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Money Velocity and the Natural Rate of Interest

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Abstract

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M1 velocity is, approximately, the permanent component of the short-term rate. This implies that agents—in deciding how much wealth to allocate to non interest-bearing M1, as opposed to interest-bearing assets—almost uniquely react to permanent shocks to the opportunity cost, essentially ignoring transitory shocks. This suggests that money-demand models must be modified to allow for such distinct reaction to permanent and transitory variation in the opportunity cost of holding M1. Under monetary regimes making inflation stationary, permanent fluctuations in M1 velocity uniquely reflect, to a close approximation, permanent shifts in the natural rate of interest.

Money demand; unit roots; cointegration; structural VARs; natural rate of interest.

E30; E32

1 Introduction

Two new stylized facts pertaining to the relationship between M1 velocity (defined as the ratio between nominal GDP and nominal M1) and short-term nominal interest rates are documented in this paper: since WWI, (I) M1 velocity has been, to a close approximation, the permanent (i.e., unit root) component of the short-term nominal rate, and (II) the cointegration residual between M1 velocity and a short-term rate has uniformly exhibited a strong negative correlation with the spread between a short- and a long-term nominal interest rate.

A simple and intuitive way of restating stylized fact (I) is the following. By definition, M1 velocity is the inverse of the demand for M1 balances as a fraction of GDP. The fact that M1 velocity only reacts, to a first approximation, to permanent shocks to the short-term rate therefore means that economic agents, in deciding how much of their wealth to allocate to non interest-bearing M1, as opposed to interest-bearing assets, almost uniquely react to permanent shocks to the opportunity cost of holding M1 balances,[1] whereas they essentially ignore transitory shocks. To put it differently, to a close approximation *the demand for M1 balances only reacts to permanent shocks to the opportunity cost of M1*, whereas it does not react to its transitory variation. To the very best of my knowledge, no existing model of money demand exhibits this property: in fact, no model of money demand—from the classic analyses of Baumol and Tobin on—even distinguishes between permanent and transitory variation in the opportunity cost of money.[2]

On the other hand, this result is conceptually in line with some conjectures made by John Hicks in his classic paper 'A Suggestion for Simplifying the Theory of Money', where he stated that[3]

> '[...] a person is deterred from investing money for short periods, partly because of brokerage charges and stamp duties, partly because it is not worth the bother. [...] [S]ince the expected interest increases both with the quantity of money to be invested and with the length of time for which it is expected that the investment will remain untouched, while the costs of investment are independent of the length of time, [...] it will not pay to invest money for less than a certain period.'

Intuitively, consider the following two polar cases, assuming for the sake of simplicity perfect foresight on the part of economic agents. In the first instance, the Treasury bill rate increases by 5% for one single day, and it reverts to its previous value after that. In the second case, on the other hand, the increase is permanent. Quite obviously, in the former instance nobody would direct her broker to convert M1 balances into Treasury bills in order to take advantage of such one-day investment opportunity. In the latter case, on

[1] The reason is that M1, being the sum of currency and non interest-bearing bank deposits, does not pay interest, and a short-term rate can therefore be regarded as a meaningful measure of its opportunity cost.

[2] Uribe (2018) studies the impact of permanent and transitory interest rate shocks within a New Keynesian model, but he does not derive a money demand schedule (in fact, the model does not even feature a monetary aggregate).

[3] See Hicks (1935, p. 6).

the other hand, we can safely assume that most, or even all agents would switch from M1 into government bonds. Stylized fact (I) therefore suggests that an important avenue for future research is to develop a model of money demand allowing for a distinction between permanent and transitory variation in the opportunity cost of money, in order to replicate such (near) lack of responsiveness of the demand for M1 to transitory variation in nominal interest rates.

Stylized fact (II) is conceptually related to—and, in fact, is a direct implication of—stylized fact (I). The intuition for this is straightforward. Empirically, as it is well known, short- and long-term nominal interest rates are cointegrated, so that their I(1) components are driven by a common permanent shock. Since the long rate is much closer to the common stochastic trend than the short rate—in the sense that the latter contains a sizeable transitory component, whereas the former typically has a (near-) negligible one—the long rate is, in fact, a good proxy for the common stochastic trend, i.e, for the permanent component of the short rate. The implication is that when the short rate is above (below) its permanent component, it is typically above (below) *both* M1 velocity *and* the long-term rate, simply because *both* of them are good proxies for such permanent component. As a result, the cointegration residual between M1 velocity and a short-term nominal rate tends to negatively and strongly co-move with the spread between a short- and a long-term rate.

The two stylized facts have the following *logical* implication. Since the unit root in short-term nominal rates originates from either permanent inflation shocks, or permanent shocks to the real rate, the fact that M1 velocity is the unit root component of the short rate logically implies that, under monetary regimes which cause inflation to be I(0), permanent fluctuations in velocity uniquely reflect, to a close approximation, permanent shifts in the natural rate of interest, which—conceptually in line with Laubach and Williams (2003)—is defined as the permanent component of the real rate.[4] To put it differently, under these regimes M1 velocity

[4]Within DSGE models the natural rate is defined as the real rate that would prevail with full price flexibility—let's call it r^*. To the extent that variation in r^* is mainly driven by its permanent component,

is essentially a function of the natural rate of interest—e.g., $V_t = \alpha + \beta R_t^N + \epsilon_t$, where the notation is obvious, and ϵ_t is a 'small' noise component—so that the natural rate is, to a first approximation, and up to a scale factor, *observed.* A corollary of all this is that a consistent decrease in M1 velocity under a monetary regime causing inflation to be I(0)—such as the protracted fall in velocity which has been going on in several inflation-targeting countries since the early 1990s—provides *direct evidence* of a fall in the natural rate of interest.

The fact that this implication holds under monetary regimes which make inflation I(0) would appear, at first sight, to circumscribe its practical relevance. In fact, this is not the case. As I have documented—see Benati (2008)—with the single, notable exception of the Great Inflation episode, inflation has consistently been I(0) throughout the entire recorded history. This implies that, far from pertaining to an unlikely set of circumstances, the fact that M1 velocity is, to a first approximation, a linear transformation of the natural rate of interest should be regarded as the normal state of affairs. In particular, if and only if were inflation to acquire, once again, a unit root, would then permanent inflation shocks distort the relationship between M1 velocity and the natural rate of interest.

The paper is organized as follows. The next section provides an illustration of this paper's main findings for the United Kingdom, for which evidence is so stark that it can be seen essentially *via* the naked eye. Section 3 discusses the evidence for other countries. Section 4 provides evidence for monetary regimes which cause inflation to be I(0). Section 5 concludes.

Finally, Appendix A describes the dataset, and reports results from unit root tests. In brief, evidence of a unit root in the series is very strong, with the p-values being almost uniformly greater than the 10 per cent level which is taken as the benchmark throughout the entire paper, often significantly so.

defining the natural rate as the permanent component of the real rate is approximately correct.

2 A Stark Illustration: The United Kingdom

Although my results are qualitatively the same for the vast majority of countries,[5] for some of them they are especially stark, as they can be seen essentially with the naked eye. This is the case for the United Kingdom over the post-World War II period. This section therefore illustrates this paper's main findings by drawing on the post-WWII U.K. experience.

2.1 Stylized facts

We start by discussing simple visual evidence, and then move to the econometric results.

2.1.1 The time-series relationship between M1 velocity and the short rate

The first panel of Figure 1*a* shows M1 velocity and the short rate for the post-WWII U.K.. Visual impression clearly suggests the following three facts: (*i*) M1 velocity and the short rate are both I(1); (*ii*) the two series are cointegrated; and, crucially, (*iii*) up to a linear transformation, M1 velocity is, essentially, the stochastic trend of the short rate. (Econometric evidence on (*i*) is provided in Appendix A.1; the corresponding evidence on (*ii*) and (*iii*) will be discussed in Section 2.2.) The implication is that when the system is out of equilibrium, adjustment takes place *via* movements in the short rate towards its stochastic trend—i.e., (rescaled) velocity—rather than *via* movements in velocity. To put it differently, velocity is *always* (approximately) in equilibrium: it is rather the short rate which, featuring a sizeable transitory component, is typically out of equilibrium.

Interpretation A simple way of interpreting these results is the following. Assume that the nominal short-term interest rate, R_t, is equal to the sum of two orthogonal components,

[5] The main exception is Japan.

a random walk, R_t^P, and a stationary AR(1) process, R_t^T:

$$R_t = R_t^P + R_t^T \tag{1}$$

$$R_t^P = R_{t-1}^P + u_t \tag{2}$$

$$R_t^T = \rho R_{t-1}^T + v_t \tag{3}$$

with $0 \leq \rho < 1$, and u_t and v_t white noise. (Section 2.2.2 will provide evidence that in the United Kingdom the short rate is indeed not a pure unit root process, and it rather features a sizeable transitory component. Section 3 will provide analogous evidence for the other countries.) Then, consider the following two linear specifications for money velocity, corresponding to what Benati, Lucas, Nicolini, and Weber (2019) label as the 'Selden-Latané' money-demand specification, from Richard Selden (1956) and Henry Allen Latané (1960):[6]

$$V_t = \alpha + \beta R_t + \epsilon_t \tag{4}$$

$$V_t = \alpha + \beta R_t^P + \epsilon_t \tag{5}$$

The key difference between (4) and (5) is that whereas in the former specification—in line with standard money-demand literature—velocity (and therefore its inverse, money balances as a fraction of GDP) depends on the nominal interest rate, in the latter one it depends on its *permanent component*. It can be shown that whereas (4) implies the following VECM

[6] As discussed by Benati *et al.* (2019), the key reason for considering this long-forgotten specification is that for several low-inflation countries the data seem to quite clearly prefer it over the traditional log-log and semi-log ones. As I show in Online Appendix B, the Selden-Latané specification is a special case of the 'money in the utility function' framework pioneered by Miguel Sidrauski.

representation for ΔV_t and ΔR_t:

$$\begin{bmatrix} \Delta V_t \\ \Delta R_t \end{bmatrix} = \text{Constants} + \begin{bmatrix} 0 & \beta\rho \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \Delta V_{t-1} \\ \Delta R_{t-1} \end{bmatrix} - \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\text{Loadings}} \underbrace{\begin{bmatrix} 1 & -\beta \end{bmatrix}}_{\text{Cointegration vector}} \begin{bmatrix} V_{t-1} \\ R_{t-1} \end{bmatrix} + \text{Shocks} \tag{6}$$

(5) implies the following one:

$$\begin{bmatrix} \Delta V_t \\ \Delta R_t \end{bmatrix} = \text{Constants} + \underbrace{\begin{bmatrix} 0 \\ \frac{1-\rho}{\beta} \end{bmatrix}}_{\text{Loadings}} \underbrace{\begin{bmatrix} 1 & -\beta \end{bmatrix}}_{\text{Cointegration vector}} \begin{bmatrix} V_{t-1} \\ R_{t-1} \end{bmatrix} + \text{Shocks} \tag{7}$$

In plain English, the 'traditional' specification[7] (4) implies that the VECM's adjustment towards its long-run equilibrium takes place *via* movements in velocity, with *no* reaction of the short rate to disequilibria. Specification (5), on the other hand, implies that—in line with the evidence in the first panel of Figure 1*a*—the adjustment takes place *via* movements in the short rate, with *no* reaction of velocity. This is a consequence of the fact that, according to (5), velocity is (up to a linear transformation) the stochastic trend of the short rate.

2.1.2 The short-long spread and the cointegration residual between velocity and the short rate

The second panel of Figure 1*a* provides evidence on another remarkably robust stylized fact which has held for all countries and periods in my dataset.[8] The panel shows the cointegration residual between M1 velocity and the short rate, together with the difference between the short rate and a long rate. A striking negative correlation between the two series is readily apparent. Interestingly, the period following the collapse of Lehman Brothers—which

[7] I label (4) as 'traditional'—in spite of the fact that Selden and Latané's work had been forgotten for six decades—because velocity is a function of the nominal rate, rather than of its permanent component.

[8] To be precise: for all countries for which I could find data on a long-term nominal interest rate. Evidence is reported in Figure 2, and it is discussed in Section 3.

featured the most violent phase of the recent financial crisis—does not exhibit any obvious difference with the rest of the sample. This suggest that such strong correlation originates from some deep, structural feature of the economy, so that it is not thrown out of kilter even by the largest macroeconomic shock since the Great Depression.

Interpretation The simple model in sub-section 2.1.1 points towards the following natural interpretation for this stylized fact. Assume that the long-term nominal interest rate, r_t, is equal to the permanent component of the short rate:

$$r_t = R_t^P \tag{8}$$

This specification is designed to capture, in an extreme fashion, the robust stylized facts that (i) short- and long-term rates are cointegrated, and (ii) the long rate consistently behaves as a low-frequency trend for the short rate,[9] with (e.g.) its first-difference systematically exhibiting a lower volatility than the first-difference of the short rate.[10] Equations (1) and (8) imply that the short-long spread is equal to the transitory component of the short rate, $R_t - r_t = R_t^T$. In turn, (5) implies that the cointegration residual between V_t and R_t is equal to $[V_t - \beta R_t] = \alpha - \beta R_t^T + \epsilon_t$, so that

$$[V_t - \beta R_t] = \alpha - \beta[R_t - r_t] + \epsilon_t \tag{9}$$

In plain English, the cointegration residual between velocity and the short rate is perfectly negatively correlated with the short-long spread, as documented in the second panel of Figure 1a. On the other hand, under the 'traditional' specification (4) the cointegration residual would be equal to $[V_t - \beta R_t] = \alpha + \epsilon_t$.

[9] This fact was especially apparent during the metallic standards era (i.e., before World War I), when long-term rates typically exhibited a very small extent of low-frequency variation, and short-term rates systematically fluctuated around long rates, following the ups and downs of the business cycle.

[10] E.g., for the post-WWII U.K. the standard deviations of the first-differences of the short and long rates used to compute the spread shown in Figure 1a have been equal to 0.906 and 0.567 per cent.

2.2 Econometric evidence

We now turn to the econometric evidence.

2.2.1 Cointegration tests

Table 1 reports results from Johansen's maximum eigenvalue tests[11] between velocity and the short rate based on three alternative functional forms for the demand for real money balances: (*i*) the Selden-Latané specification, in which both series enter the system in levels, i.e., $Y_t = [V_t \; R_t]'$; (*ii*) the semi-log specification, with $Y_t = [\ln(V_t) \; R_t]'$; and (*iii*) the log-log specification, with $Y_t = [\ln(V_t) \; \ln(R_t{+}1)]'$. The models feature no deterministic time trend (so, to be clear, the VECM estimator which is used is the one described in pages 643-645 of Hamilton (1994)), reflecting my judgement that, for strictly conceptual reasons, neither series should be expected to exhibit a deterministic trend.[12] The VAR lag order is selected as the maximum between the lag orders chosen by the Schwartz and the Hannan-Quinn criteria[13] for the VAR in levels, and the tests are bootstrapped *via* the procedure proposed by Cavaliere *et al.* (2012, henceforth, CRT).[14] Finally, for reasons of robustness, I also consider an alternative test for cointegration proposed by Wright (2000),[15] whose results are reported in Table 2. A key reason for considering this tests is that, as discussed by Wright (2000), it works equally well both when the data are I(1), and when they are local-to-unity.

[11] Results from the trace tests are in line with those from the maximum eigenvalue tests, and they are available upon request.

[12] For the short rate, the reason for not including a time trend is obvious: the notion that nominal interest rates may follow an upward path (the possibility of a downward path is ruled out by the zero lower bound), in which they grow over time, is manifestly absurd. For M1 velocity things are less obvious. The reason for not including a trend originates from the fact that what I am here focusing on is a demand for money *for transaction purposes* (so this argument holds for M1, but it would not hold for broader aggregates). The resulting natural assumption of unitary income elasticity logically implies that, if the demand for M1 is stable, velocity should inherit the stochastic properties of the opportunity cost of money. In turn, this implies that the unit root tests we run for velocity should be the same as those we run for the short rate.

[13] I do not consider the Akaike Information Criterion since, as discussed by Luetkepohl (1991), for systems featuring I(1) series the AIC is an inconsistent lag selection criterion.

[14] CRT's procedure is based on the notion of computing p-values by bootstrapping the model which is relevant under the null hypothesis. This means that, within the present context, the model which is being bootstrapped is a non-cointegrated VAR in differences. All of the technical details can be found in CRT.

[15] My implementation of the test exactly follows Wright (2000), which the reader is referred to.

For the United Kingdom, cointegration between velocity and the short rate is detected based on either Johansen's or Wright's test for the Selden-Latané and semi-log specifications, whereas it is detected only based on Wright's test based on the log-log. Since data for low-inflation, low-interest rates countries tend to prefer the Selden-Latané specification—see Benati *et al.* (2019)—what follows will mainly focus on this functional form, and will only mention some of the results based on the other two specifications.

Robustness issues In those cases in which Johansen's tests fail to reject no cointegration (such as, within the present case, for the log-log), a possible explanation is that this might be the figment of a short sample length and/or a highly persistent cointegration residual.[16] In order to explore how plausible this explanation is, Table A.3 in the Online Appendix reports evidence from the following Monte Carlo experiment. For all those cases for which, in Table 1, Johansen's tests do not reject the null at the 10% level, the VECM is estimated by imposing one cointegration vector. Then, the VECM is stochastically simulated 2,000 times for samples of length equal to the actual sample length, and based on each simulated sample I perform the same tests I have previously performed based on the actual data. Table A.3 reports the empirical rejections frequencies (ERFs) at the 10% level, i.e. the fractions of times, out of the 2,000 simulations, for which maximum eigenvalues tests[17] reject no cointegration. For the post-WWII U.K. based on the log-log, the ERF is equal to just 0.229, suggesting that if cointegration truly were in the data, Johansen's tests would detect it only between one-fifth and one-fourth of the time.

Online Appendix C reports results from Hansen and Johansen's (1999) tests for stability in either the cointegration vector, or the vector of loading coefficients, in the VECMs estimated conditional on one cointegration vector, as well as extensive Monte Carlo evidence on the performance of such tests. For the post-WWII U.K. there is no evidence of instability

[16] This was originally discussed by Engle and Granger (1987) with reference to ADF-based cointegration tests. Benati *et al.* (2019) provide extensive Monte Carlo evidence on this.

[17] Results for the trace tests are near-numerically identical.

in either feature of the VECM. More generally, evidence of time-variation is near-uniformly weak to non-existent for any of the countries studied in the present work, and in what follows this issue will therefore be entirely ignored.

2.2.2 IRFs and variance decompositions

Expression (5) implies that (i) assuming that ϵ_t is small, shocks to the permanent component of the short rate explain the bulk of the (forecast error) variance of velocity; and (ii) velocity only reacts to permanent shocks to the short rate, whereas it does not react to transitory shocks. The first panel in the first row of Figure 1b provides evidence on (i), whereas the corresponding panel in the second row reports evidence on (ii). I identify permanent and transitory shocks to the *short rate*—rather than velocity—by imposing upon the VECM estimated conditional on one cointegration vector the restriction that the permanent shock is the only disturbance impacting upon the short rate in the infinite long run. Confidence bands for either IRFs or fractions of FEV are computed by bootstrapping the estimated VECM *via* CRT's (2012) procedure.

Two features stand out from the two panels in the first column of Figure 1b:

first, in line with (5) and (7), the permanent shock to the short rate explains nearly all of the FEV of velocity at all horizons. On the other hand, the corresponding evidence for the short rate shows that this shock only explains between about 25 and 30 per cent of the FEV of the short rate at horizons up to five years ahead, and slightly more than 60 per cent ten years ahead. It is important to stress that these results have been obtained in spite of the fact that the shock has been identified as the one driving the permanent component of the *short rate*, rather than of velocity.

Second, V_t does not react to transitory shocks an any horizon, whereas the response of R_t is strongly statistically significant.

Both features stand in sharp contrast to the corresponding predictions of specification

(4), which implies that velocity is also driven by the transitory component of the short rate.

2.2.3 How does the system adjusts towards equilibrium?

The two panels in the third column of Figure 1*b* provide evidence on a stylized fact discussed in Section 2.1.1: when the system is out of equilibrium, adjustment takes place *via* movements in the short rate towards its stochastic trend—i.e., (rescaled) velocity—rather than *via* movements in velocity. The third column of Figure 1*b* provides clear evidence on this, by showing the bootstrapped distributions of the two series' loading coefficients on the cointegration residual in the estimated VECM. Whereas the estimate of the loading coefficient for velocity, at 0.002, is negligible and not significantly different from zero, the corresponding estimate for the short rate, at -0.185, is strongly statistically significant. In particular, the bootstrapped p-values for testing the null hypothesis that the two coefficients are equal to zero (reported in Tables A.7 in the Online Appendix) are equal to 0.149 and 0.006, respectively.

Expressions (6)-(7) provide a straightforward, and natural interpretation for the evidence in the third column of Figure 1*b*: the dynamics of M1 velocity in the post-WWII U.K. is well described by (5), rather than by the traditional specification (4), thus implying that velocity has been systematically reacting to the *permanent* component of the short rate, rather than to the short rate itself. As we will see in Section 3, this has been a robust feature of macroeconomic fluctuations in nearly all of the countries in my dataset, first and foremost, in the United States and the United Kingdom since World War I.

2.3 Implications

We now turn to discussing the implications of these findings.

2.3.1 Implications for the theory of money demand

The fact that U.K. M1 velocity only reacts, to a first approximation, to permanent shocks to the short-term nominal rate implies that economic agents, in deciding how much of their wealth to allocate to non interest-bearing M1, as opposed to interest-bearing assets, almost uniquely react to permanent shocks to the opportunity cost of holding M1 balances, whereas they essentially ignore transitory shocks. Another way of putting this is that to a close approximation, the demand for M1 only reacts to permanent shocks to the opportunity cost, whereas it does not react to its transitory variation. As already pointed out in the Introduction, to the very best of my knowledge, no existing model of money demand exhibits this property: in fact, no model of money demand even distinguishes between permanent and transitory variation in the opportunity cost of money. My findings therefore suggest that an important avenue for future research is to develop a framework allowing for a distinction between permanent and transitory variation in the opportunity cost of money, in order to replicate such (near) lack of responsiveness of the demand for M1 to transitory variation in nominal interest rates.

2.3.2 Implications for models of 'disequilibrium money'

Since M1 velocity is the inverse of the demand for M1 expressed as a fraction of GDP, these findings have the following implication. The money demand literature has routinely interpreted deviations from the long-run equilibrium between the short rate and velocity (or money balances as a fraction of GDP) as signalling future inflationary or deflationary pressures. The *implicit assumption* behind such interpretation is that the presence of a disequilibrium in the system implies that real money balances are out of equilibrium.[18] As they adjust towards equilibrium, pent-up inflationary (deflationary) pressures are released, and inflation increases (decreases). Although this interpretation is intuitively appealing, my results

[18] See e.g. the discussion in Section 4 of Goldfeld and Sichel (1990). For a critical perspective on models of 'disequilibrium money', see White (1981).

show that it is incorrect (at least, for M1). The reason is that, as pointed out, M1 velocity (and therefore the demand for M1 balances) is always approximately in equilibrium: it is rather the short rate which is typically out of equilibrium. This means that a disequilibrium in the relationship between velocity and the short rate (i.e., the cointegration residual being different from zero) does *not* signal future inflationary or deflationary pressures: rather, it signals future movements of the short rate towards equilibrium.

2.3.3 The informational content of M1 velocity for the natural rate of interest

Another logical implication of these findings is the following. Basic economic logic suggests that R_t^P should be driven by (i) permanent inflation shocks (*via* the Fisher effect) and (ii) permanent shocks to the real rate, i.e., shocks to the natural rate of interest,

that is, $R_t^P = \pi_t^P + r_t^N$, where π_t^P is the permanent component of inflation, and r_t^N is the natural rate of interest.[19] This implies that, under monetary regimes which cause inflation to be I(0)—so that π_t^P=0—permanent shifts in M1 velocity should *uniquely* reflect permanent fluctuations in the natural rate of interest, so that, e.g., $V_t = \alpha + \beta r_t^N + \epsilon_t$.

The two panels in the third column of Figure 1a provide simple evidence on this for the U.K. inflation-targeting regime. The upper panel shows GDP deflator inflation: visual evidence suggests that—in line with the evidence reported in Benati (2008)—under inflation-targeting U.K. inflation has been essentially white noise,[20] thus implying that shifts in ve-

[19] This assumption requires some discussion. The logical implication is that *all* other disturbances impacting upon nominal rates are transitory, including (e.g.) shocks to the risk premium. Now suppose—just for the sake of the argument—that shocks to the risk premium were partly transitory, and partly permanent (the argument applies to any other shock impacting upon nominal rates). Under these circumstances, my assumption would interpret permanent risk premium shocks as natural rate shocks. The key issue here, however, is that—*especially* for monetary policy purposes—this is *exactly* what we would want. The fact that, for a given equilibrium inflation rate, the equilibrium nominal Federal Funds rate increases by x per cent because of (say) a permanent risk premium shock—as opposed to a supposedly 'authentic' shock to the natural rate (due, e.g., to an acceleration of productivity growth)—is for monetary policy purposes, completely irrelevant. What matters is that the Funds rate has *permanently* increased by x per cent.

[20] In fact, as I discuss in Section 4, the null of a unit root is very strongly rejected, with p-values from Elliot *et al.*'s (1996) tests equal to or close to zero, and Hansen's (1999) bias-corrected estimate of the sum of the autoregressive coefficients in an AR(p) representation for inflation is equal to -0.32, with the 90 per cent-coverage confidence interval equal to [-0.75; 0.10].

locity should have uniquely reflected fluctuations in the natural rate of interest. In turn, this implies that the protracted fall in velocity experienced by the United Kingdom under inflation-targeting should have been driven by a decline in the natural rate.

The lower panel presents simple evidence compatible with this notion. As discussed by Laubach and Williams (2003, p. 1063), within a vast class of theoretical models changes in the economy's trend growth rate automatically map into changes in the natural rate of interest.[21] This implies that if changes in the other determinants of the natural rate (e.g., agents' rate of time preference) had been second-order compared to changes in trend GDP growth, we should see a strong correlation between velocity and trend growth in the United Kingdom under inflation-targeting. Section 4 estimates a time-varying trend for real GDP growth for the United Kingdom and several other countries based on Stock and Watson's (1996, 1998; henceforth, SW) TVP-MUB methodology. Here we report a much simpler estimate—a linear time trend for GDP growth estimated *via* OLS—which is however in line with the results produced by SW's methodology (this can be seen by comparing the linear trend in the third panel of Figure 1*a* with the TVP-MUB trend in Figure 5). The correlation between velocity and trend GDP growth, although not perfect, is very strong, with the former falling from 1.28 in 1992Q4 to 0.60 in 2015Q4, and the latter decreasing from about 2.3 to about 1.8 per cent over the same period. Although by no means does this evidence represent a hard proof that my argument is correct, it is, at the very least, compatible with such position. This implies that, in principle, it should be possible to estimate the natural rate by exploiting the informational content of velocity (in Online Appendix D I do this for Canada and the United Kingdom under inflation targeting).

[21] Several recent papers have raised doubts about the existence of a strong link between trend GDP growth and the natural rate. E.g., the evidence in Leduc and Rudebusch (2014), Hamilton *et al.* (2016), and Lunsford and West (2017) suggests that real rates do not exhibit a strong correlation with output growth at the low frequencies. In my own view, these results should be regarded as a cautionary note to the standard position—rather than a fundamental counter-argument—simply because a mapping between trend GDP growth and the natural rate is a robust prediction of standard growth models, and therefore it cannot be easily dismissed.

We now discuss the evidence for other countries.

3 Evidence for Other Countries

Figures A.0a and A.0b in the Online Appendix show the raw data for M1 velocity and the short-term rate for all countries other than the United Kingdom. In line with the evidence for the U.K., in several cases visual evidence clearly suggests that velocity and the short rate are cointegrated, and that the former is, essentially, the permanent component of the latter. As we will see, econometric evidence does indeed confirm such visual impression.

Figure 2 shows the cointegration residuals between velocity and the short rate (i.e., the series in Figures A.0a-A.0b), together with the difference between the short- and a long-term rate (due to data limitations for the long rate, the figure only shows evidence for a few countries, and the evidence for Switzerland starts in 1960, rather than in 1914 as in Figure A.0a). In line with the evidence for the United Kingdom, in nearly all cases the cointegration residual exhibits a strong, negative correlation with the short-long spread.[22] Interestingly, in the United States the correlation had been thrown temporarily out of kilter by the introduction of MMDAs in 1982, but it then reasserted itself in the second half of the 1980s, and it has consistently held since then. Further, in *all* cases[23] the period following the collapse of Lehman Brothers—which featured the most violent phase of the recent financial crisis—does not exhibit *any* obvious difference with the rest of the sample. This provides additional support to my conjecture that such strong correlation reflects a deep structural feature of the economy. In particular, the fact that the relationship has been holding at least since World War I, in spite of dramatic shifts in the monetary regime,[24] suggests that such

[22] The only exception is Korea since the beginning of the new millennium. It is to be noticed, however, that the breakdown of the correlation over the last 15-20 years has been due to the anomalous behaviour of the *spread*, which has significantly increased compared to previous years, rather than to any obvious change in the behaviour of the cointegration residual. This means that for the purpose of this paper, whose focus is the relationship between velocity and the short rate, such a breakdown is immaterial.

[23] With the just-mentioned exception of Korea.

[24] The partial reintroduction, and then the disintegration of the Gold Standard in the interwar period; the

relationship might well be structural in the sense of the Lucas critique.

3.1 Econometric evidence

We start with cointegration tests, and we then show evidence from a permanent-transitory decomposition.

3.1.1 Cointegration tests

Based on the Selden-Latané specification—which, as mentioned, appears to be the preferred functional form for low-inflation, low-interest rate countries (see Benati *et al.*, 2019)—Johansen's tests detect cointegration, based on post-WWII quarterly data, for all countries except South Africa and Hong Kong, whereas Wright's tests detect it for all countries except Australia, Taiwan, and South Africa. Based on annual data, Johansen's test rejects no cointegration in eight cases out of thirteen, whereas Wright's detects cointegration in eleven cases. Finally, as for Johansen's tests, the ERFs for the cases in which cointegration is not detected (see Table A.3 in the Online Appendix) are uniformly low, or very low, thus implying that lack of rejection of the null is, in fact, compatible with the presence of cointegration.

Based on these results, in what follows we will therefore proceed under the assumption that a cointegration relationship between velocity and the short rate based on the Selden-Latané specification does exist for all countries.[25]

Bretton Woods regime and its collapse; the introduction, in several instances, of inflation-targeting regimes in the 1990s; and the adoption of quantitative easing (QE) policies during the financial crisis.

[25] Results based on the semi-log and log-log specifications are uniformly weaker (this is especially clear for the latter specification), but for our purposes they are essentially irrelevant: as mentioned, the data tend to quite clearly prefer the Selden-Latané specification (which is, in fact, one way of interpreting the results in Tables 1-2) and in what follows I will therefore uniquely focus on this functional form.

3.1.2 Evidence from the permanent-transitory decomposition

Figures $3a$-$3b$ and $4a$-$4b$ show results from the same permanent-transitory decomposition implemented for the United Kingdom. Specifically, Figures $3a$ and $4a$ show, based on quarterly and annual data, respectively, the fractions of FEV explained by the permanent shock, whereas figures $3b$ and $4b$ show the IRFs to the transitory shock.[26]

Based on quarterly data, evidence is consistently in line with that for the United Kingdom for all countries with the single exception of Taiwan. In particular, the fractions of FEV of velocity explained by the permanent shock are consistently very high, and most of the time close to one at nearly all horizons. By contrast, the fractions of FEV of the short rate are systematically lower than those of velocity at all horizons, and in several cases they are quite remarkably low, especially at the short horizons. As for the IRFs, the response of the short rate to transitory shocks is strongly statistically significant for all countries except (again) Taiwan. As for velocity, the response is statistically insignificant at (nearly) all horizons for Canada, Australia, Korea, South Africa, Hong Kong and Mexico. As for the United States, it is insignificant (and, in fact, close to zero) on impact, whereas it is strongly significant further out. Summing up, with the single exception of Taiwan,[27] the evidence based on quarterly data is in line with that for the United Kingdom.

Turning to the annual data, support for this paper's main thesis is provided by the evidence for the United States, the United Kingdom, Switzerland, New Zealand, Australia, the Netherlands, and Finland. In all of these cases, permanent shocks explain very high fractions of the FEV of velocity (in several cases, very close to one) at all horizons, whereas they consistently explain lower fractions of the FEV of the short rate. By the same token, for

[26]Figures A.1a-A.1b in the appendix report the IRFs to the permanent shock, whereas Figures A.2a-A.2b show scatterplots of the permanent and transitory components of the two series.

[27]The evidence for Taiwan should be discounted for the following reason. Visual evidence based on the raw series in Figure A.0b suggests that velocity is *smoother* than the short rate. Indeed, once the two series have been rescaled so that they have the same sample standard deviation, the variance of the first difference of the short rate is 2.85 times the variance of the first difference of velocity. Since the series are cointegrated, and therefore driven by the same permanent shock, this is hard to square with the variance decomposition in Figure $3a$, suggesting that the *short rate* is the stochastic trend in the system.

either of these seven countries the reaction of the short rate to transitory shocks is strongly statistically significant, whereas the corresponding IRF for velocity is insignificant at all horizons for the United Kingdom, Switzerland, New Zealand, the Netherlands, and Finland; it is insignificant on impact, and at short horizons, for Australia; and it is instead mostly strongly significant for the United States. For Canada, Japan, and Belgium the fraction of FEV of the short rate explained by permanent shocks is (based on point estimates) consistently greater than the corresponding fraction for velocity (although for Belgium the difference is quite small). As for the IRFs to transitory shocks, they are uniformly insignificant at all horizons for either variable, and either country. Overall, the evidence based on annual data appears somehow weaker than that based on quarterly data, with three countries out of ten failing to support this paper's main thesis.[28]

3.1.3 Summing up

Overall, the evidence in this section provides substantial—although by no means perfect—support to my thesis that M1 velocity is, to a close approximation, the permanent component of the short rate. Evidence is strong for eight countries out of nine based on quarterly data, and for seven countries out of ten based on annual data. On the other hand, evidence is negative—but, for the reasons given in footnotes 27 and 28, it should arguably be discounted—for Taiwan based on quarterly data, and for Canada and Belgium based on annual data. This leaves us with only one country, Japan, for which evidence appears to quite clearly contradict my argument.[29]

[28] As for Taiwan, results for Canada and Belgium should be discounted, for the same reason I gave in footnote 27. First, the visual evidence in Figure 2*a* suggests that for both countries velocity is appreciably smoother than the short rate. Second, once the series are rescaled so that they have the same sample standard deviation, for both countries the first difference of the short rate is markedly more volatile than the first difference of velocity (for Belgium and Canada, respectively, the variance of the first difference of the short rate is 11.01 times, and 8.30 times greater than the variance of the first difference of rescaled velocity). Again, since the two series share the same stochastic trend, this is hard to square with the notion that the short rate might be closer to such trend than velocity. If we accept this argument, this leaves us with Japan as the single country which truly seems to contradict this paper's argument.

[29] There are a few additional countries for which evidence supports my thesis, but whose results I do not report because the sample periods are quite short (these results are available upon request). This is the

We now turn to discuss evidence for monetary regimes which have caused inflation to be I(0), such as inflation-targeting regimes. As mentioned, the most interesting feature of these regimes is that, by eliminating permanent inflation shocks, they cause velocity—if my argument is correct—to be essentially a linear transformation of the natural rate of interest.

4 Evidence for Regimes Making Inflation I(0)

As discussed in Section 2.3, within a vast class of models changes in trend output growth automatically map into changes in the natural rate of interest. This implies that, if my argument is correct, under regimes causing inflation to be stationary we should see a strong correlation between velocity and trend GDP growth. Figure 5 provides evidence for eight such regimes, specifically: four inflation-targeting countries (United Kingdom, Canada, Australia, and New Zealand);[30] European Monetary Union (EMU); Switzerland under the post-1999 'new monetary policy concept' (which is conceptually akin to EMU); West Germany/Germany up until the beginning of EMU (i.e., December 1998);[31] and Denmark, which has consistently followed a policy of pegging the Krone first to the Deutsche Mark, and then to the Euro, thus importing the strong anti-inflationary stance of the Bundesbank, and then of the European Central Bank (ECB). In line with the evidence reported in Benati (2008) based on Hansen's

case for Denmark and Sweden (for either country the sample period is 1993Q1-2017Q1): in both countries permanent shocks induce an insignificant response of velocity at all horizons, and a statistically significant response in the short rate. Results for the Euro area since 1999Q1 (data for the pre-EMU period have been reconstructed *ex post*, and so I eschewed them) exhibit the same pattern as Denmark and Sweden.

[30] Inflation targeting was introduced in the U.K., Canada, and New Zealand in October 1992, February 1991, and February 1990. As for Australia, since there never was an explicit announcement of the introduction of the new regime, I follow Benati and Goodhart (2011) in taking 1994Q3 as the starting date. I do not report results for Sweden (the available sample is 1998Q1-2017Q2) because they are manifestly puzzling. Both a linear trend estimated *via* OLS, and simple averages computed for the first and second halves of the sample, clearly suggest that trend GDP growth has progressively decreased, which would be in line with the steady decrease in M1 velocity since 1998. SW's estimate of trend growth, on the other hand, is essentially flat over the entire period.

[31] I also consider the period after unification in order to have a longer sample period. Reunification caused a discontinuity in both nominal GDP and M1, but, from a conceptual point of view, this is not a problem for the computation of velocity (i.e., their ratio). As for real GDP growth, I treat the very large observation for the quarter corresponding to reunification as an outlier, and, as in Stock and Watson (2002), I replace it with the median value of the six adjacent quarters.

(1999) estimator of the sum of the autoregressive coefficients, for all countries—with the exception of Switzerland and the Euro area—Elliot *et al.*'s. (1996) tests strongly reject the null of a unit root for inflation.[32] In spite of the results from unit root tests, I have chosen to also consider Switzerland and the Euro area for the following reasons. As for Switzerland, results from unit root tests are most likely a figment of the short sample period: in fact, for the sample starting in 1980Q1 (when GDP deflator data start being available) a unit root is rejected very strongly. This is in line with Switzerland's reputation as a hard-currency, low-inflation country.[33] As for the Euro area, visual evidence clearly suggests that the collapse of Lehman Brothers, which unleashed the most violent phase of the Great Recession, was associated with a downward shift in mean inflation, from 2.01 per cent over the period 1999Q1-2008Q3, to 0.99 per cent over the period 2008Q4-2016Q4. Once controlling for this break in the mean, a unit root is very strongly rejected, thus showing that the previous lack of rejection was a simple illustration of Perron's (1989) well-known argument. My decision to also consider the Euro area reflects the fact that, in spite of such downward shift in the mean of inflation, inflation expectations (as measured by the ECB's Survey of Professional Forecasters) have remained well-anchored,[34] thus suggesting that agents have interpreted such shift as *temporary*. Finally, I also show evidence for the United States for the period following the Volcker disinflation[35] for the following reason. Although Elliot *et al.*'s (1996) tests for sample periods following the end of the Volcker disinflation typically do not reject the null of a unit root,[36] this evidence does not square well with the fact that during this period inflation has been broadly stable. A likely explanation is that, after the Volcker stabilization, U.S.

[32] Results are in Table A.9 in the Online Appendix. By the same token, Hansen's (1999) estimate of the sum of the AR coefficients in AR(p) representations for inflation clearly suggest that in all cases (again, with the exception of the Euro area and Switzerland) inflation is (close to) white noise.

[33] Over the entire period since World War I, Swiss inflation has been equal, on average, to 1.9 per cent.

[34] Figure A.3 in the appendix shows the inflation forecasts from the ECB's Survey of Professional Forecasters at three alternative horizons, 1-, 2-, and 5-years ahead. Over the entire period since 1999Q1, the 5-years ahead forecast has fluctuated between 1.8 and 2.0 per cent.

[35] Following Clarida *et al.* (2000), I take 1983Q4 as marking the end of the disinflation.

[36] For samples starting in the first quarter of each year from 1984 to 1999, a unit root in inflation can be rejected, at the 10% level, only for those starting in 1988, 1990, and 1992.

inflation has still exhibited a small unit root component.[37] This should introduce a small permanent 'wedge' between actual M1 velocity, and the value velocity would have taken in the absence of permanent variation in inflation. Such wedge should however be quite small, so that the same argument I made for monetary regimes under which inflation has been I(0) should also approximately apply to the United States.

Figure 5 shows M1 velocity together with a SW (1996, 1998) TVP-MUB estimate of trend GDP growth, based on a time-varying parameters AR(p).[38] The correlation between the two series is strong for all countries, with the exception of Australia. For the U.S. the correlation with trend GDP growth is stronger for the velocity series based on the M1 aggregate also including MMFAs, whereas it is weaker based on the aggregate only including MMDAs. To be sure, this does not represent a hard proof that my thesis is correct. At the very least, however, it is compatible with my argument. This implies that it should be possible to exploit the informational content of M1 velocity to estimate the natural rate of interest. Online Appendix D presents estimates of the natural rate for the U.K. and Canada under inflation targeting, based on cointegrated structural VARs. In either country, the natural rate has been consistently declining since the early 1990s.

5 Conclusions

Since WWI, M1 velocity has been, to a close approximation, the I(1) component of the short rate. This implies that economic agents, in deciding how much wealth to allocate to non interest-bearing M1, as opposed to interest-bearing assets, almost uniquely react to permanent shocks to the opportunity cost of money, whereas they essentially ignore tran-

[37] Results from Cochrane's (1988) variance ratio estimator provide support to this conjecture: over the period 1984Q1-2017Q2, the size of the unit root in U.S. GDP deflator inflation has been slightly below 10 per cent (see Figure A.4 in the appendix; bootstrapped confidence bands have been computed *via* spectral bootstrapping of the first-difference of inflation as in Benati (2007)).

[38] My implementation of SW's methodology is exactly the same as Benati's (2007), which the reader is referred to for details.

sitory shocks. Since no existing money demand model exhibits this property, this implies that an important avenue for future research is to develop a framework allowing for such a distinct reaction, on the part of economic agents, to permanent and transitory shocks to the opportunity cost of money.

A The Data

Online Appendix A describes the data and their sources in detail.[39] All data are from official sources, i.e., either central banks or national statistical agencies. Almost all of the annual data are from the dataset assembled by Benati *et al.* (2019).[40]

All of the series are standard, with the single exception that, for the United States, I consider three of the alternative adjustments to the Federal Reserve's standard M1 aggregate which had originally been suggested by Goldfeld and Sichel (1990, pp. 314-315) in order to restore the stability of the long-run demand for M1, which had vanished around the mid-1980s. Specifically, I augment the standard M1 aggregate with either Money Market Deposits Accounts (MMDAs), as in Lucas and Nicolini (2015);[41] Money Market Mutual Funds (MMDFs); or both MMDAs and MMFAs. For reasons of robustness, for either of the three 'expanded' U.S. M1 aggregates I also consider an alternative version, in which currency has been adjusted along the lines of Judson (2017), in order to take into account of the fact that, since the early 1990s, there has been a sizeable expansion in the fraction of U.S. currency held by foreigners. For reasons of space, in what follows I only report results for the aggregate including MMDAs, and for that including MMDAs and MMFAs. Results for the aggregate just including MMFAs are qualitatively the same, and they are available upon request.

Online Appendix A discusses in detail a few countries in Benati *et al.*'s dataset which I have chosen not to analyze herein because the data exhibit puzzling features (this is the case in particular for Italy and Norway), and the reasons why I have decided to eschew high- and very high-inflation countries, and to exclusively focus on low-to-medium inflation countries.[42]

[39] The online appendix is available at: https://sites.google.com/site/lucabenatiswebpage.

[40] In several cases (South Africa, Taiwan, South Korea and Hong Kong, and Canada since 1967) I was able to find quarterly data for the same sample periods analyzed by Benati *et al.* (2019).

[41] As discussed by Lucas and Nicolini (2015), the reason for including MMDAs in M1 is that they perform an economic function similar to that of the 'checkable deposits' included in the official M1 series.

[42] A very partial exception to this is Mexico, for which, for one year and a half at the very beginning of

A.1 Results from unit root tests

Tables A.1a and A.1b in the Online Appendix report bootstrapped p-values[43] for Elliot *et al.* (1996) unit root tests for (the logarithms of) M1 velocity and the short rate. All tests are with an intercept, but no time trend. For the short rate, R_t, I also report results for $\ln(R_t+1)$, in which the series has been corrected conceptually in line with Alvarez and Lippi (2009), by adding to it a 1 per cent cost of either losing cash, or having it stolen.[44] In nearly all cases, evidence of a unit root in the series is very strong, with the p-values being almost uniformly greater than the 10 per cent level I take as the benchmark throughout the entire paper, often significantly so. For Switzerland a unit root is rejected for $\ln(R_t)$, but not for $\ln(R_t+1)$. Because of the reason mentioned in the previous footnote, in what follows the analysis for the 'log-log' specification will be performed based on $\ln(R_t+1)$, rather than $\ln(R_t)$, and these results are therefore ultimately irrelevant.[45] For Korea the alternative lag orders produce contrasting evidence for velocity. In this cases I regard the null of a unit root as not having been convincingly rejected, and I therefore proceed under the assumption that the series is I(1). Finally, for Taiwan a unit root is rejected for velocity based on either lag order. In the light of the evidence in Figure A.0b in the Online Appendix—showing that velocity has been consistently declining since 1961—I regard this result as a statistical fluke.[46]

Tables A.2a and A.2b in the Online Appendix report bootstrapped p-values for Elliot *et al.* (1996) unit root tests for either the first differences, or the log-differences, of velocity and

the sample, inflation exceeded 100 per cent.

[43] p-values have been computed by bootstrapping 10,000 times estimated ARIMA(p,1,0) processes. As for the lag order, p, since, as it is well known, results from unit root tests may be sensitive to the specific lag order which is being used, for reasons of robustness I consider two alternative lag orders based on annual data (either 1 or 2), and four based on quarterly data (either 1, 2, 3, or 4).

[44] A key rationale is that this correction delivers a finite satiation level of real money balances at $R_t = 0$.

[45] On the other hand, there is no point in implementing Alvarez and Lippi's (2009) correction for the other two specifications I consider, since in both cases the short rate enters in levels.

[46] When performing a large number of statistical tests, such as it the case here, a certain number of flukes should be expected. To be sure, the series I am analyzing here are not independent stochastic processes generated (e.g.) in MATLAB, but the same logic should approximately apply.

the short rate. In all cases the null of a unit root is strongly rejected, thus suggesting that the series' order of integration is not greater than one.

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Table 1 Bootstrapped p-values[a] for Johansen's maximum eigenvalue[b] tests for (log) M1 velocity and (the log of) a short-term rate

Country	Period	Money demand specification: *Selden-Latané*	*Semi-log*	*Log-log*
		I: Long-run annual data		
United States				
standard M1 + MMDAs	1915-2017	0.063	0.112	0.218
standard M1 + MMDAs + MMMFs	1915-2017	0.001	0.217	0.778
standard M1 + MMDAs#	1926-2017	0.092	0.006	0.170
standard M1 + MMDAs + MMMFs#	1926-2017	0.003	0.011	0.874
United Kingdom	1922-2016	0.003	0.010	0.334
Switzerland	1914-2015	0.007	0.015	0.383
New Zealand	1934-2017	0.099	0.108	0.044
Canada	1935-2006	0.023	0.247	0.485
Japan	1955-2017	0.567	0.326	0.205
Australia	1941-1989	0.642	0.973	0.709
Belgium	1946-1990	0.361	0.016	0.010
Netherlands	1950-1992	0.349	0.286	0.401
Finland	1914-1985	0.622	0.659	0.839
		II: Post-WWII quarterly data		
United States				
standard M1 + MMDAs	1959Q1-2017Q4	0.015	0.028	0.427
standard M1 + MMDAs + MMMFs	1959Q1-2017Q4	0.001	0.001	0.026
standard M1 + MMDAs#	1959Q1-2017Q4	0.035	0.062	0.338
standard M1 + MMDAs + MMMFs#	1959Q1-2017Q4	0.001	0.001	0.026
United Kingdom	1955Q1-2017Q2	0.054	0.099	0.596
Canada	1967Q1-2017Q4	0.008	0.233	0.003
Australia	1969Q3-2017Q4	0.070	0.062	0.531
Taiwan	1961Q3-2017Q4	0.008	0.230	0.274
South Korea	1964Q1-2017Q4	0.001	0.448	0.156
South Africa	1985Q1-2017Q4	0.354	0.338	0.371
Hong Kong	1985Q1-2017Q4	0.183	0.074	0.025
Mexico	1985Q4-2017Q1	0.051	0.041	0.444

[a] Based on 10,000 bootstrap replications. [b] Null of 0 *versus* 1 cointegration vectors.
Adjusting for currency held by foreigners. MMDAs = Money market deposit accounts; MMMFs = Money market mutual funds

Table 2 Results from Wright's tests: 90% confidence interval for the second element of the normalized cointegration vector, based on systems for (log) M1 velocity and (the log of) a short rate[a]

Country	*Period*	Money demand specification: *Selden-Latané*	*Semi-log*	*Log-log*
		I: Long-run annual data		
United States				
standard M1 + MMDAs	1915-2017	[0.331; 0.560]	[0.082; 0.146]	[0.298; 0.535]
standard M1 + MMDAs + MMMFs	1915-2017	[0.068; 0.329]	[-0.141; 0.083]	[-0.668; 0.418]
standard M1 + MMDAs#	1926-2017	[0.361; 0.598]	[0.076; 0.136]	[0.313; 0.529]
standard M1 + MMDAs + MMMFs#	1926-2017	[0.118; 0.258]	[-0.108; 0.076]	[-0.523; 0.367]
United Kingdom	1922-2016	[0.413; 0.533]	[0.113; 0.113]	[-0.071; 1.540]
Switzerland	1914-2015	[0.253; 0.422]	[-0.290; 0.219]	[-0.390; 1.610]
New Zealand	1934-2017	NCD	NCD	[0.138; 0.423]
Canada	1935-2006	[1.049; 1.510]	[0.132; 0.140]	NCD
Japan	1955-2017	[0.320; 0.517]	NCD	[0.056; 0.709]
Australia	1941-1989	NCD	NCD	NCD
Belgium	1946-1990	[0.333; 0.445]	NCD	[0.854; 0.946]
Netherlands	1950-1992	[0.258; 0.430]	NCD	NCD
Finland	1914-1985	[0.380; 0.757]	NCD	[1.856; 3.067]
		II: Post-WWII quarterly data		
United States				
standard M1 + MMDAs	1959Q1-2017Q4	[0.321; 0.549]	[0.075; 0.119]	[0.253; 0.486]
standard M1 + MMDAs + MMMFs	1959Q1-2017Q4	[0.503; 0.692]	NCD	NCD
standard M1 + MMDAs#	1959Q1-2017Q4	[0.274; 0.522]	[0.057; 0.113]	[0.206; 0.443]
standard M1 + MMDAs + MMMFs#	1959Q1-2017Q4	[0.498; 0.666]	NCD	NCD
United Kingdom	1955Q1-2017Q2	[0.075; 0.139]	[0.093; 0.145]	[0.421; 0.817]
Canada	1967Q1-2017Q4	[0.470; 0.607]	NCD	NCD
Australia	1975Q1-2017Q4	NCD	[0.065; 0.073]	[0.530; 0.798
Taiwan	1961Q3-2017Q4	NCD	[0.159; 0.183]	NCD
South Korea	1964Q1-2017Q4	[0.131; 0.143]	[0.083; 0.099]	[0.704; 0.769]
South Africa	1985Q1-2017Q4	NCD	[0.056; 0.072]	[0.818; 1.299]
Hong Kong	1985Q1-2017Q4	[0.148; 0.212]	[0.204; 0.248]	[0.711; 0.843]
Mexico	1985Q4-2017Q1	[0.190; 0.226]	[0.016; 0.020]	[0.261; 0.477]

[a] Based on 10,000 bootstrap replications. # Adjusting for currency held by foreigners. NCD = No cointegration detected. MMDAs = Money market deposit accounts; MMMFs = Money market mutual funds.

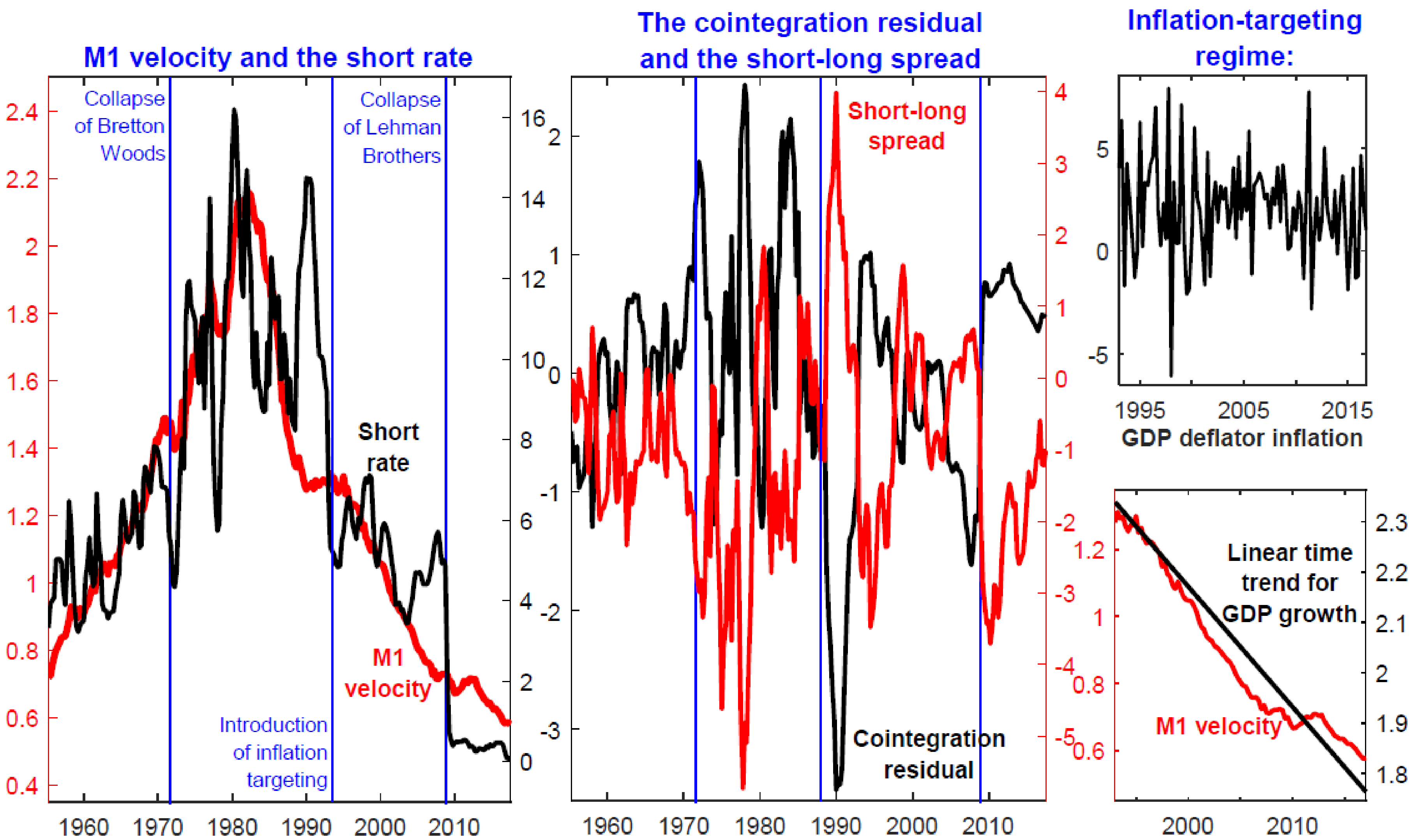

Figure 1a Evidence for the United Kingdom, 1955Q1-2017Q2

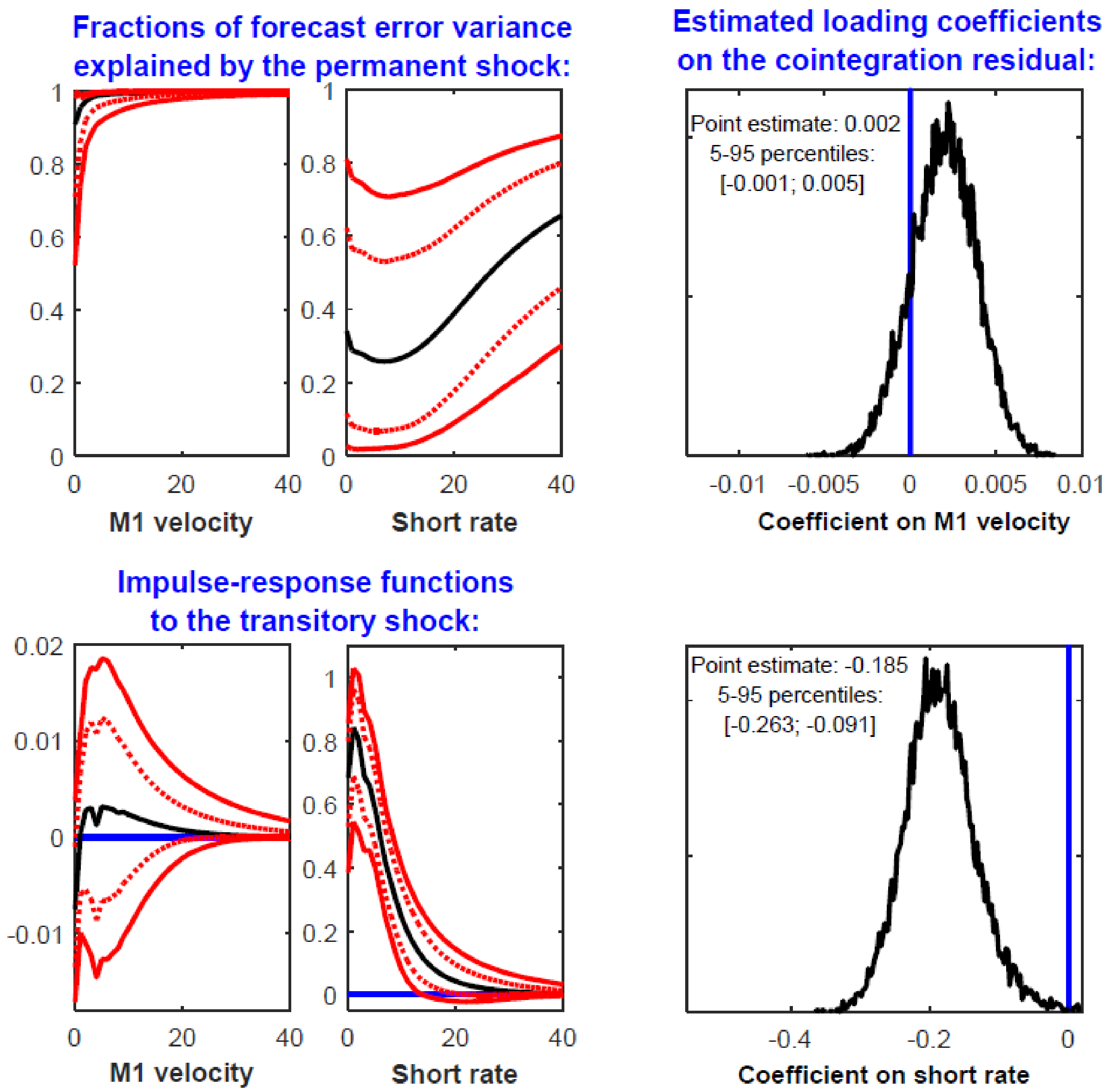

Figure 1b Evidence for the United Kingdom, 1955Q1-2017Q2

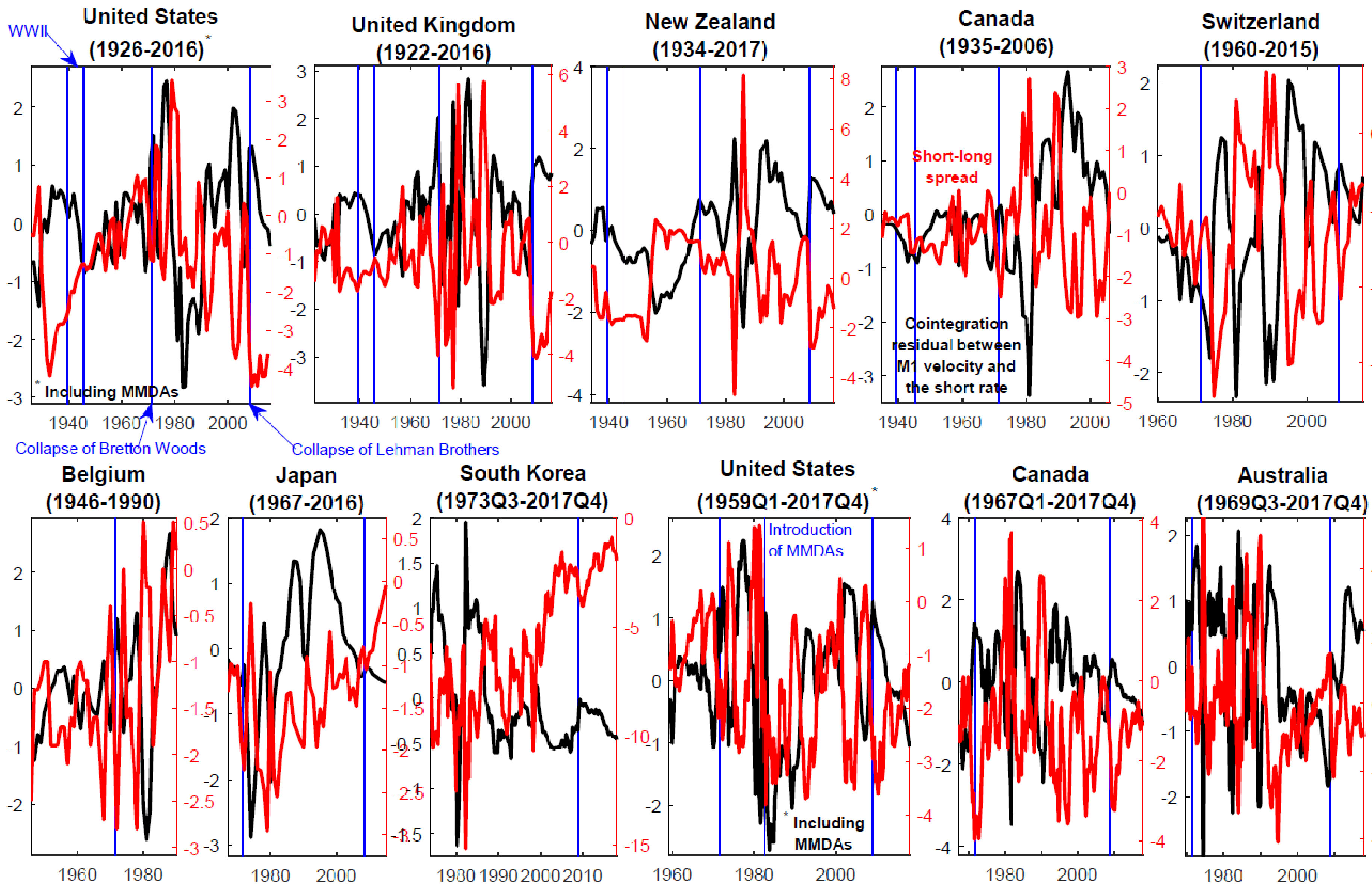

Figure 2 The cointegration residual between M1 velocity and the short rate, and the short-long spread

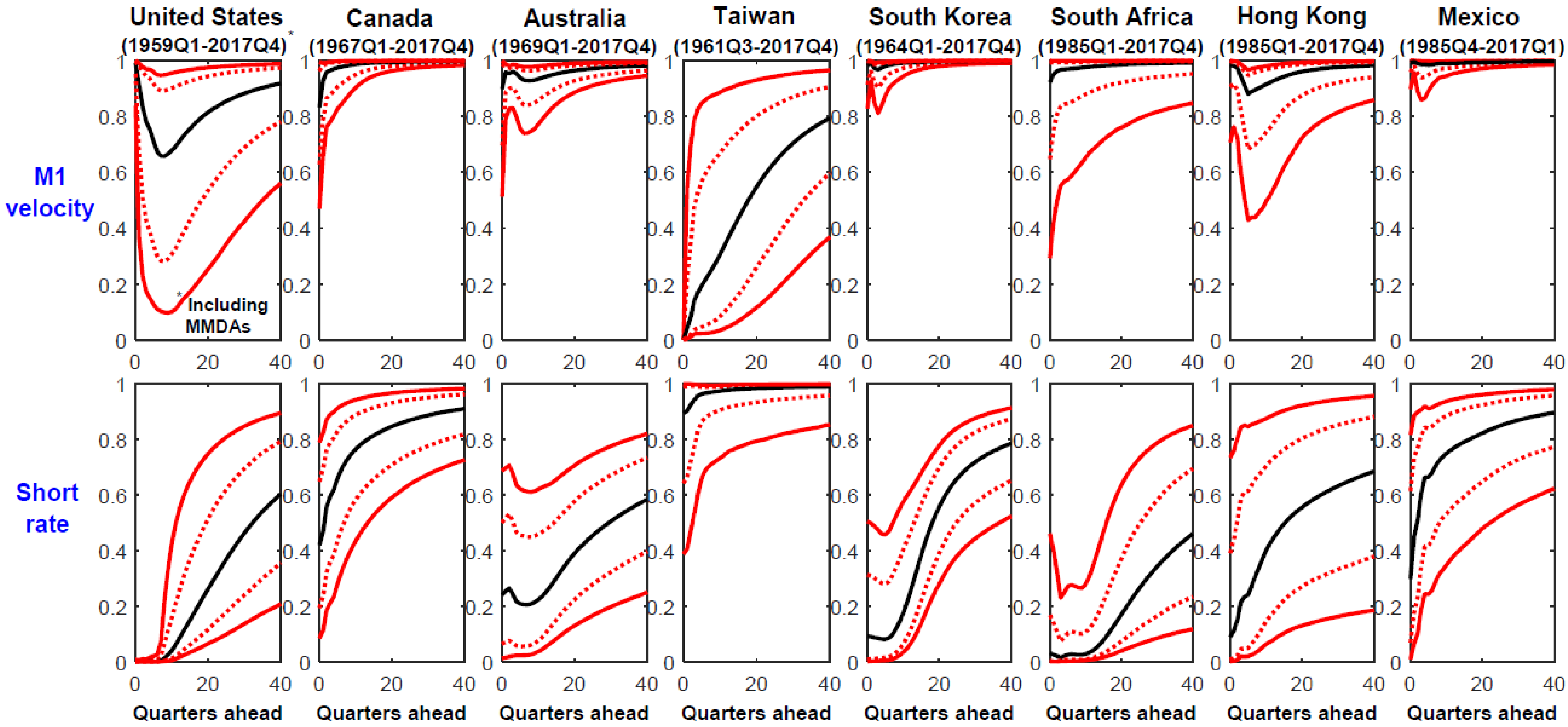

Figure 3a Results from bivariate structural VECMs for M1 velocity and the short rate: fractions of forecast error variance explained by the permanent shock (based on quarterly data)

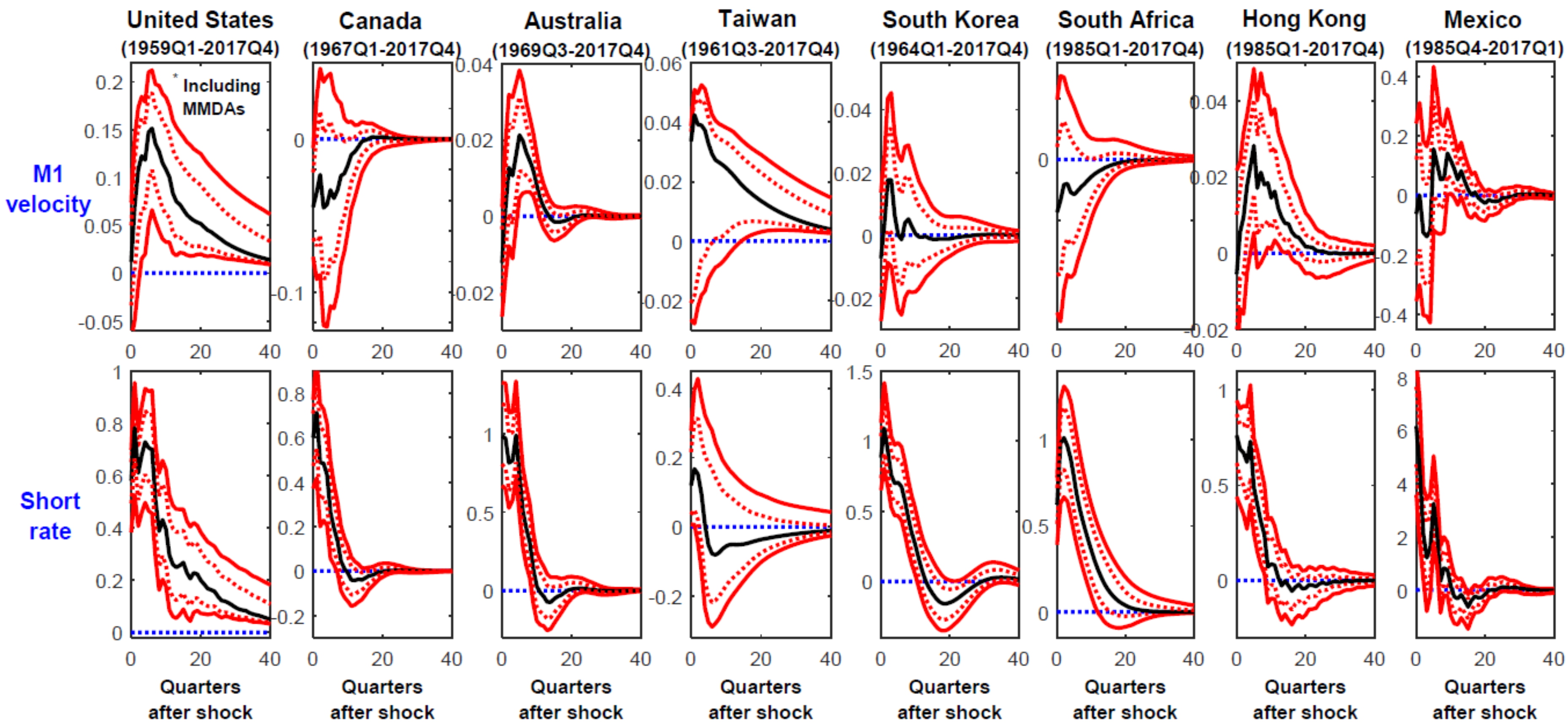

Figure 3b Results from bivariate structural VECMs for M1 velocity and the short rate: impulse-response functions to the transitory shock (based on quarterly data)

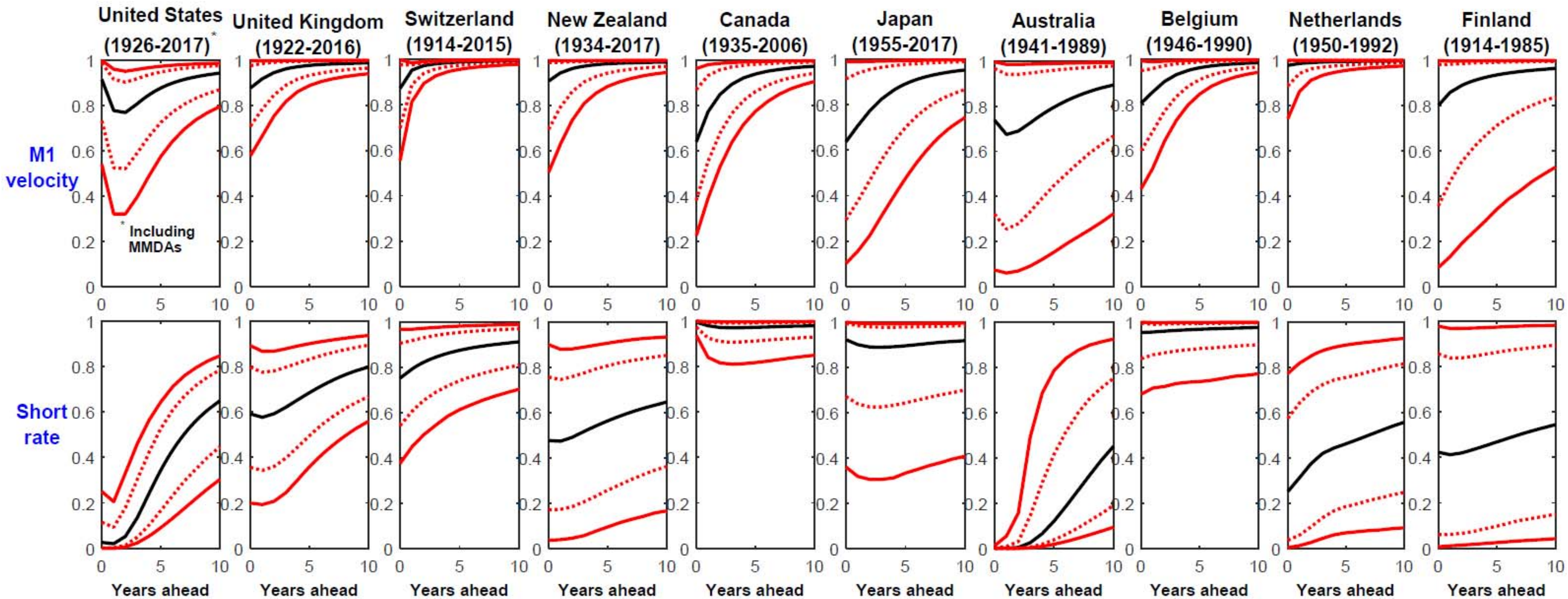

Figure 4a Results from bivariate structural VECMs for M1 velocity and the short rate: fractions of forecast error variance explained by the permanent shock (based on annual data)

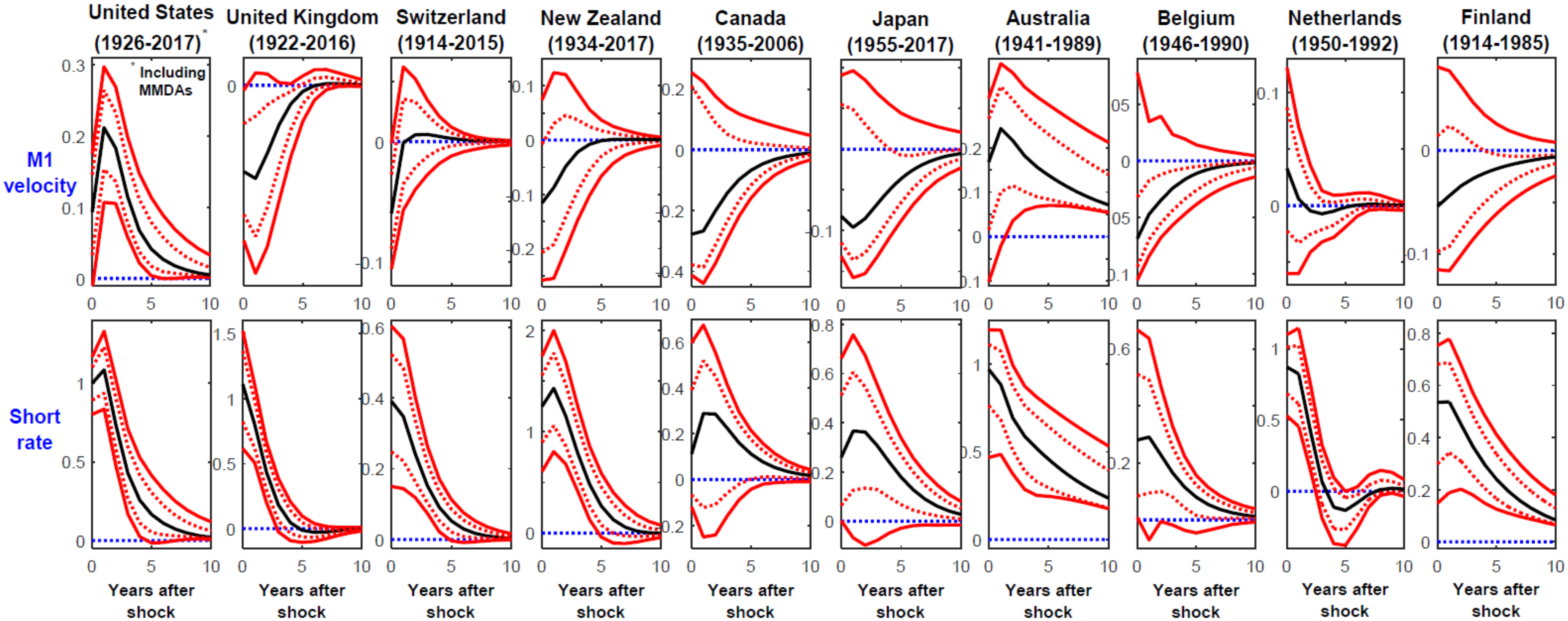

Figure 4b Results from bivariate structural VECMs for M1 velocity and the short rate: impulse-response functions to the transitory shock (based on annual data)

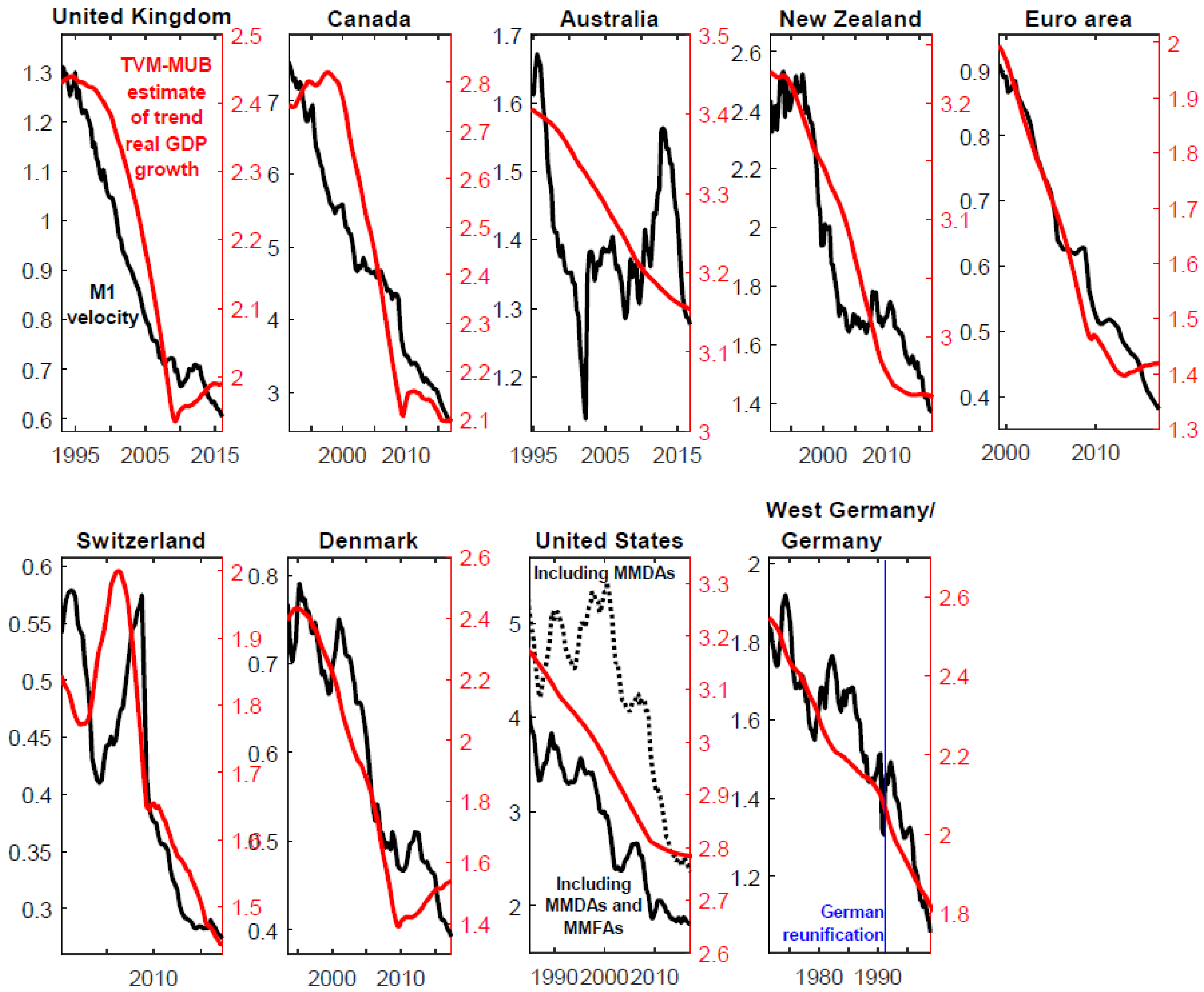

Figure 5 Evidence from monetary regimes causing inflation to be I(0): M1 velocity and Stock and Watson (1996, 1998) TVP-MUB estimate of trend real GDP growth

Online appendix for: International Evidence on Long-Run Money Demand

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A The Data

Here follows a detailed description of the dataset. Almost all of the data used in this paper are from original sources. Specifically, they are from either (*i*) original hard copy (books or, in the case of West Germany's M1, scanned PDFs of the Bundesbank's *Monthly Reports*, which are available from the Bundesbank's website), in which case we have entered the data manually into Excel; or (*ii*) central banks' or national statistical agencies' websites (these data are typically available in either Excel or simple text format). The few exceptions are discussed below. In those cases, we were not able to find the data we were looking for in original documents, and therefore we took them from either the International Monetary Fund's *International Financial Statistics* (henceforth, IMF and IFS, respectively) or the World Bank.

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A.1 Annual data

A.1.1 Argentina

All of the series are from the *Banco Central de la República Argentina* (Argentina's central bank, henceforth, *Banco Central*). Specifically, a series for M1, available for the period 1900-2014, is from *Banco Central*'s Table 7.1.4 ("Agregados Monetarios"). A series for a short-term nominal interest rate, available for the period 1821-2018, is from Banco Central's Table 7.1.4 ("Tasas activas"). Interestingly, among all of the countries we consider in this paper, Argentina is the *only* one that directly provides an estimate of (the inverse of) the velocity of circulation of monetary aggregates. Specifically, Banco Central's Table 7.1.4 provides the ratios between either M1 and M3 and nominal GDP ("M1 % PBI" and "M3 % PBI", respectively; "PBI" is the Spanish acronym for GDP). Based on the ratio between M1 and GDP, and the series for M1, we then reconstructed a nominal GDP series.

A.1.2 Australia

An annual M1 series for the period 1900-2017 has been constructed in the following way. A series for the period 1900-1973 has been kindly provided by Cathie Close of the *Reserve Bank of Australia* (henceforth, *RBA*). A monthly seasonally unadjusted series, available since 1975, is from the RBA's website ("M1, $ billion, RBA, 42216"; the series' acronym is DMAM1N), and we converted it to the annual frequency by taking annual averages (since for the year 1975 the series is available from February, the average for that year has been computed for the period February-December). The missing observation for 1974 has been interpolated as in Bernanke, Gertler, and Watson (1997), using as interpolator series the IMF's IFS series labeled as "Money", which, over the periods of overlapping, closely comoves with both M1 series. A series for a 'short rate', available for the period 1941-1989, is from Table 79 of Homer and Sylla (2005). A 90-day nominal interest rate for bank accepted bills and negotiable certificates of deposit is from the RBA's website ("90-day BABs/NCDs, Bank Accepted Bills/Negotiable Certificates of Deposit-90 days, Monthly, Original, Per cent, AFMA, 42156, FIRMMBAB90"). It is available since 1969. A series for nominal GDP, available since 1960, is from the Australian Bureau of Statistics ("Gross domestic product: Current prices; A2304617J; $ Millions"). An alternative series for nominal GDP, available for the period 1870-2012, is from the website of the *Global Price and Income History Group* at the University of California at Davis, at: http://gpih.ucdavis.edu/.

A.1.3 Austria

A monthly seasonally unadjusted M1 series, available since January 1970, is from the *European Central Bank*. The series has been converted to the annual frequency by taking simple annual averages. An annual series for the discount rate of the *Oesterreichische Nationalbank* (Austria's central bank), available for the period 1957-1998, is from the *IMF*'s *IFS*. An annual series for nominal GDP is from *Statistics Austria* since 1995, and from the *IMF*'s *IFS* before then. Over the period of overlapping the two nominal GDP series are near-identical, which justifies their linking.

A.1.4 Bahrain

An annual series for M1, available since 1965, is from the website of the Financial Stability Directorate of the Central Bank of Bahrain. An annual series for 'Interest Rates on BD Deposits & Loans', available since 1976, is from the central bank's Statistical Bulletin, available at: https://www.cbb.gov.bh/page-p-statistical_bulletin.htm. An annual series for nominal GDP is from the website GCC-Stat, a statistical database for Persian Gulf countries (at: http://dp.gccstat.org/en/DataAnalysis?215Jv283P0CFmaBBdivhQ) since 2008. Before that, it is from the World Bank.

A.1.5 Barbados

An annual series for nominal GDP in million Barbados dollars, available since 1975, is from Tables I7A and I7B of the *Barbados Statistical Service*. An annual series for M1 in million Barbados dollars, available since 1973, is from Table C1 from the *Central Bank of Barbados*. An annual series for the 3-month time deposits rate starting in 1961 has been computed as the average of the two series '3 month Time Deposits - Lower FIDR_TD3L' and '3 month Time Deposits - Upper FIDR_TD3U', from the *Central Bank of Barbados*.

A.1.6 Belgium

An annual M1 series ("Stock monétaire (milliards de francs)"), available for the period 1920-1990, is from the Séries rétrospectives, Statistiques 1920-1990 from *Banque Nationale de Belgique*'s (Belgium's central bank, henceforth *BNB*), Statistiques Economiques Belges, 1980-1990. For the period 1991-1998, M1 data are from the BNB's *Bulletin Statistique*. An annual series for nominal GDP ("Value Added at Market Prices in Current Prices, billion of francs"), available for the years 1920-1939 and 1946-1990 is from Smits, Woltjer, and Ma (2009). An annual series for the BNB's discount rate available for the period 1920-1990 is from the Séries rétrospectives, Statistiques 1920-1990 from the BNB's Statistiques Economiques Belges 1980-1990. For the period 1991-1998, the discount rate is from several issues of the BNB's *Annual Report*.

A.1.7 Belize

Annual series for M1 and for Belize's Treasury bill rate, both available since 1977, are from the *Central Bank of Belize*. An annual series for nominal GDP, available since

1970, is from the *Penn World Tables* Mark 7.0 until 2001, and from the *Central Bank of Belize* after that. Over the period of overlapping the two nominal GDP series are near-identical, which justifies their linking.

A.1.8 Bolivia

Series for nominal GDP, M1, and a short-term nominal interest rate, all available for the period 1980-2013, are from the Unidad de Analisis de Politicas Sociales y Economicas (Bolivia's national statistical agency, known as UDAPE for short).

A.1.9 Brazil

Series for nominal GDP, M1, and GDP deflator inflation, all available for the period 1901-2000, are from IBGE's (the Brazilian Institute of Geography and Statistics) *Estatisticas do Seculo XX* (Statistics of the XX Century). The URL is http://seculoxx.ibge.gov.br/economicas. A series for nominal GDP for the period 2000-2017 is also from IBGE. A series for M1 for the period 2000-2017 is from the *Banco Central do Brasil* (Brazil's central bank, henceforth *Banco Central*). A series for a short-term nominal interest rate for the period 1974-2012 is from the *Banco Central*. Two series for a nominal government bond yield (period: 1901-1913 and 1929-1959) and the *Banco Central*'s discount rate (period: 1948-1989) are both from Homer and Sylla (2005)'s Table 81, pages 629-631.

A.1.10 Canada

An annual series for nominal GDP, available since 1870, has been constructed by linking the Urquhart series (available from *Statistics Canada* (henceforth, *SC*), which is Canada's national statistical agency), for the period 1870-1924; series 0380-0515, v96392559 (1.1) from *SC*, for the period 1925-1980; and series 0384-0038, v62787311 (1.2.38) from *SC*, for the period 1981-2013. A short-term interest rate for the period 1871-1907 (specifically, the "Montreal call loan rate") is from Furlong (2001). A series for the official discount rate, available since 1926, has been constructed as follows. Since 1934, when the *Bank of Canada* (Canada's central bank) was created, it is simply the official bank rate ("Taux Officiel d'Escompte") from the *Bank of Canada*'s website. Before that, we use the Advance Rate, which had been set by the Treasury Department for the discounting of bills, from Table 6.1 of Shearer and Clark (1984).[1] As for the latter period, we use a series for the 3-month Treasury bill rate, which has been constructed by linking the series from the Historical Statistics

[1] To be precise, Shearer and Clark (1984) do not provide the actual time series for the Advance Rate, but rather the dates at which the rate had been changed (starting from August 22, 1914), together with the new value of the rate prevailing starting from that date. Based on this information, we constructed a daily series for the rate starting on January 1, 1915, *via* a straightforward MATLAB program, and we then converted the series to the annual frequency by taking annual averages.

of Canada, available for the period 1934-1935, to the series "Treasury Bill Auction - Average Yields - 3 Month, Per cent / en pourcentage" from the *Bank of Canada*. A monthly series for M1 starting in January 1872 is from Metcalf, Redish, and Shearer (1996). This series is available until December 1952. After that, we link it *via* splicing to the series labelled as "Currency and demand deposits, M1 (x 1,000,000), v37213" from *SC* until November 1981. Finally, from December 1981 until December 2006, we use the series from *SC* labelled as "M1 (net) (currency outside banks, chartered bank demand deposits, adjustments to M1 (continuity adjustments and inter-bank demand deposits) (x 1,000,000), v37200". An important point to stress is that over the periods of overlapping, the three series are nearly-identical (up to a scale factor), which justifies their linking. On the other hand, for the period after December 2006, we were not able to find an M1 series that could be reliably linked to the one we use for the period December 1981-December 2006 (over the last several decades, Canada's monetary aggregates have undergone a number of redefinitions, which complicates the task of constructing consistent long-run series for either of them). As a result, for the most recent period we have decided to use another series that we consider in isolation (that is, without linking it to any other M1 aggregate). The series is "M1B (gross) (currency outside banks, chartered bank chequable deposits, less inter-bank chequable deposits) (x 1,000,000), v41552787", which is available since January 1967 from *SC*. Finally, we convert all monthly series to the annual frequency by taking simple annual averages.

A.1.11 Chile

Annual series for nominal GDP, the GDP deflator, and M1 are from Braun-Llona *et al.* (1998) for the period 1940-1995. As for the period 1996-2012, they are from the *Banco Central de Chile,* Chile's central bank (specifically, nominal GDP and the GDP deflator are from the *Banco Central*'s Anuarios de Cuentas Nacionales, whereas M1 is from *Banco Central*'s Base Monetaria y Agregados Monetarios Privados). A short-term nominal interest rate ("1-day interbank interest rate, financial system average (annual percentage)") from *Banco Central* is available for the period 1940-1995. In order to extend our analysis to the present as much as possible, we therefore also consider, as an alternative measure of the opportunity cost of money, GDP deflator inflation.

A.1.12 Colombia

Data for Colombia have been kindly provided by David Perez Reyna. Annual series for nominal GDP and a short-term nominal interest rate for the period 1905-2003 are from Junguito and Rincón (2007). As for the period 2004-2012, they are from Colombia's *Ministerio de Hacienda y Credito Publico*. An annual series for M1 for the period 1905-2012 is from the *Banco de la Republica*, Colombia's central bank.

A.1.13 Ecuador

All of the data for Ecuador are from the website of *Banco Central del Ecuador* (henceforth, *BCE*), Ecuador's central bank. Most of them are from '85 Años, 1927-2012: Series Estadísticas Históricas', a special publication celebrating *BCE*'s 85th anniversary. Specifically, a series for annual CPI inflation ('Variación Anual del Indice Ponderado de Precios al Cunsumidor por Ciudades y por Categorias de Divisiones de Concumo, Nacional'), available for the period 1940-2011, is from Chapter 4 of '85 Años'. An annual series for a nominal interest rate has been constructed by linking the series 'Tasas, Máxima Convencional, En porcentajes', available for the period 1948-1999; 'Tasas de Interés Referenciales Nominales en Dólares, Máxima Convencional', available for the period 2000-2007; and 'Tasas de Interés Referenciales Efectivas en Dólares, Máxima Convencional', available for the period 2007-2011. All of them are from from Chapter 1 of '85 Años'. An annual series for nominal M1 in U.S. dollars has been constructed by linking the M1 aggregate ('Oferta Monetaria M1, En millones de dólares al final del período'), available for the period 2000-2011, which is expressed in U.S. dollars, and the M1 aggregate ('Medio Circulante (M1), Saldos en millones de sucres'), available for the period 1927-1999, which is expressed in Ecuador's national currency, the *sucre* (both series are from Chapter 1 of '85 Años'). The latter M1 aggregate has been converted in U.S. dollars based on the series for the *sucre*/dollar nominal exchange rate found in Chapter 2 of '85 Años', which is available for the period 1947-1999. Specifically, the exchange rate series (*sucre* per dollar) has been computed as the average between the 'Compra' (i.e., buy) and the 'Venta' (i.e., sell) series. An annual series for nominal GDP in U.S. dollars ('Producto interno bruto (PIB), Miles de dólares'), available for the period 1965-2011, is from Chapter 4 of '85 Años'. An important point to stress is that since we are working with M1 *velocity*—defined as the *ratio* between nominal GDP and nominal M1—the specific unit in which the two series are expressed (U.S. dollars, or Ecuadorian *sucres*) is irrelevant.

A.1.14 Finland

Long-run monthly data for M1 for the period January 1866-December 1985 have been generously provided by Tarmo Haavisto. The data come from his Ph.D. dissertation (see Haavisto (1992)) and have been converted to the annual frequency by taking simple annual averages. A series for Finland's monetary policy rate (labeled as the "Base rate"), available since January 1867, is from *Suomen Pankki Finlands Bank* (Finland's central bank, henceforth, *Suomen Pankki*).[2] Finally, an annual series for

[2] To be precise, *Suomen Pankki* does not provide the actual time series for the base rate, but rather the dates at which the rate had been changed (starting from January 1, 1867), together with the new value of the base rate prevailing starting from that date. Based on this information, we constructed a daily series for the base rate starting on January 1, 1867, *via* a straightforward MATLAB program, and we then converted the series to the annual frequency by taking annual averages.

nominal GDP, available since 1860, is from Finland's Historical Statistics, which are available from the web page of *Statistics Finland* (Finland's national statistical agency). (To be precise, from the homepage of Statistics Finland, look at Home > Statistics > National Accounts > Annual national accounts > Tables.) Specifically, the nominal GDP series is B1GMHT ("Gross domestic product at current prices, 1860-1960, million. mk").

A.1.15 France

Annual series for nominal GDP, nominal M1, and the short rate are all from SaintMarc (1983). Specifically, the series for nominal GDP is the Toutain Index from Annexe I: Revenu national, Produit Interieur Brut, pages 99-100 of Saint Marc (1983), and it is available for the period 1815-1913. The series for M1 is from the table "Vitesse-Revenu, Vy, et taux de liquidite, TL", pages 74-75 of Saint Marc (1983), and it is available for the period 1807-1913. The series for the short rate is from Section 7, 'Evaluation des taux de l'interet', pages 93-96, of Saint Marc (1983), and it is available for the period 1807-1913. In our analysis, however, we focus on the period 1851-1913 because for the entire period 1820-1851, the short rate had been fixed at 4%.

A.1.16 Guatemala

All of the data are from the *Banco de Guatemala*'s website. A series for nominal GDP is available for the period 1950-2017. A series for M1 ("M1 Medio Circulante-Millones de quetzales") is available for the period 1980-2018. A series for a nominal short rate ("Interest rate, Domestic currency, borrowing (passive)") is available for the period 1980-2018.

A.1.17 Hong Kong

An annual series for nominal GDP for the period 1961-2017 is from the *Hong Kong Monetary Authority*'s (henceforth, *HKMA*) website (it is labeled as "Nominal GDP, HK$ million"). The series is from Table031 ("GDP and its main expenditure components at current market prices"). An annual series for M1 for the period 1985-2017 is from the *HKMA*'s website (the series is labeled as "M1, Total, HK$ million"). An annual series for a short-term interest rate for the period 1982-2018 is from the *HKMA*'s website. The series is labeled as "Overnight rate, Table 6.3: Hong Kong Interbank Offered Rates").

A.1.18 Japan

Sources for Japanese data are as follows. A monthly series for the *Bank of Japan*'s (henceforth, *BoJ*) discount rate, available since January 1883, is from the *BoJ*'s long-run historical statistics, which are available at its website (the series is labeled as “BJ’MADR1M: The Basic Discount Rate and Basic Loan Rate”). Annual series for nominal GNP and M1 for the period 1885-1940 are from Table 48 of Tamaki (1995). As for the period since 1955, data for nominal GDP and M1 are as follows. Series for nominal GDP are from the *Economic and Social Research Institute* (henceforth, *ESRI*), Cabinet Office, Government of Japan. (The key URLs are http://www.stat.go.jp/english/data/chouki/03.htm and

http://www.stat.go.jp/english/data/nenkan/1431-03.htm.) An important point to stress here is the following. For the period before 1970, *ESRI* only provides tables for gross domestic *expenditure*, rather than gross domestic *product*. However, over the period of overlapping (that is, 1970-1998), the relevant series coming from Table 3-1 (“Gross Domestic Expenditure (At Current Prices, At Constant Prices, Deflators) - 68SNA, Benchmark year = 1990 (C.Y.1955–1998, F.Y.1955–1998), Value in billions of yen”) and Table 3-3*b* (“3-3-b Gross Domestic Product Classified by Economic Activities (Medium Industry Group), (At Current Prices, At Constant Prices, Deflators) - 68SNA, Benchmark year = 1990 (1970–1998), Value in billions of yen”) are either numerically identical (in the case of nominal series) or numerically identical up to a scale factor (in the case of real series and their deflators). This means that—as should be expected based on just simple economic logic—the series that in Table 3-1 is labeled as “Gross Domestic Expenditure” (Column Y in the Excel spreadsheet 03-01.xls) is, in fact, nominal gross domestic product. As for M1, a monthly series for the period January 1955-December 2018 was constructed by linking, *via* splicing, the following three series from the *BoJ*’s website: MA’MAMS1EN01 (“(discontinued)_M1/Amounts Outstanding at End of Period/(Reference) Money Stock (Based on excluding Foreign Banks in Japan, etc., through March 1999)”); MA’MAMS3EN01 (“(discontinued)_M1/Amounts Outstanding at End of Period /(Reference) Money Stock (from April 1998 to March 2008)”); and MA’MAM1NEM3M1MO (“M1/Amounts Outstanding at End of Period/Money Stock”). An important point to stress is that, over the periods of overlapping, the series are essentially identical (up to a scale factor), which justifies their linking. Finally, the resulting monthly M1 series was converted to the annual frequency by taking annual averages.

A.1.19 Israel

Series for nominal and real GDP, available since 1950, are from Israel’s *Central Bureau of Statistics* (henceforth, *CBS*; special thanks to Svetlana Amuchvari of the *CBS* for help with the data). Specifically, starting from 1995, the data are from Table 17 of the National Accounts. For the period 1950-1994, they are from the *CBS*’s Statistical Abstract of Israel (see columns D and J of Table 6.1, “National Income and Expenditure: Resources and Uses of Resources”). The GDP deflator has been computed as the ratio between the two series. An annual CPI inflation series (“Change in Level of Price Indices, Percentages, Annual, average”), available since 1971, is from the *CBS* website (specifically, the series is from Table 13.1 of Statistical Abstract of Israel).

For the period 1966-1975, the series for M1 is from Table 4.6, page 120, of Barkai and Liviatan (2007). For the period since April 1981, a monthly M1 series is from the *Bank of Israel*'s website (special thanks to Aviel Shpitalnik of the *Bank of Israel* for help with the data). The series is M1.M ("M1 = Money supply, Monthly (M), NIS, million, Current prices"), and it has been converted to the annual frequency by taking annual averages. A short-term interest rate for the period 1966-1974 is the "Nominal rate of return on MAKAM (3-month bills)" from Table 4.9, page 129, of Barkai and Liviatan (2007). Since 1989 it is the *Bank of Israel*'s "Actual effective rate of interest", from the *Bank of Israel*'s website. For the period 1983-1988, we use the "Discount Rate" from the IMF's IFS. Over the period of overlapping (i.e., since 1989), the *Bank of Israel*'s actual effective rate of interest and the discount rate from the IMF are virtually identical, which justifies their linking.

A.1.20 Italy

Series for nominal GDP at current market prices, real GDP in chained 2005 euros, and the implied GDP deflator, all available for the period 1861-2010, are from the sheet "Tab_03' in the Excel spreadsheet 'Data_Na150-1.1.xls", which is available at the *Banca d'Italia*'s website at http://www.bancaditalia.it/statistiche/tematiche/stat-storiche/index.html. The spreadsheet contains the estimates of the Italian National Accounts' aggregates, which are extensively discussed in Baffigi (2011). A series for M1, available for the period 1861-1991, is from the Data Appendix, pp. 49-52, of Fratianni and Spinelli (1997). Series for M1 and M2, available for the period 1948-1998, are from the table "Componenti della moneta dal 1948 al 1998" of BancadItalia (2013). In our analysis we use the M1 series from Fratianni and Spinelli (1997) for the gold standard period, and the one from *Banca d'Italia* for the post-WWII period (over the period of overlapping, however, the two series are very similar, so that in practice this choice does not entail material implications). Short- and long-term interest rates for the period 1861-1996 are from Muscatelli and Spinelli (2000). A series for the "Tasso Ufficiale di Sconto"—that is, *Banca d'Italia*'s official discount rate—is from the tables "Tassi d'interesse delle principali operazioni della banca centrale" and "Variazione dei tassi ufficiali della Banca d'Italia, 1936-2003" of BancadItalia (2013).

A.1.21 Mexico

A monthly interest rates series, available since January 1978, is from the *Banco de Mexico*'s "Indicadores de tasas de interes de Valores Publicos" (*Banco de Mexico*, henceforth *BdM*, is Mexico's central bank). It has been converted to the annual frequency by taking annual averages. Two annual interest rates series ("Interest Rate (%) Commercial loans" and "Interest Rate (%), Official discount rate", respectively) are from Table 83, pages 639-640, of Homer and Sylla (2005). The first series is available for the periods 1942-1963 and 1978-1989. The second is available for the period 1936-1978. An annual series M1 for the period 1925-2000 is from the *Instituto Nacional* *de Estadistica y Geografia* (Mexico's national statistical agency, henceforth *INEGI*), "Estadisticas Historicas de Mexico, 2014", whereas for the period 1985-2014 they are from the *BdM*'s website. The series from the *BdM* are available at the monthly frequency, and we converted them to the annual frequency by taking annual averages. Annual series for nominal GDP are from *INEGI*, "Estadisticas Historicas de Mexico 2014", for the period 1925-1970; from the IMF's IFS for the period 1970-1988; from *BdM* for the period 1988-2004; and from *INEGI* for the period since 2004. The four series have been linked *via* splicing. An annual CPI inflation series available since 1949 is from the IMF's IFS ("Mexico, Consumer Prices, All items, Percent Change over Corresponding Period of Previous Year").

A.1.22 Morocco

A monthly seasonally unadjusted series for M1, available since January 1985, is from the website of *Bank Al-Maghrib* (the central bank of Morocco, henceforth, *BAM*). The annual series has been computed by taking simple annual averages of the original monthly data. An annual series for nominal GDP, available since 1980, is from the "Comptes Nationaux" (National Accounts) from the website of the High Commission for Planning of Morocco. A series for the minimum rate applied to notebook accounts, available since January 1983, is from the website of *BAM*. *BAM* sets this interest rate two times a year, on January 1 and on July 1. The table at the central bank's website reports the values for the interest rate which have been set every January 1, and every July 1, starting from 1983. From this information we computed the annual average rates by taking a simple average within the year.

A.1.23 Netherlands

A series for the discount rate of *De Nederlandsche Bank* (the Dutch central bank, henceforth, *DNB*) for the period 1900-1992 is from Table 65 of Homer and Sylla (2005) until 1989, and from *DNB*'s website after that. Series for nominal and real net national income (NNI) and for the NNI deflator for the period 1900-1992 are from Table 1, pages 94-95, of Boeschoten (1992). A series for M1, available since 1864, has been constructed by linking the series from deJong (1967) and one from *DNB*.

A.1.24 New Zealand

A series for M1, available since 1934, is from the website of the *Reserve Bank of New Zealand* (henceforth, *RBNZ*). A series for nominal GDP in million of Australian dollars is from *Statistics New Zealand* (New Zealand's statistical agency). A series for a short-term nominal interest rate starting in 1934 has been constructed in the following way. Homer and Sylla's (2005) Table 79 contains a series for the *RBNZ*'s official discount rate for the period 1934-1989. Since 1999, the *RBNZ* has been using, as its monetary policy rate, the "Official Cash Rate", which is available from the

RBNZ's website. Since these two short-term rates have been used by the *RBNZ* as its official monetary policy rate for the periods 1934-1989 and 1999 to the present, respectively, they are in fact conceptually the same, and can therefore be linked. For the period in between (1990-1998), for which no official monetary policy rate is available, we have used the "Overnight Interbank Cash Rate" from the *RBNZ*. The rationale for doing so is that since 1999, this rate has been very close to the Official Cash Rate, which justifies the linking of the two series.

A.1.25 Norway

A series for M1, available since 1919, is from the Historical Statistics of *Norges Bank* (Norway's central bank), which are available at its website. Specifically, all historical statistics for Norway's monetary aggregates are from Klovland (2004). Series for nominal GDP and the GDP deflator; and for real GDP, real private consumption expenditures, and real gross investments (in millions of 2005 NOKs, i.e., kronas), all available since 1830, are from *Norges Bank*'s Historical Statistics (for all series, the period 1940-1945 is missing). As for the short-term nominal interest rate, ideally we would have liked to use *Norges Bank*'s discount rate. The problem is that, although the discount rate is available (from *Norges Bank*'s website) since 1819, it has missing observations for the period 1987-1990. As a result, we have resorted to using the Average Deposit Rate (again, from *Norges Bank*'s website), which is available since 1822, has no missing observations, and over the period that is analyzed herein has been quite close to the discount rate.

A.1.26 Paraguay

Annual series for CPI inflation ('Índice de Precios al Consumidor, Área Metropolitana de Asunción, Indice General'), available for the period 1951-2015, and for nominal M1 in thousands of *guaranies,* available since 1962, are both from the website of *Banco Central del Paraguay* (Paraguay's central bank, henceforth *BCP*). An annual series for nominal GDP in thousands of *guaranies*, available since 1960, is from the *International Monetary Fund*'s *International Financial Statistics*.

A.1.27 Peru

All of the data for Peru are from the website of the *Banco Central de Reserva del Perú*, Peru's central bank. An annual series for nominal GDP in million of *nuevos soles* is available since 1950. An annual series for inflation is available since 1901. An annual series for nominal M1 in million of *nuevos soles*, available since 1959, has been constructed as the sum of currency in circulation ('Billetes y Monedas en Circulación') and deposits ('Depósitos a la Vista del Sistema Bancario en Moneda Nacional').

A.1.28 Portugal

An annual series for M1 for the period 1854-1998 is from Table 5 of Mata and Valerio (2011). Annual series for real and nominal GDP for the period 1868-2008 are from Table 4 of Mata and Valerio (2011). A series for the official discount rate of the *Banco de Portugal* (the Portuguese central bank), available for the period 1930-1989, is from Table 74 of Homer and Sylla (2005).

A.1.29 South Africa

All of the data for South Africa are from the website of its central bank, the *South African Reserve Bank* (*SARB*). Specifically, a series for the "Bank rate" ("Lowest rediscount rate at *SARB*"; code is KBP1401M) is available since 1923. A series for M1 ("Monetary aggregates / Money supply: M1, R millions"; code is KBP1371J) is available since 1967. A series for nominal GDP ("Gross domestic product at market prices, R millions"; code is KBP6006J) is available since 1946.

A.1.30 South Korea

A series for M1 ("M1, Narrow Money, Average, Billion Won") is available since 1970 from the website of the *Central Bank of Korea* (henceforth, *BOK*), at: http://ecos.bok.or.kr. The series is from Table 1.1. ('Money & Banking (Monetary Aggregates, Deposits, Loans & Discounts etc.'). A series for nominal GDP ("Gross domestic product, current prices, Billion Won") is available since 1953, again from the BOK's website. A series for the central bank's discount rate ("Republic of Korea, Interest Rates, Discount Rate, Percent per Annum") is available since 1948 from the IMF's IFS.

A.1.31 Spain

An annual series for M1 for the period 1865-1998 is from Cuadro 9.16 "Agregados Monetarios, 1865-1998" of Barciela-López, Carreras, and Tafunell (2005), pp. 697-699 (the series is labeled as "M1, datos a fin de ano, en millones de pesetas"; the years 1936-1940 are missing). An annual series for nominal GDP for the period 1850-2000 is from Cuadro 17.7 of Barciela-López, Carreras, and Tafunell (2005), pp. 1338-1340 (the series is labeled as "El PIB a precios corrientes, 1850-2000, millones de pesetas"; PIB is the Spanish acronym of GDP). An annual series for the "Descuento comercial" of the *Banco de Espana* (Spain's central bank, henceforth, *BdE*) is from Cuadro 9.17 of Barciela-López, Carreras, and Tafunell (2005), pp. 699-701. The series is available for the periods 1874-1914, 1920-1935, and 1942-1985. An annual series for the official discount rate of the *BdE*, available for the period 1930-1989, is from Table 74, pp. 541-542, of Homer and Sylla (2005). A monthly series for the three-month Treasury bill rate available since March 1988 ("Tipo de interese hasta 3 meses. Conjunto del mercado. Op. simples al contado. Letras del Tesoro."), is from the *BdE*'s website, and it has been converted to the annual frequency by taking annual averages (the data for 1988 have been ignored, since the series starts in March of that year).

A.1.32 Switzerland

Annual series for M1 (based on the 1995 definition) and the official discount rate of the *Swiss National Bank* (Switzerland's central bank, henceforth *SNB*), all available at least since 1929, are from the *SNB*'s website. An annual series for nominal GDP available for the period 1948-2005 is from the website of the project *Economic History of Switzerland during the 20th century*—see at http://www.fsw.uzh.ch/histstat/main.php. (Q.16b Gross domestic product (expenditure approach) in real 1990 prices and nominal, 1948-2005 in Million Swiss Francs).

A.1.33 Taiwan

All of the data are from the *Central Bank of the Republic of China (Taiwan)*, that is, Taiwan's central bank (henceforth *CBRCT*). An annual series for nominal GDP ("GDP by expenditures at current prices") is available since 1951. An annual series for the *CBRCT*'s discount rate is available since 1962. Two annual series for M1 ("M1A (End of Period), M1A = Currency in circulation(currency held by the public)+Checking accounts and passbook deposits of enterprises, individuals and non-profit organizations held in banks and community financial institutions" and "M1B (End of Period), M1B = M1A + Passbook savings deposits of Individuals and non-profit organizations in banks and community financial institutions") are both available since 1962. In order to be sure that the series we use in this paper does not include components that go beyond a transaction purpose, we used the first one, M1A.

A.1.34 Thailand

An annual series for GDP at current prices in billions of *baht*, available for the period 1946-2005, is from Mitchell (2007). Since 1990 this series has been linked to the nominal GDP series from the Macro Economic Indicators of the *Bank of Thailand* (Thailand's central bank, henceforth *BoT*). Over the period of overlapping the two series are very close, which justifies their linking. An annual M1 series in billions of *baht*, available since 1970, has been constructed by taking, for each year, the December observation from the series 'Money supply (M1)' from Table 5 of the *BoT*'s monetary aggregates for the period up to 2005. Since then, we have taken the December observation from the monthly M1 series from the *BoT*'s Macro Economic Indicators. The reason for taking, for each year, the December observation, rather than computing the annual average, is that for the period 1970-1980 the December figure is the only one available for each year. An annual series for the 1-year maximum interest rate on fixed deposits, available since 1979, is from the *BoT*'s Macro Economic Indicators.

A.1.35 Turkey

A monthly series for M1, available since January 1964, is from the website of Turkey's central bank, *Turkiye Cumhuriyet Merkez Bankasi* (henceforth, *TCMB*). The series we use has been constructed by taking simple annual averages of the original monthly data. A series for the central bank's discount rate is from Homer and Sylla's (2005) Table 74, pages 541-542, until 1990. After that, it is from *TCMB*. Specifically, *TCMB*'s website reports the dates in which the discount rate was changed, together with the new values taken by the discount rate at each date. Based on this information, for each year since 1990 we have calculated the number of days in the year for which each value of the discount rate has been in effect, and based on this we have computed, for every year, a simple weighted average of the individual daily values of the discount rate. A series for the gross domestic product in current prices, available since 1967, is from the website of Turkey's statistical office, *TurkStat*.

A.1.36 United Kingdom

All U.K. data are from version 3.1 of the dataset 'A millennium of macroeconomic data', which is available from the *Bank of England*'s website at:

https://www.bankofengland.co.uk/statistics/research-datasets. The first version of the dataset (which was called 'Three centuries of macroeconomic data') was discussed in detail in Hills and Dimsdale (2010). Specifically, series for M1, available since 1922; the *Bank of England*'s monetary policy rate (known as the "Bank Rate"), available since 1694; and nominal GDP ("Nominal UK GDP at market prices"), available since 1700, are, respectively, from columns A.24, A.31, and A.9 of the sheet 'A1. Headline series'.

A.1.37 United States

The series for the 3-month Treasury bill rate; nominal GDP; both the 'standard' M1 aggregate and the 'New M1' one; and Money Market Deposits Accounts (MMDAs), are all from LucasJr. and Nicolini (2015). All series have been updated to 2017 based on either series' updated original data sources. The original source for the 3-month Treasury bill rate is the *Economic Report of the President* (henceforth, *ERP*), whereas the ones for nominal GDP are Kuznets and Kendrick's Table Ca184-191 before 1929, and Table 1.1.5 of the *National Income and Product Accounts* (henceforth, *NIPA*) after that. A series for Money Market Mutual Funds (MMMFs) starting in 1974 is from the Federal Reserve (the FRED II acronym is MMMFFAA027N, 'Money market mutual funds, Total financial assets, Billions of dollars'). An annual series for nominal GDP at current prices is from Officer and Williamson (2015).

Adjusting for the share of currency held by foreigners As documented, e.g., by Judson (2017), over the last several decades the fraction of U.S. currency held by foreigners has significantly increased, and it stood, at the end of 2016, at around 50-60 per cent of total currency, depending on the methodology which was used in order to estimate it. Since the demand for M1 which is being investigated in the present work is a demand on the part of U.S. nationals, this raises the issue of how to adjust U.S. currency in order to purge it of the fraction held by foreigners. This could be done in several ways, none of them ideal. One possibility would be, following Judson (2017), to estimate a model for the demand of *Canadian* currency as a function of Canadian nominal GDP and interest rates, and then to apply the estimated coefficients to U.S. nominal GDP and interest rates in order to back out a predicted level of U.S. currency demanded by U.S. nationals. As extensively discussed by Judson (2017), the rationale for doing this is that—most likely as a consequence of the similarity between the U.S. and Canadian economies—up until about 1990 the ratios between currency and nominal GDP in the two countries had tended to closely co-move. Only since then the demand for U.S. currency on the part of non-U.S. nationals has skyrocketed, thus causing the traditional relationship between the demands for U.S. and Canadian currency, as fractions of their respective GDPs, to go out of kilter. For our own purposes, this approach suffers from the limitation that, by definition, it produces a 'fundamental', predicted value for the demand for U.S. currency on the part of U.S. nationals which does not reflect idiosyncratic, transitory factors which are not captured by either nominal GDP or the short rate. Because of this, we have adopted an alternative approach in which we estimate the fraction of U.S. currency held by foreigners as the simple difference between the ratios between currency and nominal GDP in the U.S. and Canada. One problem with this approach is that since, during the early years of the Great Depression, Canada did not experience banking collapses of a magnitude comparable to the U.S., the 'flight to currency' there was much more muted. As a result, our approach mechanically interprets the increase in the demand for U.S. currency as a fraction of GDP between the crash of 1929 and the inauguration of F.D. Roosevelt's Presidency as an increase in demand on the part of foreigners. Our counterargument to this is that the spike in the demand for currency, although sizeable, was very short-lived, as it only pertained to four years, from 1930 to 1933. As a result, since this only pertains to currency—which, in 1929, was just 14.6 per cent of overall M1—it is reasonable to assume that the impact of this on our estimates should be negligible.

Constructing an own rate of return for Lucas and Nicolini's (2015) 'New M1' aggregate We construct an own rate of return for Lucas and Nicolini's (2015) 'New M1' aggregate as follows. New M1 is equal to the standard M1 aggregate until 1982Q3, and it is equal to the standard aggregate plus MMDAs starting from 1982Q4. The standard M1 aggregate, in turn, is defined as the sum of currency, which pays no interest, and checking accounts, which pay instead some small interest. We compute the own rate of return for New M1 as the weighted average of the rate on checking accounts and, since 1984Q1, MMDAs, where the weights are computed as the fractions of checking accounts and MMDAs in the overall New M1 aggregate. The rate on checking accounts is available since 1987, whereas the rate on MMDAs is available for the period 1987-2000. Since both rates are available, for these two periods, at both the annual and the quarterly frequency, for the missing periods we proceed as follows.

Working at the quarterly frequency for the period 1987Q1-2018Q3 (for the rate on checking accounts) and for the period 1987Q1-2000Q4 (for the rate on MMDAs), we estimate *via* OLS simple linear regression models linking the dynamics of the first-difference of either of the two rates to the *present* and *past* dynamics of a number of series which are available for the entire post-WWII period. The regressors we use are the vacancy rate (from Regis Barnichon's web page); the unemployment rate (the St. Louis FED's FRED II acronym is UNRATE); the rate of capacity utilization in manufacturing (CUMFNS); the first two principal components extracted from the panel of the first-differences of the 3- and 6-month Treasury bill rates, and of the 1-, 3-, 5-, and 10-year Treasury constant maturity rates (TB3MS, DTB6, GS1, GS3, GS5, and GS10, respectively);[3] and the first three principal components extracted from the panel of the successive spreads[4] among the same six interest rate series.[5] The regressions (with two lags for the MMDAS' rate, and four for the rate on checking accounts, for which the available period is longer) produce R-squared equal to 0.919 and 0.612, respectively. Then, based on the estimated model for the first-difference of either of the two rates of interest, we compute predicted values for the missing quarters, and based on them we reconstruct predicted values for their levels. Based on the predicted quarterly rates for checking accounts and MMDAs, we then compute the corresponding predicted annual rates by taking annual averages.

A.1.38 Venezuela

Annual data for nominal GDP ("Producto Interno Bruto, Millones de Bolívares a Precios Corrientes"), M1 ("Circulante, (M1), I.1, Circulante, Liquidez Monetaria y Liquidez Ampliada, Saldos al final de cada período en millones de bolívares"), and a short-term rate ("Tasas de Interes Activas Anuales Nominales Promedio, Ponderadas de los Bancos Comerciales y Universales, Porcentajes") are from the *Banco Central de Venezuela* (Venezuela's central bank). GDP is available since 1957, whereas M1 is available since 1940. The interest rate is available for the period 1962-1999. An alternative monthly interest series, available since July 1997 ("Tasa de Interés Aplicable al Cálculo de los Intereses Sobre Prestaciones Sociales (Porcentajes)") cannot be linked to the other interest rate series because, over the period of overlapping, the two series are different. As a consequence, we limited our analysis to the period 1962-1999.

[3] The first two principal components explain almost 99 per cent of the variance of the panel.

[4] That is, 6-month minus 3-month, 1-year minus 6-month, ..., 10-year minus 5-year.

[5] The first three principal components explain about 97 per cent of the variance of the panel.

A.1.39 West Germany

Although data for post-WWII Germany are available, in principle, for the entire period 1950-1998, in the empirical work we have decided to only use data for West Germany for the period 1960-1989. The reason is that we are skeptical about the possibility of meaningfully linking the various series for nominal GDP in order to create a single series for the period 1950-1998 because (*i*) before 1960, GDP data did not include West Berlin and the Saarland, which, in 1960, jointly accounted for about 6% of overall GDP; and (*ii*) the reunification of 1990 created discontinuities in both GDP and M1 (we thought the problem could be side-stepped by focusing on M1 velocity, i.e. their ratio, but in fact this series also seems to exhibit a discontinuity around the time of reunification). Entering into details, an annual series for the *Bundesbank*'s monetary policy rate for the period 1949-1998 has been constructed by taking annual averages of the monthly series "BBK01.SU0112, Diskontsatz der Deutschen Bundesbank / Stand am Monatsende, % p.a.", which is available from the *Bundesbank*'s website. As for nominal GDP, the original annual series are from Germany's Federal Statistical Office, and they are available for the period 1950-1960 ("Gross domestic product at current prices, Former Territory of the Federal Republic excluding Berlin-West and Saarland"); 1960-1970 ("Gross domestic product at current prices, Former Territory of the Federal Republic"); and 1970-1991 ("Gross domestic product at current prices, Former Territory of the Federal Republic, (results of the revision 2005)"). There is also a fourth series available for reunified Germany, but, as mentioned, it cannot be meaningfully linked to the series for the period 1970-1991 because of the discontinuity induced by the 1990 reunification. The second and third series can be linked because the difference between them is uniquely due to changes in the accounting system, rather than to territorial redefinitions. Linking the first and second series, on the other hand, is problematic because, as mentioned, before 1960 GDP data did not include West Berlin and the Saarland. Our decision has been to ignore the first GDP series, and therefore to start the sample in 1960, for the following two reasons. *First*, the dimension of West Berlin and the Saarland was not negligible. The value taken by nominal GDP in 1960 according to the first and second series was equal to 146.04 and 154.77, respectively, a difference equal to 6%. *Second*, this problem might be ignored if we had good reasons to assume that, during those years, West Berlin and the Saarland's nominal GDP was growing exactly at the same rate as in the rest of Germany. This, however, is pretty much a heroic assumption—especially for West Berlin. As a result, in the end we just decided to ignore the first series. Finally, turning to M1, this turned out to be the single most excruciating piece of data collection in the entire enterprise. German M1 data, which are available at the monthly frequency since 1948, can only be recovered from the Bundesbank's original *Monthly Reports*, which are available in scanned form at the Bundesbank's website. So we downloaded the scanned PDFs of the *Monthly Reports*, and we manually entered the data in Excel, one "piece" (that is, one *Monthly Report*) at a time. An important point to notice is that German monetary aggregates are *not revised*, so that it is indeed possible to link the figures coming from successive issues of the *Monthly Report*. With a few exceptions in 1940 and the early 1950s, each report contains about one year to one year and a half of data. There are a few discontinuities in the series, but other than that, the overlapping portions coming from successive issues are identical (over the entire sample we noticed about four to five exceptions, which means that those months were revised, and in those cases we took the values coming from the most recent *Monthly Report*). The discontinuities were just level shifts: we checked the log-differences of the two series pertaining to each discontinuity, and they were nearly identical. So in the end we linked the various pieces coming from the different issues of the *Monthly Report*, thus obtaining a single monthly series for the period up to December 1998. Finally, we converted the series to the annual frequency by taking annual averages.

A.2 Quarterly data

A.2.1 Australia

Nominal GDP ('Gross domestic product: Current prices, $ Millions, Seasonally Adjusted, A2304418T') is from the Australian Bureau of Statistics. The short rate ('3-month BABs/NCDs, Bank Accepted Bills/Negotiable Certificates of Deposit-3 months; monthly average, Quarterly average, Per cent, ASX, 42767, FIRMMBAB90') is from the Reserve Bank of Australia. M1 ('M1: Seasonally adjusted, $ Millions') is from the Reserve Bank of Australia.

A.2.2 Brazil

Seasonally adjusted data for nominal GDP, available since 1975Q1, are from IBGE (Brazilian Institute of Geography and Statistics). Seasonally adjusted data for M1 are from the *Banco Central do Brasil* (Brazil's central bank). A series for a short-term nominal interest rate is from the *Banco Central do Brasil*.

A.2.3 Canada

Nominal GDP ('Gross domestic product (GDP) at market prices, Seasonally adjusted at annual rates, Current prices') is from Statistics Canada. Series for the Bank rate (i.e., the Bank of Canada's monetary policy rate), the 3-month Treasury bill auction average yield, the benchmark 10-year bond yield for the government of Canada, and the government of Canada's 3-to-5 and 5-to-10 year marketable bonds average yields are from Statistics Canada. M1 ('v41552787, Table 176-0020: Currency outside banks, chartered bank chequable deposits, less inter-bank chequable deposits, monthly average') is from Statistics Canada. Data on currency are from Statistics Canada ('Table 176-0020 Currency outside banks and chartered bank deposits, monthly average, Bank of Canada, monthly').

A.2.4 Germany

A quarterly series for the Bundesbank's monetary policy rate has been constructed by taking quarterly averages of the monthly series "BBK01.SU0112, Diskontsatz der Deutschen Bundesbank / Stand am Monatsende, % p.a.", which is available from the Bundesbank's website. A monthly seasonally unadjusted series for M1 has been constructed based on the data contained in the Bundesbank's *Monthly Reports*, which are available in scanned form at the Bundesbank's website, exactly as we described in Section A.1.12. The series has then been seasonally adjusted based on ARIMA X-12, and it has been converted to the quarterly frequency by taking averages within the quarter. A quarterly seasonally adjusted series for nominal GDP, available since 1970Q1, is from Germany's Federal Statistical Office's website.

A.2.5 Hong Kong

The HIBOR (Hong Kong Inter-Bank Offered Rate) is from the Hong Kong Monetary Authority (HKMA). M1 ('M1, Total, (HK$ million)') is from HKMA, and it has been seasonally adjusted via ARIMA X-12. Nominal GDP ('GDP, HK$ million, From: Table031: GDP and its main expenditure components at current market prices, National Income Section (1)1,') is from Hong Kong's Census and Statistics Department. It has been seasonally adjusted *via* ARIMA X-12.

A.2.6 Israel

A seasonally adjusted series for M1, available since 1981 Q2, is from Israel's central bank. A seasonally adjusted series for nominal GDP is from the IMF's International Financial Statistics. A series for the central bank's discount rate is from the IMF's International Financial Statistics.

A.2.7 Japan

A series for the discount rate is from the Bank of Japan. A seasonally adjusted series for nominal GDP is from the Economic and Social Research Institute, Cabinet Office, Government of Japan. A seasonally adjusted series for M1 has been constructed based on MA'MAM1NAM3M1MO ('M1/Average amount outstanding/money stock') and MA'MAM1YAM3M1MO ('M1/Percent changes from the previous year in average amounts outstanding/Money Stock').

A.2.8 Mexico

Nominal GDP in billions of pesos nuevos is from INEGI. M1 ('Monetary Aggregates, M1, Nominal Stocks, Billions of Pesos, Levels, SF12718') is from the Banco de México, and it has been seasonally adjusted *via* ARIMA X-12. The short rate ('91 day Cetes, Monthly average rate in annual percent, SF3338') is from the Banco de México.

A.2.9 New Zealand

Data on nominal and real GDP ('Gross Domestic Product - expenditure measure, Nominal $m s.a.' and 'Gross Domestic Product - expenditure measure, Real $m', respectively) are from Statistics New Zealand. The short rate and M1 ('Overnight interbank cash rate, %pa, INM.MN.NZK' and 'M1', respectively) are from the Reserve Bank of New Zealand.

A.2.10 Norway

A seasonally adjusted series for M1, available since 1993Q1, is from Norges Bank (Norway's central bank). A series for the Norges Bank's NIBOR (Norway's interbank rate) is from Norges Bank. A series for the nominal GDP for mainland Norway, is available since 1986Q1, is from Statistics Norway. The original seasonally unadjusted series has been seasonally adjusted via ARIMA X-12.

A.2.11 South Africa

M1 ('Monetary aggregates / Money supply: M1, KBP1371M, R millions') and the central bank's monetary policy rate ('Bankrate (lowest rediscount rate at SARB), KBP1401M, Percentage') are from the South Africa Reserve Bank. Nominal GDP ('Gross domestic product at market prices, Current prices. Seasonally adjusted, GDP at market prices (current, sa) ') is from the South Africa Reserve Bank.

A.2.12 South Korea

For South Korea, all of the data are from the central bank: nominal and real GDP ('10.2.1.1 GDP and GNI by Economic Activities (seasonally adjusted, current prices, quarterly), Gross domestic product at market prices(GDP), Bil.Won' and '10.2.2.2 Expenditures on GDP (seasonally adjusted, chained 2010 year prices, quarterly), Expenditure on GDP, Bil.Won' respectively); M1 (''1.1.Money & Banking (Monetary Aggregates, Deposits, Loans & Discounts etc.), Seasonally Ajusted M1(End of), Bil.Won since 1969Q4; Before that: 1.1.Money & Banking (Monetary Aggregates, Deposits, Loans & Discounts etc.), M1(Narrow Money, End Of), Bil.Won, adjusted via ARIMA X-12); and the central bank's discount rate.

A.2.13 Switzerland

Both M1 and the short rate ('Monetary aggregate M1, Level' and 'Switzerland - CHF - Call money rate (Tomorrow next)', respectively) are from the Swiss National Bank's internet data portal. Data on both nominal and real GDP ('Gross domestic product, ESA 2010, Quarterly aggregates of Gross Domestic Product, expenditure approach, seasonally and calendar adjusted data, In Mio. Swiss Francs, at current prices' and 'Gross domestic product, ESA 2010, Quarterly aggregates of Gross Domestic Product, expenditure approach, seasonally and calendar adjusted data, In Mio. Swiss Francs, at prices of the preceding year, chained values ("annual overlap"), reference year 2010', respectively) are from the State Secretariat for Economic Affairs (SECO) at https://www.seco.admin.ch/seco/en/home.

A.2.14 Taiwan

Data on the central bank's discount rate and M1 ('Central Bank Rates (End of Period), Discount Rate' and 'M1A (End of Period), Millions of N.T. dollars', respectively) are from the central bank's website. Data on nominal GDP ('GDP by Expenditures (at Current prices,1951-1980)' and then 'GDP by Expenditures (at Current prices,1981-)', both seasonally adjusted via ARIMA X-12, 1981Q4. After that: GDP by Expenditures, Seasonally Adjusted-Quarterly by Period, Pricing, Expenditure and Type, Unit:Million N.T. At Current Prices) are from Taiwan's Directorate General of Budget, Accounting and Statistics (DGBAS) at http://eng.dgbas.gov.tw.

A.2.15 United Kingdom

Nominal GDP ('YBHA, Gross Domestic Product at market prices: Current price, Seasonally adjusted £m') is from the Office for National Statistics. A break-adjusted stock of M1 is from 'A millennium of macroeconomic data for the UK, The Bank of England's collection of historical macroeconomic and financial statistics, Version 3 - finalised 30 April 2017', which is from the Bank of England's website. Likewise, series for the Bank rate (i.e., the Bank of England's monetary policy rate), a long-term consols yield, a 10-year bond yield, and a Treasury bill rate, are all from the same spreadsheet.

A.2.16 United States

The short-term nominal interest rate is the Federal Funds rate—the acronym is 'FEDFUNDS', from FRED II at the St. Louis FED's website; nominal GDP is 'GDP' (Gross Domestic Product, GDP

Billions of Dollars, Quarterly, Seasonally Adjusted, Annual Rate) from the St. Louis FED's FRED II data portal; the standard M1 aggregate is 'M1SL' (M1 Money Stock, Billions of Dollars, Quarterly, Seasonally Adjusted); the MMDAs data are from the Federal Reserve's mainframe.

B Mathematical Derivations

B.1 Interest rate rules and money rules

Note that (6) and (7) in the text imply

$$\beta E\left[\frac{V'(\omega')}{\pi(s')}\right] = \frac{\varepsilon}{R}$$

and

$$\delta = \frac{\varepsilon}{n}\left[1 - \frac{R^m}{R}\right].$$

Substituting this in equation (4), we obtain

$$U'(x) = \frac{\varepsilon}{R} + \frac{\varepsilon}{n}\left[1 - \frac{R^m}{R}\right]$$

or

$$\begin{aligned} \varepsilon &= \frac{U'(x)}{\left[\frac{1}{n} + \frac{1}{R}(1 - \frac{1}{n}R^m)\right]} \\ &= \frac{nU'(x)}{\left[1 + \frac{1}{R}(n - R^m)\right]}. \end{aligned}$$

Now, combining (7) and (9), we obtain

$$\beta E\left[\frac{\varepsilon'(s')}{\pi(s')}\right] R = \varepsilon$$

or, using the result above and noting that $x = z(1 - \theta(n))$,

$$\beta E\left[\frac{n(s')U'\left[(z(s')(1 - \theta(n(s')))\right]}{\left[1 + \frac{1}{R(s')}(n(s') - R^m(s'))\right]}\frac{1}{\pi(s')}\right] R = \frac{nU'(z(1 - \theta(n)))}{\left[1 + \frac{1}{R}(n - R^m)\right]}.$$

But replacing the inflation rate $\pi(s') = \frac{M(s')x(s')}{Mx}\frac{n}{n(s')}$, we obtain

$$\beta E\left[\frac{U'\left[(z(s')(1 - \theta(n(s')))\right]}{\left[1 + \frac{1}{R(s')}(n(s') - R^m(s'))\right]}\frac{M}{M(s')}\right] R = \frac{z(1 - \theta(n))U'(z(1 - \theta(n)))}{\left[1 + \frac{1}{R}(n - R^m)\right]}.$$

Now, if we let

$$\Omega = \frac{U'(z(1 - \theta(n)))z(1 - \theta(n))}{\left[1 + \frac{1}{R}(n - R^m)\right]},$$

we can write the expression above as

$$\beta E\left[\Omega(s')\frac{M}{M'}\right]R=\Omega.$$

But

$$M(s')=M+\mu(s')P,$$

so

$$\frac{M}{M(s')}=1-\frac{\mu(s')}{\pi(s')m(s')}=\left(1-\frac{\mu(s')n(s')}{\pi(s')z(s')(1-\theta(n(s')))}\right).$$

Replacing the above,

$$\beta E\left[\Omega(s')\left(1-\frac{\mu(s')n(s')}{\pi(s')z(s')(1-\theta(n(s')))}\right)\right]R=\Omega$$

or

$$\beta E\left(\frac{\Omega(s')}{\Omega}\right)-\beta E\left(\frac{\Omega(s')}{\Omega}\frac{\mu(s')n(s')}{\pi(s')z(s')(1-\theta(n(s')))}\right)=\frac{1}{R}.$$

In general, there are many solutions for the growth rate of money stochastic sequence $\mu(s')$ that are consistent with a given interest rate. This is so because the nominal interest rate pins down (weighted) expected inflation, but there are many distributions of future price levels that are consistent with the same expected value of inflation. Notice, however, that there exists a unique growth rate of money that is consistent with the interest rate sequence, and that is predetermined the period before, the solution, μ^*, satisfying

$$E\left(\frac{\beta\Omega(s')}{\Omega}\right)-\mu^* E\left(\frac{\beta\Omega(s')}{\Omega}\frac{n(s')}{\pi(s')z(s')(1-\theta(n(s')))}\right)=\frac{1}{R}.$$

B.2 The Bellman equation describing the decision problem

The Bellman equation describing the decision problem is

$$\begin{aligned} V(\omega) \quad = \quad & \max_{x,n,m,b,q(s')} U(x)-\varepsilon\left[m+b+E\left[q(s')\pi(s')\widetilde{P}^Q(s')\right]-\omega\right]-\delta\left[x-mn\right] \\ & +\beta E\left[V(\frac{mR^m+bR+\left[1-\theta(n)\right]z-x}{\pi(s')}+\tau(s')+q(s'))\right], \end{aligned}$$

where, for simplicity, we omitted the dependence of current variables on the state, and where s' denotes the future state.

The first order conditions are

$$x: \quad U'(x)=\beta E\left[\frac{V'(\omega')}{\pi(s')}\right]+\delta \tag{1}$$

$$n: \quad \delta m = \beta E\left[\frac{V'(\omega')}{\pi(s')}\right]\theta_n(n)z \tag{2}$$

$$m: \quad \delta n + \beta E\left[\frac{V'(\omega')}{\pi(s')}\right]R^m = \varepsilon \tag{3}$$

$$b: \quad \beta E\left[\frac{V'(\omega')}{\pi(s')}\right]R = \varepsilon \tag{4}$$

$$q(s'): \quad \beta V'(\omega') = \varepsilon\pi(s')P^Q(s'), \tag{5}$$

and the envelope condition is

$$V'(\omega) = \varepsilon.$$

Note that (3) and (4) imply

$$\beta E\left[\frac{V'(\omega')}{\pi(s')}\right](R - R^m) = \delta n,$$

which in turn implies

$$\frac{m}{\theta_n(n)z}(R - R^m) = n.$$

In equilibrium,

$$m = \frac{x}{n} = \frac{z(1-\theta(n))}{n},$$

so if we replace the value of m in the previous equation and let $r^* \equiv (R - R^m)$, we obtain

$$r^* \equiv (R - R^m) = n^2\frac{\theta_n(n)}{1-\theta(n)}.$$

B.3 The model with heterogeneous agents

Consider a model as the one above, with a unit mass of agents that are alike in all respects, except that they differ in their productivity and in their borrowing constraints. Let idiosyncratic productivity for agent j be equal to $\xi^j \in [\xi_l, \xi^h]$, where the mean of ξ^j is equal to one. In each period, the productivity of each agent is $\xi^j z(s_t)$. We also assume agent-specific upper bounds on debt, which we denote as b^{j*}, with $b^{j*} \in [b_l, b^h]$.

The common preferences are given by

$$E_0\sum_{t=0}^{\infty}\beta^t U(x_t^j).$$

Equilibrium in the labor market and the equality of production and consumption imply

$$\begin{aligned} 1 &= \int_0^1 l_t^j dj + \gamma \nu_t \int_0^1 n_t^{j\sigma} dj \\ \int_0^1 x_t dj &= z_t(1 - \gamma \nu_t \int_0^1 n_t^\sigma dj). \end{aligned}$$

These technologies imply that the real wage, per unit of efficiency, is equal to z_t.

The portfolio decision is constrained by an agent-specific equivalent to (6),

$$m_t^j + b_t^j + E_t \left[s_t^{jt+1} \pi_t^{t+1} Q_t^{t+1} \right] \leq w_t^j. \tag{6}$$

Finally, we impose a productivity-adjusted borrowing constraint for the agent of the form

$$b_t \geq z_t b^{*j}. \tag{7}$$

The agent's wealth next period, contingent on the actions taken in the current period and the realization of the exogenous shock, is given by

$$w_t^{jt+1} \leq \frac{m_t^j + b_t^j(1 + r_t) + \left[1 - \gamma n_t^{j\sigma} \nu_t \right] z_t \xi^j - x_t}{\pi_t^{t+1}} + q_t^{t+1} + \tau_t^{t+1}, \tag{8}$$

where $\tau(s^t, s_{t+1})$ is the real value of the monetary transfer the government makes to the representative agent. Finally, the cash-in-advance constraint can be written in real terms as

$$x_t^j \leq m_t^j n_t^j. \tag{9}$$

We now consider the decision problem of a single, atomistic agent that maximizes utility subject to restrictions (6), (8),(7) and (9).

Consider now the solution given the distribution of ξ^j and given a distribution of initial wealth among the population. Using the same arguments as for the representative agent case, it is trivial to show that if the borrowing constraint does not bind for agent j, the solution is given by

$$r_t = n_t^{j2} \frac{\sigma \gamma n_t^{j\sigma - 1} \nu_t}{1 - \gamma n_t^{j\sigma} \nu_t}. \tag{10}$$

Thus, the individual money demand function can be well approximated by a log-log function with elasticity equal to $1/(1 + \sigma)$. Note that this demand function only depends on aggregates, so the aggregate money demand for the group of agents for which the borrowing constraint does not bind is also a log-log function with the same elasticity. It trivially follows that if no agent is constrained in equilibrium, the aggregate money demand is as in the representative agent economy.

In an intermediate case in which some agents are constrained, the solution for them is given by

$$m_t^j = w_t^j + z_t b^* \xi^j,$$

so for them, n_t^j is locally invariant to movements in the interest rate. In this intermediate case, then, aggregate real money demand is a combination of a mass of agents for which the elasticity is zero and the complement mass for which the elasticity is $1/(1+\sigma)$.

The size of the mass of agents for which the constraint binds is weakly decreasing with the interest rate, a property thereby inherited by the aggregate elasticity. Eventually, if the constraint becomes binding for all agents at some interest rate, the aggregate elasticity becomes zero.

B.4 Proof of Lemma 1

Proof. For the first part, consider a pair $r_l < r_h$, and let (m_h/x_h) be the solution to the equation

$$r_t = \frac{\sigma\gamma n_t^{\sigma+1}\nu_t}{1-\gamma n_t^{\sigma}\nu_t} \tag{11}$$

when the interest rate differential is r_h. Assume that constraint binds for r_h. It follows that

$$w - m_h < zb^*,$$

where we omitted the time subscripts for simplicity. Assume, toward a contradiction, that it does not bind for r_l. Then,

$$w - m_l > zb^*,$$

which then implies that

$$m_l < m_h. \tag{12}$$

However, as the ratio of money to output is decreasing on the net interest rate,

$$\frac{m_l}{x_l} > \frac{m_h}{x_h}.$$

But the number of trips to the bank is increasing with the net interest rate, so $n_l < n_h$. This implies that

$$z(1-\gamma\nu n_l^{\sigma}) = x_l > z(1-\gamma\nu n_h^{\sigma}) = x_h.$$

The last two conditions jointly imply that

$$\frac{m_l}{m_h} > \frac{x_l}{x_h} > 1$$

which contradicts (12). A symmetric argument proves the second part. QED. ■

C Integration Properties of the Data

Table C.1 reports bootstrapped p-values[6] for Elliot, Rothenberg, and Stock (1996) unit root tests for either the levels or the logarithms of M1 velocity and the short rate, and for the logarithms of nominal M1 and nominal GDP,[7] and Table C.2 reports the corresponding set of results for either the first differences or the log-differences of the series. For the logarithms of nominal GDP and nominal M1, which exhibit obvious trends, tests are based on models including an intercept and a time trend.[8] For (the logarithms of) the short rate and velocity, on the other hand, they are based on models including an intercept but no time trend. For the short rate, the rationale for not including a trend is obvious: the notion that nominal interest rates may follow an upward path,[9] in which they grow over time, is manifestly absurd.[10] For velocity, on the other hand, things are at first sight less obvious. The reason for not including a trend has to do with the fact that we are focusing here on a demand for money *for transaction purposes* (so this argument holds for M1, but it would not hold for broader aggregates). The resulting natural assumption of unitary income elasticity logically implies that, if the demand for M1 is stable, M1 velocity should inherit the stochastic properties of the opportunity cost of money. In turn, this implies that the type of tests we run for velocity should be *the same* as those for the nominal rate.

The evidence in the two tables can be summarized as follows.

First, there is overwhelming evidence of unit roots in any of the series, with the bootstrapped p-values being near-uniformly greater than the 10% threshold which, throughout the entire paper, we take as our benchmark significance level and in most cases markedly so.[11] The handful of cases in which the null of a unit root is rejected

[6] For any of the series, p-values have been computed by bootstrapping 10,000 times estimated ARIMA(p,1,0) processes. In all cases, the bootstrapped processes are of a length equal to the series under investigation. As for the lag order, p, since, as it is well known, results from unit root tests may be sensitive to the specific lag order which is being used, for reasons of robustness we consider two alternative lag orders, either 1 or 2 years.

[7] The reason for not considering tests based on the levels of nominal M1 and nominal GDP is that either series' level is manifestly characterized by exponential-type growth. This would not be a problem if Elliot *et al.*'s tests allowed for the alternative of stationarity around an *exponential* trend rather than a linear one. Since this is not the case, for both GDP and M1 we are compelled to only consider tests based on the logarithms.

[8] The reason for including a time trend is that, as discussed, for example, by Hamilton (1994, pp. 501), the model used for unit root tests should be a meaningful one also under the alternative.

[9] The possibility of a downward path is ruled out by the zero lower bound.

[10] This does *not* rule out the possibility that, over specific sample periods in which inflation exhibits permanent variation (such as post-WWII samples dominated by the Great Inflation episode), nominal interest rates are I(1), too. Rather, by the Fisher effect, we *should* expect this to be the case. Historically, however, a unit root in inflation has been the exception rather than the rule; see Benati (2008).

[11] In a few cases, results based on the two alternative lag orders we consider produce contrasting evidence. This is the case, for example, for the logarithm of nominal GDP for Austria, the Barbados islands, Hong Kong, Canada (1967-2017), Israel, and South Korea. In these cases, we regard the null

based on either lag order has been highlighted, in Table C.1, in yellow.

Second, for both the first difference and the log-difference of either velocity or the short rate, the null of a unit root can be rejected almost uniformly, with the very few cases in which this is not the case—so that the relevant series should be regarded, according to Elliot *et al.*'s (1996) tests, as I(2)—having been highlighted in yellow in Table C.2. Accordingly, for these cases we will not run cointegration tests. As for nominal M1 and especially nominal GDP, on the other hand, the opposite is true, with the null of a unit root not being rejected most of the time. In all of these cases, we will therefore eschew unrestricted specifications for the logarithms of nominal M1, nominal GDP, and a short rate.

D Details of the Bootstrapping Procedures

As for the Johansen test, we bootstrap trace and maximum eigenvalue statistics via the procedure proposed by Cavaliere *et al.* (2012; henceforth, CRT). In a nutshell, CRT's procedure is based on the notion of computing critical and p-values by bootstrapping the model that is relevant under the null hypothesis. This means that for tests of the null of no cointegration against the alternative of one or more cointegrating vectors, the model that is being bootstrapped is a simple, noncointegrated VAR in differences. For the maximum eigenvalue tests of h versus h+1 cointegrating vectors, on the other hand, the model that ought to be bootstrapped is the VECM estimated under the null of h cointegrating vectors. All of the technical details can be found in CRT, to which the reader is referred. We select the VAR lag order as the maximum[12] between the lag orders chosen by the Schwartz and the Hannan-Quinn criteria[13] for the VAR in levels.

As for the Wright (2000) test, since the test has been designed to be equally valid for data-generating processes (DGPs) featuring either exact or near unit roots, we consider two alternative bootstrapping procedures, corresponding to either of the two possible cases. (In practice, as a comparison between the results reported in Table 2 in the text and in Table E.1 in Appendix E makes clear, the two procedures produce nearly identical results.) The former procedure involves bootstrapping—as detailed in CRT and briefly recounted in the previous paragraph—the cointegrated VECM estimated (based on Johansen's procedure) under the null of one cointegration vector. This bootstrapping procedure is the correct one if the data feature exact unit

of a unit root as not having been convincingly rejected, and in what follows we therefore proceed under the assumption that these series are I(1).

[12] We consider the maximum between the lag orders chosen by the SIC and HQ criteria because the risk associated with selecting a lag order smaller than the true one (model misspecification) is more serious than the one resulting from choosing a lag order greater than the true one (overfitting).

[13] On the other hand, we do not consider the Akaike Information Criterion since, as discussed by Luetkepohl (1991), for example, for systems featuring I(1) series, the AIC is an inconsistent lag selection criterion, in the sense of not choosing the correct lag order asymptotically.

roots. For the alternative possible case in which velocity and the short rate are near unit root processes, we proceed as follows. Based on the just-mentioned cointegrated VECM estimated under the null of one cointegration vector, we compute the implied VAR in levels. By construction, this VAR has one—and only one—eigenvalue equal to 1. Bootstrapping this VAR would obviously be exactly equivalent to bootstrapping the underlying cointegrated VECM, that is, it would be the correct thing to do if the data featured exact unit roots. Since, on the other hand, here we want to bootstrap under the null of a near unit root cointegrated DGP, we turn such exact unit root VAR in levels into its near unit root correspondent, by "shrinking down" the single unitary eigenvalue to λ=1-0.5$\cdot$(1/T), where T is the sample length. In particular, we do that via a small perturbation of the parameters of the VAR matrices B_j's in the cointegrated VECM representation $\Delta Y_t = A + B_1 \Delta Y_{t-1} + ... + B_p \Delta Y_{t-p} + GY_{t-1} + u_t$, where Y_t collects (the logarithms of) M1 velocity and the short rate, and the rest of the notation is obvious. By only perturbating the elements of the VAR matrices B_j's—leaving unchanged the elements of the matrix G (and therefore both the cointegration vector and the loading coefficients)—we make sure that both the long-run equilibrium relationship between velocity and the short rate, and the way in which disequilibria in such relationship map into subsequent adjustments in the two series, remain unchanged. The bootstrapping procedure we implement for the second possible case in which the processes feature near unit roots is based on bootstrapping such near unit root VAR.

We now turn to discussing Monte Carlo evidence on the performance of the two bootstrapping procedures.

D.1 Monte Carlo evidence on the performance of the two bootstrapping procedures

D.1.1 Evidence for Johansen's test of the null of no cointegration

Table D.1 in this appendix reports Monte Carlo evidence on the performance of the bootstrapping procedure for Johansen's trace tests[14] proposed by CRT.[15] We perform the Monte Carlo simulations based on two types of DGP, featuring *no cointegration* and *cointegration*, respectively. As for the DGP featuring *no cointegration*, we simply consider two independent random walks. As for the one featuring *cointegration*, we consider the following bivariate process:

$$y_t = y_{t-1} + \epsilon_t, \text{ with } \epsilon_t \sim i.i.d.\ N(0,1) \tag{D.1}$$

[14]Numerically near-identical evidence for Johansen's maximum eigenvalue tests is not reported for reasons of space, but it is available upon request.

[15]Extensive Monte Carlo evidence on the good performance of the CRT procedure was already provided by CRT themselves in their original paper. Benati (2015) also provided some (much more limited) evidence conditional on the specific DGPs he was interested in. The rationale for providing additional evidence here is the same as Benati (2015), that is, looking at how the procedure performs conditional on the DGPs we are interested in here.

$$x_t = y_t + u_t \tag{D.2}$$

$$u_t = \rho u_{t-1} + v_t, \text{ with } 0 \leq \rho < 1,\ v_t \sim i.i.d.\ N(0,1). \tag{D.3}$$

As for ρ, we consider six possible values, corresponding to alternative ranges of persistence of the cointegration residual between the three series, that is, $\rho = 0$, 0.25, 0.5, 0.75, 0.9, and 0.95. There are two reasons for using this specific DGP. *First*, it captures the essence of the problem at hand. Here we have two I(1) series—M1 velocity and a short rate—whose long-run dynamics might obey a cointegration relationship. *Second*, by parameterizing the extent of persistence of the deviation from the long-run equilibrium relationship, we can effectively explore how the performance of the test depends on such persistence, even in very large samples. This is key because, as we document in Online Appendix H, real-world ("candidate") cointegration residuals are indeed very highly persistent. Intuitively, for the reasons discussed by Engle and Granger (1987), we would expect that, *ceteris paribus*, the higher the persistence of the cointegration residual, the more difficult it is for any statistical test to detect cointegration. As we will see, this is indeed the case.

Details of the Monte Carlo simulations are as follows. For either DGP, we consider five alternative sample lengths, $T = 50$, 100, 200, 500, and 1,000. For each combination of values of ρ and T, we generate 5,000 artificial samples of length T+100, and we then discard the first 100 observations in order to eliminate dependence on initial conditions (which we set to 0 for either series). For each individual simulation, we bootstrap the relevant test statistic based on 2,000 bootstrap replications.

Table D.1 reports the evidence for Johansen's trace test of the null of no cointegration against the alternative of one or more cointegration vectors. Specifically, the table reports, for either DGP, the sample length and (for the DGP featuring cointegration) the value of ρ; and the fraction of replications for which no cointegration is rejected at the 10% level. The following main findings clearly emerge from the table.

First, in line with the evidence reported by both CRT and Benati (2015), the procedure performs remarkably well conditional on DGPs featuring *no cointegration*. A key point that ought to be stressed is that the specific sample length used in the simulations does not appear to make any material difference for the final results, with the fractions of rejections ranging between 0.098 and 0.119 (with the ideal one being 0.1). This is testimony to the power of bootstrapping, which is capable of automatically controlling for the specific characteristics of the DGP under investigation.

Second, when the DGP does feature *cointegration*, ideally we would like the test to reject as much as possible. As the lower part of the table shows, the procedure indeed performs very well if ρ is small. If $\rho = 0$, for example, cointegration is already detected 100% of the time for $T = 100$, whereas if $\rho = 0.5$, it is detected 88.2% of the time for $T = 100$, and a sample length of $T = 200$ is already sufficient to detect cointegration 100% of the time. As ρ increases, however, the performance deteriorates. The intuition for this is straightforward: as the cointegration residual becomes more and more persistent, it gets closer and closer to a random walk (in which case there would be no cointegration), and the procedure therefore needs larger and larger samples to detect the truth (that the residual is highly persistent but ultimately stationary). In particular, as ρ increases, the fraction of rejections tends to converge, for each sample size, to the fraction of rejections under the DGP featuring no cointegration. This is especially apparent for $T = 50$ or 100, with the fractions being equal to 0.114 and 0.120, respectively. In the limit, for $\rho \to 1$, the procedure will tend to reject 10% of the time.

Comparison with the Monte Carlo evidence of Cavaliere *et al.* (2012) This evidence is qualitatively and also quantitatively in line with the Monte Carlo evidence reported in CRT's Tables I and II, pp. 1731-1732. Although the DGPs they used (either noncointegrated VARs or cointegrated VECMs featuring one cointegration vector) were different from the DGPs used herein, their results and ours turn out to be very close. Specifically, the results are as follows:

- The results in panel (b) of their Table I illustrate the excellent performance of their bootstrapping procedure for tests of the null of no cointegration when the true DGP features no cointegration. In line with the evidence reported in the first row of our Table E.1, their results illustrate how, at the 5% level, the empirical rejection frequencies (henceforth, ERF) are quite close to 5% irrespective of the sample size.

- Panel (a) in the same table reports qualitatively and quantitatively similar evidence for the maximum eigenvalue test of 1 versus 2 cointegrating vectors, conditional on DGPs featuring one cointegrating vector.

- Finally, CRT's Table II reports evidence on the ability of the sequential bootstrapped procedure to select the correct cointegration rank, which in their experiments is one (see the columns under the heading "Bootstrap (CRT)"). Those results are in line with the ones reported in our Table 1 in the main text conditional on DGPs featuring one cointegration vector. In either case, the larger the sample size, the more frequently CRT's procedure detects the truth, with ERFs converging toward 1 for sufficiently large samples. In comparatively small samples (e.g., for $T = 50$), ERFs are typically much below one—as we show, the more so, the more persistent is the cointegration residual.[16]

[16] Different from the present work, CRT do not explore how the persistence of the cointegration residual affects the performance of their procedure. The results reported in their Table II, however, are quantitatively in line with ours. We found this in the following way. We simulated their VECM conditional on one cointegration vector 10,000 times for samples of length $T = 10{,}000$, and for each simulation we computed the implied cointegration residual, and based on it we estimated an AR(4) process (in fact, given the nature of their DGP, an AR(2) would have been enough). The sum of the AR coefficients is our measure of persistence. For their benchmark case of δ=0.1, both the mean and the median of the distribution were equal to 0.61. From their Table II, we can see that for δ=0.1 and $T = 50$, the ERF is 49.0%. In Table 1 of the main text we report, for $T = 50$ and ρ=0.5, an

The bottom line is that our Monte Carlo evidence, although based on a set of DGPs that have been specifically tailored to the problem at hand, is in fact exactly in line, both qualitatively and quantitatively, with the evidence reported in CRT.

Summing up The proceeding can be summarized as follows:

- If the true DGP features *no cointegration*, CRT's procedure performs remarkably well irrespective of the sample size.
- If, however, the true DGP features *cointegration*, Johansen's tests—even bootstrapped as in CRT—perform well only if the persistence of the cointegration residual is sufficiently low and/or the sample size is sufficiently large. If, on the other hand, the cointegration residual is persistent and the sample size is small, the procedure will fail to detect cointegration a nonnegligible fraction of the time. For example, with $T = 100$, cointegration will be detected 43.3% of the time if $\rho = 0.75$ and just 12.0% of the time if $\rho = 0.95$.

All of this means that if Johansen's tests do detect cointegration, we should have a reasonable presumption that cointegration is indeed there. If, on the other hand, they do not detect it, a possible explanation is that the sample period is too short and/or the cointegration residual is highly persistent.

D.1.2 Evidence for Wright's (2000) test of the null of cointegration

Table D.2 reports evidence for the two bootstrapping procedures we use for Wright's test. Specifically, the top portion of the table reports evidence for the case of exact unit roots, with the true DGP that is being simulated being given by (D.1)-(D.3), and the bootstrapping procedure being the first one discussed in the second paragraph of this appendix, that is, being based on bootstrapping the VECM estimated conditional on one cointegration vector. The second portion of Table D.2 reports the corresponding Monte Carlo evidence for the near unit root case. Here the DGP that is being simulated is the near unit root version of (D.1)-(D.3), that is, the DGP that is obtained when the random walk (D.1) is replaced with

$$y_t = \lambda y_{t-1} + \epsilon_t, \text{ with } \epsilon_t \sim i.i.d.\ N(0,1) \tag{D.1}$$

with λ=1-0.5$\cdot$(1/T), where T is the sample length. For each single bootstrapped replication, the test statistics are bootstrapped based on the second procedure discussed in the second paragraph of this appendix, that is, by bootstrapping the near unit root VAR in levels obtained by "shrinking down" to λ=1-0.5$\cdot$(1/T) the unitary eigenvalue of the VAR in levels implied by the VECM estimated conditional on one cointegration vector.

ERF of 35%.

All of the details of the Monte Carlo simulations are exactly the same as for Johansen test (grids of values for ρ and T, number of Monte Carlo simulations, and number of bootstrap replications). As a comparison between the top and bottom portions of Table D.2 shows, evidence for the two cases of exact and near unit roots is virtually identical and can be summarized as follows:[17]

First, if the true DGP features *cointegration*, the procedure works remarkably well if the sample size is sufficiently long, the persistence of the cointegration residual is sufficiently low, or both. For example, for T = 1,000, the empirical rejection frequencies (ERFs) at the 10% level range between 0.103 and 0.116, very close to the ideal of 0.1. As the sample size decreases, however, the ERFs systematically increase. For $T = 200$, for example, the ERF is still equal to 0.105 for $\rho = 0.75$, but for $\rho = 0.95$ it becomes equal to 0.127. For $T = 50$ the ERF is still reasonably close to 0.1 for $\rho = 0.5$, but for $\rho = 0.9$ it is already equal to 0.181, and it further increases to 0.206 for $\rho = 0.95$.

Second, if the true DGP features *no cointegration* (i.e., two independent random walks), the ERFs range between 0.200 and 0.227 depending on sample size.

Summing up The proceeding can be summarized as follows:

- If the true DGP features *cointegration*, in the case of either exact or near unit roots, the respective bootstrapping procedures perform remarkably well if the sample size is sufficiently long, the persistence of the cointegration residual is sufficiently low, or both. However, if the sample is sufficiently short *and* the cointegration residual is sufficiently persistent, the null of cointegration will be incorrectly rejected, in the worst possible scenario analyzed in the Monte Carlo experiment ($T = 50$ and $\rho = 0.95$) at about twice the nominal size. The explanation for this is straightforward, and it is, in fact, in line with the previously mentioned point made by Engle and Granger (1987): when the cointegration residual is highly persistent, only sufficiently long samples allow the test to detect the truth, that is, that the deviation between the two series is ultimately transitory, so that they are in fact cointegrated. On the other hand, under these circumstances the shorter the sample period, the more likely it will be to mistakenly infer that the deviation between the two series is, in fact, permanent, so that they are not, in fact, cointegrated.

- If, on the other hand, the true DGP features *no cointegration*, in the case of either exact or near unit roots, the test will reject the null at roughly twice the nominal size.

A key implication is that, in fact, lack of rejection of the null of no cointegration does not represent very strong evidence that cointegration truly is in the data. Since in

[17] In what follows, all of the numbers mentioned pertain to the case of exact unit roots.

the case of two independent random walks (or their near unit root correspondent) the null of cointegration is rejected about one time out of five irrespective of sample length, an alternative interpretation is simply that the data do not feature cointegration, but the test is not capable of detecting this.

E Additional Results from Wright's (2000) Test for M1 Velocity and the Short Rate

Table E.1 in this appendix reports results from Wright's (2000) test based on the second bootstrapping procedure previously discussed in Appendix D, that is, based on bootstrapping the near unit root VAR, which has been obtained by perturbating the coefficients of the AR matrices of the cointegrated VECM produced by Johansen's procedure (estimated conditional on one cointegration vector) in such a way that the unitary eigenvalue is "shrunk down" to its near unit root equivalent 1-0.5$\cdot(1/T)$, where T is the sample length. Evidence is nearly identical to that reported in Table 2 in the main text, and it points towards the presence of cointegration across the board between (the logarithms of) M1 velocity and the short rate.

F Unrestricted Tests of the Null of No Cointegration

Table F.1 in this appendix reports results from Johansen's cointegration test based on unrestricted specifications featuring the logarithms of M1, nominal GDP, and a short rate. On the other hand, we do not perform the corresponding set of tests based on the *levels* of the three series, since two of them—M1 and nominal GDP—exhibit obvious exponential (that is, nonlinear) trends, which makes linear cointegration tests, such as Johansen's, Shin's (1994), or Wright's (2000), pointless within the present context. Out of 18 samples, the tests detect cointegration only in 5 cases. Results for the United Kingdom, the United States, Argentina, Canada, Switzerland, Bolivia, Belize, Australia, Spain, and South Africa are in line with those based on the corresponding restricted specification for the logarithms of velocity and the short rate in Table 1 in the main text. On the other hand, for Colombia, New Zealand, and Belgium, where the tests based on velocity and the short rate did detect cointegration, the corresponding unrestricted tests in Table F.1 do not reject the null of no cointegration. A possible and likely explanation for this contrast is that failure to impose the (true) restriction of unitary income elasticity decreases the power of the test, leading it to incorrectly not reject the null. For Israel, Portugal, and Norway, relaxation of the constraint of unitary income elasticity leads the unrestricted test to detect cointegration, in contrast to the results from the unrestricted test in Table 1. In light of the evidence of a remarkably strong correlation between the logarithms of M1 velocity and the short rate for Israel in Figure 5 of the main text, and of the corresponding lack of any correlation between the levels of the two series, we regard the results for this country in either table as a fluke, likely due to the short sample length.

G Testing for Stability in Cointegration Relationships

In Section 6.2 in the main text of the paper we test for either breaks or, more generally, time variation in cointegration relationships based on the three tests discussed by Hansen and Johansen (1999): Two Nyblom-type tests for stability in the cointegration vector and the vector of loading coefficients, respectively; and a fluctuation test, which is essentially a joint test for time-variation in the cointegration vector and the loadings.

G.1 Monte Carlo evidence on the performance of Hansen and Johansen's (1999) tests

Table G.1 in this appendix reports Monte Carlo evidence on the performance of the three tests conditional on bivariate cointegrated DGPs featuring no time variation of any kind, for alternative sample lengths, and alternative degrees of persistence of the cointegration residual, which is modelled as an AR(1). Specifically, the DGP is given by

$$y_t = y_{t-1} + \epsilon_t, \text{ with } \epsilon_t \sim i.i.d.\ N(0,1) \tag{G.1}$$

$$x_t = y_t + u_t \tag{G.2}$$

with the cointegration residual being

$$u_t = \rho u_{t-1} + v_t, \text{ with } 0 \leq \rho < 1,\ v_t \sim i.i.d.\ N(0,1). \tag{G.3}$$

We consider T = 50, 100, 200, and ρ = 0, 0.25, 0.5, 0.75, 0.9, 0.95, and for each combination of T and ρ, we stochastically simulate the DGP based on each simulation, we perform either of the three tests on $[y_t\ x_t]'$, bootstrapping them as in Cavaliere *et al.* (2012) conditional on one cointegration vector (i.e., the true number of cointegration vectors in the DGP), based on 1,000 bootstrap replications. In performing of the three tests, we set the "trimming parameter" to the standard value in the literature of 0.15. For each combination of T and ρ, we perform 1,000 Monte Carlo simulations. The table reports, for each combination of T and ρ, the fraction of Monte Carlo simulations for which stability is rejected at the 10% level.

The main results in the table can be summarized as follows. The two Nyblom-type tests exhibit an overall reasonable performance, incorrectly rejecting the null of no time variation, most of the time, at roughly the nominal size. Crucially, this is the case irrespective of the sample length and of the persistence of the cointegration residual. The fluctuation test, on the other hand, exhibits a good performance only if the persistence of the cointegration residual is low. The higher the residual's persistence, however, the worse the performance, so that, for example, when the AR root of the residual is equal to 0.95, for a sample length $T = 50$, the test rejects at roughly twice the nominal size. This is clearly problematic since—as we mentioned in the main text, and we discuss more in detail in Appendix H —cointegration residuals are typically moderately to highly persistent. Because of this, in the main text we therefore focus on the results from the two Nyblom-type tests, whereas we eschew results from fluctuations tests (these results are, however, reported in table G.4).

G.2 Evidence on the stability of cointegration relationships

Table G.2 in this appendix reports the results from Hansen and Johansen's (1999) Nyblom-type tests for breaks in either the cointegration vector or the loading coefficients in the cointegrated VECMs estimated (based on Johansen's estimator) conditional on one cointegration vector. Specifically, the table reports p-values for testing stability in either feature of the cointegrated VECM, which have been bootstrapped as in Cavaliere *et al.* (2012), that is, based on the model estimated conditional on one cointegration vector. The main result emerging from the table is that evidence of breaks in either feature is weak to nonexistent. In particular, focusing on the cointegration vector—which, for the purpose of addressing the question of whether there is a stable long-run relationship between velocity and the short rate is clearly the key feature—at the 10% level we detect a break in four cases (Canada, Thailand, Turkey, and South Africa) based on the Selden-Latané specification, and in *just two* cases (Belgium and Finland) based on the log-log. Evidence of breaks in the vector of loading coefficients is slightly stronger—seven instances based on Selden-Latané and three based on log-log—but breaks in this feature bear *no implication* for the presence of a stable long-run relationship between the two series, as they uniquely hinge upon the way the system converges toward the long-run equilibrium.

Table G.3 reports, for the handful of cases in which a break in either feature has been detected, the estimated break dates, together with the values which the model feature which has been subject to breaks—either the cointegration vector or the loading coefficients—has taken in the two subsamples.

Finally, Table G.4 reports p-values (again, bootstrapped as in Cavaliere *et al.* (2012) based on the VECM estimated conditional on one cointegration vector) for Hansen and Johansen's (1999) fluctuations tests based on the cointegrated VECM. Results are qualitatively in line with those for the two just-discussed Nyblom-type tests for breaks in either the cointegration vector or the loading coefficients, with evidence of time variation identified only in a handful of cases.

H Evidence on the Persistence of 'Candidate' Cointegration Residuals

Tables H.1 and H.2 in this appendix report Hansen's (1999) "grid bootstrap" median-unbiased (henceforth, MUB) estimates of the sum of the AR coefficients in AR(2) representations for the "candidate cointegration residuals" in our dataset.[18] By "candidate cointegration residual" (henceforth, CCR), we mean the linear combination of the I(1) variables in the system which will indeed be regarded as a cointegration residual *if* cointegration is detected.[19] For reasons of robustness, for either the Selden-Latané specification (Table H.1) or the log-log (Table H.2), we consider two alternative estimators of the cointegration residual, either Johansen's or Stock and Watson's (1993).

Evidence points toward both a nonnegligible degree of persistence of the CCRs and a wide degree of heterogeneity across countries. Focusing on results based on the log-log specification, the MUB estimate based on Johansen's estimator of the cointegration vector—let's label it as $\hat{\rho}^J_{MUB}$—ranges from a minimum of 0.27 for Belize to a maximum of 1.17 for the Barbados islands. By classifying the $\hat{\rho}^J_{MUB}$'s, in an admittedly arbitrary fashion, as "highly persistent" ($\hat{\rho}^J_{MUB} \geq 0.8$), "moderately persistent" ($0.4 < \hat{\rho}^J_{MUB} < 0.8$), and "not very persistent" ($\hat{\rho}^J_{MUB} \leq 0.4$), we end up with 22 $\hat{\rho}^J_{MUB}$'s in the first group, 14 in the second, and 4 in the third. Results based on Stock and Watson's estimator are qualitatively the same, with the three groups comprising respectively 25, 13, and 2 countries.

I Evidence on the Functional Form

Figures I.1 and I.2 provide simple, informal evidence on which specifications—Selden-Latané or log-log —provides the most plausible description of the data at low and, respectively, high interest rates. In both figures, the top row shows the levels of M1 velocity and the short rate, and the bottom row shows the logarithms of the two series. The evidence in the two rows therefore corresponds to a Selden-Latané and, respectively, a log-log specification for the demand for real M1 balances with unitary income elasticity, linearly relating either the level or the logarithm of M1 velocity to the level or, respectively, the logarithm of the short rate. Figure I.2 reports evidence for all of the countries that we classified as high-inflation countries,

[18] Results are based on 2,000 bootstrap replications for each possible value of the sum of the AR coefficients in the grid. Bootstrapping has been performed as in Diebold and Chen (1996). For reasons of robustness, we report results based on two alternative estimators of the cointegration vector, Johansen's, and Stock and Watson's (1993).

[19] We label it as the candidate cointegration residual because, as the Monte Carlo evidence in the previous section has shown, if a residual is highly persistent, cointegration might well *not* be detected *even if it is present*, which would prevent the candidate from being identified as a true cointegration residual.

arranged in descending order according to the level of the interest rate. For reasons of space, Figure I.2 only reports evidence for 10 of the remaining countries. This is however without loss of generality, as this evidence is representative of the entire set of low-inflation countries (this evidence is available upon request).

Two broad patterns emerge from Figures I.1 and I.2. First, at low interest rates the Selden-Latané specification appears to provide a more plausible description of the data than the log-log, in the sense that, in several cases, the correlation between the levels of velocity and the short rate is manifestly stronger than that between the logarithms of the two series. This is especially clear for the United States, the United Kingdom, Switzerland, the Barbados islands, and, to a certain extent, Belize, whereas evidence for Japan is slightly weaker, and it crucially hinges on the period since the beginning of the new millennium. On the other hand, for countries such as Canada, South Korea, the Netherlands, and New Zealand, neither specification is manifestly superior, as the correlation between the levels of the series appears as equally strong as that between their logarithms. Another way of stating this is that, among low-inflation rate countries, in no single case does the log-log specification provide a manifestly better description of the data than the Selden-Latané specification, whereas in several cases the data clearly prefer the latter to the former.

Second, at high or very high interest rates, the opposite is true: in no single case is the Selden-Latané specification clearly preferred by the data, whereas in a few cases, the log-log specification provides a manifestly better description of the data. This is especially clear for Israel, Argentina, and Brazil, and to a lesser extent for Chile, Bolivia, and Ecuador, whereas for Turkey and Venezuela, the correlations between the levels and, respectively, the logarithms of the two series appear as equally strong or weak, depending on the period. Interestingly, the data's preference for the log-log specification appears stronger the higher the level of the interest rate (which is reported, in the top row, in the right-hand side scale in either panel in black), whereas it becomes progressively weaker for countries characterized by comparatively lower interest rates.

J Evidence Based on Quarterly Data

As we mentioned in the text, for nine countries in our dataset,[20] Benati (2020) presents, based on post-WWII quarterly data, the same evidence as in the present work. The results there—see his Table 1 for Johansen's tests, and Table 2 for Wright's tests—are in line with those reported in the present work based on annual data. For other seven countries (Israel, Brazil, New Zealand, Switzerland, Japan, Norway, West Germany) we were able to find quarterly post-WWII data for nominal GDP, nominal M1, and a short rate. For the remaining countries in our dataset we could not find

[20]The U.S., the U.K., Canada, Australia, Taiwan, South Korea, South Africa, Hong Kong, and Mexico.

quarterly data.

Tables J.1 and J.2 report results from Johansen's and Wright's tests, respectively, for six of the additional seven countries for which we were able to find quarterly data. On the other hand, we do not consider Germany because a unit root in the short rate is strongly rejected both for the full sample period 1970Q1-1998Q4, and for the pre-unification sample. Evidence is in line with that based on annual data. Based on Wright's tests, we detect cointegration for all of the six countries. As for Johansen's tests, we detect it for Brazil, Israel, and Japan based on either specification, whereas we do not detect it for New Zealand, Norway, and Switzerland. It is to be noticed, however, that for these latter three countries the empirical reject frequencies are quite low—ranging from 0.218 to 0.325—thus implying that, if cointegration were truly there, Johansen's tests would have a hard time in detecting. Once again, all of this is exactly in line with our evidence in the main text based on annual data.

K Robustness to Using Consumption Velocity

Although our theoretical framework (in Section 2 of the paper) only models the demand for money originating from households, models like this have long been recognized in the literature as 'shortcuts' for more complex, but ultimately mathematically equivalent models also considering the demand originating from firms. Crucially, since the data we analyze also include M1 balances held by firms, in fact the most appropriate aggregate for the purpose of computing velocity is GDP, rather than consumption.

In spite of this, for reasons of robustness, in this appendix we also consider evidence based on consumption velocity—which we compute as the ratio between nominal consumption and nominal M1—for all of the countries for which we could find data on nominal consumption.

As documented by several authors—e.g., for the U.S. by Cochrane (1994)—as a consequence of the permanent income hypothesis consumption and GDP are cointegrated. Therefore, on logical grounds, computing velocity based on either consumption or GDP should make no difference. As the evidence in Tables K.1 and K.2 (based on annual data) shows, this is indeed the case. We do not comment on the results in detail. The only point to stress is that this evidence is qualitatively the same as that reported in Tables 1 and 2 in the main text, and (for quarterly data) in Tables J.1 and J.2 in Online Appendix J.

L Evidence on Weak Exogeneity and the Error-Correction Mechanism

Tables L.1 and L.2 report evidence on the weak exogeneity of either variable within the cointegrated VECMs. Specifically, the two tables report, based on annual and quarterly data, respectively, bootstrapped p-values for testing the null hypothesis that the loading coefficient on the cointegration residual for either variable is equal to zero. Therefore, e.g., the results for the Selden-Latané specification for the U.S. in Table L.2—with the p-values for velocity and the short rate being equal to 0.301 and 0.002, respectively—suggest that velocity is weakly exogenous, whereas the short rate is not. The implication is that the adjustment towards long-run equilibrium takes place mostly *via* movements in the short rate, rather than *via* movements in velocity (in Tables L.1 and L.2 all such cases have been highlighted in yellow).

The finding that, for many countries, velocity is weakly exogenous, whereas the short rate is not, has a straightforward interpretation. In line with Benati's (2020) finding that M1 velocity is essentially the permanent component of the short rate, this has the natural interpretation that economic agents, in deciding how much of their wealth to allocate to non interest-bearing M1, as opposed to interest-bearing assets, almost uniquely react to permanent shocks to the opportunity cost of holding M1 balances, whereas they essentially ignore transitory shocks. To put it differently, to a close approximation the demand for M1 balances only reacts to permanent shocks to the opportunity cost of M1, whereas it does not react to its transitory variation.

Tables L.3 and L.4 report 90 per cent bootstrapped confidence intervals for the coefficients on either lagged velocity, or the lagged short rate, for either of the two equations of the VECM. For reasons of space, for any country we report evidence based on the specification which, based on the discussion in Appendix I, we regard as the most plausible, i.e. the log-log for high-inflation (and therefore, high interest rates) countries, and the Selden-Latané specification for all other countries. Specifically, the first group of countries comprises Argentina, Brazil, Bolivia, Israel, Mexico, Ecuador, Thailand, Venezuela, Chile, Turkey, and Peru, i.e. the countries for which, in Figures I.1-I.2, we plotted the logarithms of velocity and the short rate. Consistent with the previously mentioned evidence about weak exogeneity, in the U.S. and several other countries adjustment takes place *via* movements in the short rate, rather than movements in velocity.

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Table C.1 Bootstrapped p-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Logarithm of:								*Level of:*			
	nominal GDP		nominal M1		M1 velocity		short rate		M1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Argentina, 1914-2009	0.227	0.903	0.201	0.847	0.686	0.746	0.316	0.332	0.562	0.724	0.012	0.009
Australia												
1941-1989	0.771	0.607	0.622	0.978	0.931	0.854	0.784	0.872	0.947	0.932	0.915	0.984
1969-2017	0.100	0.465	0.955	0.982	0.856	0.884	0.905	0.966	0.840	0.872	0.596	0.815
Austria, 1970-1998	0.024	0.128	0.125	0.685	0.970	0.977	0.509	0.351	0.961	0.968	0.361	0.198
Bahrain, 1980-2017	0.525	0.466	0.368	0.059	0.652	0.769	0.448	0.365	0.544	0.689	0.335	0.216
Barbados, 1975-2016	0.060	0.114	0.572	0.317	0.802	0.748	0.992	0.968	0.552	0.528	0.418	0.428
Belgium, 1946-1990	0.861	0.885	0.211	0.236	0.036	0.166	0.130	0.670	0.497	0.733	0.414	0.664
Belize, 1977-2017	0.234	0.580	0.302	0.138	0.942	0.943	0.359	0.956	0.876	0.904	0.811	0.796
Bolivia, 1980-2013	0.090	0.067	0.114	0.062	0.915	0.849	0.866	0.837	0.627	0.674	0.139	0.188
Brazil, 1934-2014	0.304	0.581	0.258	0.559	0.747	0.744	0.367[b]	0.340[b]	0.554	0.534	0.007[b]	0.054[b]
Canada												
1926-2006	0.132	0.148	0.340	0.144	0.792	0.793	0.607	0.730	0.790	0.806	0.384	0.479
1967-2017	0.087	0.330	0.022	0.032	0.971	0.969	0.792	0.824	0.854	0.837	0.617	0.681
Chile												
1940-1995	0.399	0.544	0.374	0.261	0.134	0.050	0.341	0.263	0.212	0.124	0.133	0.090
1941-2017	0.922	0.913	0.906	0.692	0.450	0.303	0.205[b]	0.280[b]	0.302	0.127	0.047[b]	0.017[b]
Colombia, 1960-2017	0.107	0.974	0.090	0.942	0.244	0.263	0.827	0.905	0.162	0.199	0.718	0.822
Ecuador, 1980-2011	0.493	0.375	0.097	0.188	0.877	0.815	0.793	0.870	0.795	0.761	0.437	0.727
Finland, 1946-1985	0.288	0.100	0.057	0.071	0.776	0.594	0.541	0.521	0.914	0.897	0.512	0.511
France, 1852-1913	0.001	0.001	0.896	0.891	0.642	0.803	0.051	0.037	0.522	0.743	0.027	0.040
Guatemala, 1980-2017	0.950	0.965	0.992	0.993	0.672	0.585	0.596	0.508	0.622	0.520	0.573	0.516
Hong Kong, 1985-2017	0.023	0.113	0.649	0.805	0.925	0.942	0.622	0.596	0.716	0.849	0.495	0.481

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and a time trend for the logarithms of nominal GDP and nominal M1, and with an intercept and no time trend for the other series.
[b] For this period we consider inflation, rather than the short rate.

Table C.1 (continued) Bootstrapped p-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Logarithm of:								*Level of:*			
	nominal GDP		nominal M1		M1 velocity		short rate		M1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Japan, 1955-2017	0.144	0.623	0.090	0.314	0.945	0.936	0.697	0.748	0.749	0.757	0.578	0.566
Israel, 1983-2016	0.000	0.144	0.001	0.001	0.792	0.040	0.477	0.065	0.318	0.257	0.106	0.057
Italy, 1949-1996	0.794	0.889	0.993	0.945	0.333	0.648	0.857	0.899	0.234	0.643	0.805	0.848
Mexico, 1985-2014	0.013	0.021	0.066	0.016	0.767	0.100	0.629	0.289	0.679	0.027	0.346	0.023
Morocco, 1985-2017	0.119	0.193	0.400	0.654	0.319	0.267	0.806	0.717	0.059	0.048	0.761	0.595
Netherlands, 1950-1992	0.985	0.996	0.703	0.783	0.100	0.194	0.194	0.450	0.232	0.297	0.243	0.347
New Zealand, 1934-2017	0.960	0.981	0.479	0.527	0.825	0.814	0.665	0.651	0.815	0.798	0.351	0.337
Norway, 1946-2014	0.955	0.995	0.123	0.153	0.898	0.880	0.802	0.755	0.846	0.837	0.774	0.723
Paraguay, 1962-2015	0.719	0.920	0.733	0.706	0.426	0.447	0.032	0.067	0.342	0.394	0.125	0.249
Peru, 1959-2017	0.767	0.857	0.738	0.794	0.599	0.427	0.488	0.564	0.600	0.419	0.112	0.116
Portugal, 1914-1998	0.634	0.614	0.209	0.145	0.594	0.407	0.716	0.714	0.607	0.430	0.596	0.469
South Africa, 1965-2015	0.995	0.995	0.751	0.839	0.918	0.927	0.289	0.484	0.875	0.882	0.283	0.332
South Korea, 1970-2017	0.080	0.245	0.101	0.417	0.664	0.610	0.643	0.745	0.384	0.290	0.061	0.258
Spain, 1941-1989	0.632	0.504	0.154	0.505	0.187	0.440	0.828	0.878	0.363	0.512	0.589	0.720
Switzerland, 1948-2005	0.949	0.930	0.498	0.712	0.425	0.359	0.156	0.177	0.453	0.417	0.186	0.120
Taiwan, 1962-2017	0.309	0.788	0.141	0.574	0.314	0.264	0.662	0.713	0.057	0.034	0.408	0.513
Thailand, 1979-2016	0.291	0.867	0.944	0.936	0.907	0.916	0.619	0.523	0.890	0.898	0.589	0.418
Turkey, 1968-2017	0.856	0.827	0.879	0.903	0.735	0.767	0.644	0.668	0.653	0.673	0.727	0.770
United Kingdom, 1922-2016	0.140	0.805	0.080	0.391	0.831	0.746	0.926	0.942	0.779	0.728	0.345	0.575
United States, 1915-2017												
M1	0.702	0.385	0.482	0.158	0.626	0.783	0.443	0.248	0.578	0.697	0.302	0.319
M1 + MMDAs	0.699	0.380	0.488	0.159	0.833	0.811	0.443	0.251	0.713	0.702	0.293	0.326
Venezuela, 1962-1999	0.521	0.752	0.738	0.817	0.574	0.729	0.744	0.730	0.543	0.786	0.691	0.706
West Germany, 1960-1989	0.844	0.963	0.662	0.840	0.752	0.739	0.067	0.137	0.721	0.719	0.069	0.138

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and a time trend for the logarithms of nominal GDP and nominal M1, and with an intercept and no time trend for the other series.

Table C.2 Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Log-difference of:								*First difference of:*			
	nominal GDP		nominal M1		M1 velocity		short rate		M1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Argentina, 1914-2009	0.038	0.050	0.023	0.035	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
Australia												
1941-1989	0.046	0.061	0.004	0.009	0.011	0.024	0.000	0.004	0.003	0.023	0.000	0.032
1969-2017	0.245	0.415	0.002	0.012	0.001	0.024	0.000	0.004	0.002	0.022	0.000	0.001
Austria, 1970-1998	0.350	0.227	0.111	0.258	0.071	0.098	0.023	0.067	0.041	0.084	0.025	0.057
Bahrain, 1980-2017	0.007	0.085	0.028	0.260	0.002	0.004	0.005	0.004	0.001	0.003	0.001	0.001
Barbados, 1975-2016	0.117	0.144	0.033	0.075	0.017	0.048	0.069	0.073	0.006	0.011	0.001	0.003
Belgium, 1946-1990	0.006	0.002	0.009	0.050	0.002	0.000	0.000	0.004	0.001	0.001	0.001	0.002
Belize, 1977-2017	0.006	0.033	0.011	0.031	0.008	0.023	0.074	0.055	0.004	0.007	0.003	0.007
Bolivia, 1980-2013	0.125	0.157	0.135	0.150	0.044	0.085	0.007	0.032	0.019	0.051	0.017	0.054
Brazil, 1934-2014	0.133	0.197	0.070	0.205	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
Canada												
1926-2006	0.005	0.003	0.012	0.019	0.002	0.011	0.000	0.000	0.003	0.018	0.000	0.000
1967-2017	0.132	0.379	0.007	0.006	0.020	0.031	0.000	0.001	0.023	0.020	0.000	0.001
Chile												
1940-1995	0.153	0.079	0.361	0.317	0.000	0.002	0.001	0.001	0.001	0.004	0.005	0.002
1941-2017	0.126	0.053	0.328	0.221	0.000	0.000	0.000[b]	0.000[b]	0.000	0.000	0.000[b]	0.000[b]
Colombia, 1960-2017	0.001	0.003	0.000	0.004	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
Ecuador, 1980-2011	0.016	0.111	0.043	0.061	0.022	0.020	0.007	0.076	0.016	0.019	0.003	0.036
Finland, 1946-1985	0.014	0.051	0.010	0.007	0.001	0.001	0.000	0.003	0.000	0.002	0.000	0.003
Guatemala, 1980-2017	0.053	0.120	0.011	0.053	0.005	0.020	0.001	0.042	0.005	0.014	0.001	0.081
Hong Kong, 1985-2017	0.251	0.282	0.023	0.095	0.041	0.156	0.010	0.015	0.045	0.133	0.002	0.005

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and no time trend.
[b] For this period we consider inflation, rather than the short rate.

Table C.2 (continued) Bootstrapped p-values for Elliot, Rothenberg, and Stock unit root tests[a]

	Log-difference of:								*First difference of:*			
	nominal GDP		nominal M1		M1 velocity		short rate		M1 velocity		Short rate	
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2
Japan, 1955-2017	0.475	0.716	0.136	0.342	0.009	0.046	0.000	0.002	0.004	0.011	0.000	0.000
Israel, 1983-2016	0.009	0.001	0.051	0.020	0.002	0.042	0.003	0.021	0.006	0.017	0.012	0.053
Italy, 1949-1996	0.205	0.565	0.152	0.394	0.002	0.031	0.002	0.120	0.001	0.057	0.000	0.031
Mexico, 1985-2014	0.239	0.002	0.100	0.129	0.009	0.029	0.009	0.036	0.013	0.019	0.006	0.009
Morocco, 1985-2017	0.019	0.279	0.094	0.207	0.027	0.329	0.020	0.083	0.013	0.216	0.022	0.097
Netherlands, 1950-1992	0.068	0.437	0.007	0.099	0.001	0.057	0.000	0.000	0.001	0.042	0.000	0.000
New Zealand, 1934-2017	0.001	0.014	0.000	0.032	0.004	0.022	0.000	0.000	0.002	0.004	0.000	0.000
Norway, 1946-2014	0.001	0.026	0.005	0.047	0.003	0.033	0.001	0.002	0.003	0.035	0.001	0.003
Paraguay, 1962-2015	0.101	0.225	0.029	0.117	0.002	0.005	0.000	0.000	0.001	0.003	0.000	0.000
Peru, 1959-2017	0.120	0.066	0.062	0.058	0.009	0.018	0.000	0.002	0.007	0.016	0.000	0.001
Portugal, 1914-1998	0.026	0.039	0.039	0.010	0.000	0.000	0.003	0.089	0.000	0.000	0.001	0.003
South Africa, 1965-2015	0.037	0.090	0.001	0.015	0.001	0.008	0.000	0.001	0.001	0.009	0.000	0.001
South Korea, 1970-2017	0.664	0.717	0.076	0.255	0.001	0.006	0.000	0.002	0.000	0.001	0.000	0.001
Spain, 1941-1989	0.001	0.006	0.011	0.011	0.011	0.051	0.000	0.000	0.027	0.095	0.000	0.002
Switzerland, 1948-2005	0.028	0.087	0.000	0.005	0.000	0.009	0.000	0.000	0.000	0.007	0.000	0.001
Taiwan, 1962-2017	0.221	0.553	0.018	0.026	0.001	0.002	0.000	0.000	0.001	0.000	0.000	0.000
Thailand, 1979-2016	0.131	0.164	0.007	0.059	0.006	0.028	0.003	0.005	0.007	0.032	0.002	0.003
Turkey, 1968-2017	0.462	0.562	0.122	0.405	0.001	0.006	0.007	0.116	0.001	0.005	0.002	0.055
United Kingdom, 1922-2016	0.008	0.060	0.007	0.020	0.001	0.006	0.000	0.001	0.000	0.004	0.000	0.000
United States, 1915-2017												
M1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
M1 + MMDAs	0.000	0.000	0.001	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.000
Venezuela, 1962-1999	0.171	0.305	0.035	0.344	0.001	0.051	0.031	0.037	0.000	0.064	0.061	0.039
West Germany, 1960-1989	0.106	0.243	0.011	0.175	0.007	0.090	0.007	0.077	0.005	0.114	0.005	0.054

[a] Based on 10,000 bootstrap replications of estimated ARIMA processes. Tests are with an intercept and no time trend.

Table D.1 Monte Carlo evidence on the performance of Johansen's tests of the null of no cointegration, bootstrapped as in Cavaliere *et al.*'s (2012):[a] fractions of replications for which no cointegration is rejected[b] at the 10 per cent level

	Sample length:				
	$T=50$	$T=100$	$T=200$	$T=500$	$T=1000$
	True data-generation process: no cointegration[c]				
	0.116	0.098	0.105	0.107	0.119
Persistence of the cointegration residual:	*True data-generation process: cointegration*				
$\rho=0$	0.774	1.000	1.000	1.000	1.000
$\rho=0.25$	0.584	0.993	1.000	1.000	1.000
$\rho=0.5$	0.350	0.882	1.000	1.000	1.000
$\rho=0.75$	0.184	0.433	0.937	1.000	1.000
$\rho=0.9$	0.117	0.167	0.328	0.958	1.000
$\rho=0.95$	0.114	0.120	0.164	0.533	0.966

[a] Based on the trace test of the null of no cointegration against the alternative of 1 or more cointegrating vectors. [b] Based on 5,000 Monte Carlo replications, and, for each of them, on 2,000 bootstrap replications. [c] Two independent random walks.

Table D.2 Monte Carlo evidence on the performance of Wright's tests of the null of cointegration:[a] fractions of replications for which cointegration is rejected at the 10 per cent level

Persistence of the cointegration residual:	Sample length: $T = 50$	$T = 100$	$T = 200$	$T = 500$	$T = 1000$
	True DGP: exact unit root processes Bootstrapped process: cointegrated VECM				
	True data-generation process: cointegration				
$\rho = 0$	0.113	0.115	0.103	0.098	0.107
$\rho = 0.5$	0.119	0.115	0.109	0.097	0.109
$\rho = 0.75$	0.143	0.115	0.105	0.102	0.103
$\rho = 0.9$	0.181	0.133	0.122	0.112	0.116
$\rho = 0.95$	0.206	0.167	0.127	0.120	0.103
	True data-generation process: no cointegration[b]				
	0.227	0.215	0.223	0.202	0.200
	True DGP: near unit root processes Bootstrapped process: near unit root VAR				
	True data-generation process: cointegration				
$\rho = 0$	0.125	0.109	0.087	0.084	0.093
$\rho = 0.5$	0.117	0.116	0.111	0.094	0.094
$\rho = 0.75$	0.146	0.121	0.111	0.099	0.091
$\rho = 0.9$	0.174	0.140	0.124	0.110	0.108
$\rho = 0.95$	0.209	0.170	0.142	0.125	0.105
	True data-generation process: no cointegration[b]				
	0.228	0.217	0.229	0.216	0.220

[a] Based on 5,000 Monte Carlo replications, and, for each of them, on 2,000 bootstrap replications. [b] Two independent random walks.

Table E.1 Additional results from Wright's (2000) tests: 90% coverage confidence intervals for the second element of the normalized cointegration vector (based on bootstrapping a near-unit root VAR)

Country	*Period*	*Selden-Latané*	*Log-log*
United Kingdom	1922-2016	[-0.537; -0.409]	NCD
U.S. - M1 + MMDAs	1915-2017	[-0.609; -0.397]	[-0.706; 0.171]
U.S. - M1	1915-2017	[-1.401; -0.837]	[-0.352; -0.108]
Argentina	1914-2009	[-0.111; -0.087]	[-0.537; -0.205]
Brazil	1934-2014	[-0.065; -0.009]	[-1.302; 0.216]
Canada	1926-2006	[-1.526; -1.021]	[-0.739; -0.579]
	1967-2017	[-0.586; -0.486]	[-0.417; -0.313]
Colombia	1960-2017	[-0.247; -0.183]	NCD
Guatemala	1980-2017	[-0.760; -0.440]	[-0.686; -0.398]
New Zealand	1934-2017	NCD	[-0.669; -0.196]
Switzerland	1948-2005	NCD	NCD
Bolivia	1980-2013	[-0.357; -0.201]	[-0.524; -0.384]
Israel	1983-2016	NCD	[-0.392; -0.316]
Mexico	1985-2014	[-0.264; -0.180]	[-0.426; -0.310]
Belgium	1946-1990	[-0.473; -0.277]	[-1.411; -0.702]
Belize	1977-2017	[-1.144; -0.388]	[-2.567; 1.433]
Austria	1970-1998	[-0.729; 0.208]	[-1.040; 0.618]
Bahrain	1980-2017	NCD	[-0.262; -0.186]
Barbados	1975-2016	[-2.115; -0.636]	[-2.899; 0.101]
Ecuador	1980-2011	NCD	NCD
Netherlands	1950-1992	NCD	[-0.495; -0.303]
South Korea	1970-2017	[-0.613; -0.525]	[-0.643; -0.334]
Thailand	1979-2016	[-0.449; -0.405]	[-0.514; -0.366]
Venezuela	1962-1999	[-0.031; -0.003]	[-0.301; 0.404]
Australia	1941-1989	[-0.695; -0.518]	[-0.848; -0.191]
	1969-2017	[-0.500; -0.388]	[-0.514; -0.302]
Chile	1940-1995	[-0.112; -0.060]	[-0.382; -0.278]
	1941-2017	[-0.110; 0.047]	[-0.235; 0.105]
Finland	1946-1985	[-0.558; -0.390]	[-2.557; -1.917]
Japan	1955-2017	[-0.544; -0.292]	[-0.537; -0.077]
Spain	1941-1989	[-0.175; -0.151]	[-0.384; -0.320]
Taiwan	1962-2017	[-0.453; -0.337]	[-0.465; -0.229]
Turkey	1968-2017	NCD	NCD
West Germany	1960-1989	[-0.991; 0.959]	[-0.581; 0.929]
Italy	1949-1996	[0.032; 0.208]	[0.123; 0.567]
Norway	1946-2014	[-1.001; 1.025]	[-0.255; 1.155]
Paraguay	1962-2015	[-0.360; 0.157]	[-0.236; 0.017]
Peru	1959-2017	[-0.038; 0.022]	[-0.533; 0.748]
Portugal	1914-1998	NCD	[-0.046; 0.246]
South Africa	1965-2015	NCD	[-0.096; 1.281]

NCD = No cointegration detected.

Table F.1 Bootstrapped *p*-values for Johansen's maximum eigenvalue tests[a] between the logarithms of M1, nominal GDP, and a short rate[b]

Country	*Period*	*Bootstrapped p value*
United Kingdom	1922-2016	0.841
U.S. - M1 + MMDAs	1915-2017	0.922
U.S. - M1	1915-2017	0.924
Argentina	1914-2009	0.004
Canada	1926-2006	0.775
Colombia	1960-2017	0.309
New Zealand	1934-2017	0.238
Switzerland	1948-2005	0.001
Bolivia	1980-2013	0.183
Israel	1983-2016	0.000
Belgium	1946-1990	0.124
Belize	1977-2017	0.020
Australia	1941-1989	0.356
Spain	1941-1989	0.556
Norway	1946-2014	0.637
Portugal	1914-1998	0.941
South Africa	1965-2015	0.362

[a] Tests of 0 versus 1 cointegration vectors.
[b] Based on 10,000 bootstrap replications.

Table G.1 Monte Carlo evidence on the performance of Hansen and Johansen's tests for time-variation in cointegrated VARs, bootstrapped as in Cavaliere *et al.*'s (2012):[a] fractions of replications for which stability is rejected[b] at the 10 per cent level

	Sample length:		
Persistence of the cointegration residual:	$T = 50$	$T = 100$	$T = 200$
	I: Nyblom test for stability in the cointegration vector		
$\rho = 0$	0.124	0.099	0.109
$\rho = 0.25$	0.140	0.106	0.115
$\rho = 0.5$	0.125	0.114	0.121
$\rho = 0.75$	0.107	0.144	0.127
$\rho = 0.9$	0.091	0.137	0.130
$\rho = 0.95$	0.102	0.119	0.146
	II: Nyblom test for stability in the loading coefficients		
$\rho = 0$	0.083	0.072	0.072
$\rho = 0.25$	0.088	0.087	0.070
$\rho = 0.5$	0.086	0.108	0.070
$\rho = 0.75$	0.093	0.097	0.099
$\rho = 0.9$	0.077	0.100	0.111
$\rho = 0.95$	0.080	0.098	0.109
	III: Fluctuation tests		
$\rho = 0$	0.105	0.119	0.117
$\rho = 0.25$	0.118	0.124	0.120
$\rho = 0.5$	0.135	0.139	0.131
$\rho = 0.75$	0.169	0.142	0.133
$\rho = 0.9$	0.206	0.166	0.156
$\rho = 0.95$	0.200	0.196	0.165

[a] Based on 1,000 Monte Carlo replications, and, for each of them, on 1,000 bootstrap replications.

Table G.2 Bootstrapped p-values[a] for testing stability in the cointegration relationship between (log) M1 velocity and (the log of) a short-term term rate

		Tests for stability in:			
		cointegration vector		*loading coefficients*	
Country	*Period*	*Selden-Latané*	*Log-log*	*Selden-Latané*	*Log-log*
United Kingdom	1922-2016	0.444	0.771	0.543	0.612
U.S. - M1 + MMDAs	1915-2017	0.811	0.348	0.250	0.592
Argentina	1914-2009	–	0.638	–	0.065
Brazil	1934-2014	–	0.506	–	0.822
Canada	1926-2006	0.048	0.489	0.187	0.463
	1967-2017	0.657	0.600	0.075	0.176
Colombia	1960-2017	0.599	0.359	0.968	0.913
Guatemala	1980-2017	0.239	0.194	0.912	–[b]
New Zealand	1934-2017	0.440	0.601	0.183	0.717
Switzerland	1948-2005	0.903	0.470	0.272	0.594
Bolivia	1980-2013	0.362	0.180	0.645	0.346
Israel	1983-2016	0.504	0.441	0.242	0.358
Mexico	1985-2014	0.107	0.350	0.883	–[b]
Belgium	1946-1990	0.632	0.049	0.611	0.225
Belize	1977-2017	0.668	0.999	0.185	0.380
Austria	1970-1998	–[b]	0.619	–[b]	–[b]
Bahrain	1980-2017	0.472	0.509	0.509	0.086
Barbados	1975-2016	0.138	–[b]	0.786	–[b]
Ecuador	1980-2011	–[b]	0.288	–[b]	–[b]
Netherlands	1950-1992	0.347	0.355	0.093	0.941
South Korea	1970-2017	0.491	0.714	0.934	0.853
Thailand	1979-2016	0.066	0.249	0.894	0.974
Venezuela	1962-1999	0.318	0.897	0.974	0.975
Australia	1941-1989	0.220	0.544	0.479	0.716
	1969-2017	0.747	0.781	0.815	0.877
Chile	1940-1995	0.430	0.301	0.090	0.166
	1941-2017	0.593	0.102	0.947	0.163
Finland	1946-1985	0.279	0.062	0.485	0.028
Japan	1955-2017	0.637	0.134	0.936	0.106
Spain	1941-1989	0.714	0.597	0.134	0.981
Taiwan	1962-2017	–	0.889	–	0.501
Turkey	1968-2017	0.073	–	0.004	–
West Germany	1960-1989	–	0.361	–	0.527
Italy	1949-1996	0.973	–	0.685	–
Norway	1946-2014	0.126	0.327	0.059	0.398
Paraguay	1962-2015	0.411	0.332	0.915	0.723
Peru	1959-2017	0.292	0.651	0.013	0.081
Portugal	1914-1998	0.745	0.800	0.017	0.909
South Africa	1965-2015	0.086	0.113	0.337	0.648

[a] Based on 10,000 bootstrap replications. [b] In these cases the test could not be run.

Table G.3 Estimated break dates for the cointegration relationship between (log) M1 velocity and (the log of) a short-term term rate[a]			
		I: Tests for stability in the cointegration vector	
Country	*Period*	Break date	$\hat{\beta}_1'$, $\hat{\beta}_2'$
			Selden-Latané
Canada	1926-2006	1979	[1 -1.013]′, [1 -1.351]′
Thailand	1979-2016	1992	[1 -0.541]′, [1 -0.370]′
Turkey	1968-2017	1994	[1 -0.073]′, [1 -0.169]′
South Africa	1965-2015	1981	[1 -0.951]′, [1 -0.476]′
			Log-log
Belgium	1946-1990	1982	[1 -0.599]′, [1 -0.680]′
Finland	1946-1985		[1 -3.048]′, [1 -2.901]′
		II: Tests for stability in the loading coefficients	
		Break date	$\hat{\alpha}_1'$, $\hat{\alpha}_2'$
			Selden-Latané
Canada	1967-2017	1980	[-0.008; 1.306]′, [-0.160; 0.900]′
Netherlands	1950-1992	1975	[0.234; 0.647]′, [-0.056; 0.862]′
Chile	1940-1995	1973	[-0.009; -3.041]′, [0.014; 1.309]′
Turkey	1968-2017	2001	[0.007; -0.309]′, [-0.137; -0.612]′
Norway	1946-2014	1980	[-0.011; 0.032]′, [-0.017; -0.004]′
Peru	1959-2017	1988	[-0.002; -4.308]′, [-0.006; 3.353]′
Portugal	1914-1998	1983	[-0.001; 0.022]′, [0.000; -0.056]′
			Log-log
Argentina	1914-2009	1987	[-0.035; -0.076]′, [-0.082; 0.513]′
Bahrain	1980-2017	2005	[-0.699; -1.704]′, [-0.845; -2.059]′
Finland	1946-1985	1930	[-0.101; 0.096]′, [-0.125; 0.119]′
Peru	1959-2017	1988	[-0.010; -0.331]′, [-0.060; 0.248]′

[a] Based on 10,000 bootstrap replications.

Table G.4 Bootstrapped p-values[a] for fluctuations tests for the cointegrated VECM for for (log) M1 velocity and (the log of) a short-term rate[a]

Country	*Period*	*Selden-Latané*	*Log-log*
United Kingdom	1922-2016	0.518	0.255
U.S. - M1 + MMDAs	1915-2017	0.544	0.183
Argentina	1914-2009	–	0.702
Brazil	1934-2014	0.454	0.521
Canada	1926-2006	0.326	0.133
	1967-2017	0.175	0.171
Colombia	1960-2017	0.056	0.032
Guatemala	1980-2017	0.652	0.512
New Zealand	1934-2017	0.013	0.530
Switzerland	1948-2005	0.693	0.732
Bolivia	1980-2013	0.502	0.692
Israel	1983-2016	0.556	0.532
Mexico	1985-2014	0.437	0.733
Belgium	1946-1990	0.068	0.421
Belize	1977-2017	0.311	0.571
Austria	1970-1998	0.082	0.071
Bahrain	1980-2017	0.398	0.391
Barbados	1975-2016	0.121	0.192
Ecuador	1980-2011	0.367	0.515
Netherlands	1950-1992	0.011	0.039
South Korea	1970-2017	0.013	0.352
Thailand	1979-2016	0.885	0.491
Venezuela	1962-1999	0.249	0.201
Australia	1941-1989	0.037	0.209
	1969-2017	0.120	0.008
Chile	1940-1995	0.548	0.307
	1941-2017	0.756	0.155
Finland	1946-1985	0.514	0.106
Japan	1955-2017	0.294	0.373
Spain	1941-1989	0.649	0.659
Taiwan	1962-2017	0.552	0.812
Turkey	1968-2017	0.192	–
West Germany	1960-1989	–	0.547
Italy	1949-1996	0.530	–
Norway	1946-2014	0.573	0.287
Paraguay	1962-2015	0.523	0.850
Peru	1959-2017	0.129	0.469
Portugal	1914-1998	0.606	0.493
South Africa	1965-2015	0.745	0.158

[a]Based on 10,000 bootstrap replications.

Table H.1 Estimates of the sum of the AR coefficients for the candidate cointegration residual based on Selden-Latanè[a]

		Estimates based on:	
Country	*Period*	*Johansen*	*Stock and Watson*
United Kingdom	1922-2016	0.55 [0.39; 0.71]	0.57 [0.41; 0.74]
U.S. - M1 + MMDAs	1915-2017	0.70 [0.58; 0.83]	0.75 [0.63; 0.87]
U.S. - M1	1915-2017	0.92 [0.85; 1.01]	1.00 [0.96; 1.02]
Argentina	1914-2009	0.33 [0.19; 0.47]	0.47 [0.32; 0.62]
Brazil	1934-2014	0.58 [0.40; 0.77]	0.86 [0.75; 1.01]
Canada	1926-2006	0.76 [0.63; 0.91]	0.81 [0.68; 0.95]
	1967-2017	0.32 [0.10; 0.53]	0.32 [0.11; 0.53]
Colombia	1960-2017	0.90 [0.75; 1.02]	0.91 [0.75; 1.02]
Guatemala	1980-2017	0.62 [0.35; 0.95]	0.63 [0.37; 1.01]
New Zealand	1934-2017	0.78 [0.67; 0.90]	0.84 [0.74; 0.95]
Switzerland	1948-2005	0.70 [0.49; 0.93]	0.78 [0.60; 0.99]
Bolivia	1980-2013	0.42 [0.12; 0.77]	0.56 [0.28; 0.98]
Israel	1983-2016	0.36 [0.33; 0.40]	0.35 [0.32; 0.39]
Mexico	1985-2014	0.46 [0.26; 0.69]	0.51 [0.29; 0.71]
Belgium	1946-1990	0.57 [0.38; 0.79]	0.63 [0.45; 0.84]
Belize	1977-2017	0.70 [0.50; 0.96]	0.74 [0.53; 1.01]
Austria	1970-1998	0.67 [0.37; 1.02]	1.01 [0.89; 1.05]
Bahrain	1980-2017	0.67 [0.50; 0.86]	0.59 [0.36; 0.84]
Barbados	1975-2016	0.62 [0.39; 0.88]	0.71 [0.52; 0.95]
Ecuador	1980-2011	0.79 [0.53; 1.03]	0.99 [0.71; 1.04]
Netherlands	1950-1992	0.60 [0.35; 0.89]	0.71 [0.49; 1.01]
South Korea	1970-2017	0.49 [0.30; 0.69]	0.51 [0.32; 0.70]
Thailand	1979-2016	0.66 [0.47; 0.87]	0.66 [0.46; 0.88]
Venezuela	1962-1999	0.91 [0.74; 1.03]	0.88 [0.69; 1.03]
Australia	1941-1989	0.80 [0.61; 1.02]	0.78 [0.58; 1.01]
	1969-2017	0.41 [0.17; 0.67]	0.42 [0.18; 0.68]
Chile	1940-1995	0.74 [0.64; 0.85]	0.75 [0.58; 0.98]
	1941-2017	0.66 [0.55; 0.78]	0.83 [0.74; 0.93]
Finland	1946-1985	0.38 [0.09; 0.67]	0.46 [0.17; 0.76]
Japan	1955-2017	0.82 [0.70; 0.97]	0.87 [0.76; 1.01]
Spain	1941-1989	0.59 [0.39; 0.82]	0.61 [0.41; 0.82]
Taiwan	1962-2017	0.90 [0.83; 0.98]	0.81 [0.72; 0.91]
Turkey	1968-2017	1.01 [0.84; 1.04]	1.01 [0.84; 1.04]
West Germany	1960-1989	0.39 [0.11; 0.71]	1.01 [0.83; 1.04]
Italy	1949-1996	0.97 [0.80; 1.03]	0.98 [0.85; 1.03]
Norway	1946-2014	1.00 [0.95; 1.02]	1.01 [0.97; 1.02]
Paraguay	1962-2015	0.70 [0.52; 0.90]	0.81 [0.66; 1.00]
Peru	1959-2017	0.36 [0.17; 0.59]	0.89 [0.76; 1.02]
Portugal	1914-1998	0.99 [0.93; 1.02]	0.99 [0.93; 1.02]
South Africa	1965-2015	0.87 [0.75; 1.01]	1.01 [0.96; 1.03]

[a] Median and 90% confidence interval (based on 10,000 bootstrap replications).

Table H.2 Estimates of the sum of the AR coefficients for the candidate cointegration residual based on log-log[a]

		Estimates based on:	
Country	*Period*	*Johansen*	*Stock and Watson*
United Kingdom	1922-2016	0.94 [0.84; 1.02]	0.93 [0.83; 1.02]
U.S. - M1 + MMDAs	1915-2017	0.86 [0.78; 0.95]	0.90 [0.81; 1.00]
U.S. - M1	1915-2017	0.94 [0.88; 1.01]	1.00 [0.97; 1.01]
Argentina	1914-2009	0.82 [0.72; 0.93]	0.86 [0.77; 0.99]
Brazil	1934-2014	0.94 [0.84; 1.02]	1.01 [0.97; 1.03]
Canada	1926-2006	0.77 [0.62; 0.95]	0.80 [0.66; 1.00]
	1967-2017	0.38 [0.18; 0.57]	0.38 [0.19; 0.57]
Colombia	1960-2017	0.89 [0.76; 1.02]	0.91 [0.79; 1.02]
Guatemala	1980-2017	0.56 [0.29; 0.85]	0.58 [0.31; 0.89]
New Zealand	1934-2017	0.88 [0.78; 1.01]	0.91 [0.81; 1.01]
Switzerland	1948-2005	0.73 [0.56; 0.91]	0.81 [0.67; 1.00]
Bolivia	1980-2013	0.73 [0.53; 0.99]	0.72 [0.53; 0.97]
Israel	1983-2016	0.60 [0.34; 0.93]	0.62 [0.35; 0.95]
Mexico	1985-2014	0.75 [0.59; 0.96]	0.74 [0.55; 0.98]
Belgium	1946-1990	0.53 [0.32; 0.76]	0.56 [0.35; 0.79]
Belize	1977-2017	0.27 [0.23; 0.44]	0.26 [0.07; 0.44]
Austria	1970-1998	0.75 [0.41; 1.02]	1.01 [0.81; 1.04]
Bahrain	1980-2017	0.60 [0.36; 0.87]	0.57 [0.31; 0.91]
Barbados	1975-2016	1.17 [1.10; 1.48]	1.04 [1.00; 1.16]
Ecuador	1980-2011	0.94 [0.75; 1.03]	0.98 [0.78; 1.04]
Netherlands	1950-1992	0.64 [0.41; 1.00]	0.68 [0.45; 1.00]
South Korea	1970-2017	1.00 [0.84; 1.03]	0.90 [0.73; 1.02]
Thailand	1979-2016	0.67 [0.48; 0.88]	0.66 [0.46; 0.90]
Venezuela	1962-1999	1.00 [0.93; 1.04]	0.94 [0.75; 1.03]
Australia	1941-1989	0.71 [0.48; 1.01]	0.71 [0.48; 1.01]
	1969-2017	0.85 [0.65; 1.02]	0.84 [0.63; 1.02]
Chile	1940-1995	0.76 [0.58; 0.94]	0.76 [0.59; 0.98]
	1941-2017	0.84 [0.72; 0.99]	0.88 [0.78; 1.00]
Finland	1946-1985	0.36 [0.08; 0.64]	0.50 [0.22; 0.82]
Japan	1955-2017	0.92 [0.84; 1.01]	0.94 [0.85; 1.01]
Spain	1941-1989	0.84 [0.69; 1.01]	0.84 [0.68; 1.01]
Taiwan	1962-2017	0.84 [0.75; 0.96]	0.86 [0.76; 0.96]
Turkey	1968-2017	1.01 [0.90; 1.04]	1.01 [0.88; 1.04]
West Germany	1960-1989	0.38 [0.09; 0.71]	1.01 [0.85; 1.09]
Italy	1949-1996	0.96 [0.80; 1.03]	0.97 [0.84; 1.02]
Norway	1946-2014	1.00 [0.94; 1.02]	1.01 [0.98; 1.03]
Paraguay	1962-2015	0.58 [0.34; 0.89]	0.69 [0.48; 0.94]
Peru	1959-2017	0.93 [0.82; 1.02]	0.98 [0.89; 1.02]
Portugal	1914-1998	0.97 [0.91; 1.01]	0.97 [0.91; 1.02]
South Africa	1965-2015	0.92 [0.81; 1.01]	1.01 [0.98; 1.04]

[a] Median and 90% confidence interval (based on 10,000 bootstrap replications).

Table J.1 Bootstrapped p-values[a] for Johansen's maximum eigenvalue[b] tests, and empirical rejection frequencies of the tests under the null

		I: Bootstrapped p-values		II: Empirical rejection frequencies	
Country	*Period*	*Selden-Latané*	*Log-log*	*Selden-Latané*	*Log-log*
Brazil	1975Q1-1994Q2	0.004	0.002	0.991	0.995
Israel	1982Q1-2019Q2	0.006	0.042	0.856	0.746
Japan	1955Q2-2019Q2	0.027	0.018	0.587	0.646
New Zealand	1988Q2-2016Q4	0.525	0.673	0.325	0.248
Norway	1986Q1-2017Q1	0.413	0.700	0.230	0.280
Switzerland	1980Q1-2019Q2[c]	0.748	0.807	0.235	0.218

[a] Based on 10,000 bootstrap replications. [b] Null of 0 *versus* 1 cointegration vectors.
[c] Based on the log-log the sample period is 1980Q1-2011Q2, as we exclude observations for which the short rate is negative.

Table J.2 Results from Wright's (2000) tests

		Confidence interval			
		90% coverage		95% coverage	
Country	*Period*	*Selden-Latané*	*Log-log*	*Selden-Latané*	*Log-log*
Brazil	1975Q1-1994Q2	[-0.003; 0.001]	[-0.429; -0.393]	[-0.003; 0.001]	[-0.437; -0.385]
Israel	1982Q1-2019Q2	[-0.248; -0.051]	[-0.392; -0.332]	[-0.264; -0.035]	[-0.400; -0.324]
Japan	1955Q2-2019Q2	[-0.531; -0.327]	[-0.573; -0.124]	[-0.567; -0.294]	[-0.625; -0.052]
New Zealand	1988Q2-2016Q4	[-0.857; -0.221]	[-0.620; -0.072]	[-0.926; -0.149]	[-0.656; -0.004]
Norway	1986Q1-2017Q1	[-0.437; -0.157]	[-0.434; -0.122]	[-0.465; -0.113]	[-0.458; -0.046]
Switzerland	1980Q1-2019Q2[a]	[-0.657; -0.189]	[-0.393: -0.084]	[-0.709; -0.137]	[-0.433; -0.040]

[a] Based on the log-log the sample period is 1980Q1-2011Q2, as we exclude observations for which the short rate is negative.

Table K.1 Bootstrapped p-values[a] for Johansen's maximum eigenvalue[b] tests, based on consumption velocity

Country	*Period*	*Selden-Latané*	*Log-log*
United Kingdom	1922-2016	0.016	0.551
US – M1 + MMDAs	1929-2016	0.154	0.035
Argentina	1935-2004	0.023	0.145
Brazil	1947-2016	0.081	0.153
Canada	1967-2016	0.016	0.036
	1926-2006	0.160	0.472
Colombia	1960-2017	0.075	0.033
Guatemala	1980-2017	0.041	0.212
Switzerland	1948-2015	0.052	0.252
Israel	1983-2017	0.000	0.315
Belgium	1953-1990	0.071	0.131
Bahrain	1980-2014	0.862	0.848
Ecuador	1980-2007	0.389	0.455
South Korea	1970-2017	0.121	0.790
Venezuela	1962-1998	0.879	0.953
Australia	1969-2017	0.194	0.514
Chile	1941-2017	0.001	0.021
	1940-1995	0.111	0.488
Finland	1946-1985	0.136	0.143
Japan	1955-2016	0.585	0.514
Spain	1941-1989	0.071	0.198
Taiwan	1962-2017	0.057	0.540
West Germany	1960-1989	0.252	0.254
Italy	1949-1996	0.125	0.147
Norway	1946-2014	0.034	0.017
Paraguay	1962-2015	0.248	0.193
Peru	1959-2017	0.130	0.045
South Africa	1965-2015	0.237	0.397

[a] Based on 10,000 bootstrap replications.
[b] Null of 0 *versus* 1 cointegration vectors.

Table K.2 Results from Wright's (2000) tests based on consumption velocity: 90% confidence interval for the second element of the normalized cointegration vector, based on systems for (log) M1 velocity and (the log of) a short rate[a] (based on annual data)

		Confidence interval			
		90% coverage		95% coverage	
Country	*Period*	*Selden-Latané*	*Log-log*	*Selden-Latané*	*Log-log*
United Kingdom	1922-2016	[-0.273; -0.177]	NCD	[-0.281; -0.161]	[-0.327; 0.330]
US – M1 + MMDAs	1929-2016	[-0.297; -0.093]	[-0.316; -0.027]	[-0.317; -0.069]	[-0.356; 0.029]
Argentina	1935-2004	[-0.022; 0.022]	[-0.828; -0.375]	[-0.022; 0.022]	[-0.916; -0.287]
Brazil	1947-2016	[-0.043; 0.0007]	NCD	[-0.051; 0.009]	[-0.256; 0.721]
Canada	1967-2016	[-0.299; -0.243]	[-0.371; -0.318]	[-0.307; -0.235]	[-0.383; -0.302]
	1926-2006	[-0.737; -0.397]	[-0.613; -0.393]	[-0.793; -0.341]	[-0.645; -0.321]
Colombia	1960-2017	[-0.145; -0.049]	[-0.379; -0.023]	[-0.161; -0.033]	[-0.399; 0.033]
Guatemala	1980-2017	[-0.623; -0.355]	[-0.553; -0.329]	[-0.655; -0.323]	[-0.573; -0.301]
Switzerland	1948-2015	[-0.274; 0.071]	NCD	[-0.294; 0.119]	NCD
Israel	1983-2017	NCD	[-0.352; -0.336]	NCD	[-0.364; -0.324]
Belgium	1953-1990	[-0.226; -0.186]	NCD	[-0.238; -0.162]	NCD
Bahrain	1980-2014	[-0.286; 0.146]	[-0.277; 0.388]	[-0.314; 0.194]	[-0.313; 0.488]
Ecuador	1980-2007	NCD	[-0.348; 0.197]	NCD	[-1.048; 0.429]
South Korea	1970-2017	[-0.419; -0.371]	NCD	[-0.427; -0.355]	NCD
Venezuela	1962-1998	[-0.082; -0.050]	[-0.254; -0.010]	[-0.098; -0.038]	[-0.302; 0.074]
Australia	1969-2017	[-0.273; -0.212]	[-0.465; -0.253]	[-0.281; -0.204]	[-0.485; -0.165]
Chile	1941-2017	[-0.103; 0.030]	[-0.375; 0.158]	[-0.119; 0.046]	[-0.407; 0.230]
	1940-1995	NCD	[-0.389; -0.293]	NCD	[-0.409; -0.257]

[a] Based on 10,000 bootstrap replications. NCD = No cointegration detected.

Table K.2 (continued) Results from Wright's (2000) tests based on consumption velocity: 90% confidence interval for the second element of the normalized cointegration vector, based on systems for (log) M1 velocity and (the log of) a short rate[a] (based on annual data)

		Confidence interval			
		90% coverage		95% coverage	
Country	*Period*	*Selden-Latané*	*Log-log*	*Selden-Latané*	*Log-log*
Finland	1946-1985	[-0.542; -0.402]	[-2.445; -2.025]	[-0.562; -0.386]	[-2.549; -1.925]
Japan	1955-2016	[-0.248; -0.132]	NCD	[-0.260; -0.104]	[-0.340; -0.276]
Spain	1941-1989	[-0.171; -0.155]	NCD	[-0.187; -0.139]	[-0.392; -0.304]
Taiwan	1962-2017	NCD	NCD	NCD	[-0.407; -0.259]
West Germany	1960-1989	[-0.583; 0.638]	[-0.417; 0.836]	[-0.783; 0.843]	[-0.481; 1.013]
Italy	1949-1996	[0.000; 0.204]	[0.238; 0.814]	[-0.028; 0.232]	[0.194; 0.926]
Norway	1946-2014	[-0.464; 0.629]	NCD	[-0.692; 0.853]	NCD
Paraguay	1962-2015	NCD	[-0.287; -0.127]	NCD	[-0.307; -0.102]
Peru	1959-2017	[-0.030; 0.014]	[-0.446; 0.799]	[-0.034; 0.022]	[-0.567; 0.959]
South Africa	1965-2015	NCD	[0.152; 0.817]	[-0.064; 0.165]	[0.052; 1.229]

[a] Based on 10,000 bootstrap replications. NCD = No cointegration detected.

Table K.3 Bootstrapped p-values[a] for Johansen's maximum eigenvalue[b] tests for (log) M1 velocity and (the log of) a short-term rate, based on consumption velocity

Country	*Period*	Money demand specification: *Selden-Latané*	*Log-log*
United States	1959Q1-2017Q4	0.041	0.322
United Kingdom	1955Q1-2017Q2	0.083	0.707
Canada	1967Q1-2017Q4	0.025	0.005
Australia	1969Q3-2017Q4	0.035	0.447
Taiwan	1961Q3-2017Q4	0.000	0.083
South Korea	1964Q1-2017Q4	0.060	0.837
South Africa	1985Q1-2017Q4	0.162	0.222
Hong Kong	1985Q1-2017Q4	0.430	0.277
Israel	1982Q1-2019Q2	0.017	0.013
Japan	1955Q2-2019Q2	0.027	0.027
New Zealand	1988Q2-2016Q4	0.470	0.700
Norway	1986Q1-2017Q1	0.319	0.298
Switzerland	1980Q1-2019Q2[c]	0.669	0.690

[a] Based on 10,000 bootstrap replications. [b] Null of 0 *versus* 1 cointegration vectors. [c] Based on the log-log the sample period is 1980Q1-2011Q2, as we exclude observations for which the short rate is negative.

Table K.4 Results from Wright's (2000) tests based on consumption velocity: 90% confidence interval for the second element of the normalized cointegration vector, based on systems for (log) M1 velocity and (the log of) a short rate[a] (based on quarterly data)

		Money demand specification:	
Country	*Period*	*Selden-Latané*	*Log-log*
United States	1959Q1-2017Q4	[-0.307; -0.147]	[-0.241; -0.060]
United Kingdom	1955Q1-2017Q2	[-0.081; -0.044]	[-0.504; -0.208]
Canada	1967Q1-2017Q4	[-0.471; -0.355]	[-0.401; -0.321]
Australia	1975Q1-2017Q4	[-0.070; -0.066]	[-0.661; -0.393]
Taiwan	1961Q3-2017Q4	NCD	NCD
South Korea	1964Q1-2017Q4	[-0.105; -0.093]	[-0.698; -0.550]
South Africa	1985Q1-2017Q4	NCD	NCD
Hong Kong	1985Q1-2017Q4	[-0.134; -0.081]	[-0.410; -0.226]
Israel	1982Q1-2019Q2	[-0.030; -0.010]	[-0.346; -0.346]
Japan	1955Q2-2019Q2	[-0.215; -0.054]	NCD
New Zealand	1988Q2-2016Q4	[-0.182; -0.038]	[-1.327; 0.539]
Norway	1986Q1-2017Q1	[-0.047; -0.043]	[-0.598; -0.321]
Switzerland	1980Q1-2019Q2[c]	[-0.092; -0.056]	[-0.353; -0.177]

[a] Based on 10,000 bootstrap replications. NCD = No cointegration detected.

Table L.1 Bootstrapped p-values[a] for testing weak exogeneity, based on annual data

		Selden-Latané		*Log-log*	
		Testing weak exogeneity of:			
Country	*Period*	Velocity	Short rate	Velocity	Short rate
United Kingdom	1922-2016	0.042	0.003	0.065	0.332
US – M1 + MMDAs	1915-2017	0.437	0.002	0.124	0.023
Argentina	1914-2009	0.000	0.000	0.000	0.115
Brazil	1934-2014	0.002	0.030	0.014	0.021
Canada	1967-2017	0.311	0.004	0.300	0.004
	1926-2006	0.015	0.125	0.252	0.041
Colombia	1960-2017	0.002	0.122	0.002	0.407
Guatemala	1980-2017	0.000	0.493	0.001	0.328
New Zealand	1934-2017	0.159	0.007	0.003	0.097
Switzerland	1948-2005	0.001	0.173	0.017	0.243
Bolivia	1980-2013	0.064	0.010	0.055	0.380
Israel	1983-2016	0.000	0.000	0.031	0.007
Mexico	1985-2014	0.014	0.143	0.009	0.094
Belgium	1946-1990	0.427	0.001	0.092	0.032
Belize	1977-2017	0.496	0.021	0.458	0.457
Austria	1970-1998	0.189	0.007	0.101	0.007
Bahrain	1980-2017	0.178	0.099	0.070	0.097
Barbados	1975-2016	0.260	0.017	0.000	0.000
Ecuador	1980-2011	0.175	0.028	0.269	0.024
Netherlands	1950-1992	0.350	0.010	0.383	0.015
South Korea	1970-2017	0.478	0.002	0.324	0.175
Thailand	1979-2016	0.003	0.117	0.012	0.170
Venezuela	1962-1999	0.109	0.490	0.089	0.294
Australia	1969-2017	0.204	0.020	0.155	0.335
	1941-1989	0.499	0.030	0.274	0.023
Chile	1941-2017	0.011	0.199	0.012	0.495
	1940-1995	0.300	0.261	0.003	0.027
Finland	1946-1985	0.147	0.002	0.441	0.001
Japan	1955-2017	0.157	0.019	0.177	0.019
Spain	1941-1989	0.440	0.021	0.375	0.018
Taiwan	1962-2017	0.017	0.193	0.015	0.266
Turkey	1968-2017	0.047	0.095	0.076	0.096
West Germany	1960-1989	0.089	0.043	0.084	0.044
Italy	1949-1996	0.173	0.307	0.240	0.470
Norway	1946-2014	0.087	0.418	0.038	0.153
Paraguay	1962-2015	0.008	0.056	0.013	0.073
Peru	1959-2017	0.000	0.000	0.004	0.486
Portugal	1914-1998	0.002	0.336	0.004	0.104
South Africa	1965-2015	0.483	0.004	0.429	0.004

[a] Based on 10,000 bootstrap replications.

Table L.2 Bootstrapped p-values[a] for testing weak exogeneity, based on quarterly data

		Selden-Latané		*Log-log*	
		Testing weak exogeneity of:			
Country	*Period*	Velocity	Short rate	Velocity	Short rate
United States	1959Q1-2017Q4	0.301	0.002	0.047	0.045
United Kingdom	1955Q1-2017Q2	0.077	0.003	0.264	0.075
Canada	1967Q1-2017Q4	0.016	0.000	0.072	0.000
Australia	1969Q3-2017Q4	0.130	0.002	0.237	0.012
Taiwan	1961Q3-2017Q4	0.000	0.230	0.000	0.175
South Korea	1964Q1-2017Q4	0.435	0.001	0.011	0.026
South Africa	1985Q1-2017Q4	0.102	0.003	0.064	0.004
Hong Kong	1985Q1-2017Q4	0.304	0.012	0.165	0.010
Israel	1982Q1-2019Q2	0.002	0.003	0.249	0.026
Japan	1955Q2-2019Q2	0.006	0.098	0.004	0.182
New Zealand	1988Q2-2016Q4	0.182	0.021	0.227	0.034
Norway	1986Q1-2017Q1	0.075	0.043	0.036	0.050
Switzerland	1980Q1-2019Q2[b]	0.255	0.056	0.474	0.067

[a] Based on 10,000 bootstrap replications. [b] Based on the log-log the sample period is 1980Q1-2011Q2, as we exclude observations for which the short rate is negative.

Table L.3 90 per cent bootstrapped[a] confidence intervals for the error-correction coefficients, based on annual data

Country	*Period*	Equation for ΔV_t: coefficient on: V_{t-1}	Equation for ΔV_t: coefficient on: R_{t-1}	Equation for ΔR_t: coefficient on: V_{t-1}	Equation for ΔR_t: coefficient on: R_{t-1}
United Kingdom	1922-2016	-0.062 [-0.126; -0.002]	0.027 [0.001; 0.052]	0.858 [0.458; 1.341]	-0.387 [-0.605; -0.197]
US – M1 + MMDAs	1915-2017	0.079 [-0.016; 0.164]	-0.040 [-0.081; 0.007]	0.910 [0.598; 1.318]	-0.445 [-0.612; -0.313]
Argentina	1914-2009	-0.110 [-0.204; -0.039]	0.060 [0.030; 0.093]	0.289 [0.121; 0.523]	-0.166 [-0.279; -0.070]
Brazil	1934-2014	-0.010 [-0.082; 0.018]	0.028 [-0.001; 0.049]	0.036 [-0.269; 0.245]	-0.137 [-0.294; -0.033]
Canada	1967-2017	-0.075 [-0.236; 0.071]	0.043 [-0.041; 0.127]	1.195 [0.610; 1.806]	-0.682 [-1.027; -0.335]
	1926-2006	-0.111 [-0.162; -0.060]	0.141 [0.084; 0.189]	0.134 [0.030; 0.288]	-0.172 [-0.365; -0.037]
Colombia	1960-2017	-0.195 [-0.316; -0.078]	0.069 [0.034; 0.102]	0.067 [-0.386; 0.675]	-0.026 [-0.275; 0.118]
Guatemala	1980-2017	-0.517 [-0.735; -0.315]	0.295 [0.186; 0.419]	-0.031 [-0.596; 0.603]	0.017 [-0.387; 0.313]
New Zealand	1934-2017	-0.048 [-0.139; 0.002]	0.032 [-0.006; 0.063]	0.242 [0.003; 0.572]	-0.181 [-0.355; -0.043]
Switzerland	1948-2005	-0.051 [-0.188; 0.053]	0.021 [-0.024; 0.069]	1.183 [0.391; 2.236]	-0.501 [-0.863; -0.166]
Bolivia	1980-2013	-0.278 [-0.580; 0.015]	0.132 [-0.024; 0.274]	0.157 [-0.718; 0.998]	-0.085 [-0.553; 0.297]
Israel	1983-2016	0.435 [0.032; 0.840]	-0.180 [-0.323; -0.041]	2.493 [0.440; 4.678]	-1.018 [-1.769; -0.360]
Mexico	1985-2014	-0.535 [-0.891; -0.189]	0.174 [0.051; 0.305]	-0.902 [-2.075; 0.527]	0.281 [-0.237; 0.668]
Belgium	1946-1990	-0.003 [-0.152; 0.143]	0.002 [-0.100; 0.101]	0.924 [0.469; 1.471]	-0.633 [-0.988; -0.321]
Belize	1977-2017	-0.007 [-0.273; 0.151]	0.004 [-0.140; 0.152]	0.511 [0.110; 0.937]	-0.399 [-0.690; -0.113]
Austria	1970-1998	-0.006 [-0.141; 0.026]	0.006 [-0.009; 0.017]	0.370 [-3.297; 3.405]	-0.436 [-0.781; -0.137]
Bahrain	1980-2017	-0.264 [-0.725; 0.107]	0.137 [-0.095; 0.347]	0.528 [-0.220; 1.168]	-0.310 [-0.682; 0.061]
Barbados	1975-2016	-0.051 [-0.156; 0.019]	0.098 [-0.043; 0.227]	0.254 [0.083; 0.502]	-0.483 [-0.782; -0.233]
Ecuador	1980-2011	0.124 [-0.218; 0.334]	-0.079 [-0.215; 0.079]	0.751 [0.165; 1.379]	-0.430 [-0.769; -0.157]
Netherlands	1950-1992	-0.064 [-0.317; 0.087]	-0.042 [-0.088; 0.014]	-0.170 [-1.014; 1.025]	-0.217 [-0.524; -0.005]

[a] Based on 10,000 bootstrap replications.

Table L.3 (continued) 90 per cent bootstrapped[a] confidence intervals for the error-correction coefficients, based on annual data

Country	*Period*	Equation for ΔV_t: coefficient on V_{t-1}	Equation for ΔV_t: coefficient on R_{t-1}	Equation for ΔR_t: coefficient on V_{t-1}	Equation for ΔR_t: coefficient on R_{t-1}
South Korea	1970-2017	0.074 [-0.074; 0.214]	-0.048 [-0.132; 0.045]	1.101 [0.649; 1.654]	-0.686 [-0.971; -0.448]
Thailand	1979-2016	-0.336 [-0.743; -0.033]	0.199 [0.027; 0.382]	0.322 [-0.299; 0.865]	-0.220 [-0.543; 0.111]
Venezuela	1962-1999	-0.324 [-0.667; -0.014]	0.063 [-0.094; 0.224]	-0.203 [-0.697; 0.302]	0.013 [-0.271; 0.155]
Australia	1969-2017	-0.132 [-0.305; -0.002]	0.063 [-0.004; 0.114]	0.272 [-0.221; 0.823]	-0.157 [-0.429; 0.052]
	1941-1989	-0.164 [-0.360; 0.006]	0.132 [-0.007; 0.274]	0.382 [0.109; 0.708]	-0.317 [-0.579; -0.086]
Chile	1941-2017	-0.067 [-0.135; -0.014]	0.043 [0.015; 0.066]	0.229 [0.040; 0.560]	-0.160 [-0.349; -0.031]
	1940-1995	-0.507 [-0.782; -0.282]	0.100 [0.037; 0.175]	-0.926 [-1.533; -0.339]	0.176 [0.048; 0.316]
Finland	1946-1985	-0.094 [-0.229; 0.012]	0.049 [-0.008; 0.104]	1.078 [0.547; 1.772]	-0.558 [-0.856; -0.298]
Japan	1955-2017	-0.043 [-0.108; 0.006]	0.024 [-0.005; 0.052]	0.301 [-0.006; 0.693]	-0.179 [-0.386; -0.004]
Spain	1941-1989	-0.197 [-0.360; -0.063]	0.031 [0.012; 0.051]	1.488 [0.318; 3.029]	-0.245 [-0.505; -0.049]
Taiwan	1962-2017	-0.278 [-0.455; -0.142]	0.091 [0.045; 0.148]	-0.175 [-0.584; 0.214]	0.055 [-0.085; 0.173]
Turkey	1968-2017	-0.268 [-0.493; -0.078]	0.108 [0.000; 0.226]	-0.220 [-0.475; 0.147]	0.074 [-0.129; 0.172]
West Germany	1960-1989	-0.060 [-0.325; 0.023]	-0.034 [-0.090; 0.008]	-0.760 [-2.565; 0.503]	-0.428 [-0.742; -0.193]
Italy	1949-1996	0.024 [-0.044; 0.065]	0.008 [-0.006; 0.018]	-0.848 [-2.241; 0.022]	-0.228 [-0.416; -0.112]
Norway	1946-2014	0.039 [-0.035; 0.081]	-0.044 [-0.078; 0.005]	0.139 [-0.007; 0.396]	-0.139 [-0.299; -0.041]
Paraguay	1962-2015	-0.091 [-0.227; -0.003]	0.037 [0.003; 0.066]	0.622 [0.093; 1.362]	-0.281 [-0.527; -0.080]
Peru	1959-2017	-0.035 [-0.103; 0.011]	0.044 [0.027; 0.066]	0.076 [-0.072; 0.330]	-0.124 [-0.326; -0.017]
Portugal	1914-1998	-0.119 [-0.252; -0.006]	0.031 [-0.024; 0.098]	-0.060 [-0.181; 0.096]	0.006 [-0.084; 0.049]
South Africa	1965-2015	-0.003 [-0.081; 0.026]	0.002 [-0.025; 0.033]	0.382 [-0.150; 0.951]	-0.283 [-0.441; -0.171]

[a] Based on 10,000 bootstrap replications.

Table L.4 90 per cent bootstrapped[a] confidence intervals for the loading coefficients, based on quarterly data

		Equation for ΔV_t: coefficient on:		Equation for ΔR_t: coefficient on:	
Country	*Period*	V_{t-1}	R_{t-1}	V_{t-1}	R_{t-1}
United States	1959Q1-2017Q4	0.011 [-0.015; 0.031]	-0.006 [-0.016; 0.007]	0.296 [0.169; 0.464]	-0.148 [-0.212; -0.102]
United Kingdom	1955Q1-2017Q2	-0.011 [-0.031; 0.003]	0.001 [-0.001; 0.003]	0.878 [0.436; 1.464]	-0.110 [-0.177; -0.055]
Canada	1967Q1-2017Q4	-0.037 [-0.055; -0.022]	0.038 [0.027; 0.051]	0.066 [0.022; 0.1296]	-0.070 [-0.131; -0.023]
Australia	1969Q3-2017Q4	-0.025 [-0.068; -0.001]	0.012 [0.000; 0.024]	0.107 [-0.027; 0.256]	-0.061 [-0.127; -0.006]
Taiwan	1961Q3-2017Q4	-0.051 [-0.086; -0.025]	0.006 [0.003; 0.009]	-0.020 [-0.322; 0.322]	0.002 [-0.040; 0.033]
South Korea	1964Q1-2017Q4	-0.009 [-0.044; 0.022]	0.001 [-0.003; 0.006]	1.039 [0.668; 1.569]	-0.144 [-0.212; -0.096]
South Africa	1985Q1-2017Q4	-0.026 [-0.080; 0.003]	0.008 [-0.002; 0.021]	0.224 [0.072; 0.432]	-0.073 [-0.128; -0.034]
Hong Kong	1985Q1-2017Q4	-0.009 [-0.047; 0.018]	0.002 [-0.005; 0.010]	1.030 [0.356; 1.956]	-0.254 [-0.438; -0.113]
Israel	1982Q1-2019Q2	-0.027 [-0.071; 0.017]	0.011 [-0.008; 0.028]	0.332 [0.147; 0.592]	-0.138 [-0.239; -0.061]
Japan	1955Q2-2019Q2	-0.016 [-0.030; -0.006]	0.008 [0.005; 0.013]	0.077 [0.030; 0.153]	-0.042 [-0.073; -0.021]
New Zealand	1988Q2-2016Q4	-0.014 [-0.068; 0.009]	0.003 [-0.003; 0.009]	0.433 [-0.125; 1.065]	-0.095 [-0.183; -0.025]
Norway	1986Q1-2017Q1	-0.051 [-0.150; 0.000]	0.003 [0.000; 0.007]	0.992 [-0.292; 2.600]	-0.072 [-0.151; -0.010]
Switzerland	1980Q1-2019Q2	-0.010 [-0.062; 0.008]	0.004 [-0.005; 0.015]	0.173 [-0.225; 0.525]	-0.086 [-0.194; 0.008]

[a] Based on 10,000 bootstrap replications.

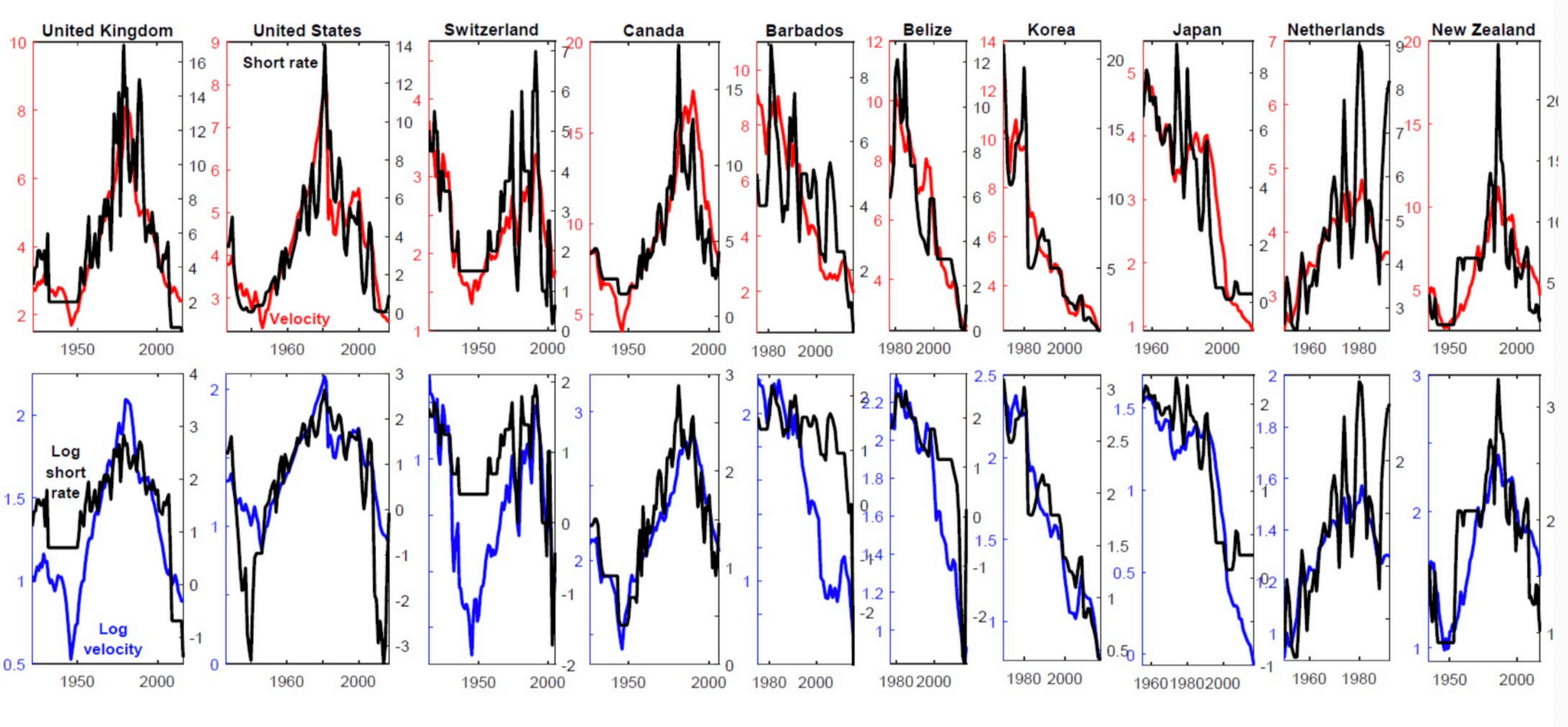

Figure I.1 Comparing the Selden-Latané and log-log specifications: selected evidence for low-inflation countries

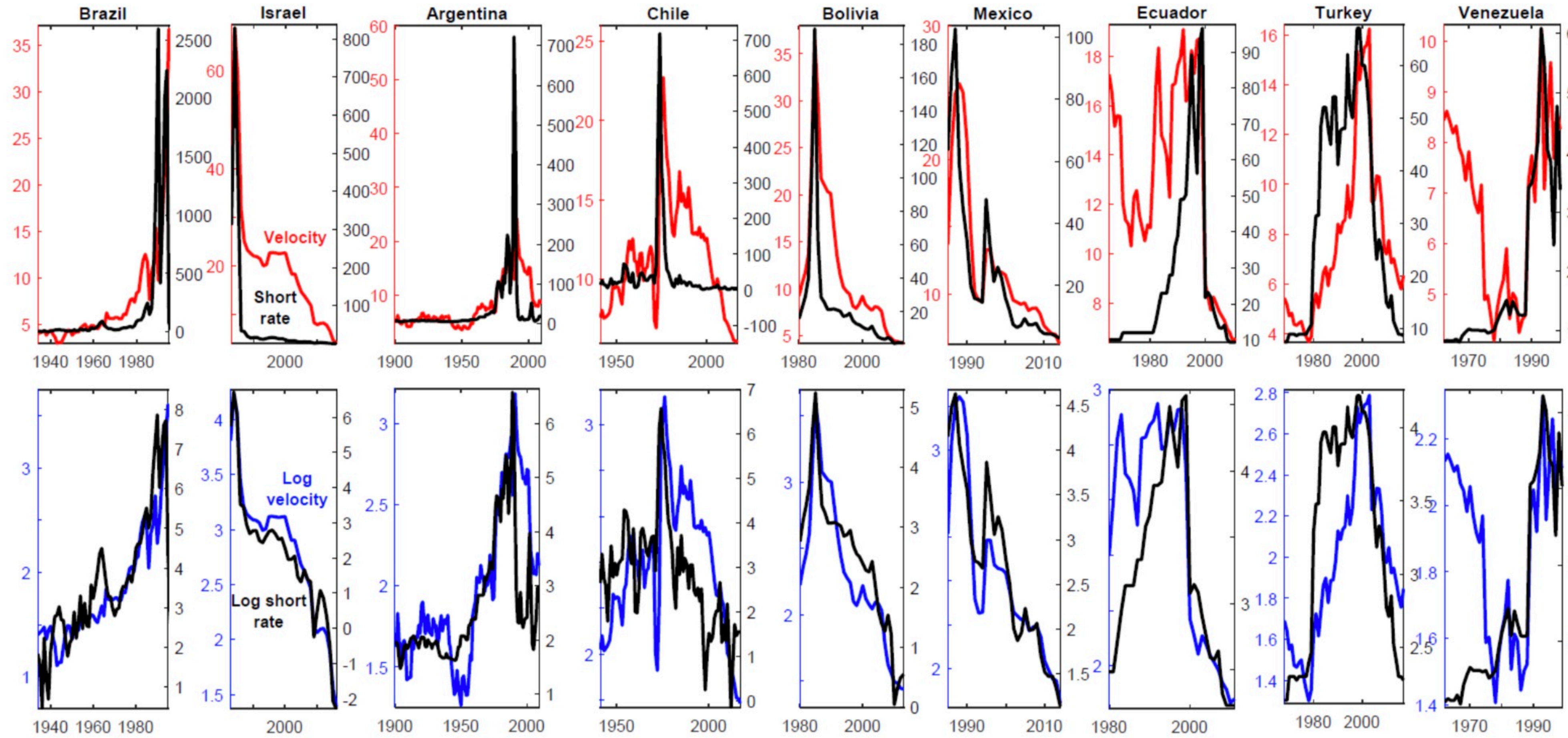

Figure I.2 Comparing the Selden-Latané and log-log specifications: selected evidence for high-inflation countries