b
UNIVERSITÄT
BERN

Faculty of Business, Economics
and Social Sciences

Department of Economics

Imperfect Competition with Costly Disposal

Severin Lenhard

21-05

April, 2021

DISCUSSION PAPERS

Schanzeneckstrasse 1
CH-3012 Bern, Switzerland
http://www.vwi.unibe.ch

Imperfect Competition with Costly Disposal*

Severin Lenhard†

Version: April 2021

Abstract

This paper studies the disposal costs' effect on consumer surplus and firms' profits. The costlier disposal, the less is disposed of, firms' competition for market shares increases, thereby benefiting consumers. Yet firms decrease their production to mitigate costs, affecting consumer surplus negatively. We present a model with ex ante homogeneous firms producing inventories either early at low cost and with little information about demand, or later with more information yet at higher costs. Unsold products are disposed of. In equilibrium, firms may be asymmetric. Disposal goes down with costs but so do inventories. In our set-up, the negative effect on the trade volume dominates decreasing consumer surplus and firms' profits. We show, however, that low disposal costs substitute information about demand. Increasing disposal costs improve a firm's information advantage and may increase its profits.

Keywords: Disposal, Inventory, Uncertain Demand, Market Structure
JEL: D43, L11, L13, L50

*I thank Giacomo Calzolari, Winand Emons, Yassine Lefouili, Igor Letina, Marc Möller, Sarah Spycher, Eva Zuberbühler, and two anonymous referees for helpful discussions and suggestions.

†Universität Bern, severin.lenhard@vwi.unibe.ch. Departement Volkswirtschaftslehre, Universität Bern, Schanzeneckstrasse 1, Postfach 8573, CH-3001 Bern, Switzerland.

1 Introduction

A wide variety of commodities is produced in advance. Firms manufacture their products anticipating future demand, determining inventories while their product's popularity is unknown. Accordingly, production costs are sunk when products are put for sale. If demand turns out to be lower than expected, firms may hold back some quantity to increase the market price.

This behavior has been increasingly observed over the last years. Investigative journalists have uncovered several cases where firms have discarded new, unsold products. One of the most infamous scandals was uncovered in 2010 by the New York Times. A Hennes and Mauritz (H&M) store in New York City discarded new clothes at their back entrance, cutting them to make sure they will never be worn. The same course of action was used by a Nike store in 2017. In reaction to the negative headlines, firms usually pledge improvement, yet disposing of unsold products is an open secret in the fashion industry.[1]

Discarding new products is not confined to the apparel industry. French Amazon dumped almost 3 million unsold products in 2018. All over France, new products worth $900 million are discarded each year according to an estimate by the government.[2] The disposal of unused products is considered as a waste of resources and an environmental burden, which led the French government to legislate.

In 2016 France already passed a law prohibiting grocery stores from disposing of food as long as it is still edible. With the new *loi anti-gaspillage* (anti-waste law), regulators broaden the prohibition of disposal to non-food products, including textiles, electronics, and daily hygiene products. Unsold products must be donated or recycled.[3] The new regulation is expected to come into effect in 2023.

This paper studies how firms respond to a regulatory increase in their disposal costs and the effects on consumer surplus. The literature on this subject is scarce.[4] Environmental economists usually discuss policies to reduce waste and increase recycling.[5] The focus is usually on the failure of the first welfare theorem resulting from exter-

[1]Not all firms keep it a secret. Burberry literally burnt almost $40 million of stock in 2018. The fashion brand reported the deed in their annual report and specified that the energy was used to make the process environmentally friendly.

[2]`https://www.nytimes.com/2019/06/05/world/europe/france-unsold-products.html` (last accessed September 30, 2020).

[3]Projet de loi relatif à la lutte contre le gaspillage et à l'économie circulaire (TREP1902395L).

[4]The literature on operation research considers the costs of unsold products for the inventory's optimization. Choosing the optimal inventory is known as the newsvendor problem, e.g., Rosenfield (1989) or van der Laan and Salomon (1997).

[5]For example, Dinan (1993) argues that taxing disposal instead of a virgin tax can increase efficiency in newspaper markets. In his model, disposing of newspapers creates a cost, which is internalized by taxing total output. A virgin tax, however, only leads to the substitution of input factors, i.e., firms use recycled old newspapers instead of virgin material.

nalities. Instead of looking at an efficient mechanism to reduce disposal, we focus on the costly disposal's effect in a market with imperfect competition, thereby abstracting from externalities.

The larger disposal costs are, the less firms dispose of. Thus, given its inventory, a firm competes more aggressively for a larger market share if disposal is costly, thereby benefiting consumers. However, firms adjust their inventory strategy in response to costly disposal. They decrease their inventories to mitigate costs if demand is lower than expected. Disposal decreases, yet consumers are negatively affected. Firms, furthermore, may adjust their location choice if disposal costs increase leading to a change in the market structure.

In our model, firms first choose their manufacturing location, which determines their production timing. They produce their commodities either abroad at low costs and with little information about demand, or at home close to the market with more information yet at higher costs. If demand is lower than expected, firms can hold back quantities to increase the market price. Restrained or unsold products are not perfectly reversible; firms may even incur a per-unit cost to dispose of commodities. Each unsold unit is, therefore, not only a loss in revenue, it also increases costs.

There exist three different types of equilibria in pure strategies: (i) If demand is highly uncertain, both firms produce close to the market. They delay their production until they have full information. Consequently, firms dispose of nothing, and an increase in disposal costs has no effect. (ii) If demand is reasonably predictable, both firms produce abroad at low costs. An increase in disposal costs decreases the expected disposal yet also expected consumer surplus and firms' expected profits.

(iii) For intermediate levels of demand uncertainty, one firm produces abroad, while the other one close to the market. The firm abroad manufactures at lower costs, yet the firm close to the market has an information advantage. The higher disposal costs are, the lower is the expected disposal. Consumer surplus, however, is also lower. The abroad firm's expected profits decrease, while the other one's increase. The information advantage is more valuable, the higher disposal costs. At some point, the abroad firm may postpone its production closer to the market, too. Due to the change in the market structure, profits and consumer surplus may change discontinuously.

The subgame perfect equilibrium in pure strategies is unique, except for type (iii), where an equivalent equilibrium exists with the firms' label interchanged. Although firms are ex ante symmetric, they may choose an asymmetric market structure. The ordering of firms' profits is ambiguous: The firm abroad has a cost advantage, while the other an information advantage. The former's reaction to a demand below expectations is expensive if disposal costs are high. Due to this costly reaction to new information, the latter's information advantage is more valuable if disposal is expensive.

By contrast, without disposal costs, the firm producing abroad does not incur additional costs. Any reaction to new information comes for free. Low disposal costs substitute information about demand. To be more specific, take the example of a monopolist. If demand realization is known, the firm produces the monopoly quantity. Now suppose demand is uncertain, yet the firm's inventory is perfectly reversible, i.e., disposal costs are zero. The monopoly produces its inventory equal to the monopoly quantity for the greatest possible demand. If a lower demand materializes, the monopolist reverses parts of its inventory and sells again the monopoly quantity, equivalent as if the firm had known its demand. Information about demand is, therefore, more valuable if disposal is costly.

In our set-up, a regulatory increase in disposal costs fulfills its purpose to decrease expected disposal. Yet, consumers are generally worse off. Although competition for market shares increase, firms decrease their production and thereby the trade volume. We extend our model to test the robustness of the negative effect's domination over the positive competitive effect. First, we allow firms to produce in both locations. Firms manufacture first their inventory at low costs while demand is uncertain. Both firms can then react to new information about demand either by disposing of or producing additional quantities. In the unique, symmetric equilibrium, the disposed of amount decreases. Yet consumer surplus and firms' profits also decrease.[6]

Second, we study the same model when firms observe their competitor's quantity. Companies announce their targeted sales to inform investors. These targets are publicly announced. Thus, competitors may not directly observe the inventory but can infer its size. If inventories are observed, there may exist additional asymmetric equilibria. Extensive stocks send the message of large intended sales. The firm with a larger inventory can only credibly commit to selling large parts if disposal is costly. This effect benefits the larger firm. However, costly disposal increases the costs to adjust to demand below expectations. Due to the two opposing effects, the larger firm's expected profits are ambiguous, precisely, U-shaped in disposal costs. The smaller firm produces mainly after the demand's realization. Its information advantage becomes more valuable with costly disposal, resulting in higher expected profits. Consequently, both firms' profits may increase in disposal costs, yet consumer surplus decreases.

Furthermore, we discuss different forms of competition, namely, perfect competition and price competition. In general, a regulatory increase in disposal costs decreases the disposed of amount. Yet the expected trade volume decreases, thereby putting consumers in a worse position.

[6]In the supplementary materials, we study an extension to $N \geq 2$ firms - consumer surplus increases in the number of firms, yet the disposal, too. Competition increases at the cost of a larger number of disposers. Policymakers concerned about the disposed of amount face a trade-off between competition and the disposed of amount.

Related Literature. The literature on inventory is closely linked to the capacity literature. Technically, firms first choose an upper bound of their next stage's sales volume. In a capacity game, firms have to incur additional costs in the second stage, while production costs are sunk in an inventory game. Whenever the costs in the second stage are normalized to zero, the games are therefore formally equivalent. In their seminal paper, Kreps and Scheinkman (1983) show that capacity choice followed by price competition yields an outcome equivalent to the Cournot outcome.

Their result depends on the rationing rule (see, e.g., Davidson and Deneckere (1986)), furthermore a pure strategy equilibrium may not exist if uncertainty is introduced (e.g. Hviid (1991) or Reynolds and Wilson (2000)). Nonetheless, Young (2010) confirms the Cournot equivalence with a multiplicative demand shock and relatively high capacity costs. He avoids a rationing rule by introducing product differentiation. A recent paper by Montez and Schutz (2018) studies a similar game, in which firms can not observe their competitor's inventory and price. Firms' may hold back some of their inventories, which are at most partly reversible.[7] When production costs tend to full reversibility, the outcome ends to Bertrand competition. Conclusively, the inventories' observability determines the difference between the Cournot and Bertrand outcome.

None of the previous papers allow for additional costs to discard unsold products. By contrast, Saloner (1986) provides in his seminal paper a model allowing for additional costs to dispose of. Inventory is not fully reversible, and firms may even incur additional costs for unsold products. First, firms choose (simultaneously or sequentially)[8] their inventory, which is observed by their competitor and later compete in sales volume. Since there is no demand uncertainty, in the end, firms dispose of nothing. The higher the disposal costs, the more credible it is for a firm to dispose of nothing. Inventories indicate, therefore, intended sales. We rule out this effect by assuming that inventories are unobserved by the competitors. However, we relax our assumption in an extension.[9]

Mitraille and Moreaux (2013) study a two-period model where firms can manufacture in each period. Storing is costly and observed by the competitors. The stored commodities' production costs are sunk, resulting in zero effective marginal costs. When

[7]Pashigian (1988) showed that clearance sale prices are below marginal costs in the apparel industry, presenting empirical evidence for the imperfect reversibility.

[8]Maggi (1996) studies a reduced form model with demand uncertainty, which predicts a sequential outcome in pure strategies. By contrast, Pal (1993) studies the Saloner model with mixed strategies and argues that the sequential outcome is just a realization of a symmetric equilibrium in mixed strategies.

[9]Recently, different effects of inventory have been studied in the literature. Antoniou and Fiocco (2019) analyze the inventory's impact on future prices to prevent stockpiling of consumers. Similar to Mitraille and Thille (2014), who study the presence of speculators: demand may be higher, but later competition increases due to the resellers. Dana and Williams (2019) and Qu et al. (2018) discuss the effect of inventory on intertemporal price discrimination.

firms compete in the second stage, firms with storage have thus a cost advantage.[10] Firms seek leadership at the cost of storing the commodity. Sales volume increases resulting in a lower market price if firms can store.

By contrast, Thille (2006) finds in an infinitely repeated game that storage does not affect prices in the absence of depreciation. With depreciation, firms incur higher costs to maintain their stock of inventory, resulting in lower sales are higher market prices.

We do not explicitly model storage costs. In our set-up, storing costs would decrease the cost advantage of early production. Since inventories are not observable and demand is uncertain, firms postpone their production. Similar to the latter, sales go down and the price increases.

Dada and van Mieghem (1999) also study production timing. A monopolist chooses inventory, sales volume, and prices, either before or after demand realization. The monopolist restrains some products to affect the market clearing price. Anupindi and Jiang (2008) extend the model to a three-stage version with competing firms, whereby firms invest in capacity before demand materializes. Flexible firms produce after the demand realization, while inflexible firms produce ex ante. Firms trade-off the value of commitment to flexible production. None of these papers studies costly disposal nor competition with unobserved inventories.

We follow Hamilton and Slutsky (1990) and have a pre-game stage where firms choose their production timing without committing to quantities. By contrast, Liu (2005) and Wang and Xu (2007) study a sequential move game where demand uncertainty decreases over time. In their set-up, the firm producing in the second stage makes higher profits if its information advantage is large. However, the authors implicitly assume infinitely high disposal costs. If the other firm can adjust its sales volume to the demand realization, their result remains only partly. We show that low disposal costs substitute information about demand.

In our set-up, ex ante symmetric firms may choose different strategies, as in Robson (1990). According to van Damme and Hurkens (1999), a sequential equilibrium is only stable if the first mover has a cost advantage. This is also a necessary condition in our model. Additionally, we require a follower's information advantage and no free disposal for the existence of an asymmetric equilibrium.

The rest of the paper is structured as follows. The next section presents the model. We derive the equilibrium in section 3. Section 4 extends the model to multiple manufacturing locations. Furthermore, we discuss the effect of observable inventories; a formal analysis is contained in the supplementary materials. Finally, we discuss other forms of competition. Section 5 concludes. All proofs are relegated to the appendix.

[10] Technically, a disposal cost is a negative production cost expanding the cost advantage.

2 Model

Two firms produce a homogeneous commodity. Demand is linear. The inverse demand function's slope is random, reflecting an unknown number of identical consumers. Formally, let the inverse demand function in state $s \in \{l, h\}$ be $P_s(Q) = a - b_s Q$, where Q is the total sales volume. The intercept $a > 0$ denotes the maximal willingness to pay and is commonly known. The slope b_s takes on one of two values, $b_l = 1 + \beta$ or $b_h = 1 - \beta$, each with equal probability. The difference between the two is measured by $\beta \in [0, 1)$, which we refer to as demand uncertainty. If b_h materializes, the willingness to pay is higher for any quantity. Therefore, we refer to b_h (b_l) as the high (low) demand state. With this set-up, the expected inverse demand function is independent of β.[11]

First, firms choose their location, determining their timing of production. They either produce the quantity $\bar{q}_1$ in the first period at marginal costs normalized to zero or postpone their production until the demand has materialized and manufacture $q_{2,s}$ at marginal costs $c \in [0, a/2]$. Producing at an early stage is less expensive, e.g., firms have time to adjust processes to substitute input factors. Firms can off-shore their production to decrease their costs. However, products manufactured abroad need to be shipped to the home market. Production, therefore, has to precede in time. Firms trade-off costs and uncertainty: producing at a lower cost with less information or defer the manufacturing until more information is available, yet production is more expensive.

We denote the strategy to produce (abroad) before the demand materializes by A and the strategy to produce in the second stage by H (home). If a firm chooses strategy A, it can hold back its goods after the demand has materialized. Let a firm's sales volume be $q_{1,s}$, thus $\bar{q}_1 - q_{1,s}$ is the quantity held back. We denote its marginal cost as $d > 0$, reflecting costs to dispose of products.[12] We restrict parameters such that the consumers' willingness to pay is relatively large, formally $a \geq 2c + d$. This assumption guarantees that both firms are active in equilibrium. Figure 1 summarizes the timing strategies.[13]

[11]Commonly, demand uncertainty is modeled with a linear demand curve and a random intercept (e.g. Gilpatric and Li (2015) or a random slope (e.g. Daughety and Reinganum (1994) or Malueg and Tsutsui (1996)). In Klemperer and Meyer (1986), firms facing a random slope prefer to fix quantities and let prices adjust depending on the demand realization. In our model, firms also prefer to fix quantities and let prices adjust if disposal costs are high.

[12]The model can be generalized to costs of production in the first stage $\tilde{c} \geq 0$, with costs of production in the second stage $c \geq \tilde{c}$ and disposal costs $d \geq -\tilde{c}$. Note that for $d \in [-\tilde{c}, 0)$ parts of the inventory is reversible. For example, if products are reused or sold in a clearance sale. Our main results do qualitatively not change.

[13]We assume that only one timing strategy is feasible. We extend the model in section 4. For a model where firms can produce and sell in both periods, see, for example, Arvan (1985) or Mitraille and Moreaux (2013).

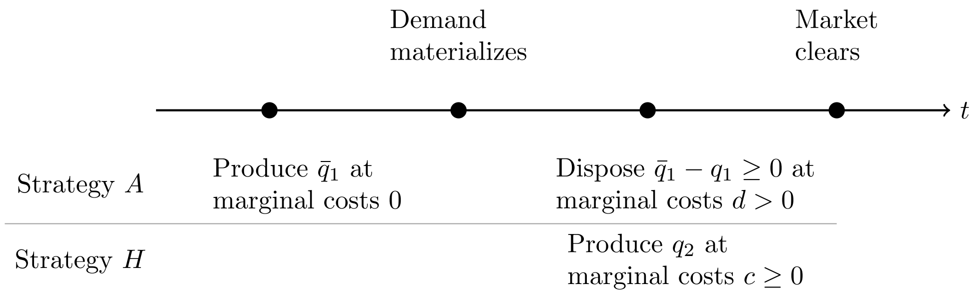


Figure 1: Timeline and production strategies.

Henceforth, we suppress the state index s for quantities. Formally, the two strategies' expected profits are

$$\mathbb{E}[\pi_1(q_1, \bar{q}_1)] = \mathbb{E}[P_s(Q)q_1 - d(\bar{q}_1 - q_1)], \tag{1}$$

$$\mathbb{E}[\pi_2(q_2)] = \mathbb{E}[P_s(Q)q_2 - cq_2], \tag{2}$$

where Q is the sum of the firms' sales volume; (1) refers to the strategy A and (2) to strategy H.

Firms simultaneously choose their location, which is observed by the competitor. Thus, four cases exist in pure strategies: both firms choose strategy A (A,A), both choose strategy H (H,H), and one chooses strategy A and the other strategy H (A,H) and (H,A). By symmetry of the game, the latter two differ only in the firms' labels.

3 Equilibrium

We derive the subgame perfect equilibrium in pure strategies. Thus, we solve the game by backward induction. We first derive all equilibria following symmetric location strategies. Second, we derive all equilibria following the asymmetric ones. Each location strategy pair has a unique subgame equilibrium, which we then use to determine the firms' equilibrium location strategies.

Symmetric Subgames. Given firms choose the same location strategy, the subgame is symmetric and we denote one firm by i and the other by j. Furthermore, we use the index A to indicate equilibrium quantity and profits of the (A,A) game and H for the (H,H) game. We start by deriving the subgame equilibrium of the (A,A) game, i.e., both firms produce abroad.

After the demand realization, production costs are sunk, both firms take their inventory as given. Both choose their sales volume $q_i \in [0, \bar{q}_i]$ to maximize

$$\pi_{i,s}(q_i|\bar{q}_i) = P_s(Q)q_i - d(\bar{q}_i - q_i), \tag{3}$$

q_A	high demand	low demand
$d < \beta a$	$\frac{a-d}{3(1-\beta)}$	$\frac{a+d}{3(1+\beta)}$
$d \geq \beta a$	$\frac{a}{3}$	$\frac{a}{3}$

Table 1: Inventory and sales volume of the (A,A) subgame. Inventory equals the sales volume in the high demand state.

where $Q = q_i + q_j$. The best response function of firm i can be written as

$$q_i(q_j|\bar{q}_1) = \min\left\{\max\left\{\frac{a+d}{2b_s} - \frac{1}{2}q_j, 0\right\}, \bar{q}_i\right\}. \tag{4}$$

The best response function weakly decreases in the competitor's sales volume, and it can maximally sell its total inventory.

A firm's sales volume given by (4) increases in d for any q_j and $\bar{q}_i$. The costlier disposal, the less is disposed of. If inventories are fixed, increasing disposal costs make firms compete more aggressively for large market shares.

Firms do not observe their competitor's inventory. Nonetheless, each firm anticipates its own disposal behavior. Formally, the firms choose their inventory $\bar{q}_i \geq 0$ to maximize (1) subject to (4). The optimal inventory strategy of firm i can be written as

$$\bar{q}_i(q_{j,l}, q_{j,h}) = \max\left\{\frac{a}{2} - \frac{1+\beta}{4}q_{j,l} - \frac{1-\beta}{4}q_{j,h}, \frac{a-d}{2(1-\beta)} - \frac{1}{2}q_{j,h}, 0\right\}. \tag{5}$$

The best response function is explicitly derived in the appendix. We show that in the first part of the maximum operator, firm i sells its inventory regardless of the demand state. In the second, it disposes of if demand is below expectations. The inventory decreases with d. Firms decrease their inventory to mitigate costs if demand is lower than expected, thereby giving up profits in the high demand state.

We summarize the subgame's equilibrium sales volume in Table 1. In the high demand state, firms sell their total inventory, which decreases in d. In the low demand state, firms dispose of if $d < \beta a$. Firms dispose of less, the higher disposal costs are, the sales volume in the low demand state increases in d. If $d \geq \beta a$, nothing is thrown away, firms sell their total inventory regardless of the demand's size.[14]

Expected price, disposal, firm's profit, and consumer surplus are summarized in Lemma 1.

[14]For general $\tilde{c} > 0$, the threshold can be rearranged to $\tilde{c} \geq (\beta a - d)/(1+\beta)$. Products expensive in manufacturing are not disposed of.

Lemma 1. *The (A,A) game's unique subgame perfect equilibrium implies an expected market price*

$$\mathbb{E}[P] = \frac{a}{3},$$

and expected disposal of

$$\mathbb{E}[\bar{q}_A - q_A] = \begin{cases} \frac{2(\beta a - d)}{3(1-\beta^2)}, & \textit{if } d < \beta a; \\ 0, & \textit{if } d \geq \beta a. \end{cases}$$

Furthermore, a firm's expected profits are

$$\mathbb{E}[\pi_A] = \begin{cases} \frac{(a-d)^2}{18(1-\beta)} + \frac{(a+d)^2}{18(1+\beta)}, & \textit{if } d < \beta a; \\ \frac{a^2}{9}, & \textit{if } d \geq \beta a, \end{cases} \tag{6}$$

and expected consumer surplus is

$$\mathbb{E}[CS_A] = \begin{cases} \frac{1(a-d)^2}{9(1-\beta)} + \frac{1(a+d)^2}{9(1+\beta)}, & \textit{if } d < \beta a; \\ \frac{2a^2}{9}, & \textit{if } d \geq \beta a. \end{cases} \tag{7}$$

Disposal costs decrease expected disposal, firm's profits, and consumer surplus.

Expected prices are independent of d, while expected disposal decreases with its costs. Firms' profits decrease with d because any adjustment to a lower than expected demand is costly and firms compete more fiercely. However, expected consumer surplus also decreases: firms decrease their inventory and, thus, the expected trade volume. The lower inventories' negative effect dominates the positive effect of fiercer competition.

Next, we derive the equilibrium of the (H,H) game. Firms manufacture close to the market, thus delay their production until demand has materialized. Both firms choose $q_i \geq 0$ to maximize (2), technically, the firms play a Cournot game. Their best response can be written as

$$q_i(q_j) = \max\left\{\frac{a-c}{2b_s} - \frac{1}{2}q_j, 0\right\}. \tag{8}$$

By contrast to the best response in (4), the sales volume is not restricted anymore. Firms incur marginal costs of production c instead of disposal costs d. Formally, the disposal costs are likewise a negative production cost, as can be seen by comparing (4) to (8). Instead of incurring a cost for each unit sold, firms with disposal costs incur a cost for each unit unsold.

q_1	high demand	low demand
$d < \beta\frac{a+c}{2}$	$\frac{a-2d+c}{3(1-\beta)}$	$\frac{a+2d+c}{3(1+\beta)}$
$d \geq \beta\frac{a+c}{2}$	$\frac{a+c}{3}$	$\frac{a+c}{3}$

Table 2: Inventory and leader's sales volume depending on the demand state. Inventory equals the sales volume in the high demand state.

The subgame's equilibrium sales volume is $q_{H,s} = (a-c)/(3b_s)$. Since firms produce with full information, nothing is disposed of. The following Lemma summarizes the expected price, firm's profits, and consumer surplus.

Lemma 2. *The (H,H) game's unique subgame perfect equilibrium implies an expected market price*

$$\mathbb{E}[P] = \frac{a+2c}{3}.$$

Furthermore, a firm's expected profits are

$$\mathbb{E}[\pi_H] = \frac{(a-c)^2}{9(1-\beta^2)}. \tag{9}$$

and expected consumer surplus is

$$\mathbb{E}[CS_H] = \frac{2(a-c)^2}{9(1-\beta^2)}. \tag{10}$$

Firms produce with full information about demand, therefore, nothing is disposed of. Expected prices, profits, and consumer surplus are independent of disposal costs.

Asymmetric Subgame. Suppose one firm has chosen strategy A and the other strategy H. We denote the former as leader (she) and the latter as follower (he). The leader's expected profit is given by equation (1) and the follower's expected profit by equation (2), with $Q = q_1 + q_2$.

In the second stage, the leader's production costs are sunk. She chooses her sales volume $q_1 \in [0, \bar{q}_1]$ to maximize (3) yielding the best response function given by (4). The follower does not observe the leader's inventory. Yet, the leader anticipates her optimal disposal behavior. She maximizes (1) subject to (4), yielding the optimal inventory strategy in (5).

The follower produces after the demand realization. His best response function is given by equation (8). The leader's inventory and sales volume are given in Table 2.

q_2	high demand	low demand
$d < \beta\frac{a+c}{2}$	$\frac{a+d-2c}{3(1-\beta)}$	$\frac{a-d-2c}{3(1+\beta)}$
$d \geq \beta\frac{a+c}{2}$	$\frac{a-2c}{3(1-\beta)} + \beta\frac{a+c}{6(1-\beta)}$	$\frac{a-2c}{3(1+\beta)} - \beta\frac{a+c}{6(1+\beta)}$

Table 3: The follower's sales volume depending on the demand state.

The inventory of the leader decreases with disposal costs. She sells her total inventory if demand is high and disposes of if demand is low. Disposal costs increase the sales volume in the low demand state so that the disposed of amount decreases. She gives up some profits in the high demand state to mitigate costs if demand is low. If $d \geq \beta(a+c)/2$, nothing is thrown away.[15]

The follower's sales volume given in Table 3 moves in the opposite direction: if the leader decreases her sales volume, the follower increases his and vice versa. The assumption $a \geq 2c + d$ ensures that the follower's sales volume is positive. Entry blocking is thus not possible due to relatively low costs. Lemma 3 summarizes the expected price, disposal, firms' profits, and consumer surplus.

Lemma 3. *The asymmetric location game's unique subgame perfect equilibrium implies an expected market price*

$$\mathbb{E}[P] = \frac{a+c}{3},$$

and expected disposal of

$$\mathbb{E}[\bar{q}_1 - q_1] = \begin{cases} \frac{\beta(a+c)-2d)}{3(1-\beta^2)}, & \text{if } d < \beta\frac{a+c}{2}; \\ 0, & \text{if } d \geq \beta\frac{a+c}{2}. \end{cases}$$

Furthermore, the leader's expected profits are

$$\mathbb{E}[\pi_1] = \begin{cases} \frac{(a-2d+c)^2}{18(1-\beta)} + \frac{(a+2d+c)^2}{18(1+\beta)}, & \text{if } d < \beta\frac{a+c}{2}; \\ \frac{(a+c)^2}{9}, & \text{if } d \geq \beta\frac{a+c}{2}, \end{cases} \tag{11}$$

the follower's expected profits are

$$\mathbb{E}[\pi_2] = \begin{cases} \frac{(a+d-2c)^2}{18(1-\beta)} + \frac{(a-d-2c)^2}{18(1+\beta)}, & \text{if } d < \beta\frac{a+c}{2}; \\ \frac{4(a-2c)^2+\beta^2(a+c)(5a-7c)}{36(1-\beta^2)}, & \text{if } d \geq \beta\frac{a+c}{2}, \end{cases} \tag{12}$$

[15] For general $\tilde{c} > 0$, the threshold can be rearranged to $\tilde{c} \geq (\beta a + \beta c - 2d)/(2(1+\beta))$. Similar as above, products expensive in manufacturing are not disposed of.

and expected consumer surplus is

$$\mathbb{E}[CS_{AH}] = \begin{cases} \frac{(2a-d-c)^2}{36(1-\beta)} + \frac{(2a+d-c)^2}{36(1+\beta)}, & \textit{if } d < \beta\frac{a+c}{2}; \\ \frac{(4a-2c-\beta(a+c))^2}{144(1-\beta)} + \frac{(4a-2c+\beta(a+c))^2}{144(1+\beta)}, & \textit{if } d \geq \beta\frac{a+c}{2}. \end{cases} \tag{13}$$

Expected disposal, leader's profits, and consumer surplus decrease in disposal costs, while the follower's profits increase.

The expected price is independent of d, similar to the symmetric subgames. It only reflects production costs. In Lemma 1 both firms have zero production costs, in Lemma 2 both incur costs c, and in Lemma 3 one has zero costs, while the other has costs c. The realized price, however, depends on d: The price is higher in the high demand state and the difference between the materialized prices increases in d. Firms incur larger costs to adjust their sales volume to the demand realization. If d is small, firms inexpensively adjust to the materialized demand, thereby absorbing the effect of demand on the price.

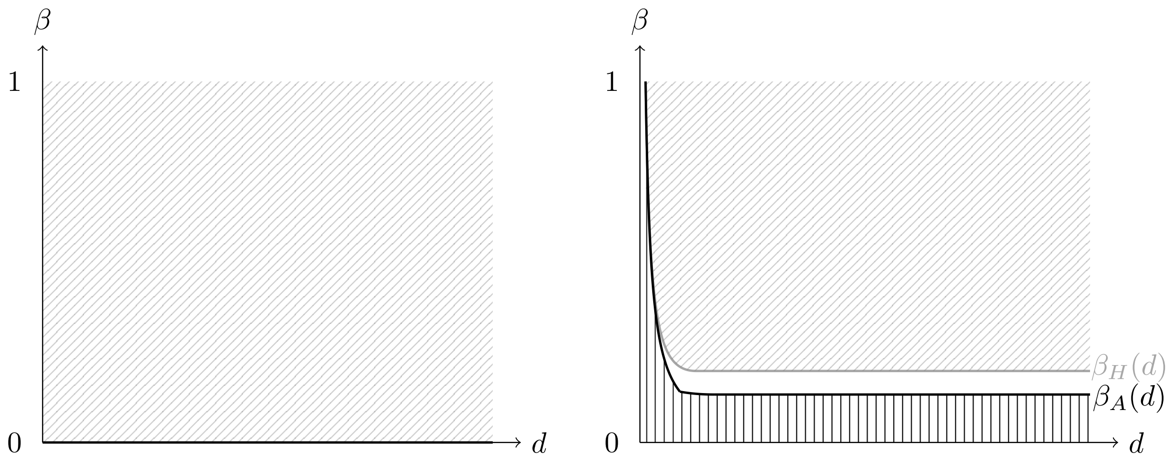
Expected disposal decreases with its costs. The leader's reaction to a low demand becomes costlier if d increases. The follower's information advantage becomes more valuable. The competitors' profits respond, therefore, opposed to an increase in disposal costs. If inventory is, however, observable, disposal costs may increase both firms' profits. We discuss this case in section 4.

By assumption, the leader has lower production costs, while the follower has superior information about demand. The ordering of profits is thus ambiguous. The leader has an advantage if either disposal costs are low, demand uncertainty is low, or both.

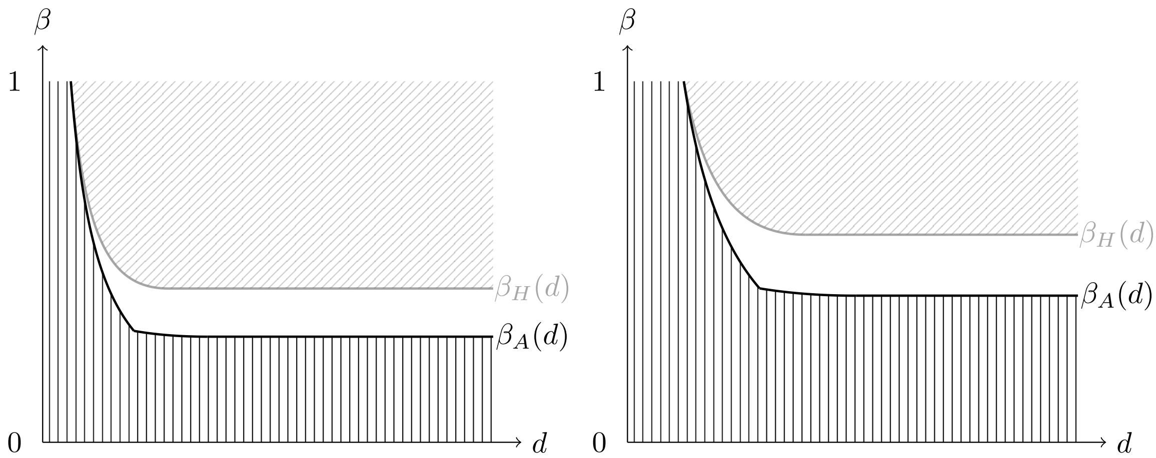
With fully reversible inventories, the follower's information advantage is worthless. The leader bears no cost to decrease her quantity in response to low demand while having a cost advantage in production. By contrast, if the leader has no cost advantage, the follower expects higher profits than the leader. In the knife edge case of no cost advantage and fully reversible inventory, both firms expect the same profit.

Location Game. A firm's optimal location strategy maximizes its expected profits. Since the competitor observes the location, firms can anticipate their competitive behavior, respectively, the subsequent market structure. We use the expected profits given in Lemmas 1, 2, and 3 to derive the equilibrium.

It is useful to define the following two threshold functions. We first define the threshold function where the leader is indifferent between producing abroad or close to the market, let $\beta_H(d) := \{\beta|\mathbb{E}[\pi_H] = \mathbb{E}[\pi_1]\}$. Similarly, we define the threshold function $\beta_A(d) := \{\beta|\mathbb{E}[\pi_A] = \mathbb{E}[\pi_2]\}$ such that the follower is indifferent between strategy A and H. We derive explicit expressions of the functions in the appendix and show that they weakly decrease with d and $\beta_H(d) \geq \beta_A(d)$.



(a) No cost advantage ($c = 0$) (b) Low cost advantage ($c = 0.1$)



(c) Medium cost advantage ($c = 0.5$) (d) High cost advantage ($c = 1$)

Figure 2: Equilibrium. In the diagonally gray (vetically black) shaded area, both firms produce with strategy H (A). In the white area firms choose an asymmetric strategy, one chooses A and the other H. (Demand intercept $a = 10$.)

Proposition 1. *Generically, there exists a unique subgame perfect equilibrium.*

(i) If demand is highly uncertain, i.e. $\beta \geq \beta_H(d)$*, both firms produce in the first stage by choosing strategy* H*.*

(ii) If demand is fairly predictable, i.e. $\beta \leq \beta_A(d)$*, both firms produce in the second stage by choosing strategy* A*.*

(iii) Otherwise, i.e. $\beta \in [\beta_A(d), \beta_H(d)]$*, one firm chooses strategy* A *and the other strategy* H*.*

Figure 2 illustrates Proposition 1. Depending on the choice of parameters, each type of equilibrium can be supported. Moreover, types do not coexist, except by definition on the threshold function $\beta_A(d)$ and $\beta_H(d)$.[16] Due to the monotonic expected profit functions, the β functions are all decreasing: The information advantage is less crucial the lower the disposal costs. Low disposal costs enable an inexpensive reaction to the demand realization. Thus, there is a negative relationship between β and d.[17]

Firms face a trade-off between costs and information. They either produce at an early stage with low costs yet with uncertain demand or postpone their production until demand has materialized, yet production costs are higher. Without the early production cost advantage, the trade-off does not exist: both firms produce at a later stage. Whereas, if the cost advantage is large, firms may produce at an early stage. Production abroad is an equilibrium either if demand uncertainty is low, disposal costs are low, or both. An asymmetric equilibrium only exists if there is a cost advantage from early production, disposal is costly, and demand is uncertain.

Comparative Statics. Lemma 1, 2, and 3 already show that although disposal goes down with its costs, there is generally a negative effect on consumer surplus and profits. Firms dispose of less and compete stronger for market shares, yet inventories are lower. Firms give up some profits in the good demand state to mitigate costs in the bad demand state. The expected trade volume, therefore, decreases with d and accordingly, so does consumer surplus.

The only market participant profiting from an increase in disposal costs is the follower in the asymmetric equilibrium. Costlier disposal increases the valuation of his information advantage.

It remains to analyze how consumers and firms are affected if an increase in disposal costs leads to a change in the market structure. Firms may change their manufacturing location in response to an increase in disposal costs. For low disposal costs, both produce abroad. One firm may change its location close to the home market if d increases. Furthermore, the firm producing abroad may also change its location if d is costly.

Proposition 2 shows how expected disposal, expected consumer surplus, and firm's expected profits are affected by a regulatory increase in disposal costs. It is useful to invert the threshold function from above by $d_H(\beta) := \min\{d|\beta_H(d) = \beta\}$, thus, at $d_H(\beta)$ the leader is indifferent between producing abroad or close to the market. Similarly, the follower is indifferent between strategy A and H at $d_A(\beta) := \min\{d|\beta_A(d) = \beta\}$.

[16]By definition all equilibrium types can only be supported if $\beta_H(d) = \beta_A(d)$. The only exception where all equilibria coexist is at $d = c$ and $\beta \to 1$.

[17]In (*iii*) there also exists a symmetric equilibrium in mixed strategies. Strategy H's probability being played increases in d. This follows directly since expected profits with strategy A, precisely $\mathbb{E}[\pi_A]$ and $\mathbb{E}[\pi_1]$, decrease while expected profits under strategy H, precisely $\mathbb{E}[\pi_2]$ increase. In the supplementary materials, we show that the mixed equilibrium's expected profits are non-monotonic in d.

Proposition 2. *A regulatory increase in disposal costs*

(i) decreases the expected disposal;

(ii) decreases expected consumer surplus except;

1. *the leader postpones her production close to the market, i.e. $d = d_H(\beta)$, expected consumer surplus increases discontinuously.*

(iii) decreases firms' expected profits except;

1. *one firm postpones its production becoming a follower, i.e. at $d = d_A(\beta)$, the leader's expected profits increase discontinuously;*
2. *in the asymmetric equilibrium, the follower's expected profit increases.*

We now discuss the disposal costs' effect throughout all types of equilibria. Suppose that d is low, such that both firms manufacture abroad. An increase in disposal costs decreases both firms' profits: If demand is below expectations, an increase in d induces firms to dispose of less, thereby, compete more intensively for a larger market share. Furthermore, firms decrease their inventory to mitigate costs if demand is low. Thus, the trade volume in the high demand state decreases.

Less disposal and fierce competition benefit consumers. However, lower trade volume decreases consumer surplus. The latter effect is stronger than the former, leading to an expected loss in consumer surplus.

At some point, $d = d_A(\beta)$, a firm, labeled as firm 2, expects the same profit from postponing its strategy close to the market. By definition of the equilibrium, firm 2's expected profits are continuous: he decides to postpone his production at $d_A(\beta)$, which is simply the inverse of $\beta_A(d)$ where $\mathbb{E}[\pi_2] = \mathbb{E}[\pi_A]$. The leader's profits, however, increase discontinuously. The change in the market structure results in a cost advantage for her. Although the follower has superior information about demand, the leader has an advantage if disposal costs are low. Low disposal costs enable an inexpensive reaction to the state of demand; the follower's information advantage is thus not crucial.

Expected consumer surplus decreases discontinuously due to the change of the market structure. While both firms produce abroad, they compete at equal strength and sell their total inventory if demand is above expectations. With the market structure change, the leader still offers her total inventory in the high demand state. However, the follower can increase his production and has monopoly power on the residual demand.

The leader's inventory decreases with costlier disposal resulting in an increase for the follower's residual demand. Expected consumer surplus, therefore, decreases further with d. The follower's market power increases and additionally, his information advantage becomes more valuable, resulting in higher expected profits. At the same time, the leader's profits decrease due to her cost increase. Consequently, at some point, the follower expects higher profits than the leader.

At $d = d_H(\beta)$, the leader's expected profit is continuous by the same argument as above. She postpones her production close to the market and gives up her cost advantage to gain information about demand. The follower's profits decrease discontinuously due to the change in the market structure. He loses his information advantage, which is relatively valuable for high d. Additionally, he loses his monopoly power on the residual demand. Firms become equal and compete for the total demand, benefiting consumers. Consequently, consumer surplus increases discontinuously at $d_H(\beta)$.

When both firms produce close to the market, disposal costs have no effect. Firms only produce after demand has materialized, therefore, no products are disposed of and disposal costs are irrelevant. Note that an increase in d does not always lead to a change in the market structure, as can be seen in figure 2: for low demand uncertainty, it is not possible to influence the market structure such that both firms (or even one) postpone its production.

4 Extensions

We now discuss several extensions to test our result's robustness. First, we relax the assumption of only one production location. Next, we allow firms to observe their competitor's inventory. Finally, we discuss alternative forms of competition, namely perfect competition or price competition.

Multiple Manufacturing Sites. We relax the assumption of only one production location and allow firms to manufacture at both locations. We show that expected disposal decreases in its costs yet also expected profits and consumer surplus decrease.

First, firms choose their inventory $\bar{q}_i \geq 0$ at zero marginal costs. After the demand's realization, firms choose either to dispose of at marginal costs $d > 0$ or to produce additional quantity at marginal costs $c > 0$. Firm i's expected profits can be written as

$$\mathbb{E}[\pi(q_i, \bar{q}_i)] = \mathbb{E}[P_s(Q)q_i - c\max\{q_i - \bar{q}_i, 0\} - d\max\{\bar{q}_i - q_i, 0\}], \tag{14}$$

with $Q = q_i + q_j$. The first term is the revenue, the second are the costs of additional production to sell more than the inventory, and the third are the disposal costs.

In the second stage, firms take their inventories as given. They choose their sales volume $q_i \geq 0$ to maximize

$$\pi(q_i|\bar{q}_i) = P_s(Q)q_i - c\max\{(q_i - \bar{q}_i, 0\} - d\max\{(\bar{q}_i - q_i, 0\}.$$

q_i	high demand	low demand
$d < \min\{\beta a, c\}$	$\frac{a-d}{3(1-\beta)}$	$\frac{a+d}{3(1+\beta)}$
$\beta a \leq \min\{c, d\}$	$\frac{a}{3}$	$\frac{a}{3}$
$c < \min\{\beta a, d\}$	$\frac{a-c}{3(1-\beta)}$	$\frac{a+c}{3(1+\beta)}$

Table 4: Inventory and sales volume with multiple manufacturing sites. Inventory equals the sales volume in the high (low) demand's state if $d < \min\{\beta a, c\}$ ($c < \min\{\beta a, d\}$).

The optimal strategy can be written as

$$q_i(q_j|\bar{q}_i) = \max\left\{\min\left\{\max\left\{\frac{a+d}{2b_s} - \frac{1}{2}q_j, 0\right\}, \bar{q}_i\right\}, \frac{a-c}{2b_s} - \frac{1}{2}q_j\right\}. \tag{15}$$

Similar as before, sales volume are increasing in disposal costs. The costlier it is to dispose of, the more firms compete for larger market share. We maintain the assumption that inventories are not observed by the competitor. Firm i therefore chooses her inventory $\bar{q}_i$ to maximize (14) subject to (15). The optimal inventory strategy of firm i can be written as

$$\bar{q}_i(q_{j,l}, q_{j,h}) = \begin{cases} \frac{a+c}{2(1+\beta)} - \frac{1}{2}q_{j,l}, & \text{if } d > c \text{ and } q_{j,h} - q_{j,l} < \frac{2(\beta a - c)}{1-\beta^2}; \\ \frac{a-d}{2(1-\beta)} - \frac{1}{2}q_{j,h}, & \text{if } c > d \text{ and } q_{j,h} - q_{j,l} < \frac{2(\beta a - d)}{1-\beta^2}; \\ \frac{a}{2} - \frac{1+\beta}{4}q_{j,l} - \frac{1-\beta}{4}q_{j,h}, \text{else,} \end{cases} \tag{16}$$

whenever it is larger than zero.

The best response function's derivation is in the appendix. If marginal production costs are larger than the disposal costs, i.e. $c > d$, the optimal inventory strategy is equivalent to (5). With multiple manufacturing sites, firms can produce in the second stage and thus lower their inventory.

A firm's equilibrium inventory and sales volume are shown in Table 4. With low disposal costs, firms sell their inventory in the high demand state and dispose of if demand is below expectations. Note that the equilibrium is the same as in the last section: firms decrease their inventories as a response to an increase in disposal cost to mitigate costs if demand is below expectations. By contrast, if production costs in the second period are low, firms sell their inventory in the low demand state and produce additional quantities if demand is higher than expected. If disposal and production are costly, firms sell their inventory regardless of the demand.

We summarize expected prices, disposal, firms' profits, and consumer surplus in the following Proposition.

Proposition 3. *The unique, symmetric equilibrium implies an expected market price*

$$\mathbb{E}[P] = \frac{a}{3}$$

and expected disposal of

$$\mathbb{E}[\bar{q}_i - q_i] = \begin{cases} \frac{2(\beta a - d)}{3(1-\beta^2)}, & \textit{if } d < \min\{\beta a, c\}; \\ 0, & \textit{else.} \end{cases}$$

Furthermore, a firm's expected profits are

$$\mathbb{E}[\pi_i] = \begin{cases} \frac{(a-d)^2}{18(1-\beta)} + \frac{(a+d)^2}{18(1+\beta)}, & \textit{if } d < \min\{\beta a, c\}; \\ \frac{a^2}{9}, & \textit{if } \beta a \leq \min\{c, d\}; \\ \frac{(a-c)^2}{18(1-\beta)} + \frac{(a+c)^2}{18(1+\beta)}, & \textit{if } c < \min\{\beta a, d\}. \end{cases} \tag{17}$$

and expected consumer surplus is

$$\mathbb{E}[CS] = \begin{cases} \frac{(a-d)^2}{9(1-\beta)} + \frac{(a+d)^2}{9(1+\beta)}, & \textit{if } d < \min\{\beta a, c\}; \\ \frac{2a^2}{9}, & \textit{if } \beta a \leq \min\{c, d\}; \\ \frac{(a-c)^2}{9(1-\beta)} + \frac{(a+c)^2}{9(1+\beta)}, & \textit{if } c < \min\{\beta a, d\}. \end{cases} \tag{18}$$

Expected disposal, firm's profits, and consumer surplus decrease in disposal costs.

With this set-up, firms always choose a symmetric strategy. None of the two gives up the production abroad. Thus the results are comparable to the ones in the last section. In the previous section, whenever it is less expensive to dispose of manufactured products compared to produce new ones, $d \leq c$, both firms produce abroad. The models' equilibrium is equivalent.

The expected price is independent of d. It reflects again only the first period production costs that we normalized to zero. Disposal decreases in its cost, yet profits and consumer surplus decrease, too. Firms decrease their inventories due to costly disposal and, therefore, the expected trade volume is lower.

In the supplementary materials, we extend the model to N firms. Expected consumer surplus increases in the number of firms, yet also expected disposal. An increasing number of firms results in a competitive market but also a rise in the number of disposers. Total expected disposal decreases stronger in its costs, the larger the number of firms.

However, firms expect a lower profit if disposal costs increase. Accordingly, some firms may leave the market, resulting in a lower number of firms. Competition is lowered, thereby additionally decreasing consumer surplus. The firms staying in the market are negatively affected by higher disposal costs, yet benefit from fewer competitors. Their expected profit may thus increase. The positive effect of lower disposal and, thereby, more intense competition for market shares is not noticeable for consumers. With this set-up, competition may even decrease in response to an increase in disposal costs.

Observable Inventories. We discuss in this section an extension to observable inventories. A formal analysis can be found in the supplementary materials.[18] In reality, firms may not perfectly observe their competitor's inventories. Public companies, yet, announce their targeted sales to inform investors. Those announcements are observed by the competitors, who can infer the inventory from it. For simplicity, we assume in our model that inventories are perfectly observable.

If inventories are observable, an additional, opposing effect exists. With higher disposal costs, firms sell large parts of their inventories even if demand is lower than expected. Extensive inventories, therefore, send the message of large intended sales. A firm can only credibly commit to selling its total inventory if disposal is expensive.

Generally, disposal costs decrease inventories, yet, the neglected effect works in the opposite direction. If firms observe their competitors' inventories, the equilibrium may not be unique, nor is it monotone, due to the opposing effects. Although firms are ex ante symmetric, there may exist asymmetric equilibria, where one firm has a larger inventory than the other. The firm with the smaller inventory produces additional quantities if demand is higher than expected, while the other disposes of if demand is lower than expected. Expected disposal decreases in its costs. Furthermore, inventories still decrease in disposal costs. Due to the lower trade volume, consumer surplus decreases.

A regulatory increase in disposal costs fulfills its purpose to decrease the disposed of quantity yet at the consumers' cost. Competition for market shares is not achieved by this policy, even if inventories are observable.

Similar to the main model, the firm producing (primarily) abroad is negatively affected by an increase in disposal costs, since any reaction to new information about demand becomes more costly. By contrast, observability strengthens its dominant position in terms of market share. It can signify large targeted sales with a large inventory. The costlier disposal, the less does a firm dispose of its inventory. The inventory's credibility to indicate targeted sales increases with d, strengthening the firm's competitive advantage. Profits are, thus, ambiguously affected by an increase in disposal costs. Precisely, the larger firm's expected profits are U-shaped.

[18] An earlier version of this paper also contained an extension of the location game in section 2. Results are qualitatively similar. Please contact the author to access it.

The other firm has a small inventory and manufactures large parts of its production after the demand's realization. Accordingly, it has an information advantage, which is more valuable if the other firm's reaction to new information is costly. Similar to the main model, the smaller firm's expected profits increase with d.

To sum up, the increasing credibility benefits the larger firm if demand is lower than expected, while the information advantage benefits the smaller firm if demand is higher than expected. By contrast to unobserved inventories, both firms' expected profits may increase in disposal costs.

H&M and Zara[19], the two most prominent players in the European fashion market increased their recycling standards over the last years. According to our model, this leads to higher costs, which may increase profits. Our model is consistent with the market structure: H&M mainly produces in Asia and ships its product to the European market; Zara manufactures mostly in Europe. Zara manufactures close to the market. The firm claims that within two weeks of the original design, clothes are in retail. The shipment from Asia to Europe already takes more time. Consequently, H&M's clothes are manufacture earlier. In the fast fashion industry, multiple products are introduced in a single week to stay on-trend. In order to compete trendily, according to our model, H&M produces large parts of its inventory abroad and has larger expected disposal than Zara. This is consistent with the fact that Zara only discards 10% of its products, which is half of the industry average.

Besides the discussed asymmetric equilibria, there always exists the same symmetric equilibrium described in Proposition 3.[20] The difference between observable and unobservable inventories is, therefore, the asymmetric equilibria's existence. We use numerical simulations to compare the equilibria and find that both firms' expected profits may be higher in the symmetric equilibrium. Note that the firms can guarantee to be in the symmetric equilibrium if inventories are unobservable. However, there exists parameters, where one firm, either the smaller or the larger, expects higher profits in an asymmetric equilibrium, i.e. prefers if inventories are observable.

Perfect Competition. New firms may enter a profitable market in the long run, resulting in a perfectly competitive market. Firms make zero expected profits with both timing strategies. Higher disposal costs decrease firms' inventories and consumer surplus: Suppose there are many firms in both timing allocations and expected profits are zero. Firms with strategy A may dispose of if demand is below its expectation, i.e., incur costs. They have, therefore, to turn positive profits if demand is above expectations. An increase in disposal costs forces those firms to decrease their inventory. Otherwise, firms turn negative expected profits because the loss in the bad state out-

[19]Zara is part of the Inditex holding, which also includes Pull&Bear, Massimo Dutti, Oysho, and others. Although we mean Inditex in lieu we refer to Zara because it is the flagship of Inditex.

[20]Dubey and Shubik (1981) show generally that any pure strategy equilibrium with unobservable inventories is also an equilibrium if inventories are observed.

weighs the gains in the good state. Due to the lower inventory, firms in the second period increase their production, but these quantities come at a higher production cost. Introducing additional costs in an efficient market decreases consumer surplus.
Price Competition. Instead of quantity competition in the second stage, Kreps and Scheinkman (1983) and Montez and Schutz (2018) used price competition. Both firms choosing timing strategy H results in zero profits à la Bertrand. Both choosing timing strategy A results in a model similar to de Frutos and Fabra (2011): firms end up with different capacity/inventory levels. Given an asymmetric timing, the leader has to set prices weakly below the follower's marginal costs, or else the latter undercuts the price. It depends on the rationing rule how demand is shared with equal prices. For example, one could use equal demand sharing as de Frutos and Fabra (2011). If the leader's inventory is not large enough to satisfy total demand, the follower becomes a monopolist for the residual demand. The follower sets prices strictly above marginal costs and the leader tries to undercut them. No pure strategy equilibrium may exist.

5 Conclusion

For each unit not sold firms incur a cost if inventory is not fully reversible. An unsold unit is not only a loss in revenue, it also causes additional costs. As we show in this paper, firms thus discard less of their commodities if disposal costs increase. Therefore, competition for sales increases. Accordingly, one would expect consumer surplus to increase and firms' expected profits to decrease.

Although correct, this expectation is shortsighted. Firms adjust their inventories if disposal costs increase. The higher the disposal costs, the costlier it is for a firm to adjust to a demand below expectations. To mitigate costs, a firm lowers its inventory, which leads to lower profits if demand is high.

In our model, firms either produce their inventory earlier, at a low cost and little information about demand or later, with more information yet at higher costs. Although firms are ex ante symmetric, firms may choose asymmetric production strategies. We derive three necessary conditions for an asymmetric equilibrium: First, early production has to yield a strict cost advantage. Second, disposal has to be strictly costly. Third, demand has to be uncertain, yet, not too much. If demand uncertainty is considerable, firms jointly produce with more information, yet at higher costs. If demand uncertainty is low, firms jointly produce at low costs with little information about demand.

We showed that a regulatory increase in disposal costs decreases the expected disposal. Yet consumers do not benefit from increasing competition for market sales. The lower trade volume impairs them. In general, consumers do not benefit from an increase in disposal costs. There is, however, an exception. In an asymmetric equilibrium, the firm manufacturing close to the market has monopoly power over the residual demand. Increasing disposal costs may change the market structure, and the competitor postpones its production close to the market, too. Firms become equal and competition increases, benefiting consumers.

Generally, firms expect a lower profit, the costlier disposal is. However, there are also some exceptions. With an increase in disposal costs, information about demand become more valuable. Disposing of products as response to a demand below expectations becomes costlier. Firms may, therefore, postpone their production with increasing disposal costs. Changes in the market structure may benefit a firm. Furthermore, in the asymmetric equilibrium, one of the two firms has an information advantage. Since costlier disposal increases the information's value, the firm expects a higher profit.

We also discussed the case when firms observe their competitor's inventory. This gives rise to another effect: a firm's inventory sends the message of its intended sales. However, a company can only credibly commit to selling large parts of its inventory if disposal is costly. Due to this opposing effect, each firm's profits may increase separately in disposal costs. Firms may profitably agree on costlier disposal, e.g., in the form of higher recycling standards. Expected disposal decreases, yet consumer surplus, too.

In our set-up, a regulatory increase in disposal costs impairs firms and consumers. We discuss some exceptions, whereby firms may benefit more often than consumers. Our model is consistent with the market structure in the fashion market. Furthermore, our model explains the 'reshoring' of firms. If cost advantages abroad decline or disposal costs increase, information about demand becomes more valuable. Thus, firms produce closer to their home market.

We studied demand uncertainty. However, in some markets, demand is relatively predictable, but costs may vary due to input factor prices. Commodities that are expensive in production are less often disposed of. Studying cost uncertainty may, therefore, be of interest.[21] Another interesting question is how disposal costs affect collusive behavior. Paha (2017) studies collusion with capacities; Rotemberg and Saloner (1989) study the use of inventory for strategic collusion. US data of the aluminum industry analyzed in Rosenbaum (1989) reveals markups' negative correlation with inventory, but a positive with excess capacity. Low disposal costs allow an inexpensive firm's adjustment, it is thus easier to deviate, and strategic collusion may be aggravated.

[21] In Thille (2006) the prediction of the model crucially depends on the primary uncertainty. Less competitive market structures have a relatively low price variance when uncertainty is primarily due to uncertain cost and relatively high price variance when uncertainty is mainly due to uncertain demand.

References

Antoniou, F., Fiocco, R., 2019. Strategic inventories under limited commitment. The RAND Journal of Economics 50, 695–729.

Anupindi, R., Jiang, L., 2008. Demand uncertainty and sales: A study of fashion and markdown pricing. Management Science 54, 1876–1890.

Arvan, L., 1985. Some examples of dynamic cournot duopoly with inventoy. The RAND Journal of Economics 16, 569–578.

Dada, M., van Mieghem, J., 1999. Price versus production postponement: Capacity and competition. Management Science 45, 1631–1649.

Dana, J., Williams, K., 2019. Intertemporal price discrimination in sequential quantity-price games. Working Paper available at https://cowles.yale.edu/sites/default/files/files/pub/d21/d2136-r2.pdf.

Daughety, A., Reinganum, J., 1994. Asymmetric information acquisition and behavior in role choice models: An endogenously generated signaling game. International Economic Review 35, 795–819.

Davidson, C., Deneckere, R., 1986. Long-run competition in capacity, short-run competition in price, and the cournot model. The RAND Journal of Economics 17, 404–415.

de Frutos, M., Fabra, N., 2011. Endogenous capacities and price competition: The role of demand uncertainty. International Journal of Industrial Organization 29, 399–411.

Dinan, T., 1993. Economic efficiency effects of alternative policies for reducing waste disposal. Journal of Environmental Economics and Management 25, 242–256.

Dubey, P., Shubik, M., 1981. Information conditions, communication and general equilibrium. Mathematics of Operations Research 6, 186–189.

Gilpatric, S., Li, Y., 2015. Information value under demand uncertainty and endogenous market leadership. Economic Inquiry 53, 589–603.

Hamilton, J., Slutsky, S., 1990. Endogenous timing in duopoly games: Stackelberg or cournot equilibria. Games and Economic Behavior 2, 29–46.

Hviid, M., 1991. Capacity constrained duopolies, uncertain demand and non-existence of pure strategy equilibria. European Journal of Political Economy 7, 183–190.

Klemperer, P., Meyer, M., 1986. Price competition vs. quantity competition: The role of uncertainty. The RAND Journal of Economics 17, 618–638.

Kreps, D., Scheinkman, J., 1983. Quantity precommitment and bertrand competition yield cournot outcomes. The Bell Journal of Economics 14, 326–337.

Liu, Z., 2005. Stackelberg leadership with demand uncertainty. Managerial and Decision Economics 26, 498–516.

Maggi, G., 1996. Endogenous leadership in a new market. The RAND Journal of Economics 27, 641–659.

Malueg, D., Tsutsui, S., 1996. Duopoly information exchange: The case of unknown slope. International Journal of Industrial Organization 14, 119–136.

Mitraille, S., Moreaux, M., 2013. Inventories and endogenous stackelberg leadership in two-period cournot oligopoly. Journal of Economics & Management Strategy 22, 852–874.

Mitraille, S., Thille, H., 2014. Speculative storage in imperfectly competitive markets. International Journal of Industrial Organization 35, 44–59.

Montez, J., Schutz, N., 2018. All-pay oligopolies: Price competition with unobservable inventory choices. Working Paper available at `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3190205`.

Paha, J., 2017. The value of collusion with endogenous capacity and demand uncertainty. The Journal of Industrial Economics LXV, 623–653.

Pal, D., 1993. Endogenous stackelberg equilibria with identical firms. Games and Economic Behavior 12, 81–94.

Pashigian, P., 1988. Demand uncertainty and sales: A study of fashion and markdown pricing. The American Economic Review 78, 936–953.

Qu, Z., Raff, H., Schmitt, N., 2018. Incentives through inventory control in supply chains. International Journal of Industrial Organization 59, 486–513.

Reynolds, S., Wilson, B., 2000. Bertrand-edgeworth competition, demand uncertainty, and asymmetric outcomes. Journal of Economic Theory 92, 345–350.

Robson, A., 1990. Duopoly with endogenous strategic timing: Stackelberg regained. International Economic Review 31, 263–274.

Rosenbaum, D., 1989. An empirical test of the effect of excess capacity in price setting, capacity-constrained supergames. International Journal of Industrial Organization 7, 231–241.

Rosenfield, D., 1989. Disposal of excess inventory. Operations Research 37, 404–409.

Rotemberg, J., Saloner, G., 1989. The cyclical behavior of strategic inventories. The Quarterly Journal of Economics 104, 486–513.

Saloner, G., 1986. The role of obsolescence and inventory costs in providing commitment. International Journal of Industrial Organization 4, 333–345.

Thille, H., 2006. Inventories, market structure, and price volatility. Journal of Economic Dynamics & Control 30, 1081–1104.

van Damme, E., Hurkens, S., 1999. Endogenous stackelberg leadership. Games and Economic Behavior 28, 105–129.

van der Laan, E., Salomon, M., 1997. Production planning and inventory control with remanufacturing and disposal. European Journal of Operational Research 102, 264–278.

Wang, J., Xu, J., 2007. Information advantage in stackelberg duopoly under demand uncertainty. Working Paper available at https://mpra.ub.uni-muenchen.de/6409/.

Young, D., 2010. Endogenous investment and pricing under uncertainty. The B.E. Journal of Theoretical Economics 10, 1–27.

A Proofs

This section contains all Lemmas' and Propositions' proofs and derives the best response (5) and (15).

Proof Equation (5). Firm i maximizes (1) subject to (4). Note that by (4) the sales volume equals not always the inventory. To simplify notation, we write $\bar{q}_i$ explicitly whenever $q_i = \bar{q}_i$ and use $\hat{q}_i$ when the sales volume is lower than the inventory.

Suppose first, $q_{j,h} - q_{j,l} \leq 2\beta(a+d)/(1-\beta^2)$, thus, firm i's expected profits can be written as

$$\mathbb{E}[\pi_i] = \begin{cases} \frac{1}{2}(a-(1-\beta)(\bar{q}_i+q_{j,h})+d)\bar{q}_i + \\ \frac{1}{2}(a-(1+\beta)(\bar{q}_i+q_{j,l})+d)\bar{q}_i - d\bar{q}_i, & \text{if } \bar{q}_i < \frac{a+d}{2(1+\beta)} - \frac{q_{j,l}}{2} =: \tau; \\ \frac{1}{2}(a-(1-\beta)(\bar{q}_i+q_{j,h})+d)\bar{q}_i + \\ \frac{1}{2}(a-(1+\beta)(\hat{q}_i+q_{j,l})+d)\hat{q}_i - d\bar{q}_i, & \text{if } \tau \leq \bar{q}_i < \frac{a+d}{2(1-\beta)} - \frac{q_{j,h}}{2}; \\ \frac{1}{2}(a-(1-\beta)(\hat{q}_i+q_{j,h})+d)\hat{q}_i + \\ \frac{1}{2}(a-(1+\beta)(\hat{q}_i+q_{j,l})+d)\hat{q}_i - d\bar{q}_i, & \text{else.} \end{cases}$$

In the first part firm i sells its total inventory regardless of the demand's state, yielding the interior solution $\bar{q}_i = a/2 - (1+\beta)q_{j,l}/4 + (1-\beta)q_{j,h}/4$. In the second part, firm i disposes of if demand is below expectations, yielding the interior solution $\bar{q}_i = (a-d)/(2(1-\beta)) - q_{j,h}/2$. The third part is strictly decreasing. Whenever $2(\beta a - d) \leq (1-\beta^2)(q_{j,h} - q_{j,l})$, the second part is decreasing and the first interior solution is the global maximum. Otherwise, the first part is increasing and the second interior solution is the global maximum. This can be rewritten as the best response function given in (5).

It remains to show that for $q_{j,h} - q_{j,l} > 2\beta(a+d)/(1-\beta^2)$ no other maximum exists. The second part of the expected profit function changes: firm i disposes of in the high state and sells its inventory in the low demand state. This implies that the sales volume in the high state is lower than in the low state, $q_{i,h} \leq q_{i,l}$. However, the right-hand side in the inequality above is weakly positive, yielding a contradiction. Thus (5) is indeed firm i's best response. □

Proof Lemma 1. We prove that the unique equilibrium is given in Table 1. Therefore, we look for fixpoints for the best response function of (4) and (5). We go step by step through all three parts in (5) and derive thereby all equilibria.

First we show that $\bar{q}_i = 0$ is never an equilibrium. Therefore, we show that $q_{i,s} = 0$ is never an equilibrium. Both firms' sales volumes are strictly positive in both demand states. Firm j's best response to $q_{i,s} = 0$ is

$$q_{j,s} = \min\left\{\frac{a+d}{2b_s}, \max\left\{\frac{a}{2}, \frac{a-d}{2(1-\beta)}\right\}\right\},$$

implying that $q_{j,s} \leq \min\{(a+d)/b_s, (a-d)/(1-\beta)\}$. Accordingly, firm i's sales volume $q_{i,s} > 0$. This proofs that both firms sell a strictly positive quantity. Hence, $\bar{q}_i > 0$.

Next, suppose firm i sells its inventory in both demand state. Hence, $\bar{q}_i = a/2 - (1+\beta)q_{j,l}/4 - (1-\beta)q_{j,h}/4$ and $q_{i,l} = q_{i,h} = \bar{q}_i$. Firm j's best response is $\bar{q}_j = \max\{a/2 - \bar{q}_i/2, (a - d/(2(1-\beta)) - \bar{q}_i/2\}$.

Suppose first $d < \beta a$, hence, $\bar{q}_j = q_{j,h} = (a - d/(2(1-\beta)) - \bar{q}_i/2$ and $q_{j,l} = (a+d)(2(1+\beta)) - \bar{q}_i/2$. Direct calculation imply $\bar{q}_i = a/3$, and $q_{j,h} = (\beta a - d)/(2(1-\beta))$. A necessary condition is that $a/3$ is the maximum of (5), which simplifies to $3d \geq 2a + \beta a$, which yields a contradiction, thus there is no equilibrium where firm i sells its inventory in both states but firm j disposes of.

Suppose now that also firm j sells its inventory in both states, $d \geq \beta a$. Direct calculation imply that $\bar{q}_i = q_{i,l} = q_{i,h} = a/3$, which forms a symmetric equilibrium if $d \geq \beta a$. This proofs the second part in Table 1.

Finally, suppose firm i disposes of if demand is below expectations, thus $\bar{q}_i = (a - d)/(2(1-\beta)) - q_{j,h}/2$. By the arguments above $\bar{q}_j = (a-d)/(2(1-\beta)) - q_{i,h}/2$. Direct calculations imply the symmetric equilibrium candidate $\bar{q}_i = q_{i,h} = (a-d)/(3(1-\beta))$ and $q_{i,l} = (a+d)/(3(1+\beta))$. The necessary condition that $(a-d)/(3(1-\beta))$ is the maximum of (5) simplifies to $d < \beta a$. This proofs the first part in Table 1.

Hence, the equilibrium is unique. Plugging in the sales volume yields the expected expressions in Lemma 1. Since $\beta \in [0, 1)$, the negative effect of d is more weighted than the positive. Thus, d's negative effect follows directly. □

Proof Lemma 2. Firms know the demand's state and maximize (2). Note that similar as in Lemma 1, $q_{i,s} = 0$ is never an equilibrium. Therefore, best response functions are linearly and strictly decreasing in the relevant part and a unique equilibrium exists. Exploiting the symmetry directly implies $q_{H,s} = (a-c)/(3b_s)$. Plugging in the sales volume yields the desired result. □

Proof Lemma 3. The leader's best response function is given by (5), the follower's by (8). Similar to the proof of Lemma 1, we analyze the best response function step by step to find all equilibria.

Suppose $\bar{q}_1 = 0$, directly implying $q_{1,g} = q_{1,b} = 0$ and thus $q_{2,s} = (a-c)/(2b_s)$. However, the leader's best response is $\bar{q}_1 > 0$, hence there is no equilibrium with zero inventory.

Next, suppose $\bar{q}_1 = a/2 - (1+\beta)q_{2,b}/4 + (1-\beta)q_{2,g}/4$, i.e. the leader sells her inventory in both states. Combining this with the best response of the follower yields $\bar{q}_1 = (a+c)/3$, which is indeed an equilibrium if $\beta(a+c)/2 \leq d$.

Lastly suppose $\bar{q}_1 = (a-d)/(2(1-\beta)) - q_{2,g}/2$, combining this with (8) yields $\bar{q}_1 = (a - 2d + c)/(3(1-\beta))$, which is indeed an equilibrium if $\beta(a+c)/2 > d$.

No other equilibria exist since it would not be on firm 1's best response function. Therefore, the equilibrium is unique. Plugging in the sales values yields directly the expected expressions. Similar to the proof of Lemma 1, d's effect follows directly. □

Proof Proposition 1. Both firms choose strategy H if $\mathbb{E}[\pi_H] \geq \mathbb{E}[\pi_1]$, where the expressions are given in (9) and (11). We show that

$$\beta_H(d) = \begin{cases} \frac{2ac+2d^2}{d(a+c)}, & \text{if } d < \beta\frac{a+c}{2}; \\ \frac{2\sqrt{ac}}{(a+c)}, & \text{if } d \geq \beta\frac{a+c}{2}, \end{cases}$$

with $\beta \geq \beta_H(d)$ the strategy pair (H,H) is an equilibrium. First, for $d < \beta\frac{a+c}{2}$ the inequality can be rearranged to $\beta d(a+c) \geq ac + d^2$, hence, the first part directly follows. Note that the left-hand side increases stronger in d than the right-hand side, since $d \leq \beta(a+c)$. By definition, the first part of $\beta_H(d)$ therefore decreases. For $d \geq \beta\frac{a+c}{2}$ the inequality simplifies to $\beta^2(a+c)^2 \geq 4ac$, which concludes the proof of (i).

Both firms choose strategy A if $\mathbb{E}[\pi_A] \geq \mathbb{E}[\pi_2]$, where the expressions are given in (6) and (12). We show that

$$\beta_A(d) = \begin{cases} \frac{c}{d}, & \text{if } d < \beta\frac{a+c}{2}; \\ \frac{\sqrt{a^2d^2+4(a+c)(5a-7c)(d^2+4(a-c))}-4ad}{(a+c)(5a-7c)}, & \text{if } \beta\frac{a+c}{2} \leq d < \beta a; \\ 4\sqrt{\frac{c}{9a+7c}}, & \text{if } d \geq \beta a, \end{cases}$$

with $\beta \leq \beta_A(d)$ the strategy pair (A,A) is an equilibrium. First, note that $\beta a \geq \beta(a+c)/2$. For $d < \beta(a+c)/2$ the inequality simplifies to $\beta d \leq c$, which concludes the first part. Obviously, it decreases in d. For $d \in [\beta(a+c)/2, \beta a]$, the inequality can be written as $\beta^2(a+c)(5a-7c) + 8\beta ad - 16c(a-c) - 4d^2 \leq 0$. The left-hand side is convex and at $\beta = 0$ negative and increasing. Hence, the larger root is the relevant one, which is explicitly given in the second part.

We use the implicit function theorem to show $\beta_A(d)$'s second part has a negative slope. The left-hand side's derivative with respect to d is $8(\beta a - d) > 0$; the derivative with respect to β is $2\beta(a+c)(7a-7c) \geq 0$. Hence, the implicit function theorem implies that $\beta_A(d)$ decreases.

For $d \geq \beta a$ the inequality simplifies to $\beta^2(9a+7c) \leq 16c$, which concludes the proof of (ii).

Inverting the inequalities proves (iii). It remains to show that $\beta_A(d) \leq \beta_H(d)$ to proof the generical uniqueness. We show in the following proof that the inequality holds. □

Proof Proposition 2. We first compare the leader's and follower's profits. The leader's profits are larger if and only if $\mathbb{E}[\pi_1] \geq \mathbb{E}[\pi_2]$, where the expressions are given in (11) and (12). Equating the two expressions and rearranging yields

$$\beta_{AH}(d) = \begin{cases} \frac{2ac-c^2+d^2}{2ad}, & \text{if } d < \beta\frac{a+c}{2}; \\ \sqrt{\frac{4c(2a-c)}{(3a-c)(a+c)}}, & \text{if } d \geq \beta\frac{a+c}{2}. \end{cases}$$

Thus, $\beta \leq \beta_{AH}(d)$ implies $\mathbb{E}[\pi_1] \geq \mathbb{E}[\pi_2]$ and $\beta \geq \beta_{AH}(d)$ implies $\mathbb{E}[\pi_1] \leq \mathbb{E}[\pi_2]$.

Hence, it is sufficient to show that $\beta_A(d) \le \beta_{AH}(d) \le \beta_H(d)$. If this is the case, a firm's profits increase discontinuously if the other delays its production. Technically, at $\beta_A(d)$ by definition $\mathbb{E}[\pi_A] = \mathbb{E}[\pi_2]$ and by the inequality above $\mathbb{E}[\pi_1] \ge \mathbb{E}[\pi_2]$. Similarly at $\beta_H(d)$ by definition $\mathbb{E}[\pi_H] = \mathbb{E}[\pi_1]$ and by the inequality above $\mathbb{E}[\pi_2] \ge \mathbb{E}[\pi_1]$. Thus, the leader's profit jumps up at $\beta_A(d)$ while the follower's jumps down at $\beta_H(d)$.

For $d < \beta(a+c)/2$, the two inequalities above can be simplified to $2ac^2 \le ac^2 - c^3 + ad^2 + cd^2 \le 2ad^2$, i.e. for the existence of an asymmetric equilibrium it has to hold that $c^2 \le d^2$. The two inequalities can be rewritten as $(a+c)(d^2 - c^2) > 0$ and $(a-c)(c^2 - d^2) < 0$, which both are true whenever an asymmetric equilibrium exists.

Since $\beta_H(d)$ and $\beta_{AH}(d)$ are continuous and constant while $\beta_A(d)$ decrease for $d \ge \beta(a+c)/2$ the relevant inequality holds. This directly concludes the proof for the firms' part.

For the consumer surplus, we first show a discontinuous decrease at $\beta = \beta_A(d)$ and finally we prove a discontinuous increase at $\beta = \beta_H(d)$.

First, at $\beta = \beta_A(d)$, $\mathbb{E}[CS_A] > \mathbb{E}[CS_{AH}]$, where the expressions are given by (7) and (13). For $d < \beta(a+c)/2$ the inequality simplifies to $8ac - 2c^2 + 6d^2 > 4\beta d(c - 6a)$, where the left-hand side is strictly positive while the right-hand side is strictly negative. For $d \in [\beta(a+c)/2, \beta a)$, the inequality simplifies to $32ac - 64ad - 8c^2 + 32d^2 + \beta(a+c)(16a - 8c) - 2\beta^2(a+c)^2 > 0$. The left-hand side is a concave function in β. Suppose $\beta \to 0$, this implies $d \to 0$, hence, the inequality is satisfied for low β. The left-hand side is positive and increasing at $\beta = 0$, furthermore, its maximal value is at $\beta = (4a - 2c)/(a+c)$, which is strictly larger than 1. Hence, the inequality holds for any $\beta \in [0, 1)$. Since the expected consumer surplus is continuous and constant for $d \ge \beta a$, this concludes the first part.

Next we show that at $\beta = \beta_H(d)$, $\mathbb{E}[CS_H] > \mathbb{E}[CS_{AH}]$, where the expressions are given by (10) and (13). For $d \le \beta(a+c)/2$ the inequality simplifies to $4\beta d(2a - c) > 2(4ac - 3c^2 + d^2)$. Plugging in $\beta_H(d)$ we can simplify the expression to $(d^2 - c^2)(a - c) > 0$. This is a necessary condition for the existence of an asymmetric equilibrium as discussed above. Since both functions are continuous and constant for $d \ge \beta(a+c)/2$ this concludes the proof for the consumer surplus.

It remains to prove that expected disposal decreases in its cost. First note that in the (H,H) subgame, expected disposal is always zero. Next, using Lemma 1 and 3, we derive that expected disposal in the (A,A) subgame equilibrium is larger than in the (A,H) if and only if $\beta \ge c/(3a)$. This is independent of d. From Proposition 1 it follows directly that the minimal uncertainty for (A,H) to form an equilibrium is $\beta = \sqrt{16c/(9a + 7c)}$. The direct comparison yields that whenever (A,H) may be an equilibrium, the expected disposal is lower since $9ac(a - c) + 135a^2c - 7c^3 > 0$. This concludes the proof. □

Proof Equation (15). Similar as in Equation (5)'s proof we use again $\bar{q}_i$ if the sales volume equals the inventory and else $\hat{q}_i$. To simplify notation let

$$\vartheta_1 := \frac{a-c}{2(1-\beta)} - \frac{1}{2}q_{j,h},$$
$$\vartheta_2 := \frac{a+d}{2(1+\beta)} - \frac{1}{2}q_{j,l},$$
$$\vartheta_3 := \frac{a+d-(1-\beta)q_{j,h}}{2(1-\beta)},$$

and

$$\vartheta_4 := \frac{a-c-(1+\beta)q_{j,l}}{2(1+\beta)}.$$

With this expected profits can be written as

$$\mathbb{E}[\pi_i] = \begin{cases} \frac{1}{2}(a-(1-\beta)(\bar{q}_i+q_{j,h}))\bar{q}_i + \\ \frac{1}{2}(a-(1+\beta)(\bar{q}_i+q_{j,l}))\bar{q}_i, & \text{if } \vartheta_1 \leq \bar{q}_i < \vartheta_2; \\ \frac{1}{2}(a-(1-\beta)(\bar{q}_i+q_{j,h}))\bar{q}_i + \\ \frac{1}{2}(a-(1+\beta)(\hat{q}_i+q_{j,l})+d)\hat{q}_i - d\bar{q}_i, & \text{if } \max\{\vartheta_1,\vartheta_2\} \leq \bar{q}_i < \vartheta_3; \\ \frac{1}{2}(a-(1-\beta)(\hat{q}_i+q_{j,h})-c)\hat{q}_i + \\ \frac{1}{2}(a-(1+\beta)(\bar{q}_i+q_{j,l}))\bar{q}_i + c\bar{q}_i, & \text{if } \vartheta_4 < \bar{q}_i < \min\{\vartheta_1,\vartheta_2\}; \\ \frac{1}{2}[(a-(1-\beta)(\hat{q}_i+q_{j,h}))\hat{q}_i - \\ \max\{d(\bar{q}_i-\hat{q}_i), c(\hat{q}_i-\bar{q}_i\}] + \\ \frac{1}{2}[(a-(1+\beta)(\hat{q}_i+q_{j,l}))\hat{q}_i - \\ \max\{d(\bar{q}_i-\hat{q}_i), c(\hat{q}_i-\bar{q}_i\}], & \text{else.} \end{cases}$$

In the first part, firm i sells its inventory in both states, yielding the interior solution $\bar{q}_i = a/2 - (1+\beta)q_{j,l}/4 + (1-\beta)q_{j,h}/4$. In the second part firm i disposes of if demand is below expectations, yielding the interior solution $\bar{q}_i = (a-d)/(2(1-\beta)) - q_{j,h}/2$. In the third part, firm i produces additional quantities if demand is above expectations, yielding the interior solution $\bar{q}_i = (a+c)/(2(1+\beta)) - q_{j,l}/2$. The fourth part is strictly in or decreasing, depending on c and d. If $c = d$ there may exists multiple maxima, where the firm disposes of if demand is below expectations or produces if it is above expectations. However, all yield the same expected profit as if the firm only produces, or only disposes of instead of doing both, since the expected profit function is continuous.

If $\max\{2(\beta a - c), 2(\beta a - d)\} \leq (1-\beta^2)(q_{j,h} - q_{j,l})$, the first part is indeed an interior solution and the second part is strictly decreasing; the third strictly increasing, hence it is the unique maximum. Furthermore, for the region's existence we need that $q_{j,h} - q_{j,l} \geq 2\beta a - (1+\beta)c - (1-\beta)d$, which is satisfied in equilibrium.

If $d < c$, and $2(\beta a - c) \leq (1-\beta^2)(q_{j,h} - q_{j,l}) \leq 2(\beta a - d)$, the first part is strictly increasing, the second has an interior solution and the third is strictly increasing. Thus the second part is the global maximum.

Finally, if $d > c$ and $(1-\beta^2)(q_{j,h} - q_{j,l}) \leq 2(\beta a - d)$, the first and second part are strictly decreasing and the third part is an interior solution. The global maximum is thus the third part. This yields the best response function (15). □

Proof Proposition 3. By the similar argument as in Proposition 1's proof firm i's sales volume is strictly positive in both demand states. The best response simplifies therefore to

$$q_i(q_j) = \max\left\{\min\left\{\frac{a+d}{2b_s} - \frac{1}{2}q_j, \bar{q}_i\right\}, \frac{a-c}{2b_s} - \frac{1}{2}q_j\right\},$$

where $\bar{q}_i$ is given by (15). The inventory is strictly positive: production is costly in the second stage, firms produce at least the low demand's state sales volume in the first period.

To derive all equilibria, we analyze the best response function step by step.

We start with (15)'s first part: suppose $\bar{q}_i = (a+c)(2(1+\beta)) - q_{j,l}/2$, which implies $q_{i,h} = (a-c)/(2(1-\beta)) - q_{j,h}/2$ and $q_{i,l} = \bar{q}_i$.

First, suppose $\bar{q}_j = q_{j,l} = (a+c)(2(1+\beta)) - q_{i,l}/2$ and $q_{j,h} = (a-c)/(2(1-\beta)) - q_{i,h}/2$. By symmetry we directly get $q_{i,h} = (a-c)/(3(1-\beta))$ and $\bar{q}_i = q_{i,l} = (a+c)/(3(1+\beta))$, which indeed forms a symmetric equilibrium if $c < \min\{\beta a, d\}$.

Second, suppose $\bar{q}_j = q_{j,l} = q_{j,h} = \frac{a}{2} - \frac{1+\beta}{4}q_{i,l} - \frac{1-\beta}{4}q_{i,h}$. We directly get that $\bar{q}_j = a/3$ and $q_{i,h} = (a+c)/(2(1+\beta)) - a/6$ and $q_{i,l} = (a-c)/(2(1-\beta)) - a/6$. Hence, $q_{i,h} - q_{i,l} = (\beta a - c)/(1-\beta^2)$, which has to be positive. Yet in order to have firm j play a best response it has to be negative, hence, a contradiction.

Lastly, note that $\bar{q}_j = (a-d)(2(1-\beta)) - q_{i,h}/2$ is never a best response since it contradicts $d > c$. This concludes the first part of (15).

Next, suppose $\bar{q}_i = a/2 - (1+\beta)q_{j,l}/4 - (1-\beta)q_{j,h}/4$, which implies $q_{i,l} = q_{i,h} = \bar{q}_i$.

First, suppose $\bar{q}_j = q_{j,l} = q_{j,h} = \frac{a}{2} - \frac{1+\beta}{4}q_{i,l} - \frac{1-\beta}{4}q_{i,h}$, by symmetry we directly get $\bar{q}_i = q_{i,h} = q_{i,l} = a/3$, which indeed forms a symmetric equilibrium if $\beta a \leq \min\{c, d\}$.

Second, suppose $\bar{q}_j = q_{j,h} = (a-d)(2(1-\beta)) - q_{i,h}/2$ and $q_{j,l} = (a+d)(2(1+\beta)) - q_{i,l}/2$. Direct calculation yield $\bar{q}_i = a/3$ and $q_{j,h} = (a-d)/(2(1-\beta)) - a/6$ and $q_{j,l} = (a+d)/(2(1+\beta)) - a/6$, hence, $q_{j,h} - q_{j,l} = (\beta a - d)/(1-\beta^2)$, which has to be positive. Yet a necessary condition for firm i's strategy to be a best reply is $d \geq \beta a$, yielding a contradiction.

This concludes the second part of (15). Finally, suppose $\bar{q}_i = q_{i,h} = (a-d)(2(1-\beta)) - q_{j,h}/2$ and $q_{i,l} = (a+d)(2(1+\beta)) - q_{j,l}/2$. The remaining case is the symmetric one for $\bar{q}_j = q_{j,h} = (a-d)(2(1-\beta)) - q_{i,h}/2$ and $q_{j,l} = (a+d)(2(1+\beta)) - q_{i,l}/2$. We directly get $\bar{q}_i = q_{i,h} = (a-d)/(3(1-\beta))$ and $q_{i,l} = (a+d)/(3(1+\beta))$, which forms a symmetric equilibrium if $d < \min\{\beta a, c\}$.

Plugging in the sale volumes yields the expected values. The disposal cost's negative effect immediately follows from Lemma 3. □

B Supplementary Materials

Mixed Equilibrium. This section derives the unique symmetric equilibirum. Note that we have shown in section 3 that the equilibrium is unique and symmetric whenever (A,A) or (H,H) forms an equilibrium, see Proposition 1 for details. Whenever the asymmetric equilibirum exists, there exists a second asymmetric equilibrium with the firms label interchanged. Furthermore, there exists a symmetric equilibrium in mixed strategies, where the probability to play strategy H is

$$p = \frac{\mathbb{E}[\pi_A] - \mathbb{E}[\pi_2]}{\mathbb{E}[\pi_A] - \mathbb{E}[\pi_2] + \mathbb{E}[\pi_H] - \mathbb{E}[\pi_1]},$$

respectively, $1-p$ to play strategy A. Taking the derivative with respect to d yields

$$\frac{\partial p}{\partial d} = \frac{\frac{\partial \mathbb{E}[\pi_1]}{\partial d}(\mathbb{E}[\pi_A] - \mathbb{E}[\pi_2]) + (\frac{\partial \mathbb{E}[\pi_A]}{\partial d} - \frac{\partial \mathbb{E}[\pi_2]}{\partial d})(\mathbb{E}[\pi_H] - \mathbb{E}[\pi_1])}{(\mathbb{E}[\pi_A] - \mathbb{E}[\pi_2] + \mathbb{E}[\pi_H] - \mathbb{E}[\pi_1])^2} \geq 0.$$

The sing follows from the expected profits derived in Lemma 1-3, furthermore, existence requires $\mathbb{E}[\pi_A] \leq \mathbb{E}[\pi_2]$ and $\mathbb{E}[\pi_H] \leq \mathbb{E}[\pi_1]$. Note that p is continuous in d and the derivative exists everywhere except for $d = \beta(a+c)/2$ and $d = \beta a$. Thus, the probability to play strategy H increases in d.

The expected profits in the mixed equilibrium can be written as

$$\mathbb{E}[\pi_M] = p\mathbb{E}[\pi_H] + (1-p)\mathbb{E}[\pi_2].$$

It follows that

$$\frac{\partial \mathbb{E}[\pi_M]}{\partial d} = (1-p)\frac{\partial \mathbb{E}[\pi_2]}{\partial d} - \frac{\partial p}{\partial d}(\mathbb{E}[\pi_2] - \mathbb{E}[\pi_H]),$$

where both terms are positive. Plugging in p's derivative yields

$$\begin{aligned}\frac{\partial \mathbb{E}[\pi_M]}{\partial d} \quad = \quad & -\frac{\partial \mathbb{E}[\pi_2]}{\partial d}(\mathbb{E}[\pi_1] - \mathbb{E}[\pi_H])(\mathbb{E}[\pi_A] - \mathbb{E}[\pi_1]) + \\ & \frac{\partial \mathbb{E}[\pi_A]}{\partial d}(\mathbb{E}[\pi_1] - \mathbb{E}[\pi_H])(\mathbb{E}[\pi_2] - \mathbb{E}[\pi_H]) + \\ & \frac{\partial \mathbb{E}[\pi_1]}{\partial d}(\mathbb{E}[\pi_2] - \mathbb{E}[\pi_A])(\mathbb{E}[\pi_2] - \mathbb{E}[\pi_H]).\end{aligned}$$

By the analysis in the main text we have close to $\beta_H(d)$ that $\mathbb{E}[\pi_1] \approx \mathbb{E}[\pi_H]$ and $\mathbb{E}[\pi_2] \geq \mathbb{E}[\pi_A]$. Thus, expected profits decrease in d. However, expected profits may also increase. For example at parameters $a = 1$, $c = 1/5$, $\beta = 2/3$, and $d = 1/3$, the expected profits are 0.1417, yet at $d = 0.334$ expected profits are 0.1418.

By contrast to the main text, expected profits are continuous. The non-monotonicity follows from the same economic effect discussed in the main text. Disposal costs decrease expected profits under strategy A, precisely $\mathbb{E}[\pi_A]$ and $\mathbb{E}[\pi_1]$, thus firms play strategy A with a smaller probability if d increases. However, in the mixed equilibrium an asymmetric outcome arises with probability $p(1-p)$. In this asymmetric outcome, the second firm has an information advantage; its valuation increases in d. The probability p increases in d, and whenever it is close to 1, this positive effect is a rare event. Therefore, only the negative effect remains, and expected profits decrease.

N Firms. In this section we extend the model from section 4 in the main text to N symmetric firms. Lets repeat the set-up. Each firm produces inventory $\bar{q}_i$ at zero marginal costs. After demand's realization, firms choose their sales volume q_i. On the one hand, if the sales volume exceeds the firm's inventory, the additional quantity induces marginal costs of $c > 0$. On the other hand, if a firm's sales volume deceeds its inventory, the disposed of quantity induces marginal costs of $d > 0$. A firm's profits can be written as

$$\mathbb{E}[\pi(q_i, \bar{q}_i)] = \mathbb{E}[P_s(Q)q_i - c\max\{(q_i - \bar{q}_i, 0\} - d\max\{(\bar{q}_i - q_i, 0\}],$$

where the inverse demand is $P_s(Q) = a - b_s(Q)$. The intercept $a > \max\{c, d\}$ is common knowledge, while the slope b_s takes on the value $b_l = 1 + \beta$ or $b_h = 1 - \beta$, each with equal probability. Q is the total sales volume, i.e., the sum of q_i over all N.

As in the main text, we assume that firms do not observe their competitors' inventories. In the second stage, a firm takes its own inventory as given and chooses $q_i \geq 0$ to maximize

$$\pi(q_i|\bar{q}_i) = P_s(Q)q_i - c\max\{(q_i - \bar{q}_i, 0\} - d\max\{(\bar{q}_i - q_i, 0\}.$$

The optimal strategy can be derived as in the main text and written as

$$q_i(Q_{-i}|\bar{q}_i) = \max\left\{\min\left\{\max\left\{\frac{a+d}{2b_s} - \frac{1}{2}Q_{-i}, 0\right\}, \bar{q}_i\right\}, \frac{a-c}{2b_s} - \frac{1}{2}Q_{-i}\right\},$$

where $Q_{-i} = \sum_{j\neq i} q_j$ is the other firms' sales volume. Since firms compete with a homogeneous product, it is of now matter for firm i how Q_{-i} is complied.

By the same argument, we immediately get the optimal inventory strategy

$$\bar{q}_i(Q_{-i,l}, Q_{-i,h}) = \begin{cases} \frac{a+c}{2(1+\beta)} - \frac{1}{2}Q_{-i,l}, \text{if } d > c \text{ and } Q_{-i,h} - Q_{-i,l} < \frac{2(\beta a - c)}{1-\beta^2}; \\ \frac{a-d}{2(1-\beta)} - \frac{1}{2}Q_{-i,h}, \text{if } c > d \text{ and } Q_{-i,h} - Q_{-i,l} < \frac{2(\beta a - d)}{1-\beta^2}; \\ \frac{a}{2} - \frac{1+\beta}{4}Q_{-i,l} - \frac{1-\beta}{4}Q_{-i,h}, \text{else.} \end{cases}$$

We immediately obtain the symmetric equilibrium inventories and sales quantities summarized in Table 5.

Comparing Table 5 with Table 4 in the main text, shows that the equilibrium is similar and thus the results in Proposition 3 remain valid for any number of firms. However, an interesting trade off for policymakers arises in the number of firms. Suppose $d < \min\{\beta a, c\}$, thus, firms dispose of if demand materializes below expectations. Expected consumer surplus can be written as

$$\mathbb{E}[CS] = \left(\frac{N}{2(N+1)}\right)^2 \left(\frac{(a-d)^2}{1-\beta} + \frac{(a+d)^2}{1+\beta}\right).$$

q_i	high demand	low demand
$d < \min\{\beta a, c\}$	$\frac{a-d}{(N+1)(1-\beta)}$	$\frac{a+d}{(N+1)(1+\beta)}$
$\beta a \leq \min\{c, d\}$	$\frac{a}{(N+1)}$	$\frac{a}{(N+1)}$
$c < \min\{\beta a, d\}$	$\frac{a-c}{(N+1)(1-\beta)}$	$\frac{a+c}{(N+1)(1+\beta)}$

Table 5: N-firms' inventory and sales volume with multiple manufacturing sites. Inventory equals the sales volume in the high (low) demand state if $d < \min\{\beta a, c\}$ ($c < \min\{\beta a, d\}$).

Expected consumer surplus increases in the number of firms, $N/(N+1) < (N+1)/(N+2) \Leftrightarrow N^2 + 2N < N^2 + 2N + 1$, which generally results from increased competition. The expected disposal, however, also increases in the number of firms. It can be written as

$$\mathbb{E}[N(\bar{q}_i - q_i)] = \frac{2N}{(N+1)} \frac{\beta a - d}{(1-\beta)^2}$$

and by the same formal argument as above, expected disposal increases in the number of firms. Disposal costs decrease the disposed of quantity, as in the main text, moreover even stronger the more firms are in the market.

In this set-up, increasing competition due to the number of firms benefits consumers yet increases the disposal. Policymakers concerned about the discarded quantities, therefore, face a trade-off.

Suppose firms face fixed costs, such that there exists an upper bound on N where firms expect positive profits. Let's denote the fix cost by F, firms' expected profits can be written as

$$\mathbb{E}[\pi_i] = \frac{1}{2(N+1)^2} \left(\frac{(a-d)^2}{1-\beta} + \frac{(a+d)^2}{1+\beta} \right) - F.$$

Increasing disposal costs decrease profits. Consequently, the upper bound on N decreases, and some firms leave the market. A decrease in firms' number decreases competition and thus consumer surplus, while firms may benefit from fewer competitors. Generally, consumers are worse off if disposal is costly due to the lower inventory hold by firms and, additionally, to a decrease in the number of firms.

Observable Inventories. This section contains the formal derivation of the discussion in section 4. We use the same model as above for $N = 2$. By contrast, we assume that firms observe their competitor's inventories before choosing their sales volume. Remind that we assume $a \geq 2c + d$ to ensure that both firms are active.

In the second stage, firms take their inventory as given and maximize their profits

$$\pi(q_i|\bar{q}_i) = P_s(Q)q_i - c\max\{(q_i - \bar{q}_i, 0\} - d\max\{(\bar{q}_i - q_i, 0\},$$

yielding the best response function (15) in the main text.

$$q_i(q_j|\bar{q}_i) = \max\left\{\min\left\{\max\left\{\frac{a+d}{2b_s} - \frac{1}{2}q_j, 0\right\}, \bar{q}_i\right\}, \frac{a-c}{2b_s} - \frac{1}{2}q_j\right\}.$$

Since competitors observe inventories, we derive the sales game's subgame equilibrium following any firm's inventory choice. Lets denote the firm with the larger inventory as 1 and the other by 2, i.e., we assume without loss of generality $\bar{q}_1 \geq \bar{q}_2$. Combining the best response functions, we can derive the unique subgame equilibrium for different ranges of parameters, which we summarize in the following Lemma.

Lemma 4. *Let $\bar{q}_1 \geq \bar{q}_2$. The unique subgame equilibrium sales volumes following the inventories*

(i) $\bar{q}_1 \leq \frac{a-c}{3b_s}$ *are*

$$q_1 = q_2 = \frac{a-c}{3b_s};$$

(ii) $\bar{q}_1 \in [\frac{a-c}{3b_s}, \frac{a+c+2d}{3b_s}]$ *and* $\bar{q}_2 \leq \frac{a-c}{2b_s} - \frac{1}{2}\bar{q}_1$ *are*

$$q_1 = \bar{q}_1 \text{ and } q_2 = \frac{a-c}{2b_s} - \frac{1}{2}\bar{q}_1;$$

(iii) $\bar{q}_1 \leq \frac{a+d}{2b_s} - \frac{1}{2}\bar{q}_2$ *and* $\bar{q}_2 \geq \frac{a-c}{2b_s} - \frac{1}{2}\bar{q}_1$ *are*

$$q_1 = \bar{q}_1 \text{ and } q_2 = \bar{q}_2;$$

(iv) $\bar{q}_1 \geq \frac{a+c+2d}{3b_s}$ *and* $\bar{q}_2 \geq \frac{a-2c-d}{3b_s}$ *are*

$$q_1 = \frac{a+c+2d}{3b_s} \text{ and } q_2 = \frac{a-2c-d}{3b_s};$$

(v) $\bar{q}_1 \geq \frac{a+d}{2b_s} - \frac{1}{2}\bar{q}_2$ *and* $\bar{q}_2 \in [\frac{a-2c-d}{3b_s}, \frac{a+d}{3b_s}]$ *are*

$$q_1 = \frac{a+d}{2b_s} - \frac{1}{2}\bar{q}_2 \text{ and } q_2 = \bar{q}_2;$$

(vi) $\bar{q}_2 \geq \frac{a+d}{3b_s}$ *are*

$$q_1 = q_2 = \frac{a+d}{3b_s}.$$

Only in subgames (*ii*) and (*iii*) does firm i sell its inventory. Otherwise, it always produces additional quantities or disposes of.

To derive all equilibria, we first exclude inventory ranges that are never optimal and, therefore no candidates for an equilibrium. Start with $\bar{q}_1 < (a-c)/(3(1+\beta))$, both firms produce additional quantity even if demand is below expectation. By increasing their inventory, firms decrease their costs. Similarly if $\bar{q}_2 > (a+d)/(3(1-\beta))$, both firms dispose of even if demand is above expectations. Firms decrease their costs by decreasing their inventory.

The maximal quantity that firm 1 could sell is $(a+c+2d)/(3(1-\beta))$, thus any larger inventory is never optimal. Similarly, the minimal quantity that firm 2 could sell is $(a-2c-d)/(3(1+\beta))$, thus any lower inventory is never optimal.

If $\bar{q}_1 > (a+d)/(2(1-\beta))-\bar{q}_2/2$, firm 1 disposes of even if demand is above expectations. By decreasing its inventory the firm decreases its costs. Similarly, if $\bar{q}_2 < (a-c)/(2(1+\beta))-\bar{q}_1/2$, firm 2 decreases its costs if it increases its inventory since it produces additional quantities even if demand is below expectations.

There remain six different areas for the inventory's equilibrium strategy. We summarize them in the following Lemma.

Lemma 5. *Let $\bar{q}_1 \geq \bar{q}_2$. The following six areas may contain an equilibrium.*

(i) If $\bar{q}_1 \leq \frac{a-c}{3(1-\beta)}$ and $\bar{q}_2 \geq \frac{a-c}{2(1+\beta)} - \frac{1}{2}\bar{q}_1$, firms sell their inventory in the low demand state and produce additional quantities if demand is above expectations.

(ii) If $\bar{q}_1 \leq \frac{a+d}{2(1+\beta)} - \frac{1}{2}\bar{q}_2$ and $\bar{q}_2 \geq \frac{a-c}{2(1-\beta)} - \frac{1}{2}\bar{q}_1$, firms sell their inventories regardless of the demand's realization.

(iii) If $\bar{q}_1 \leq \frac{a+d}{2(1-\beta)} - \frac{1}{2}\bar{q}_2$ and $\bar{q}_2 \geq \frac{a+d}{3(1+\beta)}$, firms sell their inventories if demand is above expectations and disposes of otherwise.

(iv) If $\bar{q}_1 \in [\frac{a-c}{3(1-\beta)}, \frac{a+d}{2(1+\beta)} - \frac{1}{2}\bar{q}_2]$ and $\bar{q}_2 \in [\frac{a-c}{2(1+\beta)} - \frac{1}{2}\bar{q}_1, \frac{a-c}{2(1-\beta)} - \frac{1}{2}\bar{q}_1]$, firm 1 sells its inventory regardless of the demand's realization, while firm 2 sells its inventory if demand is below expectation and produces additional quantities otherwise.

(v) If $\bar{q}_1 \in [\frac{a+d}{2(1+\beta)} - \frac{1}{2}\bar{q}_2, \frac{a+d}{2(1-\beta)} - \frac{1}{2}\bar{q}_2]$ and $\bar{q}_2 \in [\frac{a-c}{2(1-\beta)} - \frac{1}{2}\bar{q}_1, \frac{a+d}{3(1+\beta)}]$, firm 1 sells its inventory if demand is above expectation and disposes of otherwise, while firm 2 sells its inventory regardless of demand's realization.

(vi) If $\bar{q}_1 \in [\frac{a+d}{2(1+\beta)} - \frac{1}{2}\bar{q}_2, \frac{a+c+2d}{3(1-\beta)}]$ and $\bar{q}_2 \in [\frac{a-2c-d}{3(1+\beta)}, \frac{a-c}{2(1-\beta)} - \frac{1}{2}\bar{q}_1]$, firm 1 sells its inventory if demand is above expectation and disposes of otherwise, while firm 2 sells its inventory if demand is below expectations and produces additional quantities otherwise.

Next, we analyze each area for an equilibrium. We use as in the proof in the main text $\hat{q}_1$ if the sales volume is not equal to the inventory and explicitly $\bar{q}_1$ if it equals the inventory. In (i) firm 1's profits are $\mathbb{E}[\pi_1] = [(a-(1-\beta)(\hat{q}_1+q_{2,h}-c))\hat{q}_1 + (a-(1+\beta)(\bar{q}_1+q_{2,l})-c)\bar{q}_1]/2 + c\bar{q}_1$, implying a unique symmetric interior solution $\bar{q}_i = (a+c)/(3(1+\beta))$ if $c < \min\{\beta a, d\}$, in (ii), $\mathbb{E}[\pi_1] = [(a-(1-\beta)(\bar{q}_1+q_{2,h}-c))\bar{q}_1 + (a-(1+\beta)(\bar{q}_1+q_{2,l}))\bar{q}_1]/2$ implying the unique symmetric equilibrium $\bar{q}_i = a/3$ if $\beta a \leq \min\{c,d\}$, and in (iii), $\mathbb{E}[\pi_1] = [(a-(1-\beta)(\bar{q}_1+q_{2,h}+d))\bar{q}_1 + (a-(1+\beta)(\hat{q}_1+q_{2,l})+d)\hat{q}_1]/2 - d\bar{q}_1$ implying the unique symmetric equilibrium $\bar{q}_i = (a+d)/(3(1+\beta))$ if $d \leq \min\{\beta a, c\}$. For the technical details see the proof of Proposition 3 in the main text; the symmetric equilibrium is equivalent. Hence, the same symmetric equilibrium exists regardless if inventory is observed or not.

Finally, we analyze asymmetric equilibria. Focus first on (iv), firm's best replies are technically already derived in the proof of Proposition 3. The unique equilibrium candidate is $\bar{q}_1 = q_{1,h} = q_{1,l} = 2a/(5+2\beta)$ and $\bar{q}_2 = q_{2,l} = (3a+5c+2\beta c)/(2(1+\beta)(5+2\beta))$, which is indeed an interior equilibrium if $\beta a \geq c$, $c(5+2\beta)+a-8\beta a \geq 0$ and $(c-2d)(5+2\beta)+a+4\beta a \leq 0$. Note that this equilibrium is independent of d, only its existence depends on the disposal costs. If disposal costs are low, this equilibrium does not exist.

We show next, that in (v) no equilibrium exists. The unique candidate is given by $\bar{q}_1 = q_{1,h} = (3a-d(5-2\beta))/(2(1-\beta)(5-2\beta))$, $q_{1,l} = (3a-4\beta a+d(5-2\beta))/(2(1+\beta)(5-2\beta))$, and $\bar{q}_2 = q_{2,h} = q_{2,l} = 2a/(5-2\beta)$. Necessary condition for its existence are $d \leq \beta a$ and $d \geq (a+8\beta a)/(5-2\beta)$, hence, the range for d only exists if $a+3\beta a+2\beta^2 a \leq 0$, which yields a contradiction.

Lastly we derive the equilibrium in (vi). The inventories' first order conditions are already derived in the proof of Proposition 3. This implies the unique equilibrium candidate $\bar{q}_1 = q_{1,h} = (a+c-2d)/(2(1-\beta))$, $q_{1,l} = (a-2c+3d)/(4(1+\beta))$, $\bar{q}_2 = q_{2,l} = (a+2c-d)/(2(1+\beta))$, and $q_{2,h} = (a-3c+2d)/(4(1-\beta))$. This indeed forms an interior equilibrium if $d \geq (a+c)/10$, $(7+\beta)d \leq a+3\beta a+4c$, and $4d \geq a-3\beta a+7c-\beta c$. We summarize the equilibrium in the following Proposition.

Proposition 4. *If* $\max\{(a+c)/10, (a+7c-3\beta a-\beta c)/4\} \leq d \leq \min\{(a+4c+3\beta a)/(7+\beta), a-2c\}$*, the firms' inventories are*

$$\bar{q}_1 = \frac{a+c-2d}{2(1-\beta)};$$
$$\bar{q}_2 = \frac{a+2c-d}{2(1+\beta)},$$

and sale volumes

$$q_{1,h} = \bar{q}_i; \qquad q_{1,l} = \tfrac{a-2c+3d}{4(1+\beta)};$$
$$q_{2,h} = \tfrac{a-3c+2d}{4(1-\beta)}; \quad q_{2,l} = \bar{q}_2.$$

Firm 1 *disposes of if demand is lower than expected; firm* 2 *produces additional quantities if demand is higher than expected, otherwise firms sell their inventories. Expected prices, disposal, profits, and consumer surplus are*

$$\begin{aligned}
\mathbb{E}[P] &= \frac{2a - c + d}{8}; \\
\mathbb{E}[\bar{q}_1 - q_1] &= \frac{a + 3\beta a + 4c - (7 + \beta)d}{4(1 - \beta^2)}; \\
\mathbb{E}[\pi_1] &= \frac{(a - 2c + 3d)^2}{32(1 + \beta)} + \frac{(a + c - 2d)^2}{16(1 - \beta)}; \\
\mathbb{E}[\pi_2] &= \frac{(a - 3c + 2d)^2}{32(1 - \beta)} + \frac{(a + 2c - d)^2}{16(1 + \beta)}; \\
\mathbb{E}[CS] &= \frac{(3a - c - 2d)^2}{64(1 - \beta)} + \frac{(3a + 2c + d)^2}{64(1 + \beta)}.
\end{aligned}$$

Expected disposal and consumer surplus decreases with disposal costs, while expected prices and firm 2's profits increase; firm 1's expected profits are ambiguous.

By contrast to the other cases, expected prices increase in disposal costs. Firms decrease their inventories, and thus expected trade volume decreases, implying higher prices. Firms' hand an increase in disposal costs over to consumers. Interestingly, expected prices decrease with c. The higher the production cost in the second period, the more firms increase their inventory, which is produced at zero costs. Parts of this reduced production costs are handed over to consumers.

Expected disposal decreases in its costs, as in the other cases. The larger firm is the one disposing of if demand is below expectations. Higher disposal costs decrease the firm's inventory and disposal. Therefore, its sales volume in the high demand state is lower, yet higher if demand is below expectations. By contrast, the smaller firm sells less if demand is below expectations and increases its sales volume if demand is above expectations.

Profits and consumer surplus are convex in d. Firm 1's profits are ambiguously affected by d. On the one hand, firm 1's cost increase if demand is below expectations. On the other hand, firm 1's sales volume also increases, resulting in a larger market share. The total effect on profits is thus ambiguous.

Firm 2 produces mainly in the second period, thus, with an information advantage: the higher disposal costs, the more severe this information advantage, increasing firm 2's profits.

Consequently, there exist parameter ranges, where both profits increase. Consumer surplus, however, decreases in d. Firms produce less inventory if disposal is costly. If demand is higher than expected, firms indeed produce additional quantity yet at higher costs. Therefore, the trade volume decreases and, thereby, consumer surplus.

For further discussion see the main text. Conclusively, we present next the numerical simulation to show that firms may oppose to observe their competitor's inventory. Suppose $a = 1$, $c = 1/4$, and $\beta = 3/4$. With $d = 1/2$ it follows $\mathbb{E}[\pi_2] = 0.231 \geq \mathbb{E}[\pi_i] = 0.1746 \geq \mathbb{E}[\pi_1] = 0.087$, thus the smaller firm prefers if inventories are observable but the larger one is worse off. With $d = 1/3$, $\mathbb{E}[\pi_i] = 0.1746 \geq \mathbb{E}[\pi_2] = 0.1536 \geq \mathbb{E}[\pi_1] = 0.1252$, thus both firms prefers if inventories are private. Finally, with $d = 1/5$, $\mathbb{E}[\pi_1] = 0.2022 \geq \mathbb{E}[\pi_i] = 0.1879 \geq \mathbb{E}[\pi_2] = 0.1132$, thus the larger firm prefers if inventories are observed.