 velocity is a linear function of the permanent component of the short rate,

[9] The unit root properties of M1 velocity and the short rate are discussed in Appendix A. In short, Elliot *et al.* (1996) tests (with an intercept, but no time trend) strongly suggest that both series are I(1) in all samples (quite obviously, this evidence is compatible with the notion that the series are in fact near-unit root processes). Table B.1 in Appendix B reports, for bivariate systems featuring M1 velocity and a short-term rate, results from either (*i*) Johansen's maximum eigenvalue test of the null hypothesis of 0 *versus* 1 cointegration vectors, or (*ii*) Wright's (2000) test of the null hypothesis that the series are cointegrated, which is designed to perform equally well when they feature either exact or near unit roots. In short, consistent with Benati (2020) and Benati *et al.* (2021), the evidence there suggests that the two series are cointegrated in all sample. Finally, Online Appendix B.4 reports results from Hansen and Johansen's (1999) tests for stability in either the cointegration vector, or the loading coefficients, in the estimated VECMs for velocity and the short rate. For all countries no break is detected in either of the two features.

i.e.

$$V_t = \alpha + \beta R_t^P + \eta_t \tag{4}$$

where η_t is a 'small' (in the sense of explaining close to *nil* of the FEV of velocity) stationary component, and the rest of the notation is obvious.

Expression (4) is obtained by replacing the short rate with its permanent component within the money demand specification originally proposed by Selden (1956) and Latané (1960), i.e.[10]

$$V_t = \alpha + \beta R_t + \eta_t \tag{5}$$

As discussed by Benati *et al.* (2021), for several low-inflation countries—first and foremost, the U.S. and the U.K.—the data seem to quite clearly prefer the Selden-Latané specification to the traditional log-log and semi-log ones.[11]

An important point to stress is that, as long as η_t in expression (4) is 'small', $V_t \simeq \alpha + \beta R_t^P$, so that $R_t^P \simeq (V_t - \alpha)/\beta$. This implies that, under these circumstances, the *nominal* natural rate is *always observed*.[12]

Expression (4) implies the following cointegrated VECM representation for ΔV_t and ΔR_t:

$$\left[\begin{array}{c} \Delta V_t \\ \Delta R_t \end{array}\right] = \text{Constants} + \underbrace{\left[\begin{array}{c} 0 \\ \frac{1-\rho}{\beta} \end{array}\right]}_{\text{Loadings}} \underbrace{\left[\begin{array}{cc} 1 & -\beta \end{array}\right]}_{\text{Cointegration vector}} \left[\begin{array}{c} V_{t-1} \\ R_{t-1} \end{array}\right] + \text{Shocks} \tag{6}$$

In plain English, this representation implies that the system's adjustment towards its long-run equilibrium takes place *via* movements in the short rate, with no reaction of M1 velocity to disequilibria. This is because in the same way as (rescaled) consumption is, to a first approximation, the common stochastic trend in the bivariate system for GDP and consumption, (rescaled) M1 velocity is, likewise, the common stochastic trend in the system for velocity and the short rate.

3.2.2 Implications

Since M1 velocity is the inverse of the demand for M1 balances as a fraction of GDP, the fact that, to a first approximation, it only reacts to permanent shocks to the short-term nominal rate implies that economic agents, in allocating their wealth between non interest-bearing M1 and interest-bearing assets, react almost exclusively to permanent shocks to the opportunity cost of M1, whereas they essentially ignore transitory shocks. The implication is that, in the same way as consumers disentangle

[10] As shown in Benati's (2020) Online Appendix B, the Selden-Latané specification is a special case of the 'money in the utility function' framework pioneered by Miguel Sidrauski (1967*a*, 1967*b*). By the same token, Benati *et al.* (2021) derive (5) within a generalized Baumol-Tobin framework in which agents are subject to an upper limit on how much they can borrow.

[11] This is also discussed in this paper's Online Appendix B.3.

[12] Notice, once again, the close similarity with consumption and GDP: as pointed out by Cochrane (1994*a*) in the previous quotations, consumption is, likewise, the (observed) stochastic trend of GDP.

permanent and transitory income shocks and only react to the former, economic agents perform a *permanent-transitory decomposition of nominal interest rates*, and only react to the permanent component.[13]

3.3 Estimating the natural rate of interest

This suggests that, defining the real natural rate of interest as the unit root component of the *ex post* real short-term rate (see below), there is in fact a simple and straightforward way of estimating it.

Estimating the nominal natural rate First, we need to estimate the nominal natural rate by exploiting the fact that M1 velocity is, up to a scale factor, its stochastic trend. This can be accomplished, e.g., in the same way as Cochrane (1994*a*) estimated the permanent component of GNP (see his Figure III), i.e. based on a cointegrated VAR for the two series identified *via* long-run restrictions. An alternative, and much simpler approach involves projecting the short-term nominal (monetary policy rate) onto M1 velocity *via* a simple OLS regression.[14] The rationale for this is that, since $R_t = R_t^P + R_t^T$ and $V_t = \alpha + \beta R_t^P + \eta_t$, the OLS regression

$$R_t = a + bV_t + e_t, \tag{7}$$

with e_t being the residual, is a cointegrating regression, which implies that the estimator of b is super-consistent. In turn this implies that, for samples of typical size, b (and therefore also a) are likely reliably estimated, which is of obvious, paramount importance within a policy context.

In practice, the resulting estimate of the nominal natural rate,

$$\hat{R}_t^N = \hat{a}_{OLS} + \hat{b}_{OLS} V_t \tag{8}$$

is typically close to that produced by the alternative approach based on cointegrated SVARs identified *via* long-run restrictions. In what follows I will therefore only report in the main text the evidence based on the simpler approach, whereas the corresponding evidence based on cointegrated SVARs is reported in the Online Appendix.

Simple evidence on the reliability of this approach is provided in Appendix C (see in particular Figure C.1), where I compute a transitory component of post-WWII U.S. GDP by projecting log real GDP onto log real consumption[15]. Two main

[13] No existing model of money demand exhibits this feature. In fact, no model of money demand—from the classic analyses of Baumol and Tobin on—even distinguishes between permanent and transitory variation in the opportunity cost of money.

[14] Another possibility is to use the DOLS estimator proposed by Stock and Watson (1993). Although in this paper I only report and discuss results based on the simple OLS estimator, a very similar set of results based on the DOLS estimator is available upon request.

[15] I.e., I estimate (7) with velocity and the short rate replaced by log real consumption and log real GDP. The two series are the same discussed in footnote 7.

findings emerge from this exercise. First, the estimated transitory component of GDP captures remarkably well the peaks and troughs of the post-WWII U.S. business cycle as established by the NBER Business-Cycle Dating Committee. Second, this methodology interprets a sizeable portion of the fall in output associated with the Great Recession as permanent. As I discuss in Appendix C, simple but powerful corroborating evidence that this may in fact had been the case is provided by a comparison between the actual evolution of GDP and consumption, which since 2012 have closely tracked each other, and the forecast of the Hodrick-Prescott GDP trend computed based on data up to the end of 2004: at the end of 2019 for both series the shortfall had been equal to about 12 per cent.[16]

Estimating the real natural rate Since conventional monetary policy involves the manipulation, on the part of the central bank, of a nominal short-term interest rate, having a reliable estimate of the nominal natural rate is of obvious interest in itself. On the other hand, the focus of much of the recent macroeconomic debate has been on the *real* natural rate, due, first and foremost, to concerns about the limitations imposed by the Zero Lower Bound (ZLB) on conventional monetary policy; secular stagnation; and the future evolution of income inequality. Based on an estimate of the nominal natural rate, in order to obtain the corresponding estimate of the real natural rate it is necessary to take a stand on the integration properties of inflation. Basic economic logic suggests indeed that, in general, R_t^P should be driven by (i) permanent inflation shocks (*via* the Fisher effect) and (ii) permanent shocks to the real natural rate of interest, that is,

$$R_t^P = \pi_t^P + r_t^N, \tag{9}$$

where π_t^P is the permanent component of inflation, and r_t^N is the real natural rate of interest. Expression (9), together with (1) and the corresponding permanent-transitory decomposition for inflation, i.e. $\pi_t = \pi_t^P + \pi_t^T$, implies that $R_t - \pi_t = r_t^N + (R_t^T - \pi_t^T)$, so that the real natural rate of interest is the permanent component of the *ex post* real short-term rate.

Further, expression (9) logically implies that, under monetary regimes that had made, or make inflation I(0)—such as those based on metallic standards, or inflation-targeting regimes (see Benati, 2008)—so that π_t^P=0, permanent shifts in M1 velocity should *uniquely* reflect permanent fluctuations in the natural rate of interest, so that, e.g., $V_t = \alpha + \beta r_t^N + \eta_t$. Under these regimes, an estimate of the real natural rate can therefore be obtained simply by subtracting from the estimated nominal natural rate either inflation's sample average, or the inflation target.[17] If, on the other hand,

[16]The fact that this has equally held for *both* GDP *and* consumption logically suggests that the shortfalls are permanent: otherwise, by the Permanent Income Hypothesis, consumption would be close to the HP trend.

[17]Which of the two is the most appropriate depends on the credibility of the central bank's inflation target. If it is very credible, it is more appropriate to subtract the target, rather than inflation's sample average.

over the sample period inflation has been I(1),[18] so that $\pi_t^P \neq 0$, in order to compute the real natural rate it is necessary to purge the nominal natural rate of permanent inflations shocks. This can be accomplished (e.g.) based on a cointegrated SVAR for M1 velocity, the short rate, and inflation (and possible other series) identified *via* long-run restrictions. Although this is in principle straightforward, in what follows I will not pursue this avenue since, as shown by Benati (2008), and as I will confirm in Section 5 based on samples extending up to 2019Q4, under the current monetary regimes, that were introduced in the 1990s,[19] inflation has been I(0).

I now turn to discussing the estimates of the nominal natural rate obtained by projecting the short rate onto M1 velocity *via* a simple OLS regression.

4 Estimating the Nominal Natural Rate

Figure 3 shows the simple estimate of the nominal natural rate obtained by projecting the short-term nominal (monetary policy) rate onto M1 velocity—i.e. $\hat{R}_t^N$ in expression (8), which is computed by estimating equation (7) *via* OLS—together with 1- and 2-standard deviations bootstrapped confidence bands. Figure 4 shows the simple estimate of the deviation of the short rate from the nominal natural rate together with the detrended unemployment rate,[20] whereas Figures A.2-A.4 in the Online Appendix show, respectively, the deviation of the short rate from the nominal natural rate with bootstrapped confidence bands; the fraction of bootstrap replications for which the deviation has been negative; and the fraction of bootstrap replications for which the nominal natural rate is estimated to have been negative (together with the corresponding fractions for the real natural rate we will discuss in Section 6).[21] Figure A.5 in the Online Appendix reports, based on Wu and Xia's (2016) 'shadow rates', the same evidence as in Figures A.2-A.3 for the Euro area, the U.K., and the U.S.. The corresponding evidence based on cointegrated SVARs is reported in Figure A.6-A.10 in the Online Appendix.[22] In both sets of figures, confidence bands have been computed by bootstrapping as in Cavaliere *et al.* (2012) the cointegrated VECM for M1 velocity and the short rate (or the shadow rate) estimated *via* Johansen's esti-

[18] Benati (2008) shows that, historically, this has been the case only for sample periods dominated by the Great Inflation episode.

[19] As I discuss in Section 4, for the United States I consider the period following the break in the mean of inflation identified by Levin and Piger (2004), in 1992Q2.

[20] I detrended the unemployment rate *via* the band-pass filter proposed by Christiano and Fitzgerald (2003), by removing all components associated with cycles slower than 30 years. In doing this I consider for each country, the longest sample for which the unemployment rate has been available: e.g., although for the Euro area my analysis focuses on the period since the start of European Monetary Union, in January 1999, I detrend the unemployment rate based on data since 1970Q1.

[21] In what follows I will interchangeably refer to such fractions as the 'probabilities that the natural rates had been negative'.

[22] In all figures, estimates have been smoothed *via* a centered 4-quarters moving-average in order to remove some high-frequency noise.

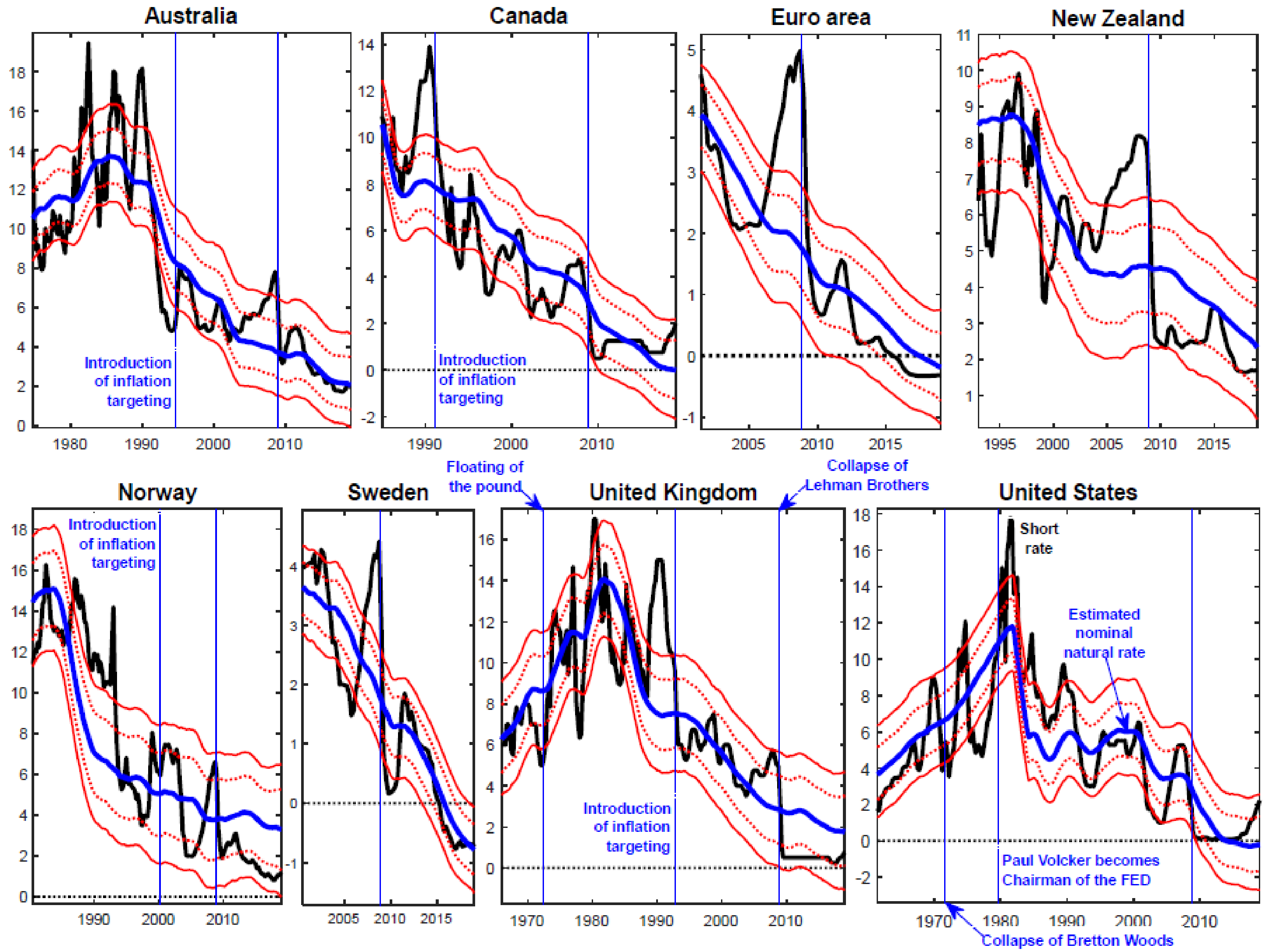

Figure 3 Estimates of the nominal natural rate computed by projecting the short rate on M1 velocity (with 1- and 2-standard deviations bootstrapped confidence bands)

mator (as described in Hamilton, 1994) imposing one cointegration vector. I set the number of bootstrap replications to 10,000. Based on each bootstrapped, artificial sample j, with $j = 1, 2, ..., 10{,}000$, I then perform exactly the same operations I previously performed based on the actual data. When estimating the nominal natural rate by projecting the short rate onto M1 velocity, I therefore estimate (7) based on the bootstrapped short rate and bootstrapped velocity, i.e. I run the OLS regression $R_t^{B,j} = a + bV_t^{B,j} + e_t^{B,j}$, where $R_t^{B,j}$ and $V_t^{B,j}$ are the bootstrapped short rate and velocity for replication j. This produces an estimate of the nominal natural rate for bootstrap replication j, i.e. $\hat{R}_t^{N,B,j} = \hat{a}_{OLS}^{B,j} + \hat{b}_{OLS}^{B,j} V_t^{B,j}$, and of the associated transitory component, $\hat{R}_t^{T,B,j} = \hat{R}_t^{N,B,j} - \hat{a}_{OLS}^{B,j} - \hat{b}_{OLS}^{B,j} V_t^{B,j}$. When working with cointegrated VARs identified *via* long-run restrictions, on the other hand, I compute $\hat{R}_t^{T,B,j}$ as in Blanchard and Quah (1989), i.e. by re-running history only conditional on transitory shocks. In this way, based on either approach I build up the bootstrapped distribution of the transitory component of the short (or shadow) rate, which I then use in order to compute confidence bands for the transitory component, and therefore, as a result, also for the permanent component.

The following main results emerge from the two sets of figures:

(i) as already mentioned, the simple, projection-based methodology produces results that are qualitatively the same, and quantitatively close to those produced by the alternative approach based on cointegrated SVARs. For reasons of simplicity and especially robustness,[23] in what follows I will uniquely focus on the results produced by the simpler approach.

(ii) Whereas for the U.S. using shadow rates does not produce materially different estimates, for the U.K. and especially the Euro area this is not the case (see Figures A.5 and A.9 in the Online Appendix). This reflects the fact that, for the latter two countries, the difference between the shadow rate and the official monetary policy rate has been significantly greater than for the U.S.. For these three countries, in what follows I will exclusively focus on the results based on the official monetary policy rate, but the full sets of results based on the shadow rates are available upon request.

(iii) As one would expect, for all countries the estimated nominal natural rate behaves as a very low-frequency component of the short-term rate.

(iv) As shown in Figure 4, the nominal rate gap—defined as the difference between the short rate and the nominal natural rate, i.e. as $R_t - \hat{R}_t^N$ in (8)—exhibits a strong negative contemporaneous correlation with the detrended unemployment rate. This

[23] If M1 velocity were *exactly* equal to the nominal natural rate, the projection-based approach would exactly capture the latter. Under these conditions the SVAR-based approach could not improve upon this estimate, because the projection-based approach would rely on an observed linear transformation of the natural rate. To the extent that, as documented by Benati (2020), in fact we are close to such ideal situation, this argument approximately holds. On the other hand, a permanent-transitory decomposition based on a cointegrated SVAR is significantly more complex than a simple OLS regression, and as such the results it produces are likely more sensitive to issues such as lag order selection, and initial conditions (i.e., when the sample starts).

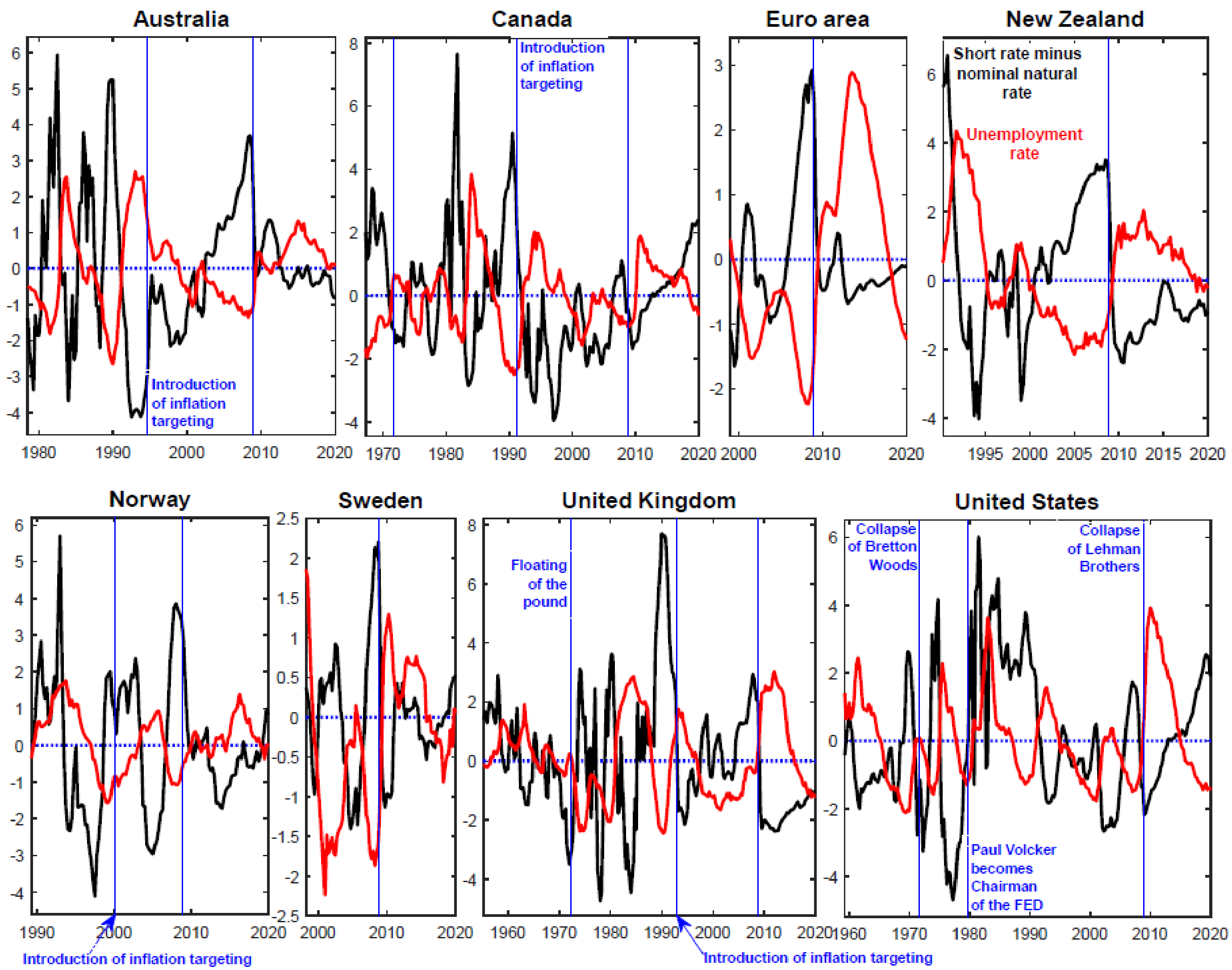

Figure 4 The nominal rate gap and the detrended unemployment rate

is in line, e.g., with the evidence in King and Watson (1996, pp. 38-39 and Figure 2) that '[the band-pass filtered cyclical components of] nominal interest rates and output are positively correlated', and it has a straightforward interpretation in terms of counter-cyclical monetary policy.[24]

(v) Before the collapse of Lehman Brothers, the probability that the nominal natural rate had been negative had consistently been (close to) *nil*. Since then, however, it has materially increased in Canada, the Euro area, Sweden, and the U.S., whereas it has exhibited little variation in the remaining countries. In particular, at the end of the sample the probability was equal to 95 per cent in Sweden, 50 per cent in Canada, and around 60 per cent in both the Euro area and the U.S..

I now turn to discussing the integration properties of inflation.

5 Monetary Regimes and the Stochastic Properties of Inflation

Table C.1 in Online Appendix C reports results from tests for multiple breaks at unknown points in the sample in the mean of inflation based on the methodology proposed by Bai and Perron (1998, 2003).[25] For Australia, Canada, New Zealand, Norway, Sweden, and the United Kingdom I focus on the sample period since the introduction of inflation targeting;[26] for the Euro area I consider the period since the start of European Monetary Union, in January 1999; and for the United States the period following the break in the mean of inflation identified by Levin and Piger (2004), in 1992Q2. The null hypothesis of no breaks in the mean of inflation cannot be rejected for any country.

[24]Interestingly, for the U.S. the relationship between the nominal rate gap and the detrended unemployment rate had been put temporarily off kilter by the introduction of Money Market Deposits Accounts (MMDAs) in 1982Q4. After a brief period of adjustment, however, the relationship strongly reasserted itself since the second half of the 1980s. This confirms the meaningfulness of working with Lucas and Nicolini's (2015) 'New M1' aggregate: what it suggests is that in fact New M1 is the equivalent, for the period since the 1980s, of the standard M1 aggregate for the previous period. Benati (2021*b*) presents additional evidence on this based on a comparison between the evolution of M1 velocity and of long-term interest rates.

[25]In performing the tests I exactly follow the recommendations of Bai and Perron (2003), with the only difference that, instead of relying on the asymptotic critical values tabulated in Bai and Perron (1998), I bootstrap the p-values *via* the procedure proposed by Diebold and Chen (1996), setting the number of bootstrap replications to 10,000.

[26]In Canada, New Zealand, Norway, Sweden, and the United Kingdom inflation targeting was was introduced, respectively, in February 1991, February 1990, March 2001, January 1993, and October 1992. As for Australia, which never formally announced an inflation target, I consider the period since mid-1994 (specifically, since 1994Q3), when the *Reserve Bank of Australia* started to target inflation *de facto*.

Table 1 Exploring inflation persistence by monetary regime						
		Bootstrapped p-values for				Hansen MUB
Country	*Period*	*p*=2	*p*=4	*p*=6	*p*=8	estimate of ρ^b
		I: Regimes with clearly-defined nominal anchors				
Australia	1994Q3-2019Q4	0.0012	0.0148	0.0908	0.3618	0.28 [0.05 0.51]
Canada	1991Q1-2019Q4	0.0000	0.0002	0.0002	0.0009	-0.08 [-0.35 0.19]
Euro area	1999Q1-2019Q4	0.1015	0.2230	0.1919	0.3032	0.66 [0.46 0.88]
New Zealand	1990Q1-2019Q4	0.0000	0.0001	0.0060	0.0029	-0.21 [-0.51 0.10]
Norway	2001Q2-2019Q4	0.0016	0.0047	0.0616	0.1582	0.22 [-0.09 0.52]
Sweden	1998Q1-2019Q4	0.0000	0.0210	0.0640	0.1115	0.30 [-0.15 0.78]
United Kingdom	1992Q4-2019Q4	0.0000	0.0001	0.0030	0.0060	-0.40 [-0.66 -0.16]
United States	1992Q2-2019Q4	0.0081	0.0501	0.0704	0.0090	0.49 [0.35 0.64]
		II: Previous periods				
Australia	1972Q2-1994Q2	0.1627	0.7399	0.5881	0.2010	1.01 [0.78 1.07]
Canada	1967Q2-1990Q4	0.2220	0.4366	0.3495	0.2957	0.90 [0.73 1.03]
Euro area	1970Q2-1998Q4	0.4598	0.4598	0.7095	0.8493	1.01 [0.92 1.04]
Norway	1978Q2-2001Q1	0.0014	0.0272	0.1735	0.1527	0.49 [0.19 0.81]
United Kingdom	1955Q2-1992Q3	0.0410	0.1942	0.1752	0.2245	0.87 [0.74 1.02]
United States	1959Q2-1992Q1	0.3491	0.3266	0.3062	0.3316	0.92 [0.85 0.99]

[a] Based on 10,000 bootstrap replications. [b] With 90% bootstrapped confidence interval.

Table 1 reports bootstrapped p-values[27] for Elliot, Rothenberg, and Stock (1996) unit root tests for inflation, together with Hansen (1999) 'grid bootstrap' median-unbiased (MUB) estimates of the sum of the autoregressive coefficients (ρ) in AR(p) representations for inflation.[28] In both cases I set the number of bootstrap replications to 10,000. As for the sample periods, I consider both the previously mentioned monetary regimes featuring clearly-defined nominal anchors,[29] and, depending on data availability, the previous periods.

The evidence in Table 1 confirms the findings in Benati (2008). In particular, for regimes with clearly-defined nominal anchors,

(i) the point estimates of ρ produced by Hansen's procedure range between -0.40 and 0.66, and the upper limits of their bootstrapped 90%-coverage confidence interval range between -0.16 and 0.88: based on Hansen's procedure there is no evidence that, under these regimes, inflation may have been I(1).

(ii) By the same token, based on Elliot *et al.*'s tests a unit root in inflation is

[27] p-values have been computed by bootstrapping 10,000 times estimated ARIMA(p,1,0) processes.

[28] For Hansen's (1999) procedure, I select the lag order p as the maximum between the lag orders selected by the Schwartz and Hannan-Quinn criteria, and I set the 'step' in the grid of possible values for ρ to 0.01.

[29] Strictly speaking, the U.S. Federal Reserve introduced an inflation target only in January 2012. In what follows I consider the entire period since 1992Q2 because, even before the introduction of a formal target, the Fed's monetary policy had been characterized since the end of the Volcker disinflation by a strong, although generic committment to price stability.

strongly rejected for Canada, New Zealand, the United Kingdom, and the United States, and for Australia, Norway, and Sweden it is rejected, at the 10 per cent level, for all lag orders except p=8. Only for the Euro area a unit root cannot be rejected for any lag order.

For the previous periods, which had been largely dominated by the Great Inflation episode, the opposite is true. Starting from Hansen's MUB estimates of ρ, the point estimate is borderline explosive for Australia and the Euro area, and for four countries (Australia, Canada, Euro area, and United Kingdom) the 90 per cent confidence interval includes 1, whereas for the United States, with an upper bound equal to 0.99, this is almost the case. Likewise, based on Elliot *et al.*'s tests the null of a unit root cannot be rejected for any lag order for Australia, Canada, the Euro area, and the United States, whereas evidence is mixed for Norway, and for the United Kingdom it can be rejected only for p=2.

These results confirm Benati's (2008) main finding that whereas for sample periods dominated by the Great Inflation experience it is typically not possible to reject the null hypothesis of a unit root in inflation, under monetary regimes, such as inflation targeting, featuring a clearly-defined nominal anchor (or, in the case of the United States before the introduction of an inflation target, a generic, but strong and credible committment to keeping inflation low and stable), inflation has consistently been I(0). Under this respect, the results from Elliot *et al.*'s tests for the Euro area should be quite heavily discounted for two reasons. First, the visual evidence in Figure A.1 in the Online Appendix clearly suggests that the collapse of Lehman Brothers, which unleashed the most violent phase of the Great Recession, was associated with a dramatic, highly persistent, but ultimately transitory fall in Euro area inflation, from an average of 2.01 per cent over the period 1999Q1-2008Q3,[30] to 1.05 per cent over the period 2008Q4-2017Q3. Over the subsequent period inflation has progressively converged towards 2 per cent. A possible, and (I would argue) plausible interpretation of the lack of rejection of a unit root for the period 1999Q1-2019Q4 is therefore that it is the figment of a very large negative transitory shock, which in a small sample can easily be confused for a permanent one. Second, in spite of such persistent downard shift in inflation, inflation expectations (as measured by the ECB's Survey of Professional Forecasters) have remained well-anchored,[31] thus suggesting that agents have correctly interpreted the shift as temporary.

In what follows I will therefore work under the assumption that, for the sample periods reported in Table 1 as 'regimes with clearly-defined nominal anchors', inflation has consistently been I(0). I now turn to discussing the estimates of the real natural

[30]In fact, in line with Benati (2008), for the period 1999Q1-2008Q3 the bootstrapped p-values for Elliot *et al.*'s tests are equal to 0.1077, 0.0416, 0.0249, and 0.0134, and Hansen's (1999) MUB estimate of ρ is 0.41 [0.02 0.82].

[31]See in particular Figure A.3 in the Online Appendix of Benati (2020). The figure shows the inflation forecasts from the ECB's Survey of Professional Forecasters at three alternative horizons, 1-, 2-, and 5-years ahead. Over the entire period since 1999Q1, the 5-years ahead forecast has fluctuated between 1.8 and 2.0 per cent.

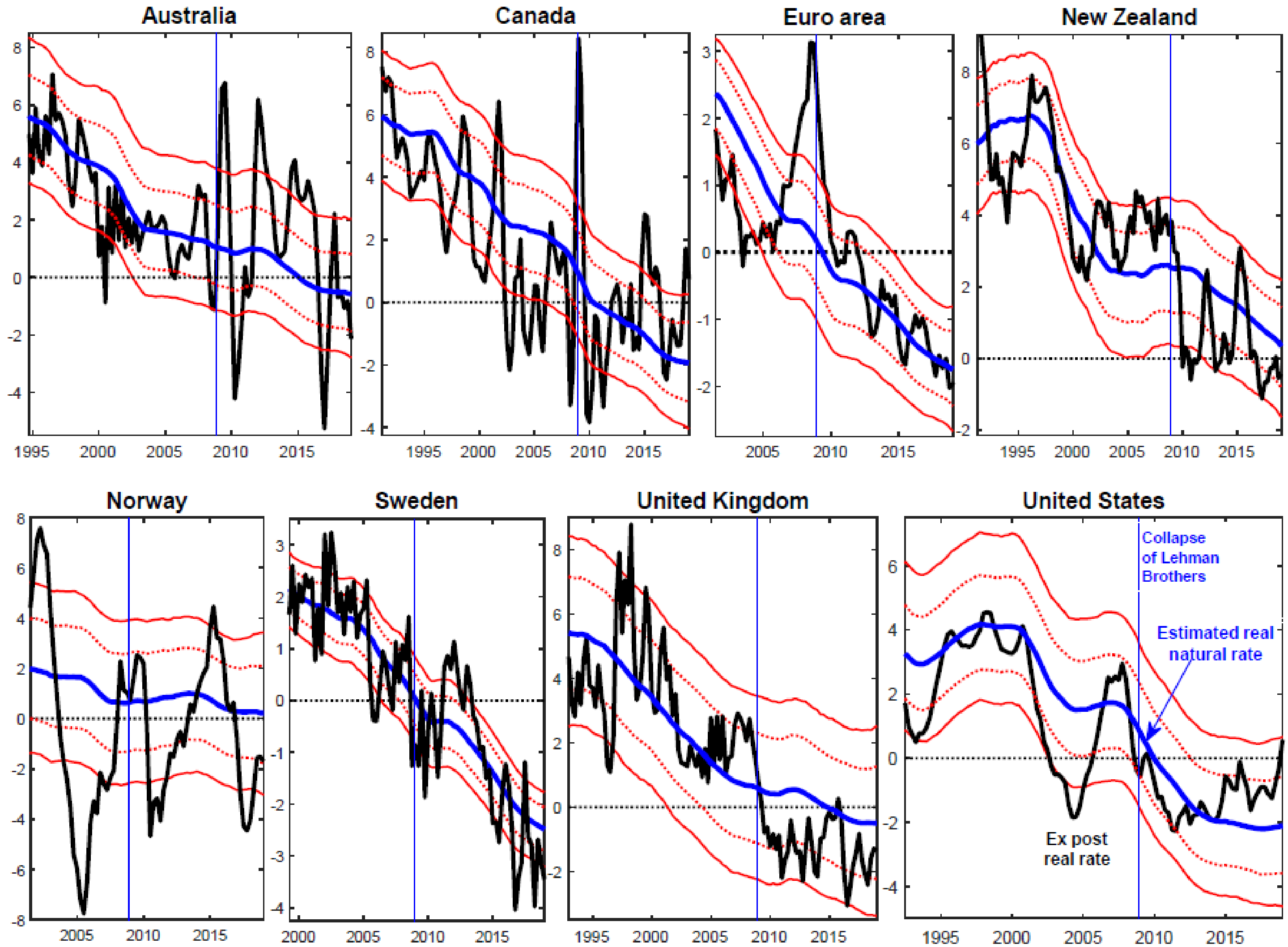

Figure 5 Estimates of the real natural rate for monetary regimes making inflation I(0) (with 1- and 2-standard deviations bootstrapped confidence bands) computed by projecting the short rate on M1 velocity

rate for such monetary regimes.

6 Estimating the Real Natural Rate

Figure 5 shows, for monetary regimes with clearly-defined nominal anchors,[32] the *ex post* short-term real rate, computed as the difference between the short-term nominal rate and inflation, together with the estimated real natural rate, which has been computed by subtracting inflation's sample average from the nominal natural rate estimates shown in Figure 3.[33] Figure A.10 in the Online Appendix shows the corresponding estimates based on cointegrated SVARs identified *via* long-run restrictions. For all countries, the estimated real natural rate behaves, as expected, as a very low-frequency component of the *ex post* short-term real rate. The following main results emerge from the two figures:

(i) consistent with both conventional wisdom, and previous evidence—see in particular Holston *et al.* (2017), and Fiorentini *et al.* (2018)—in all countries natural rate estimates have been consistently trending downwards over the entire sample period. The decrease has been especially marked for Australia, New Zealand, the U.K., and Canada: since the first half of the 1990s, the point estimate of the natural rate has fallen by about 6 per cent for the first three countries, and by about 8 per cent for the fourth. By the same token, in the U.S. it has fallen by about 6 percentage points since the peak of 4.1 per cent reached in the second half of the 1990s around the time of the 'New Economy', whereas in both the Euro area and Sweden the decrease since the start of the new millennium has been equal to about 4 percentage points.

(ii) Different from the corresponding results for the nominal natural rates discussed in point (v) of Section 4, in several countries the probability that the real natural rate had been negative had already been increasing before the collapse of Lehman Brothers. This is the case especially for the Euro area, Norway, and the U.K.. Following Lehman's collapse the probability has markedly increased in all countries except Norway. In particular, at the end of the sample the probability was equal to 100 per cent in both the Euro area and Sweden, whereas in Canada and the U.S. it was slightly greater than 90 per cent. This evidence provides support to the conjecture, first advanced by Summers (1991), that following large negative shocks the natural rate might fall below zero.

(iii) Further, in several countries the estimates are quite sobering, especially towards the end of the sample. In the U.S., for example, the point estimate has been equal to about -2 per cent since 2014, whereas in Canada, the Euro area and Sweden it has reached, in 2019, -1.9, -1.7, and -2.4 per cent, respectively. The only two countries for which in 2019 the point estimate was still (barely) positive were New

[32] The sample periods are the same reported in Table 1.

[33] Once again, in order to eliminate some low-frequency variation, all series have been smoothed *via* a 4-quarter centered moving average.

Zealand and Norway.

These estimates are very similar to those produced by Fiorentini *et al.* (2018) based on a modified (and, they argue, superior) version of the methodology originally proposed by Laubach and Williams (2003), and more recently used, e.g., by Laubach and Williams (2016) and Holston *et al.* (2017). For example, in Fiorentini *et al.*'s (2018) Figure 10, the U.S. natural rate decreased from 2.5-3 per cent in the second half of the 1990s to about -2 per cent in 2016, whereas that for the Euro area fell from 2 per cent in 2000 to about -1 per cent in 2016. These figures are very close to those in Figure 5 in the present work. On the other hand, my estimates are lower than those found in Holston *et al.* (2017), but based on Fiorentini *et al.*'s (2018) arguments those estimates should be regarded as less reliable.

Estimates of the real natural rate as low as those in Figure 5, as well as in Fiorentini *et al.* (2018), raise an obvious question: Are they plausible? Could the real natural rate truly sink *that* low? This question is best addressed by focusing on (*i*) the relationship between the natural rate and the *ex post* real rate, and (*ii*) the behavior of GDP and inflation over the sample period. Let us consider for example Sweden, with an estimated natural rate of -2.5 per cent at the end of 2019. At first sight, this number might appear to some researchers as manifestly absurd. It becomes however much less absurd, and much more plausible, when one considers that since Lehman's collapse (and in fact since the beginning of the millennium) the natural rate has closely tracked the dramatic decrease in the *ex post* real rate: this suggests that *on average* the *Riksbank*'s monetary policy has been broadly neutral, and that the fall in the *ex post* real rate it has engineered by decreasing the monetary policy rate was simply a reaction to the progressive fall in the natural rate. The evolution of prices and output is consistent with this: since the 2008-2009 recession annual inflation and GDP growth have both been broadly stable, the former slowly increasing from about 1 per cent in early 2010 to 2.5 per cent at the end of 2019, and the latter fluctuating around an average of about 2 per cent. A very similar argument can be made for the remaining countries. This suggests that central banks have been broadly tracking the natural rate, and that the progressive decreases in *ex post* real rates across the board have simply reflected the underlying fall in the natural rates. In turn, this suggests that the estimates in Figure 5 are likely plausible.[34]

Finally, it is worth highlighting how, in line with Taylor (2008, 2009), for the U.S. a comparison between the *ex post* real rate and the estimated natural rate suggests that monetary policy had been highly expansionary during the years immediately preceding the outbreak of the financial crisis. In particular, in 2004 the *ex post* real rate had been below the natural real rate, on average, by about 300 basis points.

I now turn to discussing the advantages of the methodology I am advocating compared to existing approaches.

[34]It is also worth recalling that DSGE-based estimates are often much more volatile. For example, in Barsky *et al.*'s (2014) Figure 1 the U.S. natural rate has fluctuated, since the early 1990s, between about -7 and about 12 per cent, i.e. over a range of *nearly 20 percentage points*.

7 Advantages of the Proposed Approach

Compared to existing approaches to the estimation of the natural rate, the one proposed herein features two advantages, which I discuss in turn.

7.1 Under monetary regimes making inflation I(0) the real natural rate is *observed*

As discussed in Section 3.3, under monetary regimes making inflation I(0)—so that, in (9), π_t^P=0—permanent shifts in M1 velocity *uniquely* reflect, to a first approximation, permanent fluctuations in the real natural rate of interest. In fact, as long as η_t in expression (4) is 'small', $V_t \simeq \alpha + \beta r_t^N$, so that $r_t^N \simeq (V_t - \alpha)/\beta$: in plain English, under such regimes the real natural rate of interest is, up to a linear transformation, *observed*. An immediate implication is that a consistent decrease in M1 velocity under a monetary regime causing inflation to be I(0)—such as the protracted fall in velocity that has been going on in several inflation-targeting countries since the early 1990s—provides *direct evidence* of a fall in the real natural rate of interest.

The fact that the approach I am advocating herein relies on a series that, under monetary regimes making inflation I(0), is essentially a linear transformation of the real natural rate highlights a stark difference with existing approaches (either DSGE- or non-DSGE based), none of which exploits a series with such a strong informational content for the real natural rate.

7.2 Computing high-frequency estimates of the natural rate

Since interest rates are observed on a continuous basis, and M1 is observed (at least) at the weekly frequency, all a researcher needs in order to compute high-frequency estimates of the nominal and real natural rates of interest is a corresponding high-frequency estimate of nominal GDP. Interpolating quarterly GDP to the monthly frequency[35] is routinely done in the literature (for the United States, see e.g. Bernanke, Gertler, and Watson, 1997, and Stock and Watson, 2012). The recent work of (e.g.) Lewis, Mertens, and Stock (2020) about tracking the economic impact of the COVID pandemic has shown how to perform a similar interpolation at the weekly frequency.[36] Based on a weekly estimate of nominal GDP, and weekly observations for M1 and nominal interest rates, a central bank could therefore, in principle, produce weekly estimates of nominal and real natural rates.

Figure 6 presents estimates of nominal and real natural rates at the monthly frequency for Canada, the Euro area, the United Kingdom and the United States,

[35] To the very best of my knowledge, Canada and the U.K. are the only countries producing official monthly estimates of real GDP. U.K. estimates start however in 1997, so that in the present work I have relied on the unofficial estimates from NIESR (for details, see Online Appendix A).

[36] The be precise, Lewis *et al.* (2020) focus on real GDP, but their methodology can obviously also be applied to nominal GDP.

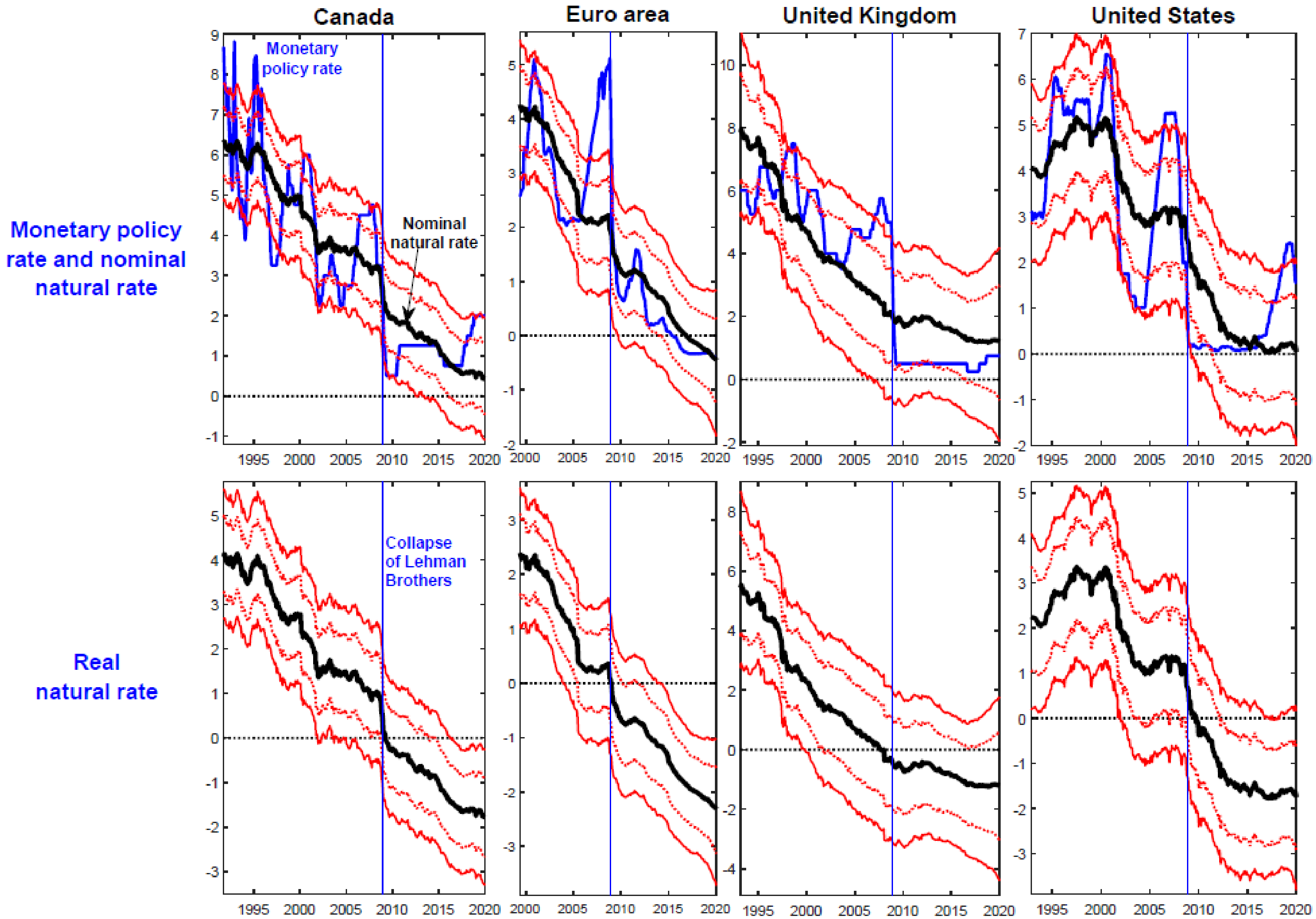

Figure 6 Monthly estimates of the nominal and real natural rate of interest for monetary regimes making inflation I(0), computed by projecting the short rate on M1 velocity (with 1- and 2-standard deviations bootstrapped confidence bands)

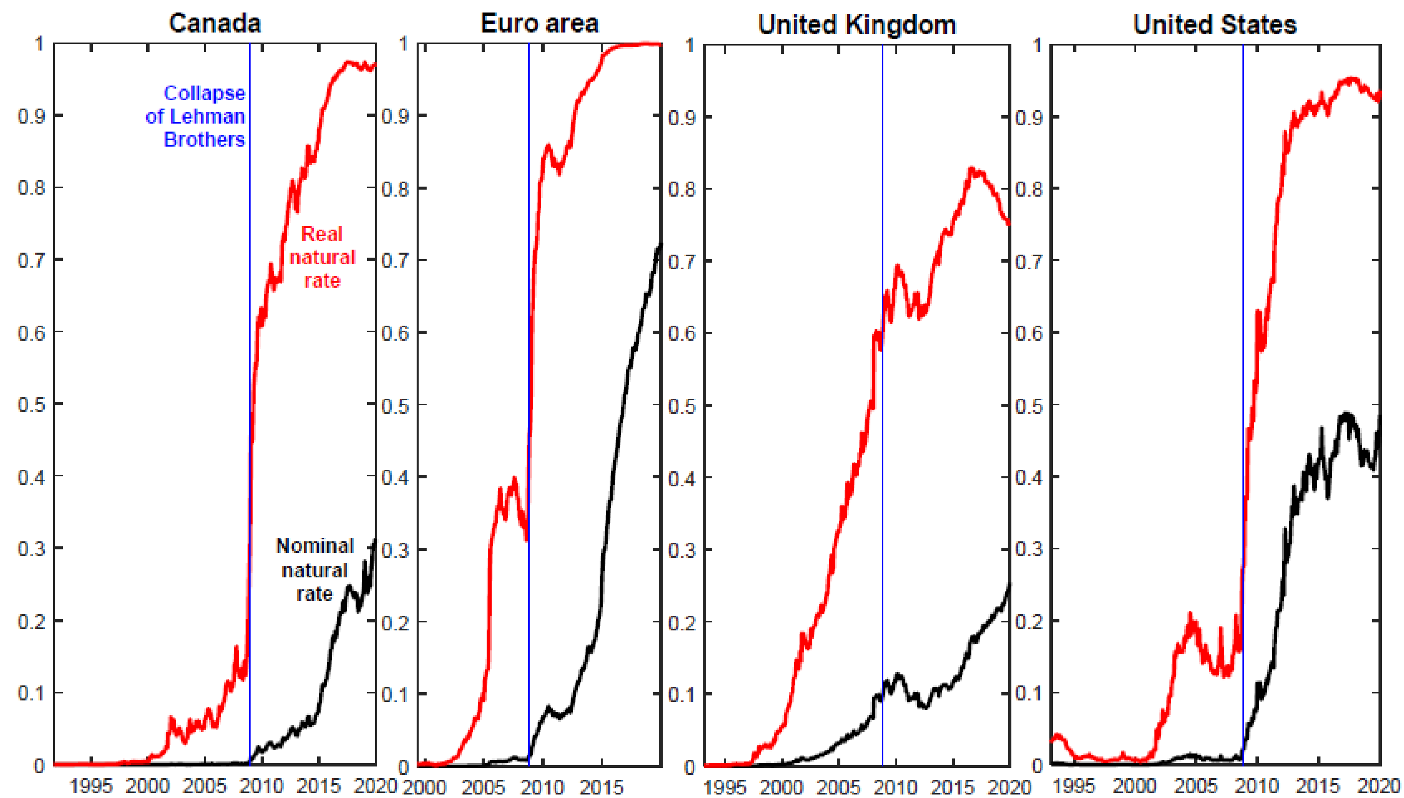

Figure 7 Evidence based on monthly data: fractions of bootstrap replications for which the nominal and the real natural rates of interest are estimated to have been negative

whereas Figure 7 reports the fractions of bootstrap replications for which the nominal and real natural rates are estimated to have been negative.[37] Table 2 reports, for selected months around the collapse of Lehman Brothers, point estimates of the real natural rate, together with the fractions of bootstrap replications for which the real natural rate is estimated to have been negative. The methodology is exactly the same used in order to produce the estimates reported in Figures 3-5. Different from those estimates, however, those in Figure 6-7 have not been smoothed in any way. On the one hand, this causes the estimates to retain a small extent of high-frequency variation, which is obviously sub-optimal from a monetary policy perspective. On the other hand, however, this highlights in an especially stark way how, following the collapse of Lehman Brothers on September 15, 2008, in three countries out of four (Canada, the Euro area, and the United States) both the nominal and the real natural rates experienced sharp and sudden declines.

Table 2 Point estimates of the real natural rate, and probability that it had been negative, in the months around the collapse of Lehman Brothers

	Point estimates of the real natural rate				Probability that the real natural rate had been negative[a]			
	CA	EA	UK	US	CA	EA	UK	US
August 2008	0.90	0.35	-0.28	1.09	0.15	0,32	0.57	0.18
October 2008	0.60	0.08	-0.34	0.67	0.23	0,46	0.59	0.28
December 2008	0.09	-0.02	-0.50	0.38	0.46	0,51	0.63	0.37
June 2009	-0.12	-0.44	-0.45	0.11	0.56	0,73	0.62	0.46
December 2009	-0.23	-0.67	-0.68	-0.38	0.61	0,83	0.68	0.63

CA = Canada; EA = Euro area; UK = United Kingdom; US = United States. [a] Fraction of bootstrap replications for which the real natural rate is estimated to have been negative.

Focusing on the real natural rate, the decline is clearly apparent both from the estimates shown in the bottom row of Figure 6, and especially from the fractions of bootstrap replications reported in Figure 7, and from the figures in Table 2. In the *two months* from August to October 2008 —i.e., from one monthy before to one month after Lehman's collapse—the real natural rate declined in Canada, the Euro area, and the U.S. by -0.30, -0.27, and -0.42 per cent: these are quite remarkable decreases, corresponding to -1.80, -1.62, and -2.52 per cent on annual basis. For the U.K. the decrease, equal to -0.06 from August to October, was comparatively minor, but it still corresponded to a fall by -0.36 per cent on an annual basis. For Canada, the Euro area, and the U.S. the decrease from August to December 2008 was equal to -0.81, -0.37, -0.71 per cent, whereas the corresponding figures for the period up to

[37]The monthly data are discussed in Appendix C, and more extensively in Online Appendix A.3.

December 2009 had been respectively equal to -1.13, -1.02, and -1.47 per cent. Even for the comparatively less affected U.K., the decline from August 2008 to December 2009 had been equal to -0.40 per cent.

By the same token, the fractions of bootstrap replications for which the real natural rate is estimated to have been negative literally skyrocketed in October 2008 for both Canada and the Euro area, and it increased very sharply, although less dramatically, for the U.S.. For the U.K., on the other hand, the increase has been continuous over the entire sample period, and it had apparently been unaffected by Lehman's collapse.

I now turn to two additional applications of the proposed methodology.

8 Two Additional Applications

In this section I present two additional applications of the proposed methodology which should be regarded with some caution: the first because of the comparatively lower quality of pre-WWII data, and the second because of the idiosyncratic nature of the COVID shock.

8.1 The evolution of the natural rate during the Great Depression

Figure 8 shows the evolution of the nominal and real U.S. natural rates during the interwar period, based on the same methodology underlying the estimates reported in Figures 3-7.[38] The sample period, 1920Q1-1941Q3 is bookended by the end of World War I, and by Japan's attack on Pearl Harbor. With the partial exception of a temporary downward fall during the deep recession of 1921, the real rate of interest had been broadly stable during the entire decade of the 1920s. Between the October 1929 stockmarket crash and Roosevelt's inauguration in March 1933, however, it collapsed from 5.6 to 2.8 per cent. Although starting in 1933 the natural rate temporarily stabilized around 2.8-3.0 per cent, following (the mistake of) 1938 it fell by nearly an additional percentage point in the period immediately preceding the United States' entry into World War II.

These results naturally lend themselves to an admittedly imperfect comparison with Eggertsson (2008). There are two key tenets of Eggertsson's analysis: first, the onset of the Great Depression was caused by a dramatic fall in the real natural rate of interest; second, the recovery that followed Roosevelt's inauguration was not caused

[38]The data are described in detail in Online Appendix A.2.8. In short, the M1 aggregate is from Friedman and Schwartz (1963), the monetary policy rate is the Federal Reserve Bank of New York's discount rate, and nominal GNP and the GNP deflator are from Balke and Gordon (1986). Bootstrapped p-values for Elliot *et al.* tests for inflation for $p = 2, 4, 6, 8$ are equal to 0.0015, 0.0425, 0.0708, 0.0814, thus strongly rejecting the null of a unit root, so that the same logic used in Sections 6 and 7 can also be applied for the interwar period.

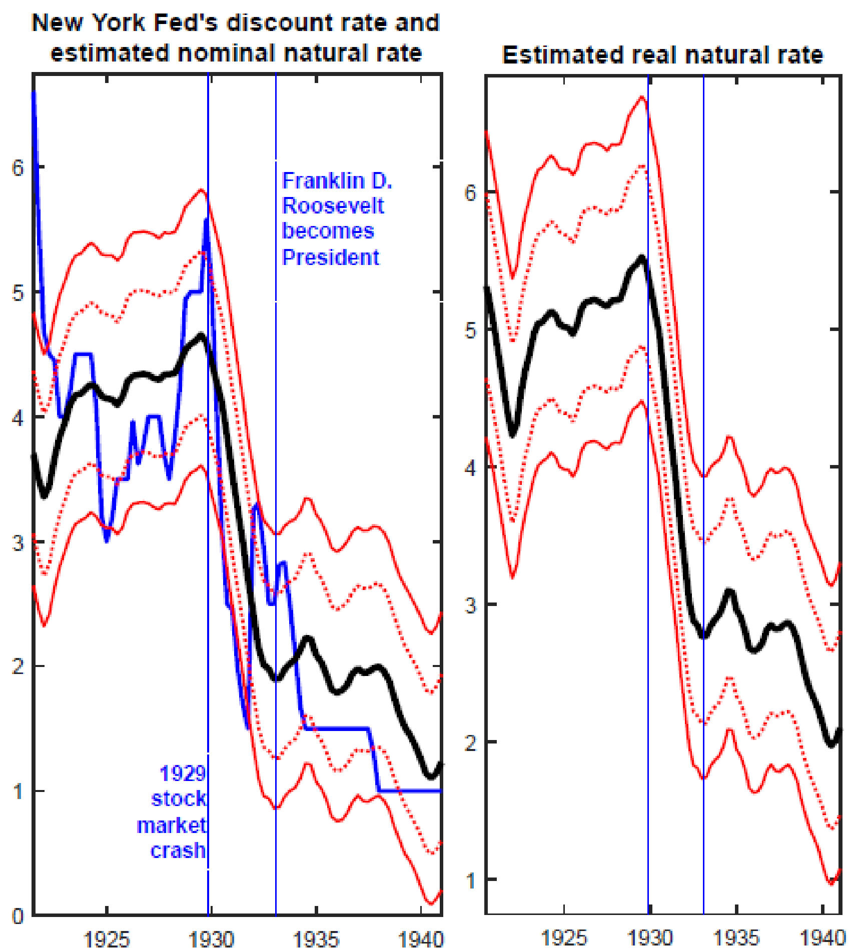

Figure 8 The evolution of the U.S. natural rate during the Great Depression

by a rebound of the natural rate around its previous level, but rather by the radical change in monetary and fiscal policies associated with the New Deal. The estimates in the right hand-side panel of figure 8 accord well with Eggertsson's analysis. On the one hand, between the crash of 1929 and Roosevelt's inauguration the natural rate decreased by about 2.8 percentage points. On the other hand, after March 1933 the natural rate did not increase, but it rather stabilized until 1938, and then it further collapsed.

8.2 Estimating the impact of the COVID shock on the natural rate

In order to avoid that the analyses of Sections 4, 6, and 7.2 be possibly distorted by the highly idiosyncratic nature of the COVID shock, I ended all of the samples there in 2019. In this section, on the other hand, I (very tentatively) attempt to apply the framework proposed herein to estimate the impact of the COVID shock on the natural rate. Although the unprecedented (in about a century) nature of the shock suggests to treat these results with significant *caveats*, on the other hand we have no reason to believe that fundamental laws of economics should somehow become 'suspended' during a pandemic. It is therefore of interest to see what the approach I am advocating has to say about the impact of COVID.

Table 3 Point estimates of the real natural rate for the months around the outbreak of COVID

	CA	EA	UK	US
January 2020	-1.63	-2.28	-1.99	-1.64
February	-1.69	-2.33	-2.03	-1.68
March	-2.08	-2.50	-2.34	-2.04
April	-2.50	-2.74	-2.88	-2.63
May	-2.45	-2.76	-2.85	-2.59
June	-2.39	-3.03	-2.43	-2.48

CA = Canada; EA = Euro area; UK = United Kingdom; US = United States.

Table 3 reports, for the same four countries in Table 2, point estimates of the real natural rate for the months around the outbreak of the pandemic. The main finding emerging from the table is that the impact of COVID on the natural rate has been broadly comparable to that of the collapse of Lehman Brothers. For the U.S., for example, the decrease between January 2020 (one month before the outbreak) and June had been equal to -0.84 per cent, whereas for Canada, the Euro area and the U.K. it had been equal, respectively, to -0.76, -0.75, and -0.44 per cent. Once again, it is important to stress the very tentative nature of these results and the significant *caveats* they are subject to. At the same time, taken at face value they suggest that the two crises had a very similar impact on the natural rate.

9 Conclusions

Since the early 1980s it has been conventional wisdom among macroeconomists and policymakers that monetary aggregates contain little useful information for monetary policy. In this paper I have shown that, in fact, a specific transformation of a monetary aggregate, the velocity of M1, contains crucial information about the evolution of the real natural rate of interest. Building upon the the insight that M1 velocity is the permanent component of nominal interest rates (see Benati, 2020), I have proposed a new and straightforward approach to estimating the natural rate of interest, which is conceptually related to Cochrane's (1994*a*) proposal to estimate the permanent component of GDP by exploiting the informational content of consumption. Under monetary regimes (such as inflation-targeting) making inflation I(0), the easiest way to implement the proposed approach is to (*i*) project the monetary policy rate onto M1 velocity—thus obtaining an estimate of the *nominal* natural rate—and then (*ii*) subtract from this inflation's sample average (or target), thus obtaining the *real* natural rate. More complex implementations based on structural VARs produce very similar estimates. Compared to existing approaches, the one proposed herein presents two key advantages: (1) under regimes making inflation I(0), M1 velocity is equal, up to a linear transformation, to the real natural rate, so that the natural rate is, in fact, *observed*; and (2) based on a high-frequency estimate of nominal GDP, the natural rate can be computed at the monthy or even weekly frequency. In the U.S., Euro area, and Canada the natural rate dropped sharply in the months following the collapse of Lehman Brothers. Likewise, the 1929 stock market crash was followed in the U.S. by a dramatic decrease in the natural rate.

References

Antolin-Diaz, J., T. Drechsel, and I. Petrella (2017), "Tracking the Slowdown in Long-Run GDP Growth", Review of Economics and Statistics, 99(2), 343-356

Bai, J. and P. Perron (1998), "Estimating and Testing Linear Models with Multiple Structural Changes", Econometrica, 66(1), 47-78

Bai, J. and P. Perron (2003), "Computation and Analysis of Multiple Structural Change Models", Journal of Applied Econometrics, 18(1), 1-22

Barsky, R. A. Justiniano and L. Melosi (2014), "The Natural Rate of Interest and Its Usefulness for Monetary Policy", American Economic Review, Papers and Proceedings, 104(5), 37-43

Benati, L. (2008), "Investigating Inflation Persistence Across Monetary Regimes", Quarterly Journal of Economics, 123(3), 1005-1060.

Benati, L. (2020), "Money Velocity and the Natural Rate of Interest", Journal of Monetary Economics, 116(December), 117-134

Benati, L. (2021*a*), "Did the Great Recession Impact Upon Trend Output Growth?", University of Bern, in progress

Benati, L. (2021*b*), "Is the Instability of Short-Run Money Demand the Figment of a Specification Error?", University of Bern, in progress

Benati, L., R.E. Lucas Jr., J.P. Nicolini, and W. Weber (2019), "Online Appendix for: International Evidence on Long-Run Money Demand", Federal Reserve Bank of Minneapolis Staff Report 588 (June 2019)

Benati, L., R.E. Lucas Jr., J.P. Nicolini, and W. Weber (2021), "International Evidence on Long-Run Money Demand", Journal of Monetary Economics, forthcoming

Bernanke, B.S., M. Gertler, and M.W. Watson (1997), "Systematic Monetary Policy and the Effects of Oil Price Shocks", Brookings Papers on Economic Behavior I:1997, 91-142

Cagan, P. (1956), "The Monetary Dynamics of Hyperinflation", in M. Friedman, ed., "Studies in the Quantity Theory of Money", The University of Chicago Press, pp. 25-120.

Cavaliere, G., A. Rahbek, and A. M. R. Taylor (2012), "Bootstrap Determination of the Cointegration Rank in Vector Autoregressive Models", Econometrica, 80(4), 1721-1740.

Christiano, L.J. and T.J. Fitzgerald (2003), "The Bandpass Filter", International Economic Review, 44(2), 435-465

Cochrane, J. H. (1994*a*), "Permanent and Transitory Components of GNP and Stock Prices", Quarterly Journal of Economics, 109(1), 241-265.

Cochrane, J. H. (1994*b*), "Shocks", Carnegie-Rochester Conference Series on Public Policy, 41, 295-364

Del Negro, M., D. Giannone, M. Giannoni, and A. Tambalotti (2017), "Safety, Liquidity, and the Natural Rate of Interest", Brookings Papers on Economic Activity, Spring 2017, 235-294

Diebold, F., and C. Chen (1996), "Testing Structural Stability with Endogenous Breakpoint: A Size Comparison of Analytic and Bootstrap Procedures", Journal of Econometrics, 70(1), 221-241

Eggertsson, G.B. (2008), "Great Expectations and the End of the Depression", American Economic Review, 98(4), 1476-1516

Elliot, G., T. J. Rothenberg, and J. H. Stock (1996), "Efficient Tests for an Autoregressive Unit Root", Econometrica, 64(4), 813-836.

Elliot, G. (1998), "On the Robustness of Cointegration Methods When Regressors Almost Have Almost Have Unit Roots", Econometrica, 66, 149-158.

Engle, R. F., and C. W. Granger (1987), "Cointegration and Error Correction: Representation, Estimation, and Testing", Econometrica, 55(2), 251-276.

Fiorentini, G, A. Galesi, G. Pérez-Quirós and E. Sentana (2018), "The Rise and Fall of the Natural Interest Rate", Documentos de Trabajo N. 1822, Banco de Espana

Goldfeld, S.M., and D.M. Sichel (1990), "The Demand for Money", in Friedman, B.M., and Hahn, F.H., eds., Handbook of Monetary Economics, Vol. I, Amsterdam, North Holland.

Hamilton, J. (1994), Time Series Analysis. Princeton, NJ, Princeton University Press.

Hansen, B. E. (1999), "The Grid Bootstrap and the Autoregressive Model", Review of Economics and Statistics, 81(4), 594-607.

Hansen, H., and S. Johansen (1999), "Some Tests for Parameter Constancy in Cointegrated VAR Models", Econometrics Journal, 2, 306-333.

Harvey, A. and A. Jaeger (1993), "Detrending, Stylized Facts and the Business Cycle", Journal of Applied Econometrics, 8, 231-247

Holston, K. T. Laubach and J.C. Williams (2017), "Measuring the Natural Rate of Interest: International Trends and Determinants", Journal of International Economics, 108, S59-S75

King, R. and S.T. Rebelo (1993), "Low Frequency Filtering and Real Business Cycles", Journal of Economic Dynamics and Control, 17, 207-231.

King, R. and M.W. Watson (1996), "Money, Prices, Interest Rates and the Business Cycle", Review of Economics and Statistics, 78(1), 35-53.

Latané, H. A. (1960), "Income Velocity and Interest Rates: A Pragmatic Approach", Review of Economics and Statistics, 42(4), 445-449.

Laubach, T., and J. Williams (2003), "Measuring the Natural Rate of Interest", Review of Economics and Statistics, 85(4), 1063-1070.

Laubach, T., and J. Williams (2016), "Measuring the Natural Rate of Interest Redux", Business Economics, 51(2), 57-67

Levin, A.T. and J.M. Piger (2004), "Is Inflation Persistence Intrinsic in Industrial Economies?", European Central Bank Working Paper n. 334, April 2004

Lewis D., K. Mertens, and J. Stock (2020), "U.S. Economic Activity During the Early Weeks of the SARS-Cov-2 Outbreak", Covid Economics, 6(April 17), 1-21

Lucas Jr., R. E., and J.-P. Nicolini (2015), "On the Stability of Money Demand", Journal of Monetary Economics, 73, 48-65.

Luetkepohl, H. (1991), Introduction to Multiple Time Series Analysis. Springer-Verlag.

Meltzer, A.H. (1963), "The Demand for Money: The Evidence from the Time Series", Journal of Political Economy, 71(3), 219-246.

Perron, P. (1989), "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis", Econometrica, 57(6), 1361-1401.

Selden, R. T. (1956), "Monetary Velocity in the United States", in M. Friedman, ed., Studies in the Quantity Theory of Money, University of Chicago Press, pp. 405-454.

Sidrauski, M. (1967*a*), "Inflation and Economic Growth", Journal of Political Economy, 75(6), 796-810.

Sidrauski, M. (1967*b*), "Rational Choice and Patterns of Growth in a Monetary Economy", American Economic Review, 57(2), 534-544.

Summers, L. (1991), "Intervention in the Panel Discussion on Price Stability: How Should Long-Term Monetary Policy Be Determined?", Journal of Money, Credit and Banking, 23(3), 625-631

Stock, J.H. and M.W. Watson (2012), "Disentangling the Channels of the 2007-2009 Recession", Brookings Papers on Economic Activity, Spring 2012.

Taylor, J.B. (2008), Housing and Monetary Policy, in Federal Reserve Bank of Kansas City, ed., 'Housing, Housing Finance, and Monetary Policy', a symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, August 30-September 1, 2007. Kansas City, Missouri: Reserve Bank of Kansas City, pp. 463-76

Taylor, J.B. (2009), "Getting Off Track", Hoover Institution Press

Wright, J.H. (2000), "Confidence Sets for Cointegrating Coefficients Based on Stationarity Tests", Journal of Business and Economic Statistics, 18(2), 211-222

Appendix

A Results from unit root tests

Table A.1 reports bootstrapped p-values for Elliot, Rothenberg, and Stock (1996) unit root tests (with an intercept, but no time trend) for M1 velocity and a short-term nominal interest rate. The p-values have been computed by bootstrapping estimated ARIMA(p,1,0) processes *via* the procedure proposed by Diebold and Chen (1996), setting the number of bootstrap replications to 10,000.

Table A.1 Bootstrapped p-values for Elliot, Rothenberg, and Stock unit root tests[a] for velocity and a short rate

Country	*Period*	p=2	p=4	p=6	p=8
		M1 velocity			
Australia	1972Q1-2019Q4	0.9557	0.9573	0.9508	0.9841
Canada	1982Q3-2019Q4	0.2122	0.3114	0.3555	0.1592
Euro area	1970Q1-2019Q4	0.0733	0.3247	0.2133	0.1701
New Zealand	1990Q1-2019Q4	0.8679	0.8850	0.8748	0.7521
Norway	1978Q1-2019Q4	0.3307	0.3386	0.2024	0.2365
Sweden	1998Q1-2019Q4	0.7846	0.7036	0.7687	0.6471
United Kingdom	1963Q1-2019Q4	0.9884	0.9647	0.9436	0.9268
United States	1959Q1-2019Q4	0.8686	0.8662	0.8576	0.8340
		Short rate			
Australia	1972Q1-2019Q4	0.4980	0.5122	0.5269	0.4098
Canada	1982Q3-2019Q4	0.0792	0.4167	0.4657	0.5730
Euro area	1970Q1-2019Q4	0.5228	0.6106	0.6629	0.5678
New Zealand	1990Q1-2019Q4	0.1123	0.0809	0.0258	0.0972
Norway	1978Q1-2019Q4	0.4458	0.5890	0.6803	0.5231
Sweden	1998Q1-2019Q4	0.3808	0.3850	0.4061	0.5515
United Kingdom	1963Q1-2019Q4	0.5847	0.4865	0.5016	0.5134
United States	1959Q1-2019Q4	0.3162	0.2854	0.2914	0.2031

[a] Based on 10,000 bootstrap replications.

In nearly all cases, evidence of a unit root for either series is very strong. The only exception is the short rate for New Zealand, for which the null of unit root is instead near-uniformly rejected. In what follows I will proceed under the assumption that *all* nominal interest rates have been I(1) over the sample periods analized herein,[39] and that the rejection of the null of a unit root for New Zealand is a statistical fluke, possibly due to small-sample issues. There are two reasons for doing so. First, even a perfectly sized test, by definition, incorrectly rejects the null hypothesis x per

[39] This is an important qualification. Under metallic standards—for which inflation had been uniformly I(0), and in fact most of the time statistically indistinguishable from white noise (see Benati, 2008)—it is often possible to reject the null hypothesis of a unit root in short-term interest rates, as one would logically expect if the natural real rate featured comparatively little variation (this evidence is available upon request).

cent of the time at the x per cent level. When performing many statistical tests, such as in the present case, a certain fraction of 'fluke rejections' of the null should therefore be logically expected. Sure enough, the data I am using herein have not been randomly generated as part of a Monte Carlo experiment,[40] but the basic logic of this argument still holds. For example, taking the argument literally—i.e., as if we were here dealing with a Monte Carlo experiment featuring *independent* random draws—the five rejections (at the 10 per cent level) of the null of a unit root reported in Table A.1 represent 7.8 per cent of the overall number of tests reported in the table, i.e. smaller than the 10 per cent of 'fluke rejections' we would expect from a perfectly sized test with independent Monte Carlo artificial samples. Second, visual evidence (see Figure 1a) strongly suggests that all nominal short rates have been non-stationary over the sample periods analized herein.

B Evidence on Cointegration Between M1 Velocity and the Short Rate

Table B.1 reports, for bivariate systems featuring M1 velocity and a short-term rate, (i) bootstrapped p-values for Johansen's maximum eigenvalue tests of the null hypothesis of 0 *versus* 1 cointegration vectors, and (ii) 90%-coverage bootstrapped confidence intervals for the second element of the normalized cointegration vector based on Wright's (2000) methodology. As for Johansen's tests, following Benati (2020) and Benati *et al.* (2021), I bootstrap them *via* the procedure proposed by Cavaliere *et al.* (2012, henceforth CRT).[41] I select the VAR lag order as the maximum[42] between the lag orders chosen by the Schwartz and the Hannan-Quinn criteria[43] for the VAR in levels, and I estimate the VECM based on Johansen's estimator as detailed in Hamilton (1994). As for Wright's (2000) test, since it has been designed to be equally valid for data-generation processes (DGPs) featuring either exact or near unit roots, following Benati (2020) and Benati *et al.* (2021) I consider two alternative bootstrapping procedures, corresponding to either of the two possible cases. The first procedure involves bootstrapping as in CRT the cointegrated VECM estimated by imposing one cointegration vector. This procedure is the correct one if the data feature *exact*

[40] In particular, the data for individual countries are not *independent* random draws, since all countries experienced common events such as the Great Inflation of the 1970s, the disinflation of the early 1980s, the spread of globalization, and the 2008-2009 financial crisis.

[41] For details see Online Appendix B.2, which also discusses Monte Carlo evidence on the performance of CRT's procedure.

[42] I consider the maximum between the lag orders chosen by the SIC and HQ criteria because the risk associated with selecting a lag order smaller than the true one (model mis-specification) is more serious than the one resulting from choosing a lag order greater than the true one (over-fitting).

[43] On the other hand, I do not consider the Akaike Information Criterion since, as discussed (e.g.) by Luetkepohl (1991), for systems featuring I(1) series the AIC is an inconsistent lag selection criterion, in the sense of not choosing the correct lag order asymptotically.

unit roots. For the alternative possibility in which the two series are *near* unit root processes, I proceed as follows. Based on the just-mentioned cointegrated VECM estimated by imposing one cointegration vector, I compute the implied VAR in levels, which by construction features one, and only one, eigenvalue equal to 1.[44] I then turn such exact unit root VAR into its corresponding near unit root VAR, by shrinking the single unitary eigenvalue to $\lambda = 1 - 0.5\times(1/T)$, where T is the sample length.[45] The bootstrapping procedure I implement for the second possible case, in which the two series are near unit root processes, is based on bootstrapping such near unit root VAR. In practice the two procedures produce near-identical results, and in Table B.1 I therefore uniquely report results based on bootstrapping the VECM estimated by imposing one cointegration vector.

Table B.1 Bootstrapped *p*-values for Johansen's maximum eigenvalue tests for M1 velocity and a short-term rate, and 90% bootstrapped confidence intervals for the second element of the normalized cointegration vector based on Wright's (2000) tests[a]

Country	*Period*	*p*-values for maximum eigenvalue tests[b]	Results from Wright's test
Australia	1969Q3-2019Q4	0.0661	[-0.9255 -0.7013]
Canada	1967Q1-2019Q4	0.0279	[-1.1642 -0.1032]
Euro area	1999Q1-2019Q4	0.0896	[-0.6013 -0.2970]
New Zealand	1990Q1-2019Q4	0.1584	[-0.1643 -0.0642]
Norway	1978Q1-2019Q4	0.0992	[-0.1268 -0.0868]
Sweden	1998Q1-2019Q4	0.1136	[-0.3642 -0.3081]
United Kingdom	1955Q1-2019Q4	0.0201	[-0.5323 -0.3441]
United States	1959Q1-2019Q4	0.0985	[-0.5634 -0.3672]

[a] Based on 10,000 bootstrap replications. [b] Null of 0 *versus* 1 cointegration vectors.

Based on Wright's (2000) tests, the null hypothesis of cointegration is never rejected. Likewise, at the 10 per cent level Johansen's tests reject the null of 0 cointegration vectors for all countries except Sweden (marginally), and New Zealand (with a p-value of 0.1584). As in Benati (2020), in what follows I will therefore proceed under the assumption that M1 velocity and the short rate are cointegrated in all samples. Online Appendix B.4 reports results from Hansen and Johansen's (1999) Nyblom-type tests for stability in either the cointegration vector, or the vector of loading coefficients, in the estimated VECMs, and discusses Monte Carlo evidence on the performance of the tests. In short, evidence of breaks in either the cointegration vector or the loading coefficients is nearly non-existent. In particular, based on either

[44] Bootstrapping this VAR would be equivalent to bootstrapping the underlying cointegrated VECM, that is, it would be correct if the data featured exact unit roots.

[45] Once again, for details see Online Appendix B.2.

the Selden-Latané specification—which, as discussed, appears to be the one preferred by the data for low-inflation (and therefore low-interest rates) countries—or the log-log, not a single break in either the cointegration vector or the loading coefficients is identified. As for the semi-log, no break in the cointegration vector is identified for any country, whereas only for Norway a break in the loading coefficients is identified, although the p-value, at 0.0935, is essentially borderline.

C Computing Permanent and Transitory GDP by Projecting GDP on Consumption

The left-hand side panel of Figure C.1 in this appendix shows the transitory component of U.S. GDP obtained by projecting log real GDP onto log real consumption, i.e. the residual from the cointegrating regression

$$\ln Y_t = a + b \ln C_t + e_t, \tag{C.1}$$

where Y_t and C_t are real GDP and real consumption, respectively (the two series are described in Online Appendix A.2.9., and their unit root and cointegration properties are discussed in footnote 7 in the main text). I estimate (C.1) *via* a simple OLS regression, but near-identical results are produced by Stock and Watson's (1993) dynamic OLS estimator. Two main findings emerge from the figure. First, the estimated transitory component of GDP captures remarkably well the peaks and troughs of the post-WWII U.S. business cycle as established by the NBER Business-Cycle Dating Committee (i.e., the vertical blue and red bars in the figure). Second, the transitory component interprets a sizeable portion of the fall in output associated with the Great Recession as permanent: this is clearly highlighted, e.g., by the fact that whereas the troughs of annual real GDP growth associated with the Volcker recession and the Great Recession had been equal to -2.6 and -3.9 per cent, respectively, the troughs of the corresponding transitory components of GDP obtained by projecting log real GDP onto log real consumption had been equal to -4.0 and -2.8 per cent, respectively. As a matter of logic, the only possible interpretation of this is that, when viewed though the lenses of consumption, the latter recession had been characterized by a significantly greater decrease in permanent GDP than the former.

The right hand-side panel of Figure C.1 provides simple, but powerful corroborating evidence that this may in fact had been the case. The figure shows log real GDP and rescaled log real consumption,[46] together with (i) up to 2004Q4, the HP-filtered trend of log real GDP,[47] and (ii) starting from 2005Q1, the forecast of the HP-filtered trend, which I computed recursively by exploiting the fact that, in the state-space

[46] I rescaled log real consumption as in Cochrane's (1994a) Figure III, i.e. by adding to it the mean log ratio between GDP and consumption.

[47] I set the smoothing parameter to the standard value of 1600 for quarterly data, but qualitatively similar results are produced by alternative plausible values of the parameter. In order to compute

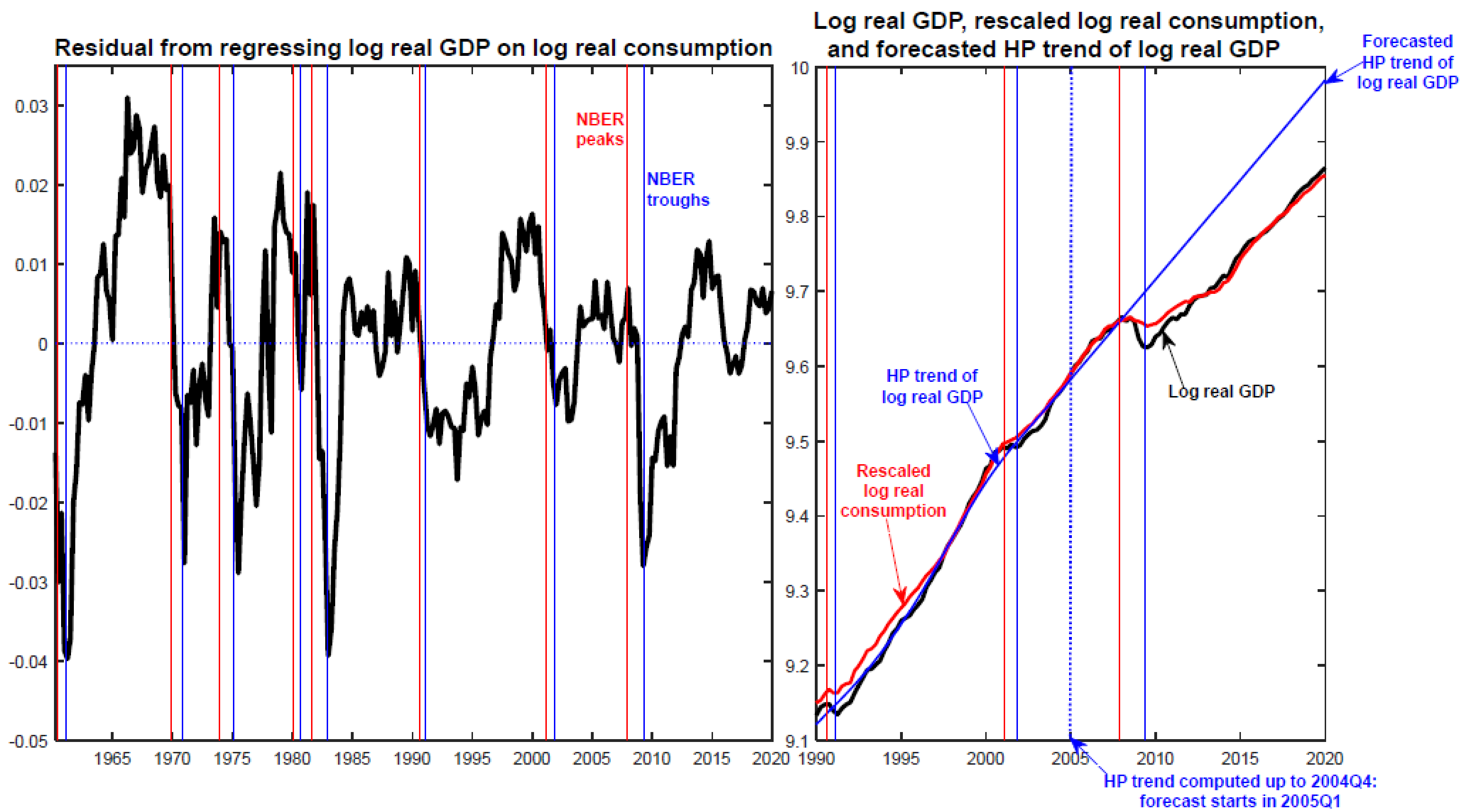

Figure C.1 Computing transitory GDP by projecting GDP on consumption representation of the Hodrick-Prescott filter, the second difference of the HP trend is white noise (see e.g. Harvey and Jaeger, 1993, and King and Rebelo, 1993), so that the trend, τ_t, evolves according to $\tau_t = 2\tau_{t-1} - \tau_{t-2} + \epsilon_t$, with ϵ_t being a shock. Following the collapse of Lehman Brothers, both GDP and consumption have significantly fallen short compared to the forecast of the HP trend. In particular, at the end of 2019 the shortfall had been for both series equal to about 12 per cent. Crucially, the fact that this has equally held for *both* GDP *and* consumption logically suggests that the shortfalls are permanent: otherwise, by the Permanent Income Hypothesis, consumption would be close to the HP trend.

D The Monthly Series Used in Section 7.2

As discussed more extensively in Online Appendix A.3, for Canada and the U.K. monthly seasonally adjusted real GDP estimates are available respectively from *Statistics Canada* and from the U.K.'s *National Institute for Economic and Social Research* (NIESR). As for the Euro area I have interpolated seasonally adjusted quarterly real GDP based on Stock and Watson's (2012) methodology, using monthly seasonally adjusted industrial production as the interpolator series. In order to compute nominal GDP, for any of the three countries I have then interpolated to the monthly frequency the quarterly seasonally adjusted GDP deflator based on Stock and Watson's (2012) methodology, using the monthly seasonally adjusted core CPI as the interpolator series.

For the United States, seasonally adjusted monthly series for real and nominal GDP are from Stock and Watson (2012) until 2010, and from *IHS Markit*, a consultancy, after that. Finally, a crucial component of Lucas and Nicolini's (2015) 'New M1' aggregate that is used herein (see the discussion in Section 2), i.e. Money Market Deposit Accounts (MMDAs), is available only at the quarterly frequency. As discussed in Online Appendix A.3.4, I have therefore interpolated MMDAs to the monthly frequency as in Stock and Watson (2012), using as monthly interpolator the seasonally adjusted series for 'Total Checkable Deposits' from the *Federal Reserve Board*. The rationale for using this interpolator series is exactly the same originally advanced by Goldfeld and Sichel (1990, pp. 314-315), and then reiterated by Lucas and Nicolini (2015), for including MMDAs within an expanded, and economically more sensible definition of M1 (see Section 2): MMDAs perform an economic function which is very similar to that of the checkable deposits included in the standard M1 series. Therefore, on the one hand it makes sense to include them within an economically sensible definition of M1; on the other hand, it makes sense to use total checkable deposits as the monthly interpolator for quarterly MMDAs.

the HP trend I only use data up to 2004Q4 because it could possibly be argued that during the years immediately preceding the financial crisis U.S. GDP had been significantly above trend. Using data up to 2008Q3, however, produces near-identical results.

Online Appendix for: A New Approach to Estimating the Natural Rate of Interest

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A The Data

Here follows a detailed description of the data. Almost all of them are from the datasets assembled by Benati (2020) and Benati *et al.* (2021), which for the post-WWII period I have updated to the most recent available observation. With a handful of exceptions (detailed below), all of the data are from official sources, that is, either central banks or national statistical agencies. In the few cases in which I was not able to find the data at central banks' or national statistical agencies' websites, I took them from the St. Louis FED's data portal, FRED II. The Wu and Xia (2016) 'shadow rates' for the United States, the Euro area and the United Kingdom are from Cynthia Wu's website, at: https://sites.google.com/view/jingcynthiawu/shadow-rates.

A.1 Annual data

As for the long-run annual data used for Figure 1*b*, the sources are as follows.

A.1.1 Argentina

All of the series are from the *Banco Central de la República Argentina* (Argentina's central bank, henceforth, *Banco Central*). Specifically, a series for M1, available for the period 1900-2014, is from *Banco Central*'s Table 7.1.4 ("Agregados Monetarios"). A series for a short-term nominal interest rate, available for the period 1821-2018, is from Banco Central's Table 7.1.4 ("Tasas activas"). Interestingly, among all of the countries I consider in this paper, Argentina is the *only* one that directly provides an estimate of (the inverse of) the velocity of circulation of monetary aggregates. Specifically, Banco Central's Table 7.1.4 provides the ratios between either M1 and M3 and nominal GDP ("M1 % PBI" and "M3 % PBI", respectively; "PBI" is the Spanish acronym for GDP).

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A.1.2 Finland

Long-run monthly data for M1 for the period January 1866-December 1985 have been generously provided by Tarmo Haavisto. The data come from his Ph.D. dissertation (see Haavisto, 1992) and they have been converted to the annual frequency by taking simple annual averages. A series for Finland's monetary policy rate (labeled as the "Base rate"), available since January 1867, is from *Suomen Pankki Finlands Bank* (Finland's central bank, henceforth, *Suomen Pankki*).[1] Finally, an annual series for nominal GDP, available since 1860, is from Finland's Historical Statistics, which are available from the web page of *Statistics Finland* (Finland's national statistical agency).

A.1.3 Japan

A monthly series for the *Bank of Japan*'s (henceforth, *BoJ*) discount rate, available since January 1883, is from the *BoJ*'s long-run historical statistics, which are available at its website (the series is labeled as "BJ'MADR1M: The Basic Discount Rate and Basic Loan Rate"). Annual series for nominal GNP and M1 for the period 1885-1940 are from Table 48 of Tamaki (1995).

A.1.4 Portugal

An annual series for M1 for the period 1854-1998 is from Table 5 of Mata and Valerio (2011). Annual series for real and nominal GDP for the period 1868-2008 are from Table 4 of Mata and Valerio (2011). A series for the official discount rate of the *Banco de Portugal* (the Portuguese central bank), available for the period 1930-1989, is from Table 74 of Homer and Sylla (2005).

A.1.5 United States

The series for the 3-month Treasury bill rate, nominal GDP, and M1 are all from Benati *et al.* (2021). The original source for the 3-month Treasury bill rate is the *Economic Report of the President* (henceforth, *ERP*), whereas nominal GDP is from Kuznets and Kendrick's Table Ca184-191. M1 is from the *Banking and Monetary Statistics, 1914 -1941* from the Federal Reserve Board.

[1] To be precise, *Suomen Pankki* does not provide the actual time series for the base rate, but rather the dates at which the rate had been changed (starting from January 1, 1867), together with the new value of the base rate prevailing starting from that date. Based on this information, I constructed a daily series for the base rate starting on January 1, 1867, *via* a straightforward MATLAB program, and I then converted it to the annual frequency by taking annual averages.

A.1.6 West Germany

Although data for post-WWII Germany are available, in principle, for the entire period 1950-1998, I have decided to only use data for West Germany for the period 1960-1989. The reason is that I am skeptical about the possibility of meaningfully linking the various series for nominal GDP in order to create a single series for the period 1950-1998 because (*i*) before 1960, GDP data did not include West Berlin and the Saarland, which, in 1960, jointly accounted for about 6% of overall GDP; and (*ii*) the reunification of 1990 created discontinuities in both GDP and M1 (I thought the problem could be side-stepped by focusing on M1 velocity, i.e. their ratio, but in fact this series also seems to exhibit a discontinuity around the time of reunification). Entering into details, an annual series for the *Bundesbank*'s monetary policy rate for the period 1949-1998 has been constructed by taking annual averages of the monthly series "BBK01.SU0112, Diskontsatz der Deutschen Bundesbank / Stand am Monatsende, % p.a.", which is available from the *Bundesbank*'s website. As for nominal GDP, the original annual series are from Germany's Federal Statistical Office, and they are available for the period 1950-1960 ("Gross domestic product at current prices, Former Territory of the Federal Republic excluding Berlin-West and Saarland"); 1960-1970 ("Gross domestic product at current prices, Former Territory of the Federal Republic"); and 1970-1991 ("Gross domestic product at current prices, Former Territory of the Federal Republic, (results of the revision 2005)"). There is also a fourth series available for reunified Germany, but, as mentioned, it cannot be meaningfully linked to the series for the period 1970-1991 because of the discontinuity induced by the 1990 reunification. The second and third series can be linked because the difference between them is uniquely due to changes in the accounting system, rather than to territorial redefinitions. Linking the first and second series, on the other hand, is problematic because, as mentioned, before 1960 GDP data did not include West Berlin and the Saarland. Our decision has been to ignore the first GDP series, and therefore to start the sample in 1960, for the following two reasons. *First*, the dimension of West Berlin and the Saarland was not negligible. The value taken by nominal GDP in 1960 according to the first and second series was equal to 146.04 and 154.77, respectively, a difference equal to 6%. *Second*, this problem might be ignored if we had good reasons to assume that, during those years, West Berlin and the Saarland's nominal GDP was growing exactly at the same rate as in the rest of Germany. This, however, is pretty much a heroic assumption—especially for West Berlin. As a result, in the end we just decided to ignore the first series. Finally, M1 data are available at the monthly frequency since 1948 from the Bundesbank's original *Monthly Reports*, which are available in scanned form at the Bundesbank's website. So I downloaded the scanned PDFs of the *Monthly Reports*, and I manually entered the data in Excel, one "piece" (that is, one *Monthly Report*) at a time. An important point to notice is that German monetary aggregates are *not revised*, so that it is indeed possible to link the figures coming from successive issues of the *Monthly Report*. Finally, I converted the series to the annual frequency by taking annual averages.

A.2 Quarterly data

A.2.1 Australia

Nominal GDP ('Gross domestic product: Current prices, $ Millions, Seasonally Adjusted, A2304418T') is from the *Australian Bureau of Statistics*. The short rate ('3-month BABs/NCDs, Bank Accepted Bills/Negotiable Certificates of Deposit-3 months; monthly average, Quarterly average, Per cent, ASX, 42767, FIRMMBAB90') is from the *Reserve Bank of Australia* (henceforth, *RBA*). M1 ('M1: Seasonally adjusted, $ Millions') is from the *Reserve Bank of Australia* since 1975Q2, and from FRED II (at the St. Louis FED's website) for the period 1972Q1-1975Q1 (over the period of overlapping, i.e. since 1975Q2, the two series are identical, which justifies their linking). 5-and 10-year government bond yields are from the *RBA*. Specifically, they are from the *RBA*'s spreadsheet 'F2.1 Capital Market Yields – Government Bond', which is available at the *RBA*'s website. A quarterly seasonally adjusted series for the 'Unemployment rate, Unemployed persons as percentage of labour force' has been computed by taking averages within the quarter of the corresponding monthly series from the *Australian Bureau of Statistics* (the series' code is GLFSURSA).

A.2.2 Canada

Nominal GDP ('Gross domestic product (GDP) at market prices, Seasonally adjusted at annual rates, Current prices') is from *Statistics Canada*. M1 ('v41552787, Table 176-0020: M1B (gross) (currency outside banks, chartered bank chequable deposits, less inter-bank chequable deposits) (x 1,000,000)') is from *Statistics Canada*. The series has been seasonally adjusted *via* ARIMA X-12 (as implemented in *Eviews*) and it has been converted to the quarterly frequency by taking averages within the quarter. Monthly series for the Bank rate (i.e., the *Bank of Canada*'s monetary policy rate), and for the 3-, 5-, and 10-year Government bond yields are all from the *Bank of Canada*, and they have been converted to the quarterly frequency by taking averages within the quarter. A quarterly seasonally adjusted series for the 'Unemployment rate, Both sexes, 15 years and over' has been computed by taking averages within the quarter of the corresponding monthly series from *Statistics Canada* (the series' code is v2091177).

A.2.3 Euro area

All of the data are from the *European Central Bank*.

A.2.4 New Zealand

Nominal GDP ('Gross Domestic Product - expenditure measure, Nominal $m s.a.') is from *Statistics New Zealand*. The short rate and M1 ('Overnight interbank cash rate, %pa, INM.MN.NZK' and 'M1', respectively) are from the *Reserve Bank of New Zealand*. A quarterly seasonally adjusted series for the 'Harmonized Unemployment Rate: Total: All Persons for New Zealand' has been computed by taking averages within the quarter of the corresponding monthly series from the *Organization for Economic Cooperation and Development* (OECD). The FRED II code is LRHUTTTTNZQ156S.

A.2.5 Norway

A quarterly seasonally unadjusted series for nominal GDP ('Gross domestic product Mainland Norway, market values, Current prices (NOK million)') is from *Statistics Norway*, and it has been seasonally adjusted via ARIMA X-12 as implemented in *Eviews*. A monthy seasonally adjusted M1 series, and a monthly series for a 5-year government bond yield, are from from *Norges Bank* (Norway's central bank), and they have been converted to the quarterly frequency by taking averages within the quarter. A quarterly series for a 90-day interbank rate ('3-Month or 90-day Rates and Yields: Interbank Rates for Norway, Percent, Quarterly, Not Seasonally Adjusted') is from the St. Louis FED's website (FRED II acronym is IR3TIB01NOM156N). A quarterly series for a 10-year government bond yield is from the St. Louis FED's website since 1989Q1 (FRED II acronym is IRLTLT01NOQ156N: 'Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Norway, Percent, Quarterly, Not Seasonally Adjusted'). Before that it is from *Norges Bank*'s spreadsheet bond_yields.xls (sheet 'p1_c4_table_A3_Monthly', column J (ST10)), which is available at *Norges Bank*'s website. A quarterly seasonally adjusted series for the 'Harmonized Unemployment Rate: Total: All Persons for Norway' has been computed by taking averages within the quarter of the corresponding monthly series from the OECD (the FRED II code is LRHUTTTTNOM156S).

A.2.6 Sweden

A quarterly seasonally adjusted series for nominal GDP ('GDP at market prices, Seasonally adjusted current prices, SEK million, BNPM') is from *Statistics Sweden*. A monthly seasonally adjusted M1 series is from is from *Statistics Sweden* ('M1, SEK millions'), and it has been converted to the quarterly frequency by taking averages within the quarter. A quarterly series for a 90-day interbank rate is from the St. Louis FED's website (acronym is IR3TIB01SEM156N, '3-Month or 90-day Rates and Yields: Interbank Rates for Sweden, Percent, Quarterly, Not Seasonally Adjusted'). Monthly series for 5-, 7-, and 10- year government bond yields are from *Statistics Sweden* (acronyms are 'SE GVB 5Y', 'SE GVB 7Y', and 'SE GVB 10Y'), and they have been converted to the quarterly frequency by taking averages within the quarter. A quarterly seasonally adjusted series for the 'Harmonized Unemployment Rate: Total: All Persons for Sweden' has been computed by taking averages within the quarter of the corresponding monthly series from the OECD (the FRED II code is LRHUTTTTSEM156S).

A.2.7 United Kingdom

Nominal GDP ('YBHA, Gross Domestic Product at market prices: Current price, Seasonally adjusted £m') is from the *Office for National Statistics*. A break-adjusted seasonally adjusted stock of M1 is from Version 3 of 'A millennium of macroeconomic data for the UK' until 2016Q4. Since then it has been updated based on the quarterly M1 series (LPQVWYT) from the *Bank of England*'s statistical database at its website. A monthly series for the Bank rate (i.e., the *Bank of England*'s monetary policy rate) available since 1694 is from from the *Bank of England*'s website. A monthly series for a 10-year government bond yield is from 'A millennium of macroeconomic data for the UK' until March 2017, and it has been updated based on the series 'Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for the United Kingdom' from the St. Louis FED's website (FRED II acronym is IRLTLT01GBM156N). A monthly series for 'Interest Rates, Government Securities, Government Bonds for United Kingdom' is from the St. Louis FED's website (FRED II acronym is IRLTLT01GBM156N). All monthly series have been converted to the quarterly frequency by taking averages within the quarter. A quarterly seasonally adjusted series for the 'administrative unemployment rate' has been computed by taking averages within the quarter of the corresponding monthly series from Version 3 of 'A millennium of macroeconomic data for the UK' until 2016, and after that of the series 'Unemployment rate (%)' from the *Office for National Statistics*' Table A02: Labour Force Survey Summary: People by economic activity for those aged 16 and over and those aged from 16 to 64 (seasonally adjusted) (the ONS code is MGSX).

A.2.8 United States

Interwar period Quarterly seasonally adjusted series for nominal GNP and the GNP deflator are from Balke and Gordon (1986). A monthly seasonally unadjusted series for M1 is from Friedman and Schwartz (1963). The series has been seasonally adjusted *via* ARIMA X-12 as implemented in *EViews*, and it has been converted to the quarterly frequency by taking averages within the quarter. A monthly seasonally unadjusted series for the Federal Reserve Bank of New York's discount rate for the United States is from the *National Bureau of Economic Research*'s Macrohistory Database (the FRED II code is M13009USM156NNBR). The series has been converted to the quarterly frequency by taking averages within the quarter.

Post-WWII period A quarterly seasonally adjusted series for nominal GDP (Gross Domestic Product, GDP Billions of Dollars, Quarterly, Seasonally Adjusted, Annual Rate) is from FRED II (acronym is GDP). Monthly series for the Federal Funds rate and for 3-, 5-, and 10-year government bond yields are also from FRED II (acronyms are FEDFUNDS, GS3, GS5, and GS10). All interest rate series have been converted to the quarterly frequency by taking averages within the quarter. As mentioned in the main text, M1 is constructed as the sum of the standard aggregate produced by the Federal Reserve and of Money Market Deposit Accounts (MMDAs). The former series is from the St. Louis FED (acronym is M1SL, 'M1 Money Stock, Billions of Dollars, Quarterly, Seasonally Adjusted'), whereas MMDAs data are from the Federal Reserve's mainframe, and they have been kindly provided by Juan-Pablo Nicolini. A quarterly seasonally adjusted series for real GDP (Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate) is from FRED II (acronym is GDPC1). A quarterly seasonally adjusted series for real chain-weighted consumption of non-durables and services has been computed based on the data in Tables 1.1.6, 1.1.6B, 1.1.6C, and 1.1.6D of the *National Income and Product Accounts* produced by the U.S. Department of Commerce's *Bureau of Economic Analysis*. A quarterly seasonally adjusted series for the 'Civilian unemployment rate, persons 16 years of age and older', has been computed by taking averages within the quarter of the corresponding monthly series from the U.S. *Bureau of Labor Statistics* (the FRED II code is UNRATE).

A.3 Monthly data

A.3.1 Canada

A monthly seasonally unadjusted series for M1 ('v41552787, Table 176-0020: M1B (gross) (currency outside banks, chartered bank chequable deposits, less inter-bank chequable deposits) (x 1,000,000)') is from *Statistics Canada*. The series has been seasonally adjusted *via* ARIMA X-12 as implemented in *Eviews*. A monthly series for the Bank rate (i.e., the *Bank of Canada*'s monetary policy rate) is from the *Bank of Canada*. A monthly seasonally adjusted series for real GDP ('Real GDP, Total economy, 1986 constant prices') is from *Statistics Canada*. I interpolated to the monthly frequency a quarterly seasonally adjusted series for the GDP deflator from *Statistics Canada* ('GDP deflator, Seasonally adjusted, 2016A000011124', acronym is v62307282) as in Stock and Watson (2012), using as interpolator series a monthly seasonally adjusted core CPI series from *Statistics Canada* ('Consumer Price Index (CPI), all-items excluding eight of the most volatile components as defined by the Bank of Canada and excluding the effect of changes in indirect taxes, seasonally adjusted', acronym is v112593706). Finally, I computed a monthly seasonally adjusted series for nominal GDP as the product of the interpolated real GDP and GDP deflator series.

A.3.2 Euro area

All of the data are from the *European Central Bank* (ECB): a monthly seasonally adjusted series for M1 ('Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs, central government and post office giro institutions reporting sector - Monetary aggregate M1, All currencies combined - Euro area (changing composition) counterpart, Non-MFIs excluding central government sector, denominated in Euro, data Working day and seasonally adjusted'; ECB code is BSI.M.U2.Y.V.M10.X.1.U2.2300.Z01.E); a monthly seasonally unadjusted series for the 3-month Euribor rate ('Euro area (changing composition) - Money Market - Euribor 3-month - Historical close, average of observations through period - Euro, provided by Reuters, average of observations through period (A)'; ECB code is FM.M.U2.EUR.RT.MM.EURIBOR3MD_.HSTA); a monthly seasonally adjusted series for the consumer price index ('Euro area (changing composition) - HICP - Overall index, Monthly Index, European Central Bank, Working day and seasonally adjusted'; ECB code is ICP.M.U2.Y.000000.3.INX); and a monthly seasonally adjusted series for industrial production ('Euro area 19 (fixed composition) - Industrial Production Index, Total Industry - NACE Rev2 Eurostat; working day and seasonally adjusted'; ECB code is STS.M.I8.Y.PROD.NS0010.4.000). Then, I interpolated to the monthly frequency quarterly seasonally adjusted series for real GDP and the GDP deflator from the *ECB* as in Stock and Watson (2012), using as interpolators the previously mentioned monthly seasonally adjusted series for industrial production and the consumer price index, respectively. Finally, I computed a monthly seasonally adjusted series for nominal GDP as the product of the interpolated real GDP and GDP deflator series.

A.3.3 United Kingdom

A monthly seasonally unadjusted series for the core CPI ('CPIH Index: Excluding Energy, food, alcoholic beverages & tobacco 2015=100', acronym is L5KB) is from the *Office for National Statistics* (henceforth, *ONS*), and it has been seasonally adjusted via ARIMA X-12 as implemented in *Eviews*. A monthly series for the Bank rate (i.e., the *Bank of England*'s monetary policy rate) available since 1694 is from from the *Bank of England*'s website. A monthly seasonally adjusted M1 series (LPMVWYT, 'Monthly amounts outstanding of monetary financial institutions' sterling and all foreign currency M1 (UK estimate of EMU aggregate) liabilities to private and public sectors (in sterling millions) seasonally adjusted') is from the *Bank of England*'s website. A monthly seasonally adjusted series for real GDP is from the *National Institute for Economic and Social Research* (*NIESR*), and it has been kindly provided by Garry Young. I interpolated to the monthly frequency a quarterly seasonally adjusted series for the GDP deflator from the *ONS* ('Implied GDP deflator at market prices: SA Index', acronym is L8GG) as in Stock and Watson (2012), using the previously mentioned monthly seasonally adjusted core CPI series as interpolator series. Finally, I computed a monthly seasonally adjusted series for nominal GDP as the product of the interpolated real GDP and GDP deflator series.

A.3.4 United States

A monthly seasonally adjusted series for the the core PCE deflator ('Personal Consumption Expenditures Excluding Food and Energy (Chain-Type Price Index)') is from the U.S *Bureau of Economic Analysis*. Seasonally adjusted monthly series for real and nominal GDP are from Stock and Watson (2012) until 2010, and from *IHS Markit*, a consultancy, after that[2] (originally, the series used to be produced by another consultancy, *Macroeconomic Advisors*). *IHS Markit*'s production notes for its monthly real GDP series states:

> '**Note**: IHS Markit's index of Monthly GDP (MGDP) is a monthly indicator of real aggregate output that is conceptually consistent with real Gross Domestic Product (GDP) in the NIPA's. The consistency is derived from two sources. First, MGDP is calculated using much of the same underlying monthly source data that is used in the calculation of GDP. Second, the method of aggregation to arrive at MGDP is similar to that for official GDP. Growth of MGDP at the monthly frequency is determined primarily by movements in the underlying monthly source data, and growth of MGDP at the quarterly frequency is nearly identical to growth of real GDP.'

A monthly series for the Federal Funds rate is from FRED II (acronym is FEDFUNDS). Finally, I interpolated to the monthly frequency a seasonally adjusted quarterly series for Money Market Deposit Accounts (MMDAS) as in Stock and Watson (2012), by using, as monthly interpolator, the seasonally adjusted series for 'Total Checkable Deposits' from the FRB H.6 release from the *Federal Reserve Board*. The rationale for using this interpolator series is exactly the same originally advanced by Goldfeld and Sichel (1990, pp. 314-315), and then reiterated by Lucas and Nicolini (2015), for including MMDAs within an expanded, and economically more sensible definition of M1 (see the discussion in Appendix A in the main text of the present work): MMDAs perform an economic function which is very similar to that of the checkable deposits included in the standard M1 series. Therefore, on the one hand it makes sense to include them within an economically sensible definition of M1; on the other hand, it makes sense to use total checkable deposits as the monthly interpolator for quarterly MMDAs.

[2]See at: https://ihsmarkit.com/products/us-monthly-gdp-index.html

B Unit Root and Cointegration Properties of the Data

B.1 Unit root tests

Tables B.1*a*-B.1*c* report bootstrapped *p*-values[3] for Elliot, Rothenberg, and Stock (1996) unit root tests for M1 velocity, a short-term nominal interest rate, up to three long-term nominal interest rates (depending on data availability for each individual country), and the corresponding long-short spreads. All tests are with an intercept, but no time trend. In nearly all cases, evidence of a unit root for M1 velocity and nominal interest rates is very strong, whereas the null of a unit root is near-uniformly rejected for the long-short spreads. The only exceptions to this pattern are (*i*) the long-short spreads for the Euro area, for which the null of a unit root is near-uniformly not rejected, and (*ii*) the short rate for the Euro area and nominal interest rates for New Zealand, for which the null of unit root is instead near-uniformly rejected. As for (*i*), as discussed in the main text, I proceed under the assumption that *all* of the long-short spreads are in fact I(0), and that the results from Elliot et *al.*'s (1996) unit root tests for the Euro area are a statistical fluke, possibly due to small-sample issues. There are two reasons for doing so. First, as discussed in the main text, basic economic theory suggests that any permanent shock to nominal interest rates—originating from either permanent inflation shocks, or permanent shocks to the Wicksellian (i.e., natural) real rate of interest—has an identical long-run impact on all nominal interest rates at all maturities. The implication is that, whatever the origin of permanent shock to nominal interest rates, the spreads will ultimately remain unaffected, and will therefore be I(0). Second, with very few exceptions, this is in fact what the data suggest: since the end of the Napoleonic wars—i.e., since when high-quality macroeconomic data start being consistently available—the null of unit root in long-short interest rates' spreads can near-uniformly be rejected. Taken together, the logical/theoretical argument, and the empirical evidence since the end of the Napoleonic wars, naturally suggest that lack of a rejection of the null of a unit root in a long-short spread is likely a statistical fluke. As for (*ii*), as discussed again in the main text, I proceed under the assumption that *all* nominal interest rates have been I(1) over the sample periods analized herein, and that the rejection of the null of a unit root is, once again, a statistical fluke possibly due to small-sample issues. There are two reasons for doing so. First, it is important to remember that even a perfectly sized test, by definition, incorrectly rejects the null hypothesis x per cent of the time at the x per cent level. When performing *many* statistical tests, such as in

[3] *p*-values have been computed by bootstrapping 10,000 times estimated ARIMA(p,1,0) processes. In all cases, the bootstrapped processes are of length equal to the series under investigation. As for the lag order, p, since, as it is well known, results from unit root tests may be sensitive to the specific lag order which is being used, for reasons of robustness I consider four alternative lag orders (either 2, 4, 6, or 8 quarters).

the present case, a certain fraction of 'fluke rejections' of the null should therefore be logically expected. Sure enough, the data I am using herein have not been randomly generated as part of a Monte Carlo experiment,[4] but the basic logic of this argument should still hold. For example, taking the argument literally—i.e., as if we were here dealing with a Monte Carlo experiment featuring *independent* random draws—the nine rejections (at the 10 per cent level) of the null of a unit root reported in Table B.1*a* represent 12.5 per cent of the overall number of tests reported in the table, not far from the 10 per cent of 'fluke rejections' we would expect from a perfectly sized test with independent Monte Carlo artificial samples. Second, for both the Euro area and New Zealand visual evidence strongly suggests that all nominal interest rate have in fact been non-stationary over the sample period analized herein. For short rates this is clearly apparent from Figure 1*a*; evidence for long rates is—as one would logically expect—even starker.

I now proceed to discuss the results from cointegration tests.

B.2 Cointegration tests

Table 1 in the main text reports, for bivariate systems featuring M1 velocity and a short-term rate, (*i*) bootstrapped *p*-values for Johansen's maximum eigenvalue tests of the null hypothesis of 0 *versus* 1 cointegration vectors, and (*ii*) 90%-coverage bootstrapped confidence intervals for the second element of the normalized cointegration vector based on Wright's (2000) methodology. As for Johansen's tests, following Benati (2020) and Benati *et al.* (2021), I bootstrap the tests *via* the procedure proposed by Cavaliere *et al.* (2012, henceforth CRT). In a nutshell, CRT's procedure is based on the notion of computing critical and *p*-values by bootstrapping the model which is relevant under the null hypothesis. This means that, within the present context, the model which is being bootstrapped is a simple, non-cointegrated VAR in differences. All of the technical details can be found in CRT (2012), which the reader is referred to. I select the VAR lag order as the maximum[5] between the lag orders chosen by the Schwartz and the Hannan-Quinn criteria[6] for the VAR in levels.

Monte Carlo evidence on the performance of CRT's procedure can be found in CRT (2012), Benati (2015), and especially Benati *et al.* (2019). Any of three papers documents the excellent performance of the procedure conditional on Data-Generation Processes (DGPs) featuring *no cointegration*, with the null incorrectly

[4] In particular, the data for the different countries are not *independent* random draws, since all countries experienced common events such as the Great Inflation of the 1970s, the disinflation of the early 1980s, the spread of globalization, and the 2008-2009 financial crisis.

[5] I consider the maximum between the lag orders chosen by the SIC and HQ criteria because the risk associated with selecting a lag order smaller than the true one (model mis-specification) is more serious than the one resulting from choosing a lag order greater than the true one (over-fitting).

[6] On the other hand, I do not consider the Akaike Information Criterion since, as discussed (e.g.) by Luetkepohl (1991), for systems featuring I(1) series the AIC is an inconsistent lag selection criterion, in the sense of not choosing the correct lag order asymptotically.

rejected at close the nominal size irrespective of the sample length. Benati *et al.* (2019), however, also show that, if the DGP features *cointegration*, the tests have a harder and harder time detecting it (*i*) the shorter the sample length, and (*ii*) the more persistent the cointegration residual. This is in line with some of the evidence reported by Engle and Granger (1987) based on the Augmented Dickey-Fuller test, and it implies that if cointegration is not detected, (*i*) and/or (*ii*) are possible explanations.

As for Wright's (2000) test, since it has been designed to be equally valid for data-generation processes (DGPs) featuring either exact or near unit roots, following Benati (2020) and Benati *et al.* (2021) I consider two alternative bootstrapping procedures, corresponding to either of the two possible cases. The first procedure involves bootstrapping—as detailed in CRT and briefly described previously—the cointegrated VECM estimated (based on Johansen's procedure) under the null of one cointegration vector. This bootstrapping procedure is the correct one if the data feature *exact* unit roots. For the alternative possible case in which velocity and the short rate are *near* unit root processes, I proceed as follows. Based on the just-mentioned cointegrated VECM estimated under the null of one cointegration vector, I compute the implied VAR in levels, which by construction features one, and only one, eigenvalue equal to 1. Bootstrapping this VAR would obviously be equivalent to bootstrapping the underlying cointegrated VECM, that is, it would be correct if the data featured exact unit roots. Since, on the other hand, here I want to bootstrap under the null of a near unit root DGP, I turn such an exact unit root VAR in levels into its corresponding near unit root VAR by shrinking down the single unitary eigenvalue to $\lambda = 1 - 0.5\times(1/T)$, where T is the sample length.[7] The bootstrapping procedure I implement for the second possible case, in which the processes feature near unit roots, is based on bootstrapping such a near unit root VAR. In practice, as shown by Benati *et al.* (2021), the two procedures produce near-identical results, and in the present work I therefore uniquely report, as Benati *et al.* (2021), results based on the first procedure (i.e., based on bootstrapping the VECM estimated conditional on one cointegration vector, as in CRT).

As discussed in the main text, based on Wright's (2000) tests, the null of cointegration is never rejected. Likewise, Johansen's maximum eigenvalue tests reject the null of 0 cointegration vectors for all countries except Sweden (marginally), and New Zealand (with a p-value of 0.1584).

[7] In particular, I do this *via* a small perturbation of the parameters of the VAR matrices B_j's in the cointegrated VECM representation $Y_t = A + B_1 Y_{t-1} + \ldots + B_p Y_{t-p} + G Y_{t-1} + u_t$, where Y_t collects (the logarithms of) M1 velocity and the short rate, and the rest of the notation is obvious. By only perturbating the elements of the VAR matrices B_j's—leaving unchanged the elements of the matrix G (and therefore both the cointegration vector and the loading coefficients)—I make sure that both the long-run equilibrium relationship between velocity and the short rate, and the way in which disequilibria in such a relationship map into subsequent adjustments in the two series, remain unchanged.

B.3 Comparing alternative money demand specifications

Table B.2 reports evidence that, in line with Benati *et al.* (2021), suggests that the data tend to 'prefer' the money demand specification proposed by Selden (1956) and Latané (1960)—featuring a *linear relationship* between velocity and the short rate—to the popular semi-log and log-log specifications proposed by Cagan (1956) and Meltzer (1963), respectively, which have long dominated research on money demand. The table reports results from Johansen's maximum eigenvalue tests[8] between M1 velocity and the short rate based on any of the three functional forms: (i) the Selden-Latané specification, in which both series enter the system in levels, i.e., $Y_t = [V_t \; R_t]'$ where V_t and R_t are M1 velocity and the short rate, respectively; (ii) the semi-log, with $Y_t = [\ln(V_t) \; R_t]'$; and (iii) the log-log, with $Y_t = [\ln(V_t) \ln(R_t)]'$.

The evidence in the table is quite clear. Out of nine countries, based on the Selden-Latané specification the null of 0 cointegration vectors is rejected six times, wheres in two cases (Norway and Sweden) the lack of rejection is borderline, with p-values equal to 0.1121 and 0.1136, respectively. Only for New Zealand the lack of rejection appears as quite solid, with a p-value equal to 0.1584. At the other end of the spectrum is Meltzer's (1963) log-log specification for which, out of six countries featuring a consistently positive nominal short-term interest rate over the sample period, the null of no cointegration is rejected in a single case, Australia. Further, in all other cases the lack of rejection appears as quite solid, with a p-value equal to 0.1431 for Canada, and the corresponding p-values for the other countries ranging between 0.3005 and 0.6209. The comparison between the results in Table B.2 for the Selden-Latané and log-log specifications provides additional, strong support to Benati *et al.*'s (2021) point that, for low-inflation (and therefore low-interest rates) countries such as those studies herein, the Selden-Latané specification provides a significantly better characterization of the data than the log-log. Turning to Cagan's (1956) semi-log specification, the null of no cointegration is rejected for just four countries out of nine. Further, only for Sweden, with a p-value of 0.1136, the lack of rejection is borderline: for the other four countries the p-values range between 0.1444 and 0.6565. Once again, a comparison between these results and those for the Selden-Latané specification clearly suggests that, between the two functional forms, the data quite clearly 'prefer' the Selden-Latané. Sure enough, an alternative interpretation of these results is also possible. Instead of interpreting them as suggesting that (i) there is indeed a stable long-run money demand, and that (ii) the correct functional form is the the Selden-Latané, a researcher could alternatively interpret them as suggesting instead that (iii) the correct functional form is (e.g.) the log-log, and (iv) that there is no stable long-run money demand. Admittedly, it is not possible to claim with certainty that the former interpretation (i.e., mine) is correct, whereas the latter is wrong. By Occam's razor, however, the former interpretation clearly appears (at

[8] Results from the trace tests are in line with those from the maximum eigenvalue tests, and they are available upon request.

least, to this author) as the most plausible and logical one.

I next turn to the issue of stability of the cointegration relationship.

B.4 Testing for stability in the cointegration relationship

Table B.3 reports results from Hansen and Johansen's (1999) Nyblom-type tests for stability in either the cointegration vector, or the vector of loading coefficients, in the estimated VECMs. The p-values reported in the table have been computed by bootstrapping, as in Cavaliere *et al.* (2012), the VECMs estimated conditional on one cointegration vector and no break of any kind, and then performing Hansen and Johansen's (1999) tests on the bootstrapped series. Before delving into the results, however, it is worth briefly discussing evidence on the performance of the tests.

B.4.1 Monte Carlo evidence on the performance of the tests

Table G.1 in Online Appendix G of Benati *et al.* (2021)—see Benati *et al.* (2019)—reports Monte Carlo evidence on the performance of the tests conditional on bivariate cointegrated DGPs for alternative sample lengths and alternative degrees of persistence of the cointegration residual, which is modeled as an AR(1). The main results can be summarized as follows. The two Nyblom-type tests exhibit an overall reasonable performance, incorrectly rejecting the null of no time variation most of the time at roughly the nominal size. Crucially, this is the case irrespective of the sample length and of the persistence of the cointegration residual. The fluctuation test, on the other hand, exhibits good performance only if the persistence of the cointegration residual is low. The higher the residual's persistence, however, the worse the performance, so that, for example, when the AR root of the residual is equal to 0.95 for a sample length $T = 50$, the test rejects at twice the nominal size. This result is clearly problematic, since as shown by Benati *et al.* (2021) cointegration residuals between (log) M1 velocity and (the logarithm of) a short-term nominal interest rate are typically moderately to highly persistent. In what follows I therefore focus on the results from the two Nyblom-type tests, and I instead eschew results from the fluctuation test.

B.4.2 Evidence

The key finding in Table B.3 is that evidence of breaks in either the cointegration vector or the loading coefficients is nearly non-existent. In particular, based on either the Selden-Latané specification—which, as discussed, appears to be the one preferred by the data for low-inflation (and therefore low-interest rates) countries—or the log-log, *not a single break* in either the cointegration vector or the loading coefficients is identified. As for the semi-log, no break in the cointegration vector is identified for any country, whereas only for Norway a break in the loading coefficients is identified, although the p-value, at 0.0935, is essentially borderline.

C Testing for Breaks in the Mean of Inflation

Table C.1 reports results from tests for multiple breaks at unknown points in the sample in the mean of inflation based on the methodology proposed by Bai and Perron (1998, 2003). Specifically, the table reports bootstrapped p-values for the double maximum test statistics *UDmax* and *WDmax* (which test the null hypothesis of no break against the alternative of at least one break). In performing the tests I exactly follow the recommendations of Bai and Perron (2003),[9] with the only difference that, instead of relying on the asymptotic critical values tabulated in Bai and Perron (1998), I bootstrap both critical and p-values *via* the procedure proposed by Diebold and Chen (1996), setting the number of bootstrap replications to 10,000. I set the maximum allowed number of structural changes to m=2. As discussed in the main text, for Australia, Canada, New Zealand, Norway, Sweden, and the United Kingdom I focus on the sample period since the introduction of inflation targeting; for the Euro area I consider the period since the start of European Monetary Union; and for the United States I consider the period following the break in the mean of inflation identified by Levin and Piger (2004), in 1992Q2. The evidence in Table C.1 is very clear: the null hypothesis of no breaks cannot be rejected for any country.

[9] See Bai and Perron (2003) section 5.5, 'Summary and Practical Recommendations'.

References

Alvarez, F. and Lippi, F. (2009), "Financial Innovation and the Transactions Demand for Cash", Econometrica, 77(2), 363-402

Andrews, D.K. and W. Ploberger (1994), "Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative", Econometrica, 62(6), 1383-1414

Bai, J. and P. Perron (1998), "Estimating and Testing Linear Models with Multiple Structural Changes", Econometrica, 66(1), 47-78

Bai, J. and P. Perron (2003), "Computation and Analysis of Multiple Structural Change Models", Journal of Applied Econometrics, 18(1), 1-22

Balke, N. and R.J. Gordon (1986), "Appendix B: Historical Data", in Robert J. Gordon, ed., "The American Business Cycle: Continuity and Change", The University of Chicago Press, 781-850

Benati, L. (2015): "The Long-Run Phillips Curve: A Structural VAR Investigation", Journal of Monetary Economics, 76(November), 15-28

Benati, L. (2020): "Money Velocity and the Natural Rate of Interest", Journal of Monetary Economics, 116(December), 117-134

Benati, L., R.E. Lucas Jr., J.P. Nicolini, and W. Weber (2019): "Online Appendix for: International Evidence on Long-Run Money Demand", Federal Reserve Bank of Minneapolis Staff Report 588 (June 2019)

Benati, L., R.E. Lucas Jr., J.P. Nicolini, and W. Weber (2021): "International Evidence on Long-Run Money Demand", Journal of Monetary Economics, 117(January)

Cagan, P. (1956), "The Monetary Dynamics of Hyperinflation", in M. Friedman, ed., "Studies in the Quantity Theory of Money", The University of Chicago Press, pp. 25-120.

Diebold, F., and C. Chen (1996), "Testing Structural Stability with Endogenous Breakpoint: A Size Comparison of Analytic and Bootstrap Procedures", Journal of Econometrics, 70(1), 221-241

Elliot, G., T.J. Rothenberg, and J.H. Stock (1996), "Efficient Tests for an Autoregressive Unit Root", Econometrica, 64(4), 813-836

Engle, R.F. and C.W.J. Granger (1987), "Cointegration and Error Correction: Representation, Estimation, and Testing", Econometrica, 55(2), 251-276

Friedman, M. and A.J. Schwartz (1963), "A Monetary History of the United States, 1867-1960", Princeton University Press

Goldfeld, S.M., and D.M. Sichel (1990), "The Demand for Money", in Friedman, B.M., and Hahn, F.H., eds., Handbook of Monetary Economics, Vol. I, Amsterdam, North Holland.

Haavisto, T. (1992), "Money and Economic Activity in Finland, 1866-1985", Lund Economic Studies

Hamilton, J. (1994), Time Series Analysis, Princeton University Press, Princeton

Hansen, H. and S. Johansen (1999), “Some Tests for Parameter Constancy in Cointegrated VAR Models”, Econometrics Journal, 2, 306-333

Homer, S. and R. Sylla (2005), “A History of Interest Rates”, John Wiley and Sons

Latané, H. A. (1960): “Income Velocity and Interest Rates: A Pragmatic Approach,” Review of Economics and Statistics, 42(4), 445-449.

Levin, A.T. and J.M. Piger (2004), “Is Inflation Persistence Intrinsic in Industrial Economies?”, European Central Bank Working Paper n. 334, April 2004

Lucas Jr., R. E., and J.-P. Nicolini (2015), “On the Stability of Money Demand”, Journal of Monetary Economics, 73, 48-65.

Luetkepohl, H. (1991), Introduction to Multiple Time Series Analysis, Springer-Verlag, Berlin

Mata, E. and N. Valerio (2011), The Concise Economic History of Portugal: A Comprehensive Guide, Coimbra: Almedina

Meltzer, A.H. (1963), “The Demand for Money: The Evidence from the Time Series”, Journal of Political Economy, 71(3), 219-246.

Tamaki, N. (1995), “Japanese Banking: A History (1859-1959)”, New York: Cambridge University Press

Selden, R. T. (1956): “Monetary Velocity in the United States,” in M. Friedman, ed., Studies in the Quantity Theory of Money, University of Chicago Press, pp. 405-454.

Stock, J.H. and M.W. Watson (2012), “Disentangling the Channels of the 2007-2009 Recession”, Brookings Papers on Economic Activity, Spring 2012.

Wu, J.C. and F.D. Xia (2016), “Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound”, Journal of Money, Credit and Banking, 48(2-3), 253-291

Tables for Online Appendix

Table B.1*a* Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a] for M1 velocity and the short rate

		M1 velocity				Short rate			
Country	*Period*	p=2	p=4	p=6	p=8	p=2	p=4	p=6	p=8
Australia	1972Q1-2019Q4	0.9557	0.9573	0.9508	0.9841	0.4980	0.5122	0.5269	0.4098
Canada	1982Q3-2019Q4	0.2122	0.3114	0.3555	0.1592	0.0792	0.4167	0.4657	0.5730
Euro area	1970Q1-2019Q4	0.0733	0.3247	0.2133	0.1701	0.5228	0.6106	0.6629	0.5678
New Zealand	1990Q1-2019Q4	0.8679	0.8850	0.8748	0.7521	0.1123	0.0809	0.0258	0.0972
Norway	1978Q1-2019Q4	0.3307	0.3386	0.2024	0.2365	0.4458	0.5890	0.6803	0.5231
Sweden	1998Q1-2019Q4	0.7846	0.7036	0.7687	0.6471	0.3808	0.3850	0.4061	0.5515
United Kingdom	1963Q1-2019Q4	0.9884	0.9647	0.9436	0.9268	0.5847	0.4865	0.5016	0.5134
United States	1959Q1-2019Q4	0.8686	0.8662	0.8576	0.8340	0.3162	0.2854	0.2914	0.2031

[a] Based on 10,000 bootstrap replications.

Table B.1*b* Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a] for long-term interest rates

Country	*Period*	First long-term rate p=2	p=4	p=6	p=8
Australia	1972Q1-2019Q4	0.9211	0.9106	0.8537	0.8794
Canada	1982Q3-2019Q4	0.0699	0.5022	0.4254	0.5377
Euro area	1970Q1-2019Q4	0.7897	0.8131	0.7688	0.6564
New Zealand	1990Q1-2019Q4	0.1581	0.0720	0.0278	0.0158
Norway	1978Q1-2019Q4	0.8696	0.9199	0.9080	0.9105
South Korea	1973Q3-2019Q4	0.7579	0.7046	0.6463	0.5849
Sweden	1998Q1-2019Q4	0.5800	0.7125	0.7276	0.8437
United Kingdom	1963Q1-2019Q4	0.8871	0.9130	0.9088	0.9146
United States	1959Q1-2019Q4	0.6337	0.6548	0.6006	0.5567
		Second long-term rate			
		p=2	p=4	p=6	p=8
Australia	1972Q1-2019Q4	0.9333	0.9415	0.8991	0.9211
Canada	1982Q3-2019Q4	0.0687	0.4569	0.3820	0.4754
Euro area	1970Q1-2019Q4	0.8705	0.8629	0.7946	0.6466
New Zealand	1990Q1-2019Q4	0.1349	0.0582	0.0272	0.0086
Norway	1978Q1-2019Q4	0.8819	0.9236	0.9142	0.9266
Sweden	1998Q1-2019Q4	0.6030	0.7577	0.7607	0.8608
United Kingdom	1963Q1-2019Q4	0.9132	0.9214	0.9079	0.9137
United States	1959Q1-2019Q4	0.7004	0.7135	0.6573	0.6302
		Third long-term rate			
		p=2	p=4	p=6	p=8
Canada	1982Q3-2019Q4	0.0396	0.4093	0.3423	0.3989
Euro area	1970Q1-2019Q4	0.9106	0.9151	0.8800	0.8356
Sweden	1998Q1-2019Q4	0.6470	0.7804	0.7880	0.8822
United States	1959Q1-2019Q4	0.7620	0.7613	0.7193	0.7182

[a] Based on 10,000 bootstrap replications.

Table B.1*c* Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a] for interest rates' spreads

Country	*Period*	First interest rate spread			
		p=2	p=4	p=6	p=8
Australia	1972Q1-2019Q4	0.0000	0.0000	0.0007	0.0026
Canada	1982Q3-2019Q4	0.0018	0.0051	0.0132	0.0047
Euro area	1970Q1-2019Q4	0.1594	0.1328	0.1623	0.1486
New Zealand	1990Q1-2019Q4	0.0160	0.0059	0.0052	0.0129
Norway	1978Q1-2019Q4	0.0004	0.0022	0.0131	0.0008
Sweden	1998Q1-2019Q4	0.0950	0.0091	0.0059	0.0076
United Kingdom	1963Q1-2019Q4	0.0139	0.0015	0.0005	0.0017
United States	1959Q1-2019Q4	0.0002	0.0000	0.0000	0.0000
		Second interest rate spread			
		p=2	p=4	p=6	p=8
Australia	1972Q1-2019Q4	0.0003	0.0004	0.0011	0.0020
Canada	1982Q3-2019Q4	0.0130	0.0087	0.0106	0.0037
Euro area	1970Q1-2019Q4	0.2622	0.1816	0.1840	0.1507
New Zealand	1990Q1-2019Q4	0.0548	0.0140	0.0111	0.0292
Norway	1978Q1-2019Q4	0.0006	0.0020	0.0150	0.0006
Sweden	1998Q1-2019Q4	0.1274	0.0049	0.0044	0.0066
United Kingdom	1963Q1-2019Q4	0.0180	0.0012	0.0023	0.0028
United States	1959Q1-2019Q4	0.0013	0.0000	0.0001	0.0002
		Third long-term rate			
		p=2	p=4	p=6	p=8
Canada	1982Q3-2019Q4	0.0481	0.0210	0.0149	0.0010
Euro area	1970Q1-2019Q4	0.2262	0.0603	0.0834	0.1110
Sweden	1998Q1-2019Q4	0.1348	0.0027	0.0012	0.0020
United States	1959Q1-2019Q4	0.0055	0.0003	0.0007	0.0007

[a] Based on 10,000 bootstrap replications.

Table B.2 Bootstrapped p-values[a] for Johansen's maximum eigenvalue[b] tests for (log) M1 velocity and (the logarithm of) a short-term rate, for three alternative money demand specifications

		Money demand specification		
Country	*Period*	Selden-Latané	Semi-log	Log-log
Australia	1969Q3-2019Q4	0.0661	0.0128	0.0617
Canada	1967Q1-2019Q4	0.0279	0.6565	0.1431
Euro area	1999Q1-2019Q4	0.0896	0.1587	–[c]
New Zealand	1990Q1-2019Q4	0.1584	0.0836	0.6209
Norway	1978Q1-2019Q4	0.1121	0.0720	0.3572
Sweden	1998Q1-2019Q4	0.1136	0.1184	–[c]
United Kingdom	1955Q1-2019Q4	0.0201	0.0764	0.5444
United States	1959Q1-2019Q4	0.0985	0.2063	0.3005

[a] Based on 10,000 bootstrap replications. [b] Null of 0 *versus* 1 cointegration vectors. [c] The last observations for the interest rate are either zero or negative.

Table B.3 Bootstrapped p-values[a] for Hansen and Johansen's (1999) tests for stability in the cointegration vector for (log) M1 velocity and (the log of) a short-term rate

		Money demand specification:		
Country	*Period*	*Selden-Latané*	*Semi-log*	*Log-log*
		I: Tests for stability in the cointegration vector		
Australia	1969Q3-2019Q4	0.7835	0.7880	0.6950
Canada	1967Q1-2019Q4	0.6900	0.7945	0.6070
Euro area	1999Q1-2019Q4	0.4880	0.2915	0.2915
New Zealand	1990Q1-2019Q4	0.5392	0.4726	0.7346
Norway	1978Q1-2019Q4	0.5590	0.1940	0.8560
Sweden	1998Q1-2019Q4	0.2335	0.1690	–[c]
United Kingdom	1955Q1-2019Q4	0.5905	0.5480	0.9365
United States	1959Q1-2019Q4	0.5875	0.8030	0.9940
		II: Tests for stability in the loading coefficients		
Australia	1972Q1-2019Q4	0.4430	0.9330	0.8635
Canada	1982Q3-2019Q4	0.6110	0.2865	0.3660
Euro area	1970Q1-2019Q4	0.1250	0.2720	–[c]
New Zealand	1990Q1-2019Q4	0.8155	0.4955	0.8525
Norway	1978Q1-2019Q4	0.6975	0.0935	0.5810
Sweden	1998Q1-2019Q4	0.1075	0.2900	–[c]
United Kingdom	1963Q1-2019Q4	0.3190	0.2770	0.6370
United States	1959Q1-2019Q4	0.1310	0.4800	0.9235

[a] Based on 10,000 bootstrap replications. [b] Null of 0 *versus* 1 cointegration vectors. [c] The last observations for the interest rate are either zero or negative.

Table C.1 Tests for multiple breaks at unknown points in the sample in the mean of inflation based on Bai and Perron (1998): bootstrapped p-values for double maximum test statistics

Country	*Period*	*UDmax*	*WDmax*
Australia	1994Q3-2019Q4	0.2348	0.2612
Canada	1991Q1-2019Q4	0.3336	0.2982
Euro area	1999Q1-2019Q4	0.2048	0.2318
New Zealand	1990Q1-2019Q4	0.3558	0.3230
Norway	2001Q2-2019Q4	0.3506	0.3212
Sweden	1998Q1-2019Q4	0.5736	0.6122
United Kingdom	1992Q4-2019Q4	0.3114	0.3548
United States	1992Q2-2019Q4	0.1572	0.1322

[a] Based on 10,000 bootstrap replications.

Figures for Online Appendix

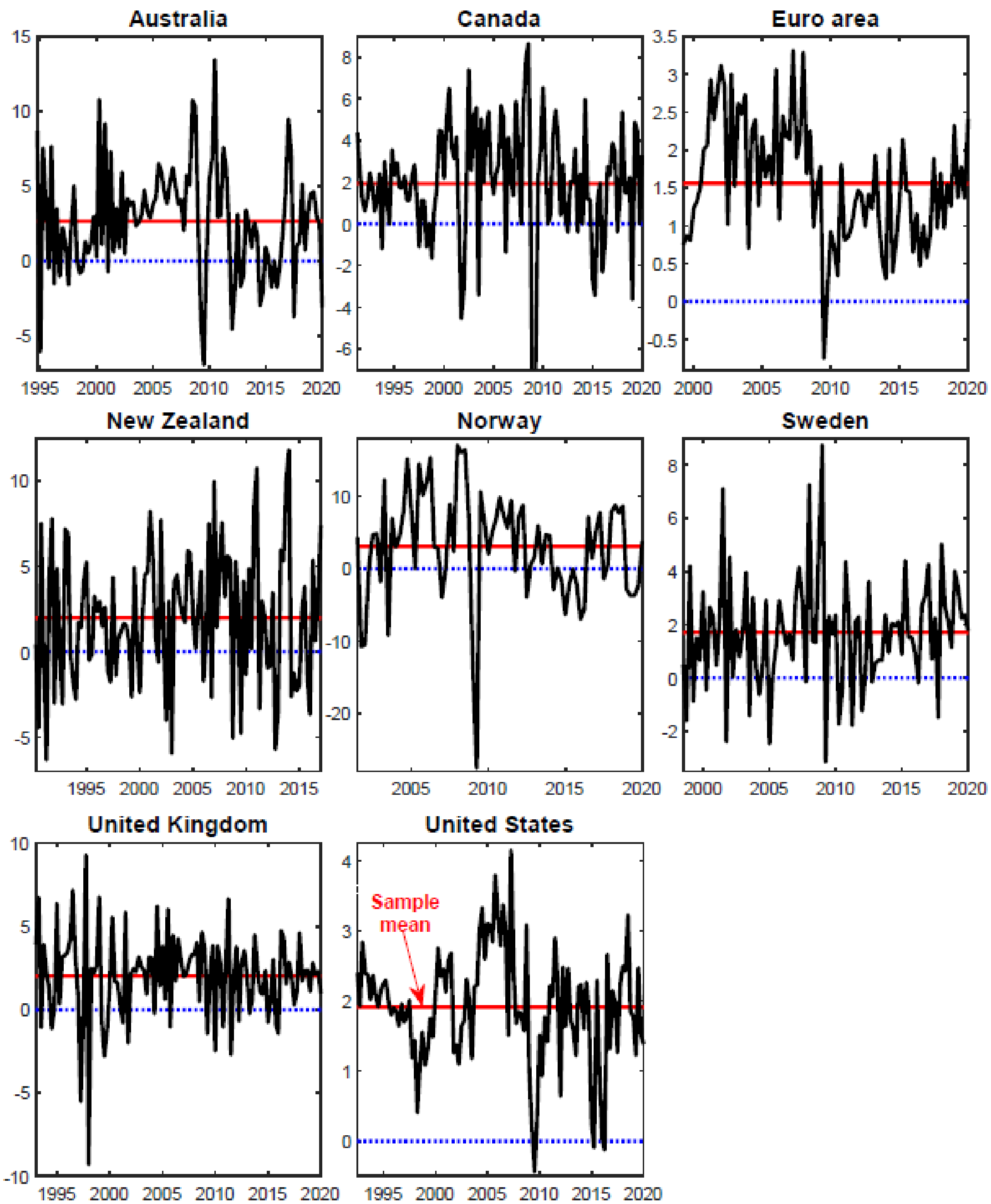

Figure A.1 Inflation under inflation-targeting regimes; European Monetary Union; and for the United States the period since the break in the mean identified by Levin and Piger (2003)

Results obtained by projecting
the short rate onto M1 velocity

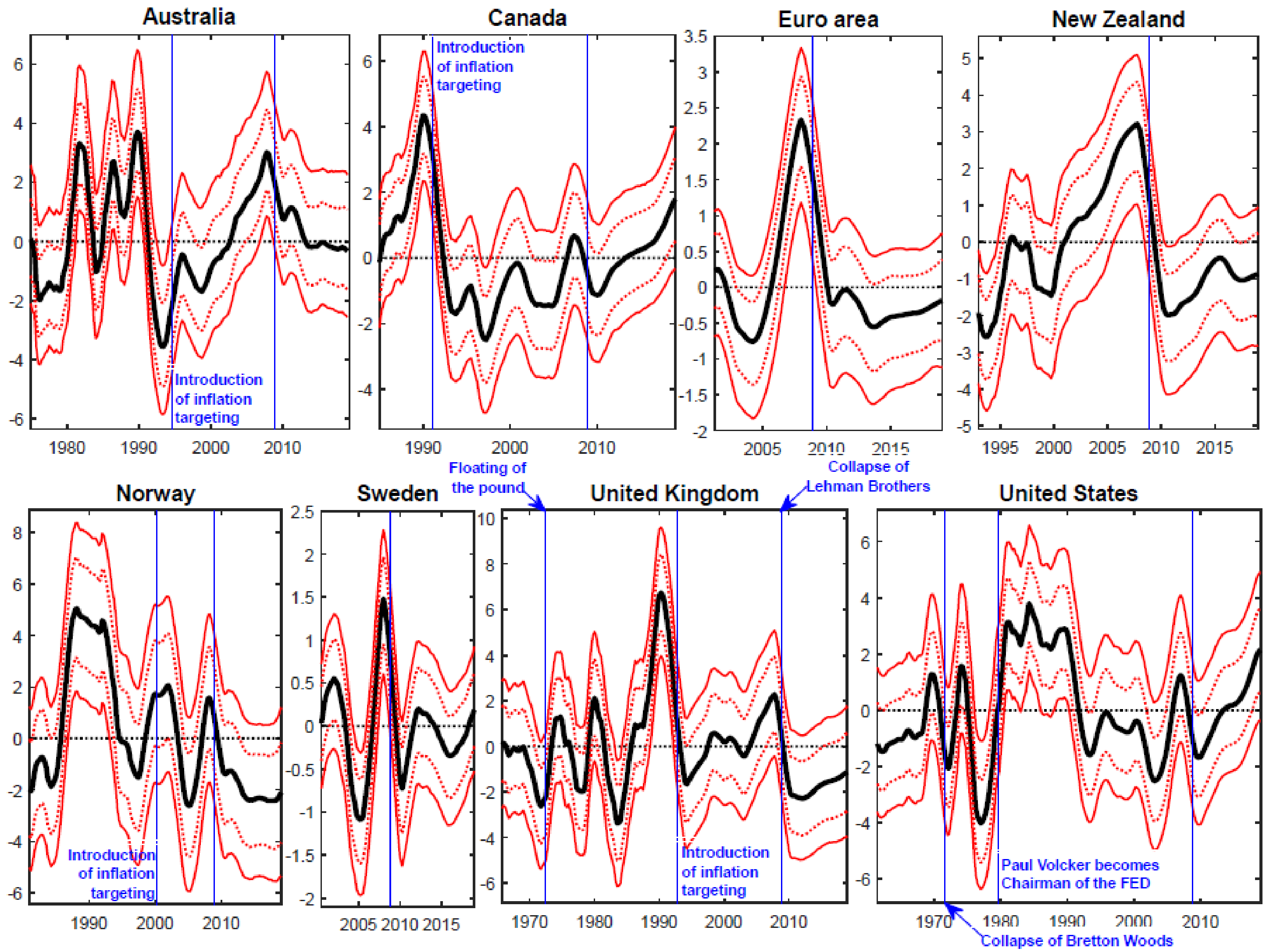

Figure A.2 Estimated deviation of the short rate from the nominal natural rate computed by projecting the short rate on M1 velocity (with 1- and 2-standard deviations bootstrapped confidence bands)

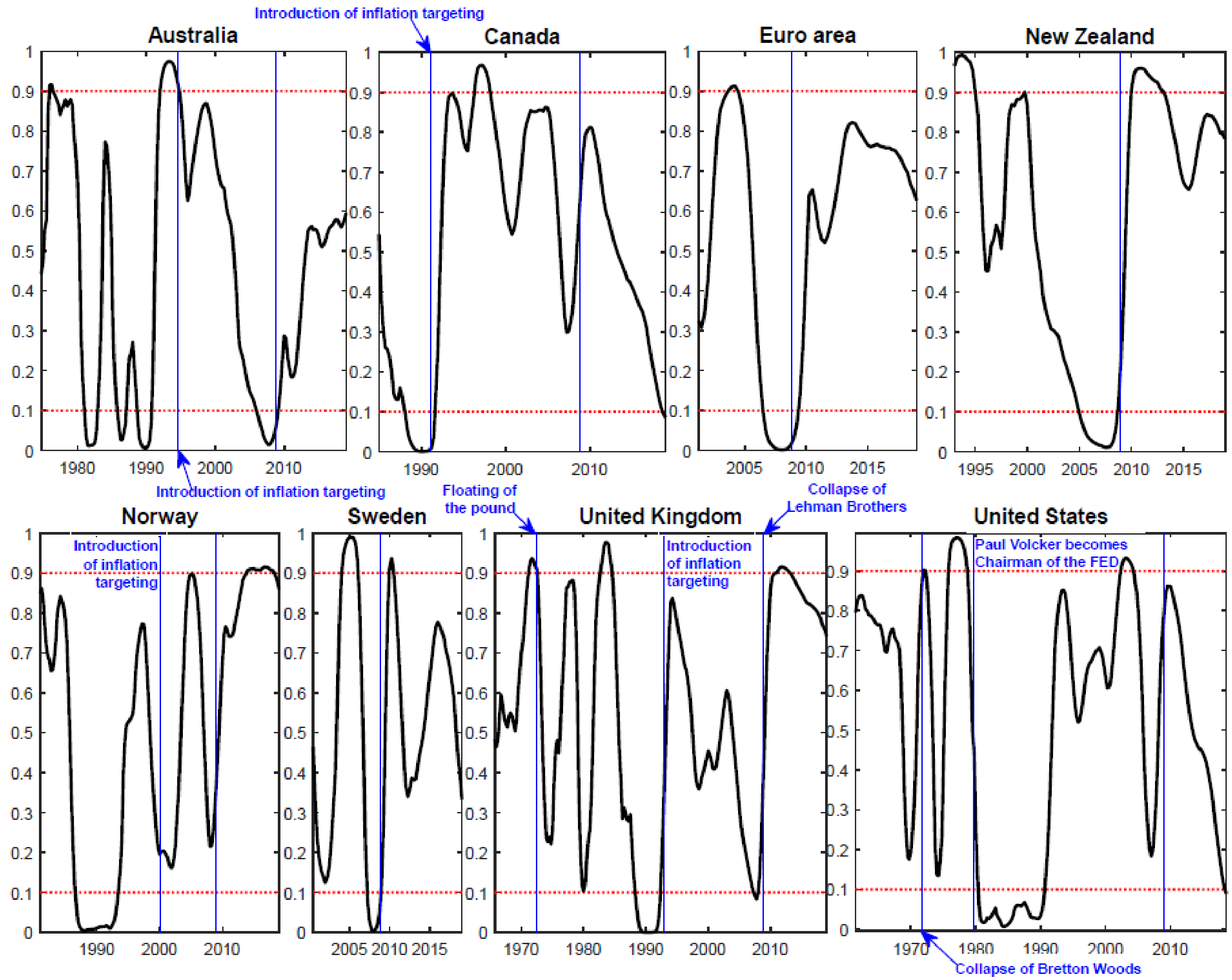

Figure A.3 Fraction of bootstrap replications for which the deviation of the short rate from the nominal natural rate computed by projecting the short rate on M1 velocity is negative

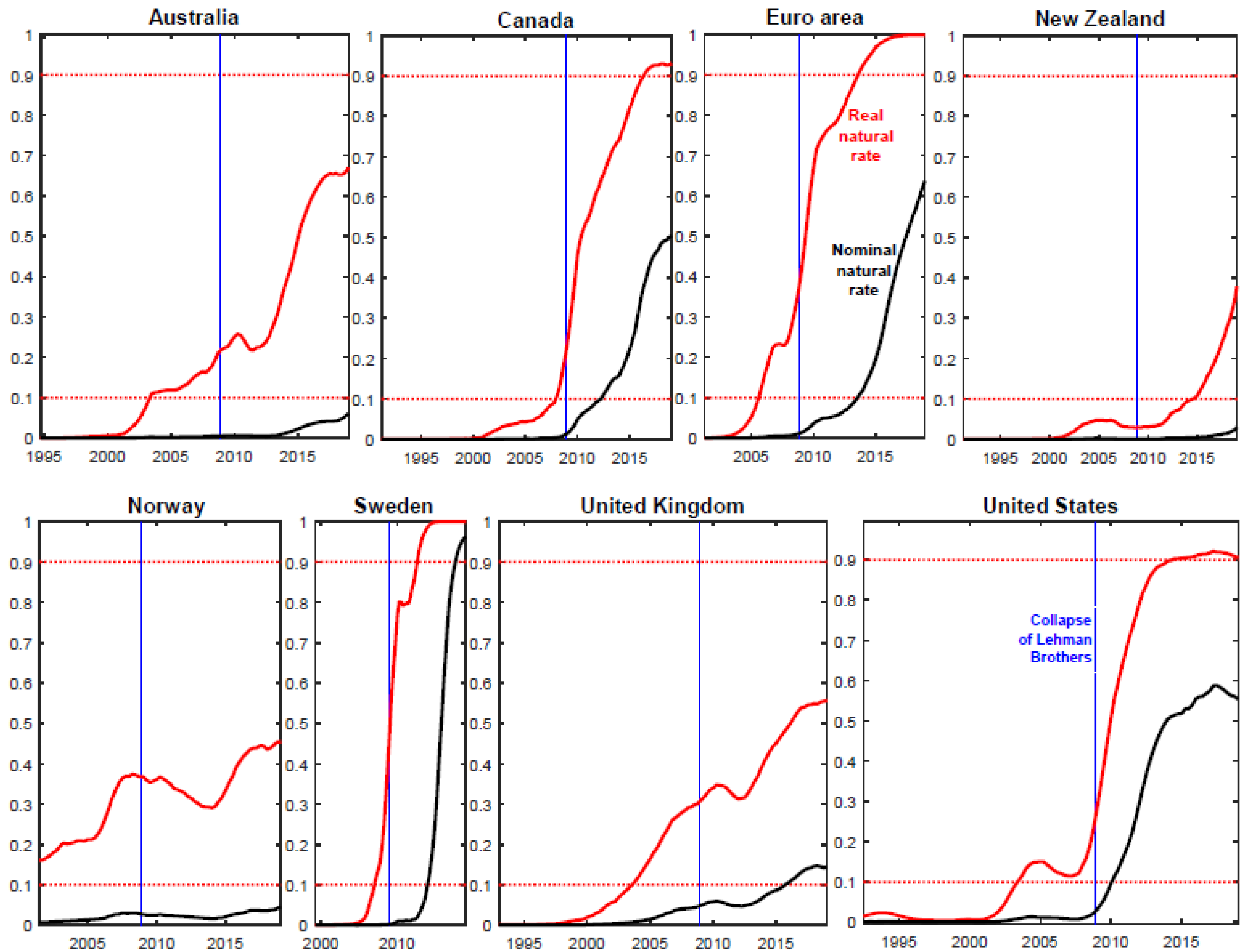

Figure A.4 Fractions of bootstrap replications for which the nominal and real natural rates of interest are estimated to have been negative, computed by projecting the short rate on M1 velocity

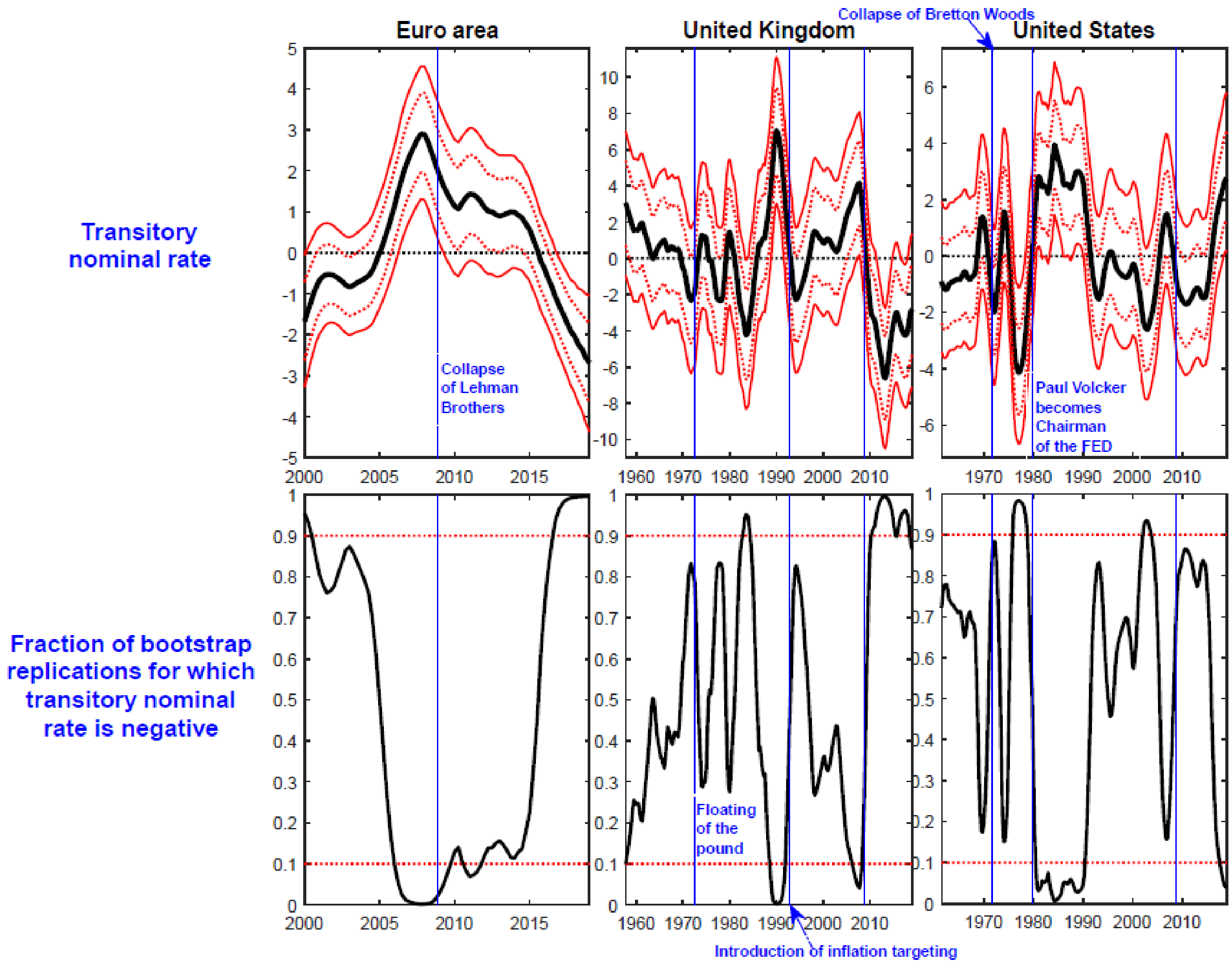

Figure A.5 Estimated deviation of Wu and Xia's 'shadow rate' from the nominal natural rate computed by projecting the shadow rate on M1 velocity (with 1- and 2-standard deviations bootstrapped confidence bands), and fraction of bootstrap replications for which the deviation is negative

Results based on cointegrated SVARs
identified via long-run restrictions

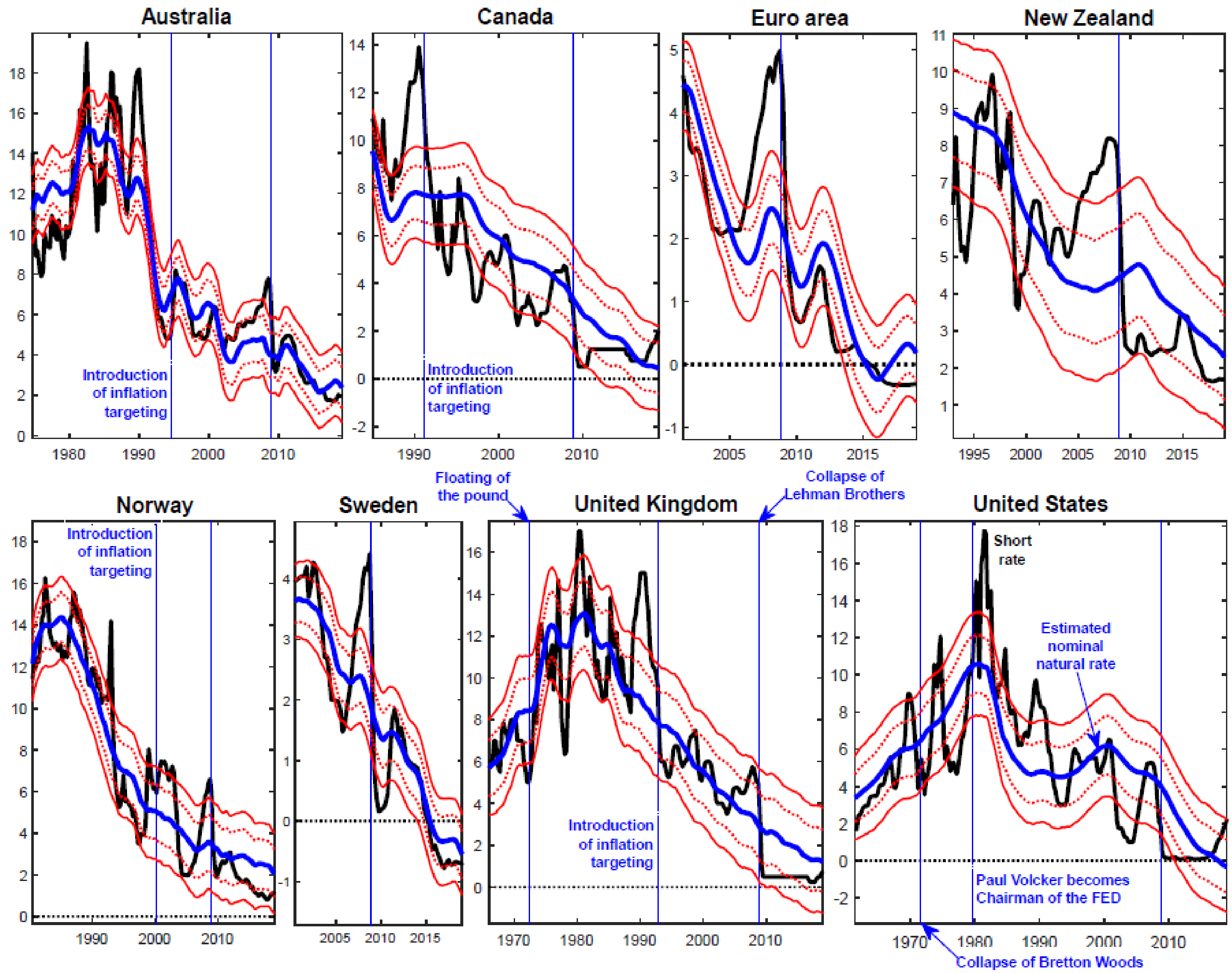

Figure A.6 Estimates of the nominal natural rate based on cointegrated SVARs (with 1- and 2-standard deviations bootstrapped confidence bands)

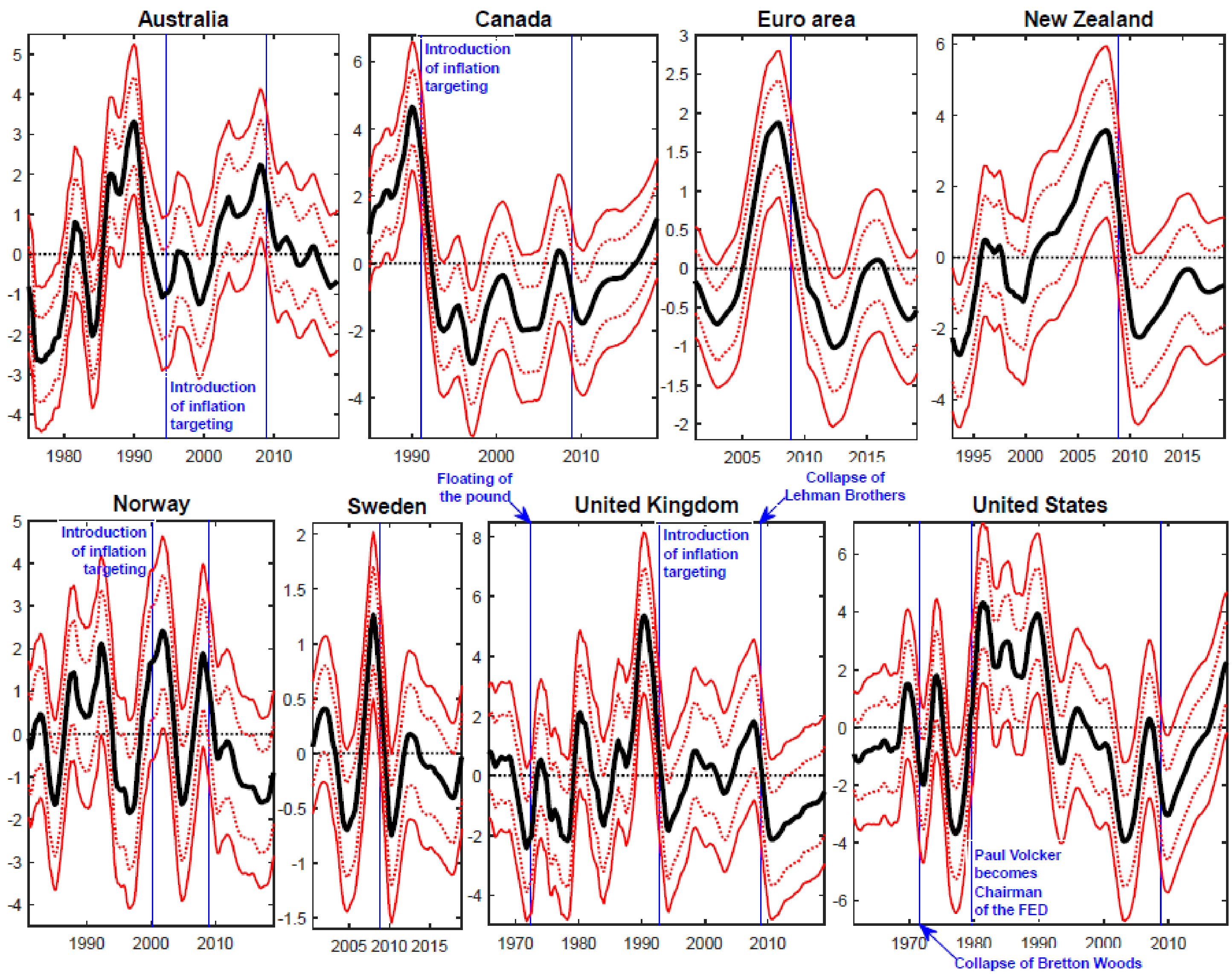

Figure A.7 Estimated deviation of the short rate from the nominal natural rate based on cointegrated SVARs (with 1- and 2-standard deviations bootstrapped confidence bands)

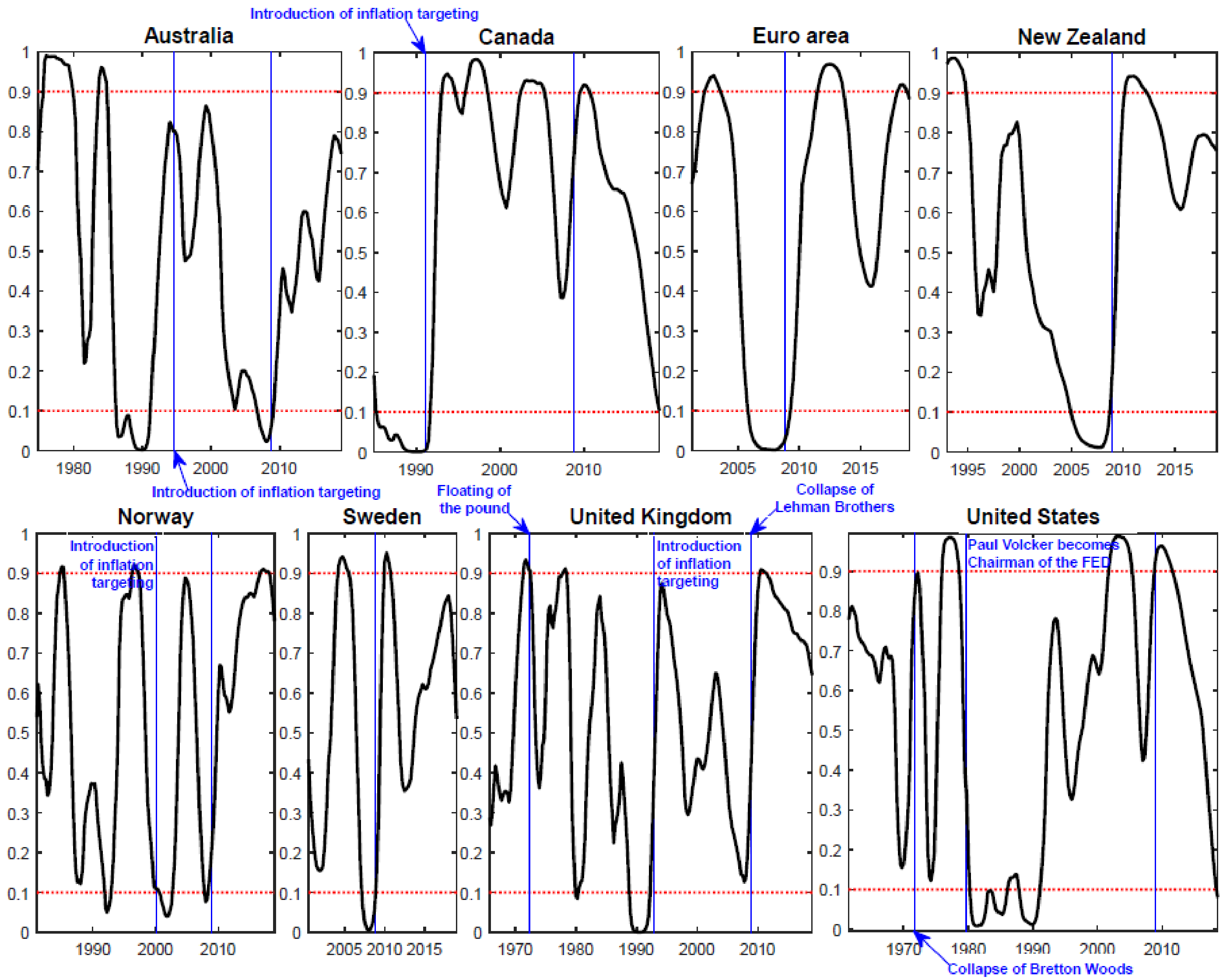

Figure A.8 Fraction of bootstrap replications for which the deviation of the short rate from the nominal natural rate is negative, based on cointegrated SVARs

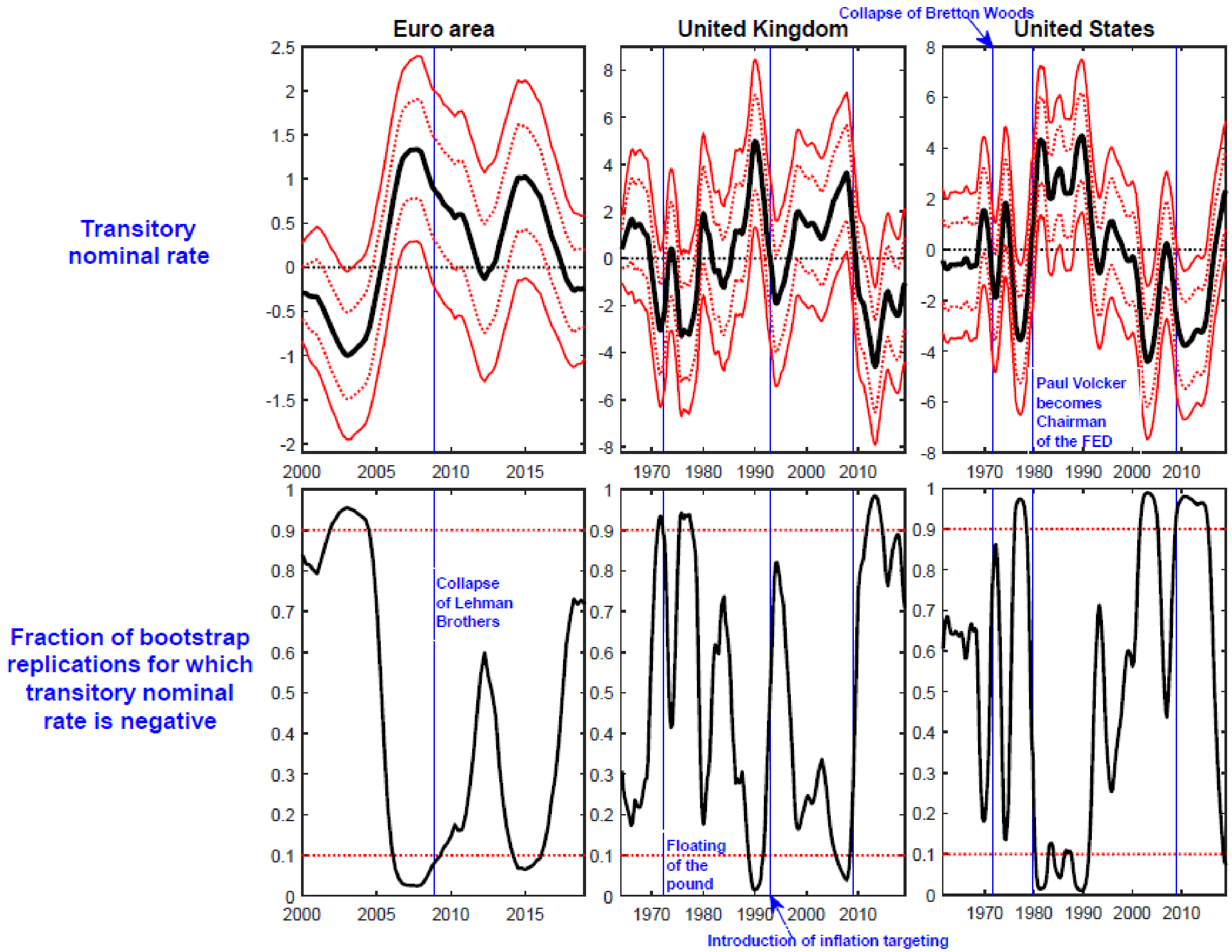

Figure A.9 Estimated deviation of Wu and Xia's 'shadow rate' from the nominal natural rate computed based on cointegrated SVARs (with 1- and 2-standard deviations bootstrapped confidence bands), and fraction of bootstrap replications for which the deviation is negative

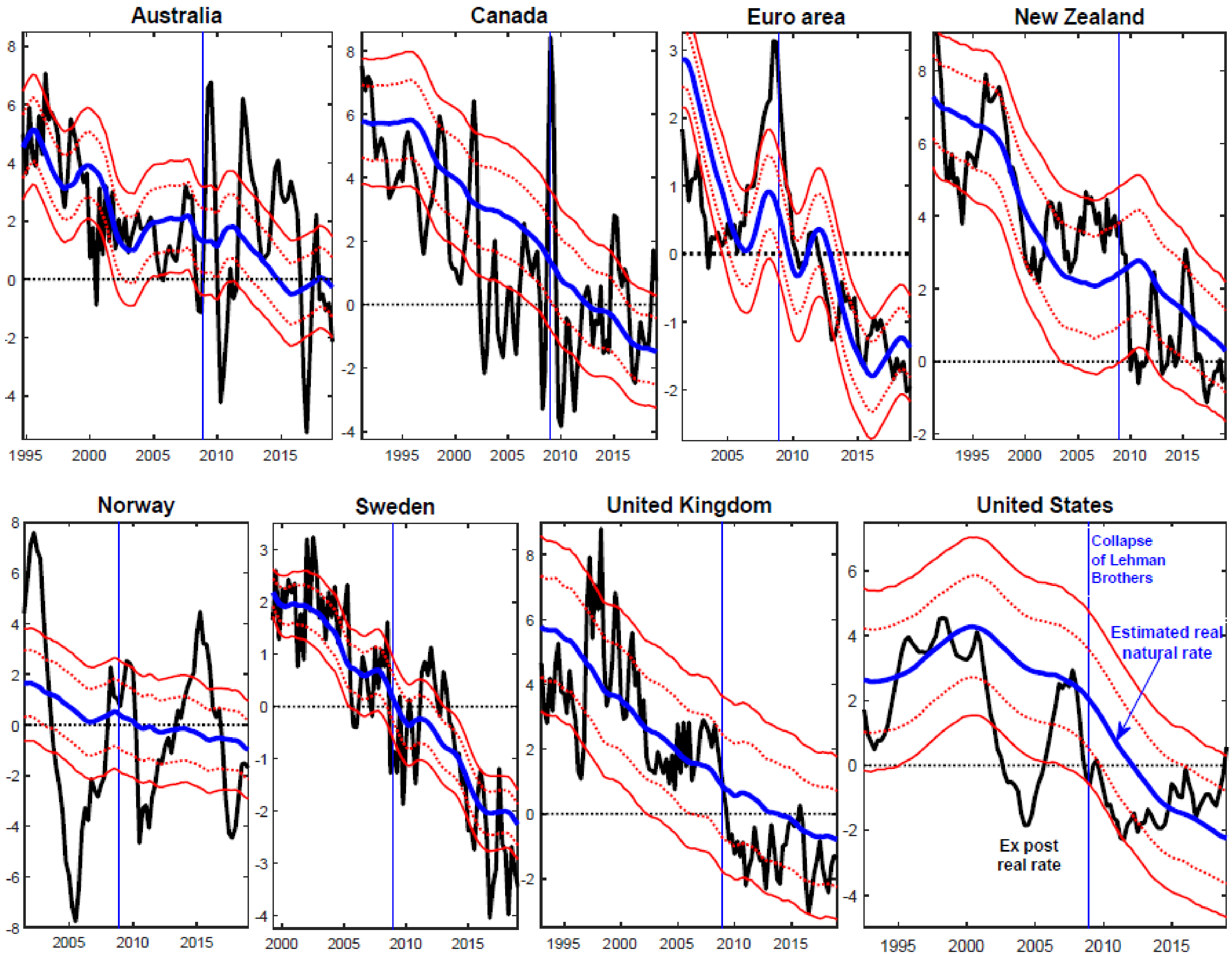

Figure A.10 Estimates of the real natural rate for monetary regimes making inflation I(0) (with 1- and 2-standard deviations bootstrapped confidence bands) based on cointegrated SVARs