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Liquidity, the Mundell-Tobin Effect, and the Friedman Rule*

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Abstract

We investigate how the Mundell-Tobin effect, i.e., a positive relation between inflation and capital investment, changes the optimal monetary policy prescription in a framework that combines overlapping generations and new monetarist models. We find that the Friedman rule is optimal if and only if there is no Mundell-Tobin effect. A Mundell-Tobin effect is more likely to occur at the Friedman rule if capital is relatively liquid, and if the agents' risk aversion is relatively low. If the Friedman rule is not optimal, the optimal money growth rate lies between the Friedman rule and a constant money stock. We also show that it is more efficient to implement deflationary monetary policies by raising lump-sum taxes on old agents only.

Keywords: *New monetarism, overlapping generations, optimal monetary policy*

JEL codes: E4, E5

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1 Introduction

When it comes to optimal monetary policy, there is a stark contrast between central bankers and monetary theorists. Most central banks in developed economies follow an inflation target of around 2% annually, and there is a general agreement among central bankers that deflation has to be avoided. Meanwhile, most theoretical models find that the Friedman rule, i.e., setting the inflation rate such that the opportunity cost of holding money balances is zero, is the optimal monetary policy. Since zero opportunity costs for holding money implies deflation in standard models, this prediction clearly differs from what central bankers believe to be optimal. The Friedman rule has been found to be optimal by Friedman himself in a model with money in the utility (Friedman, 1969), but also in a variety of other monetary models such as cash-in-advance (Grandmont and Younes, 1973; Lucas and Stokey, 1987), spatial separation (Townsend, 1980), and New Monetarism (Lagos and Wright, 2005).[1] While there have been alterations of these models that render the Friedman rule suboptimal, these usually rely on additional frictions that do not seem to be major concerns for central bankers in reality.[2]

One potential reason to run inflation above the Friedman rule is to stimulate investment. This mechanism is captured by the Mundell-Tobin effect (Mundell (1963) and Tobin (1965)). The Mundell-Tobin effect predicts that an increase in the return on nominal assets such as bonds or fiat money (e.g., a reduction in inflation) crowds out capital investment. However, inflation above the Friedman rule reduces people's willingness to hold liquid assets. If certain trades can only be settled with liquid assets, higher inflation rates thus reduce quantities traded. This implies that there is a tradeoff between the benefits of a high return on liquid assets and the costs associated with reduced capital investment due to the Mundell-Tobin effect. We investigate this tradeoff in a model that combines the overlapping generations (OLG) framework a la Wallace (1980) with a New Monetarist model a la Lagos and Wright (2005) (LW). This approach allows us to find novel results regarding both of these literatures, and settle some debates, as we explain in the literature review below. In particular, we find that the Friedman rule is optimal in our model if and only if there is no Mundell-Tobin effect. If there is a Mundell-Tobin effect at the Friedman rule, an increase in inflation leads to a first-order welfare gain from reducing hours worked, and only to a second-order welfare loss from reducing consumption in markets where liquidity matters. Further, we can show that a Mundell-Tobin effect is more likely to occur at the Friedman rule if agents care less about consumption smoothing, and if capital is relatively liquid.

[1] See Schmitt-Grohé and Uribe (2010) for an overview.

[2] E.g. for the New Monetarist literature: theft as in Sanches and Williamson (2010), incomplete tax instruments as in Aruoba and Chugh (2010), or socially undesirable activities financed by cash as in Williamson (2012).

In our model, each period is divided into two subperiods, called CM and DM. Agents are born at the beginning of the CM and live until the end of the CM of the following period; i.e., they are alive for three subperiods. There are two assets in the economy, productive capital and fiat money. When agents are born, they immediately learn whether they will be a buyer or a seller during the DM. During the first CM of their lives, all agents can work at linear disutility and accumulate capital and fiat money. In the DM, sellers can work at linear disutility and produce a DM good. Buyers cannot work during the DM, but they get concave utility from consuming the DM good. With some probability, buyers are relocated during the DM. If they are relocated, they can only use fiat money to settle trades, because we assume that capital is immobile and cannot be moved to different locations. If buyers are not relocated, they can use money and capital to purchase goods from sellers. This relocation shock follows Townsend (1987). Sellers are never relocated. During the final CM of their lives, buyers return to their original location and have access to all their remaining assets. Both buyers and sellers receive concave utility from consuming during the final CM of their lives. Monetary policy is implemented either by paying transfers to / raising taxes from the young or the old agents. The relocation shock creates a tradeoff between money and capital. Since capital pays a higher return than money unless the central bank runs the Friedman rule, capital is better suited as a store of value. However, because buyers can use money in the DM even if they are relocated, money is more liquid than capital - and the probability of relocation is a measure of the liquidity of capital, with capital being more liquid if relocation is less likely. As mentioned above, this setup blends standard OLG and LW frameworks - as in standard LW models, buyers and sellers trade with each other during the DM, and as in standard OLG models, young and old agents trade with each other during the CM. Having these two kinds of trades allows us to separate two properties of assets: Liquidity and store of value. In traditional OLG models with relocation, these two cannot easily be separated.

We first study a benchmark case where all buyers are relocated, meaning that capital is perfectly illiquid. In this version of the model, buyers face no tradeoff between money and capital, as capital can never be used to provide DM consumption, but since it (weakly) dominates in terms of rate of return, it is more useful to provide CM consumption. We show that in this case, CM consumption levels are independent of monetary policy and at the first-best level, and running the Friedman rule allows to implement the first-best consumption levels in the DM, but keeps the level of capital accumulation strictly below first best. Further, the level of capital accumulation is inversely related to aggregate hours worked in the CM, so hours worked in the CM are strictly above first best at the Friedman rule.

In the model with full relocation, there are two channels through which inflation affects capital accumulation: on the one hand, capital accumulation increases in inflation, because sellers are aware that they can sell less goods in the DM at higher inflation rates, so they accumulate more capital to provide for their CM consumption (*seller channel*); on the other hand, capital accumulation decreases in inflation since buyers hold less capital for CM consumption and tax payments if the real tax payment gets lower (*transfer channel*). The transfer channel is only active when old agents are taxed, so there is always a Mundell-Tobin effect when monetary policy is implemented by taxing the young agents, and it turns out that a constant money stock is optimal in this case. For any deflationary policy, the welfare loss from the reduction in capital accumulation (and therefore the increase in hours worked) is larger than the gains from increasing the consumption levels in the DM. If old agents are taxed instead, whether or not there is a Mundell-Tobin effect depends on the relative strength of the two channels, which is determined by the agents' preferences. If the elasticity of DM consumption is above one, there is a Mundell-Tobin effect, while if it is below one, there is actually a reverse Mundell-Tobin effect, meaning that capital accumulation decreases with inflation. Conversely, inflation reduces (increases) total hours worked in the CM if the elasticity is above (below) one. From this, it follows that the Friedman rule is optimal if the old are taxed and the elasticity of DM consumption is below one; if the elasticity is above one, the optimal money growth rate lies somewhere between the Friedman rule and one, and it is an increasing function of the elasticity of DM consumption in that interval. We also show that for any deflationary policy, welfare is higher if monetary policy is implemented over old buyers only. The reason is that under a deflationary policy, if buyers are taxed when old they can use the return on capital to pay parts of the tax.

Next, we analyze the full model with partially liquid capital, which means buyers face a trade-off between money and capital regarding DM consumption. With money, buyers can always trade in the DM, but they need to forego some return if money growth is above the Friedman rule. By running the Friedman rule, the monetary authority is able to perfectly insure agents against the relocation shock, but then all DM trades are made with money, even though capital would be accepted in some of them. This adds a third channel through which inflation affects capital accumulation, which we call the *liquidity channel*. The higher the liquidity of capital, the more willingly buyers switch to accumulating capital instead of real balances if the return on money decreases. Because the liquidity channel strengthens the Mundell-Tobin effect, the Mundell-Tobin effect is more likely to occur at the Friedman rule for lower π, and in turn this makes it less likely that the Friedman rule is the optimal monetary policy, even if the old are taxed. In the limit where

$\pi \to 0$, the Friedman rule is never optimal.

From the policymaker's point of view, the fundamental tradeoff is that the Friedman rule delivers efficiency in the DM, but a constant money stock is optimal regarding the CM. The reason for the latter point is that there are two ways to provide for (old-age) CM consumption: accumulation of capital when young, or transfers from young to old agents. The inherent return of such transfers is one, while the return of capital is larger than one; thus, the planner prefers capital accumulation whenever possible. If capital is not fully liquid, some DM trades need to be made with money, which implies that some of the sellers' CM consumption has to be financed with money. However, purchasing CM goods with money implies intergenerational transfers, as only young agents want to acquire money. Thus, setting the return on money equal to one, which can be achieved with a constant money stock, reflects the social return of using money (and thus intergenerational transfers) to acquire CM consumption. Running a constant money stock increases the price of DM consumption relative to the Friedman rule, and this increase in DM prices correctly reflects the externality of financing some of the sellers' consumption through intergenerational transfers - but this increase in the DM price also inefficiently lowers DM consumption. Thus, there is no single money growth rate that allows for (constrained) efficiency in both markets, and instead the optimal money growth rate depends on the probability that agents need to use money in the DM, and how they value DM consumption relative to labor disutility.

Existing literature. Aruoba and Wright (2003) is one of the earliest papers that includes capital in a LW framework. There, capital accumulation is independent of monetary policy if capital is fully illiquid. In our model, this is not true: because the OLG framework allows us to drop quasilinear preferences, capital accumulation is affected by monetary policy even in the version of our model with fully illiquid capital. Lagos and Rocheteau (2008) show that the Mundell-Tobin effect exists in LW models when capital is liquid. However, the Friedman rule still delivers the first-best outcome in their model, i.e., it simultaneously delivers optimal capital investment and efficient allocations in trades that require liquid assets. In Aruoba et al. (2011), capital reduces the cost of sellers to produce the DM good. They show that capital accumulation is affected more strongly by inflation if there is price-taking in the DM. Andolfatto et al. (2016) show that if taxes cannot be enforced and therefore the Friedman rule is not feasible, the first-best allocation can be implemented with a cleverly designed mechanism even if the capital stock is too small. In Wright et al. (2018, 2019), the authors study models where capital is traded in frictional markets, and they show that if money is needed to purchase capital, a reverse Mundell-Tobin effect can occur. In our model, a reverse Mundell-Tobin effect can also occur for some parameters, but due to preferences, not frictional markets. Gomis-Porqueras et al. (2020) show that there is a hump-shaped relationship between inflation and aggregate capital, as inflation affects capital accumulation negatively on the extensive margin by reducing the number of firms, besides the usual positive effect on the intensive margin.

There have been a few papers that find deviations from the Friedman rule to be optimal due to the Mundell-Tobin effect - e.g. Venkateswaran and Wright (2013), Geromichalos and Herrenbrueck (2017), Wright et al. (2018), or Altermatt (2019a). However, in these papers there is usually an additional friction that leads to underinvestment at the Friedman rule, e.g., limited pledgeability, taxes, or wage bargaining. If these frictions are shut down in the papers mentioned, the Mundell-Tobin effect still exists, but the Friedman rule is optimal.

In the OLG literature following Wallace (1980), the Mundell-Tobin effect has also been studied. Azariadis and Smith (1996) show that if there is private information about an agent's type, a Mundell-Tobin effect exists for low levels of inflation, while a reverse Mundell-Tobin effect exists for high levels of inflation. In their model, agents are either borrowers or lenders, and higher inflation makes bank deposits a relatively less attractive means of saving, which increases a savers' value of misrepresenting his type and defaulting on a loan. To prevent this, banks ration loans, which depresses the borrowers' ability to accumulate capital. In models with relocation shocks, Smith (2002, 2003) and Schreft and Smith (2002) have claimed to show that the Friedman rule is suboptimal because of the Mundell-Tobin effect.[3] However, OLG models typically find deviations from the Friedman rule to be optimal even without the Mundell-Tobin effect[4], as in Weiss (1980), Abel (1987), or Freeman (1993). Bhattacharya et al. (2005) and Haslag and Martin (2007) build on these results to show that the results in Smith (2002) and the other papers mentioned are not driven by the Mundell-Tobin effect, but by the standard properties of the OLG models. The debate whether the Mundell-Tobin effect itself can render deviations from the Friedman rule to be optimal in an OLG environment thus remained unsettled.

Zhu (2008) was the first to combine the OLG and LW structures. In his model, agents do not know their type during the first CM when they are able to accumulate assets. Therefore, the

[3]See also Schreft and Smith (1997), which focuses on positive inflation rates, but endogenizes the return on capital.

[4]There is a further complication in the welfare analysis of OLG models due to the absence of a representative agent. Freeman (1993) shows that the Friedman rule is typically Pareto optimal, but not maximizing steady state utility in OLG models. In this paper, we are going to focus on steady-state optimality when analyzing optimal policies in OLG models.

Friedman rule can be suboptimal for some parameters, as it makes saving relatively cheap and therefore reduces the sellers' willingness to produce in the DM. In contrast to this, our model follows Altermatt (2019b) by assuming that each agent knows his type. Hiraguchi (2017) extends the model of Zhu (2008) by including capital and shows that the Friedman rule remains suboptimal in this case. In another recent paper that combines OLG and LW, Huber and Kim (2020) show that the Friedman rule can be suboptimal if old agents face a higher disutility of labor than young agents.

We think that our paper contributes to the existing literature in a number of important ways. First, it is able to reconcile Smith (2002) with Bhattacharya et al. (2005) and Haslag and Martin (2007), by showing that even in a model with an OLG structure, the Friedman rule can be optimal for some parameters, but that it is never optimal when there is a Mundell-Tobin effect. Second, our paper shows that in a New Monetarist model without quasilinear preferences, capital accumulation is affected by monetary policy even if capital is illiquid, and capital accumulation can be inefficiently low at the Friedman rule due to the Mundell-Tobin effect. Third, we made some advances in understanding the frictions that arise in a model that combines OLG and LW, most importantly by showing that the timing of monetary policy implementation matters for welfare in these kind of models.

Outline. The rest of this paper is organized as follows. In Section 2, the environment and the planner's solution is explained. In Section 3, we present the market outcome for perfectly liquid capital, and in Section 4, we discuss the market outcome for perfectly illiquid capital and monetary policy implementation. Section 5 presents the results of the full model, and finally, Section 6 concludes.

2 The model

Our model is a combination of the environment in Lagos and Wright (2005), and the overlapping generations model (OLG) with relocation shocks from Townsend (1987), as used by Smith (2002). Time is discrete and continues forever. Each period is divided into two subperiods, called the decentralized market (DM) and the centralized market (CM). There are two distinct locations, which we will sometimes call islands. The two locations are completely symmetric, and everything we describe happens simultaneously on both islands. At the beginning of a period, the CM takes place, and after it closes, the DM opens and remains open until the period ends. At the beginning of each period, a new generation of agents is born, consisting of one unit mass per island each of buyers and sellers. An agent born in period t lives until the end of the CM in period $t+1$. Each

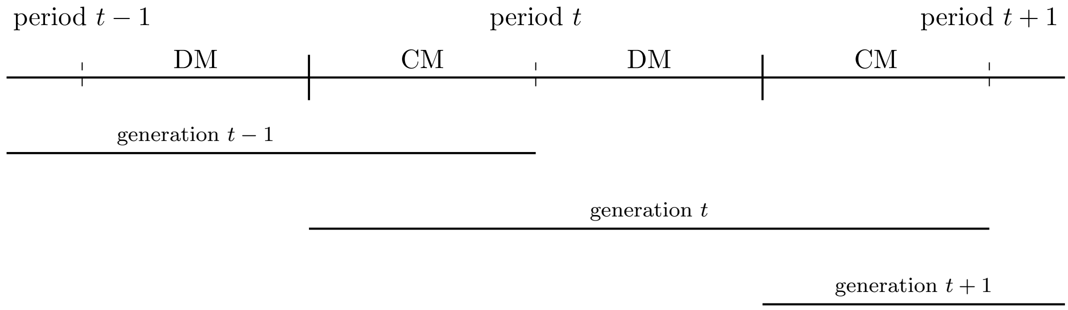

Figure 1: Timeline with lifespans of generations.

generation is named after the period it is born in. Figure 1 gives an overview of the sequence of subperiods and the lifespans of generations. There is also a a monetary authority.

Both buyers and sellers are able to produce a general good x during the first CM of their life at linear disutility h, whereas incurring the disutility h yields exactly h units of general goods; buyers and sellers also both receive utility from consuming the general good during the second CM of their life. During the DM, sellers are able to produce special goods q at linear disutility; buyers receive utility from consuming these special goods.

The preferences of buyers are given by

$$\mathbb{E}_t\{-h_t^b + u(q_t^b) + \beta U(x_{t+1}^b)\}. \tag{1}$$

Equation (1) states that buyers discount the second period of their life by a factor $\beta \in (0,1)$, gain utility $u(q)$ from consuming the special good in the DM and $U(x)$ from consuming the general good in the CM, with $u(0) = 0$, $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$, $U(0) = 0$, $U'(x) > 0$, $U''(x) < 0$, $U'(0) = \infty$, and linear disutility h from producing the general good during their first CM.[5] The preferences of the sellers are

$$-h_t^s - q_t^s + \beta U(x_{t+1}^s). \tag{2}$$

Sellers also discount the second period of their life by a factor β, gain utility $U(x)$ from consuming in the CM and disutility q from producing in the DM, with $\bar{q} = u(\bar{q})$ for some $\bar{q} > 0$.

During the CM, general goods can be sold or purchased in a centralized market. During the DM, special goods are sold in a centralized market. A fraction π of buyers are relocated during the DM, meaning that they are transferred to the other island without the ability to communi-

[5]We also assume strictly convex marginal utility in the CM, i.e. $U'''(x) > 0$. Most commonly used utility functions satisfy this assumption and it simplifies the proofs for the last section.

cate with their previous location. Sellers are not relocated, and during the CM, no relocation occurs for both types of agents. Relocated buyers are transferred back to their original location for the final CM of their life.[6] Relocation occurs randomly, so for an individual agent, the probability of being relocated is π. Buyers learn at the beginning of the DM whether they are relocated or not.

The monetary authority issues fiat money M_t, which it can produce without cost. The monetary authority always implements its policies at the beginning of the CM. The amount of general goods that one unit of fiat money can buy in the CM of period t is denoted by ϕ_t. The gross inflation rate is defined as ϕ_t/ϕ_{t+1}, and the growth rate of fiat money from period $t-1$ to t is $\frac{M_t}{M_{t-1}} = \gamma_t$. Monetary policy is implemented by issuing newly printed fiat money either to young or to old buyers via lump-sum transfers (or lump-sum taxes in the case of a decreasing money stock).[7] We denote transfers to young buyers as τ^y, and transfers to old buyers as τ^o. Furthermore, we will use an indicator variable $\mathcal{I}$ to denote the regime, i.e., which generation is taxed. If $\mathcal{I} = 1$ ($\mathcal{I} = 0$), young buyers (old buyers) are taxed, which means τ^y (τ^o) is set such that the money growth rate γ_t chosen by the monetary authority can be implemented, while $\tau^o = 0$ ($\tau^y = 0$).

Besides fiat money, there also exists capital k in this economy. During the CM, agents can transform general goods into capital. One unit of capital delivers $R > 1$ units of real goods in the CM of the following period. Capital is immobile, meaning that it is impossible to move capital to other locations during the DM. It is also impossible to create claims on capital that can be verified by other agents. We will assume throughout the paper that

$$R\beta = 1. \tag{3}$$

As we will see in the planner's problem below, this assumption implies that accumulating capital is more efficient than intergenerational transfers at financing old-age consumption.[8] It also

[6] In Smith (2002), each agent lives only for two periods. Relocation occurs during the last period of an agent's life, meaning that all assets that he cannot spend during that period are wasted from his point of view. Our model crucially differs from Smith (2002) in that regard, as our agents have access to all their assets during the final period of their life.

[7] As we will show in this paper, the exact timing of the lump-sum taxes is irrelevant for consumption allocations, but not for welfare. Assuming that only buyers are taxed is without loss of generality.

[8] For all our results to go through, $R > 1$ actually suffices. However, assuming $R\beta = 1$ has the added benefit that two common definitions of the Friedman rule coincide, i.e., the Friedman rule is given by $\gamma = \beta = \frac{1}{R}$. In LW models, the Friedman rule is typically defined as setting the money growth rate equal to the discount factor ($\gamma = \beta$), while in OLG models, the Friedman rule is typically defined as setting the rate of return on money equal to the return on other assets in the economy ($\frac{1}{\gamma} = R$). We think that the second definition is the right one in the context of our model, as it fits most closely the original definition of setting the opportunity cost of holding money to zero, and since $\frac{1}{R}$ shows up in our relevant first-order conditions, not β. But to make it clear that our results don't depend

implies that capital is a good investment, which introduces a tradeoff between liquid money and non-(or partially)-liquid capital in terms of return.

A Mundell-Tobin effect means that capital investments are increasing with inflation. Thus we speak of a Mundell-Tobin effect in the model if $\frac{\partial K}{\partial \gamma} > 0$, where $K = k^b + k^s$ are the total capital investments of buyers and sellers. Conversely, we speak of a reverse Mundell-Tobin Effect if $\frac{\partial K}{\partial \gamma} < 0$ i.e., if total capital decreases in inflation.

2.1 Planner's problem

For the planner's problem, we focus on maximizing steady-state welfare of a representative generation, while ignoring the initial old. By doing so, we follow papers like Smith (2002) and Haslag and Martin (2007), as we want to compare our results to theirs.

The planner maximizes the utility of a representative generation, which is given by

$$V^g = -h^b - h^s - q^s + \pi\left(u(q^m) + \beta U(x^m)\right) + (1-\pi)\left(u(q^b) + \beta U(x^b)\right) + \beta U(x^s)), \tag{4}$$

where a superscript m denotes consumption of relocated buyers (movers). DM-consumption of buyers must be financed by transfers from sellers, so $\pi q^m + (1-\pi)q^b = q^s$. To finance CM-consumption, the planner has two possibilities. Either the young agents work for the old and CM consumption is financed by transfers, or young agents work in order to invest in capital, and consume the returns when old. The implied return of an intergenerational transfer is 1, as when goods are taken from young agents and given to old agents, the additional goods produced by a young agent of a representative generation equal the additional goods consumed by an old agent of a representative generation. Since $R > 1$ from (3), it requires strictly less work to provide the same amount of CM consumption through capital investment instead of intergenerational transfers, so a planner wants to finance all CM consumption through capital investment. Taking this into account and using $H = h^b + h^s$ to denote total labor supply, the planner's problem is

$$\begin{aligned}
\max_{H,K,q^b,q^m,q^s,x^b,x^m,x^s} \; & -H - q^s + \pi\left(u(q^m) + \beta U(x^m)\right) + (1-\pi)\left(u(q^b) + \beta U(x^b)\right) + \beta U(x^s)) \\
s.t. \quad & \pi q^m + (1-\pi)q^b = q^s \\
& H = K
\end{aligned}$$

on a debatable definition of the Friedman rule, we assume $R\beta = 1$ throughout the paper.

$$\pi x^m + (1-\pi)x^b + x^s = RK.$$

Thus the first-best levels of DM and CM consumption q^* and x^*, hours worked H^* and capital investment K^* solve:

$$q^b = q^m = q^s = q^* \quad \text{solving} \quad u'(q^*) = 1 \tag{5}$$

$$x^b = x^m = x^s = x^* \quad \text{solving} \quad U'(x^*) = \frac{1}{\beta R} \tag{6}$$

$$H^* = K^* = \frac{2x^*}{R}. \tag{7}$$

We will later use these results as a benchmark to compare market outcomes against.[9]

2.2 Market outcomes

In the DM, special goods are sold in competitive manner.[10] Due to anonymity and a lack of commitment, all trades have to be settled immediately. Therefore, buyers have to transfer assets to sellers in order to purchase special goods. Because capital cannot be transported to other locations and claims on capital are not verifiable, relocated buyers can only use fiat money to settle trades. Nonrelocated buyers can use both fiat money and capital to purchase special goods. We will use p_t to denote the price of special goods in terms of fiat money. All buyers face the same price, regardless of their means of payment. As sellers are not relocated during the DM, all of them accept both fiat money and capital of nonrelocated buyers as payment. Because the problem is symmetric, we will only focus on one location for the remainder of the analysis.

Buyer's lifetime problem

A buyer's value function at the beginning of his life is given by

$$V^b = \max_{h_t, q_t^m, q_t^b, x_{t+1}^m, x_{t+1}^b} \; -h_t + \pi\big(u(q_t^m) + \beta U(x_{t+1}^m)\big) + (1-\pi)\big(u(q_t^b) + \beta U(x_{t+1}^b)\big)$$

$$s.t. \quad h_t + \mathcal{I}\tau_t^y = \phi m_t + k_t^b$$

$$p_t q^m \leq m_t$$

[9]With intergenerational transfers, the optimal CM-consumption levels are $x^b = x^m = x^s = x_1$ solving $U'(x_1) = \frac{1}{\beta}$, with total labor supply given by $2x_1$. However, if the same amount of CM consumption is financed by capital total labor supply would be only $\frac{2x_1}{R}$. Thus for $R > 1$ it is more efficient to finance CM consumption with capital.

[10]Zhu (2008) studies an economy with bilateral meetings and ex-ante uncertainty about an agent's type in a model that is otherwise similar to ours, and shows that these frictions can make devations from the Friedman rule optimal under some conditions. By assuming fixed types and competitive markets, we want to highlight that our results stem from different frictions than those found by Zhu.

$$p_t q^b \leq m_t + \frac{Rk^b}{\phi_{t+1}}$$

$$x_{t+1}^m = \phi_{t+1} m_t + Rk_t^b - \phi_{t+1} p_t q_t^m + (1-\mathcal{I})\tau_t^o$$

$$x_{t+1}^b = \phi_{t+1} m_t + Rk_t^b - \phi_{t+1} p_t q_t^b + (1-\mathcal{I})\tau_t^o.$$

All variables with a superscript m indicate decisions of relocated buyers (movers). Variables with superscript b indicate decisions of buyers prior to learning about relocation, or those of buyers that aren't relocated, depending on the context. The first constraint is the standard budget constraint for the portfolio choice when young. The second constraint denotes that relocated buyers cannot spend more than their money holdings during the DM, and the third constraint denotes that nonrelocated buyers cannot spend more than their total wealth for consumption during the DM.[11] The fourth and fifth constraint denote that buyers use all remaining resources for consumption when old.

We can simplify the problem by substituting some variables. Additionally, we only consider inflation rates where capital (weakly) dominates money in terms of return, i.e. $\frac{\phi_t}{\phi_{t+1}} \geq \frac{1}{R}$. In this case, the second constraint always holds at equality, as there is no reason for buyers to save money for the CM if capital pays a higher return. We also know that the third constraint never binds, because spending all wealth during the DM would imply $x_{t+1} = 0$, but this violates the Inada conditions. After simplification, the buyer's problem is

$$V^b = \max_{m_t, k_t^b, q_t^b} \quad \mathcal{I}\tau^y - \phi_t m_t - k_t^b + \pi \left(u\left(\frac{m_t}{p_t}\right) + \beta U(Rk_t^b + (1-\mathcal{I})\tau^o) \right)$$
$$+ (1-\pi)\big(u(q_t^b) + \beta U(\phi_{t+1} m_t + Rk_t^b - \phi_{t+1} p_t q_t^b) + (1-\mathcal{I})\tau^o)\big). \tag{8}$$

Seller's lifetime problem

A seller's value function at the beginning of his life is given by

$$V^s = \max_{h_t, q_t^s, x_{t+1}^s} \quad -h_t^s - q_t^s + \beta U(x_{t+1}^s)$$
$$s.t. \quad h_t^s = k_t^s$$
$$x_{t+1}^s = Rk_t^s + \phi_{t+1} p_t q_t^s.$$

Here, we already assumed that sellers do not accumulate money in the first CM, which is true in equilibrium for $\phi_t/\phi_{t+1} \geq \frac{1}{R}$. Thus, the first constraint denotes that sellers work only to

[11] The purchasing power of capital is scaled by $\frac{R}{\phi_{t+1}}$ to ensure that buyers give up the same amount of CM consumption by paying with capital and money.

accumulate capital, and the second constraint denotes that a seller's CM consumption is equal to the return on capital plus his revenue from selling the special good in the DM. Again, we can simplify the problem by substituting in the constraints. After simplification, the seller's problem is

$$V^s = \max_{q_t^s, k_t^s} \quad -k_t^s - q_t^s + \beta U(Rk_t^s + \phi_{t+1} p_t q_t^s). \tag{9}$$

3 Equilibrium with perfectly liquid capital

In this section, we solve for the market equilibrium in the special case of $\pi = 0$, which means that no relocation occurs during the DM. This case represents perfectly liquid capital, as all buyers can use capital during the DM, and abstracts from any uncertainty for all agents in the model. As money and capital are equally liquid and safe in this case, only the rate of return of the assets matters, and agents will only hold the asset with the higher rate of return. For $\phi_t/\phi_{t+1} \geq \frac{1}{R}$, capital is the asset with the (weakly) higher rate of return, and as this is the case we are most interested in, we abstract from money (and monetary policy) in this section. As we used p_t to denote the price of the DM good in terms of fiat money, we have to alter the problem slightly, as this price is undefined if money is not held in equilibrium. In this section, we therefore introduce ρ_t, which is the price of the DM good in terms of capital.

Given these alterations of the model, the buyer's problem from equation (8) becomes

$$V^b = \max_{k_t^b, q_t^b} \quad -k_t^b + u(q_t^b) + \beta U((k_t^b - \rho_t q_t^b)R),$$

and yields the following first-order conditions:

$$q^b: \quad u'(q^b) = \rho_t \beta R U'((k_t^b - \rho_t q_t^b)R) \tag{10}$$

$$k^b: \quad 1 = \beta R U'((k_t^b - \rho_t q_t^b)R). \tag{11}$$

The seller's problem is only affected by the change in notation. Solving equation (9) yields

$$q^s: \quad 1 = \beta R \rho_t U'((k_t^s + \rho_t q_t^s)R) \tag{12}$$

$$k^s: \quad 1 = \beta R U'((k_t^s + \rho_t q_t^s)R). \tag{13}$$

We already see from equations (11) and (13) that CM consumption is equal to the first-best level in this equilibrium i.e. $x^b = x^s = x^*$. Next we show that also DM consumption is at the first best level, i.e. $q^b = q^s = q^*$. Combining equations (12) and (13) gives $\rho_t = 1$ assuming optimal capital holdings of sellers are interior,[12] which means that DM prices are such that the seller is indifferent between working in the CM or the DM. Then, combining this with equations (10) and (11) yields

$$u'(q^b) = 1.$$

Furthermore, it is easily confirmed that labor supply and total capital investments are also at their first-best levels: $h^s + h^b = k^s + k^b = \frac{2x^*}{R} = H^* = K^*$. Thus, we can conclude that perfectly liquid capital allows to implement the planner's solution.

4 Equilibrium with perfectly illiquid capital

Having shown that there are no inefficiencies with perfectly liquid capital, we now want to investigate the other extreme case, which is perfectly illiquid capital. In the model, this is captured by $\pi = 1$, which means that all buyers are relocated during the DM. In this case, fiat money is the only way to acquire consumption during the DM. Thus, for $\phi_t/\phi_{t+1} \geq \frac{1}{R}$, buyers face no tradeoff between holding fiat money and capital, as only fiat money allows them to acquire DM consumption, while capital (weakly) dominates in terms of providing CM consumption.

With $\pi = 1$, the buyer's lifetime value function (8) simplifies to

$$V^b = \max_{m_t, k_t^b} \quad \mathcal{I}\tau^y - \phi_t m_t - k_t^b + u\left(\frac{m_t}{p_t}\right) + \beta U(Rk_t^b + (1-\mathcal{I})\tau^o).$$

Solving this problem yields two first-order conditions:

$$m_t : \qquad p_t \phi_t = u'\left(\frac{m_t}{p_t}\right) \tag{14}$$

[12]To implement the planner's solution with perfectly liquid capital, utility functions have to be such that sellers want to consume at least as much in the CM as they receive from selling the efficient amount of special goods at $\rho = 1$ in the DM while holding no capital. Thus we need $x_s = x^* > q^* R$ or $U'(q^* R) \geq \frac{1}{\beta R}$. We are assuming that this holds for the remainder of the paper. An alternative assumption we could make to prevent this issue is that the measure of sellers is sufficiently larger than the measure of buyers, such that individual sellers don't sell too many special goods in the DM. This friction might be interesting to study in other contexts, but it is not relevant for the points we want to make in this paper.

$$k_t: \qquad \frac{1}{\beta R} = U'(Rk_t^b + (1-\mathcal{I})\tau^o), \tag{15}$$

while solving the seller's problem yields the following first-order conditions:

$$q^s: \qquad 1 = \phi_{t+1} p_t \beta_t U'(Rk_t^s + p_t q_t^s \phi_{t+1}) \tag{16}$$

$$k^b: \qquad 1 = \beta R U'(Rk_t^s + p_t q_t^s \phi_{t+1}). \tag{17}$$

Combining equations (16) and (17) yields $p_t = \frac{R}{\phi_{t+1}}$. Plugging this into equation (14) gives

$$u'(q_t^m) = \frac{\phi_t}{\phi_{t+1}} R. \tag{18}$$

Equations (15) and (17) demonstrate that the CM consumption is always at the first-best level, independent of monetary policy $x^b = x^s = x^*$.

Given the first-order conditions, we first derive the stationary equilibrium when monetary policy is implemented over young buyers ($\mathcal{I} = 1$). In a stationary equilibrium we must have: $q^m = q^s$ (DM market clearing), $m = M$ (money market clearing) and $\phi/\phi_{+1} = \gamma$ i.e. the inflation rate must equal the growth rate of the money supply since the real value of money is constant over time, implying $\phi M = \phi_{+1} M_{+1}$. Furthermore the real value of taxes/transfers paid/received by young buyers is given by: $\tau^y = \phi(M - M_{-1}) = \frac{\gamma-1}{\gamma}\phi M$. Using this and the definitions and first-order conditions derived above for $\pi = 1$, we can then define a stationary equilibrium with perfectly illiquid capital as a list of eight variables $\{h^b, h^s, k^b, k^s, \phi_{+1}M, q^m, x^b, x^s\}$ solving:

$$u'(q^m) = \gamma R \tag{19}$$

$$x^b = x^s = x^* \tag{20}$$

$$\phi_{+1} M = q^m R \tag{21}$$

$$k^{b,\mathcal{I}=1} = \frac{x^*}{R} \tag{22}$$

$$h^{b,\mathcal{I}=1} = q^m R + \frac{x^*}{R} \tag{23}$$

$$k^s = \frac{x^*}{R} - q^m \tag{24}$$

$$h^s = k^s. \tag{25}$$

Equation (20) shows that x^b is independent of the inflation rate and thus inflation does not affect the buyer's capital accumulation. However, equation (24) shows that total capital accumulation is still indirectly affected by inflation. Sellers accumulate less capital if they expect to sell more goods in the DM - and DM consumption and thus also real balances are decreasing in the inflation rate, as we know from equation (19). Differentiating both sides of (19) with respect to γ yields:

$$\frac{\partial q^m}{\partial \gamma} = \frac{R}{u''(q^m)} < 0, \tag{26}$$

which is negative from the strict concavity of $u(q)$. In turn, this implies from equation (24) that seller's capital accumulation is increasing in inflation. This is the first channel through which capital accumulation is affected by the inflation rate, and it is active independent of the tax regime. Since it affects the sellers' capital accumulation, we call it the *seller channel*.

Total capital investment and labor supply are given by:

$$K^{\mathcal{I}=1} = k^{b,\mathcal{I}=1} + k^s = \frac{2x^*}{R} - q^m \tag{27}$$

$$H^{\mathcal{I}=1} = h^{b,\mathcal{I}=1} + h^s = \frac{2x^*}{R} + q^m(R-1) = K^{\mathcal{I}=1} + q^m R. \tag{28}$$

Next, we derive the stationary equilibrium when monetary policy is implemented over old buyers ($\mathcal{I} = 0$). The real value of taxes/transfers paid/received by old buyers is given by: $\tau^o = \phi_{+1}(M_{+1} - M) = \phi_{+1}M(\gamma - 1)$. Thus under stationarity $\tau^y = \tau^o = \tau$ for a given γ. Equilibrium consumption levels are unaffected by the different monetary policy regimes. The only changes in the equilibrium allocation affect the labor supply (equation (23)) and capital accumulation of buyers (equation (22)). These now read:

$$h^{b,\mathcal{I}=0} = \gamma q^m R + \frac{x^*}{R} - q^m(\gamma - 1) \tag{29}$$

$$k^{b,\mathcal{I}=0} = \frac{x^*}{R} - q^m(\gamma - 1), \tag{30}$$

and aggregate capital and labor supply:

$$K^{\mathcal{I}=0} = \frac{2x^*}{R} - \gamma q^m \tag{31}$$

$$H^{\mathcal{I}=0} = \frac{2x^*}{R} - \gamma q^m + \gamma R q^m = K^{\mathcal{I}=0} + \gamma R q^m. \tag{32}$$

Compare this with the equations (23) and (22) resp. (27) and (28) for the model with monetary policy implemented over young buyers. If there is inflation ($\gamma > 1$), $K^{\mathcal{I}=0} < K^{\mathcal{I}=1}$ and $H^{\mathcal{I}=0} > H^{\mathcal{I}=1}$, i.e. capital accumulation is lower when the old buyers are receiving the transfers, while the hours worked when young are higher. If there is deflation ($\gamma < 1$), $K^{\mathcal{I}=0} > K^{\mathcal{I}=1}$ and $H^{\mathcal{I}=0} < H^{\mathcal{I}=1}$, i.e., capital accumulation is higher when the old pay the taxes (a deflationary policy means $\tau < 0$), while the hours worked when young are lower. For $\gamma = 1$, the two equilibria coincide. To put this in other words, if monetary policy consists of making transfers to buyers

($\gamma > 1$), total work can be kept lower if monetary policy is implemented over young buyers, whereas the opposite is true if the monetary authority wants to implement a deflationary policy and thus raises taxes. The reason for this is that capital has a return $R > 1$; thus, if agents receive a transfer, it is better to receive it when young and invest it in capital, whereas if agents have to pay a tax, it is better to use the return on capital to pay it when old instead of paying it directly from labor income when young.

This also shows that with $\mathcal{I} = 0$, there is a second channel through which inflation affects capital accumulation: Equation (30) shows that the buyers' capital accumulation is a function of q^m, which is decreasing in inflation, and of the inflation rate itself. Since from equation (22) we know that capital accumulation of buyers is independent of inflation for $\mathcal{I} = 1$, we can conclude that this channel arises due to the monetary policy regime. We call this the *transfer channel.*

Before we characterize welfare for general inflation rates, we want to analyze what happens at the Friedman rule.

Proposition 1. *For $\pi = 1$, both DM and CM consumption are at the first best level at the Friedman rule ($\gamma = 1/R$), i.e. $q^m = q^*$ and $x^s = x^b = x^*$. Total hours worked are strictly above the first-best level, and strictly higher if the Friedman rule is implemented over taxes on the young buyers, compared to implementation through taxes on the old buyers; i.e., $H^{\mathcal{I}=1}|_{FR} > H^{\mathcal{I}=0}|_{FR} > H^*$.*

It can easily be seen from equation (19) that DM consumption is at the first-best level for $\gamma = 1/R$, and equation (20) shows that CM consumption is always at the first best level. Thus, consumption is efficient at the Friedman rule. Total capital and total hours worked however are not efficient. From (27) we see that $K^{\mathcal{I}=1}|_{FR} = 2x^*/R - q^*$ and $K^{\mathcal{I}=0}|_{FR} = 2x^*/R - q^*/R$ from (31). Thus capital investment is too low in both monetary policy regimes compared to the first-best level. This is not surprising, as sellers can only be compensated with money, which implies that their CM consumption will be partially financed through transfers from young to old agents - old sellers enter the CM with money and use it to purchase consumption goods from young buyers. At the first best, all CM consumption is financed with capital investment, so these intergenerational transfers imply an inefficiency. The inefficiency shows up in the aggregate labor supply, which is too high compared to the first best:

$$H^{\mathcal{I}=1}|_{FR} = 2x^*/R + q^*(R-1) > H^{\mathcal{I}=0}|_{FR} = 2x^*/R + q^*(R-1)/R > H^*.$$

Since $R > 1$, it can easily be seen that implementing the Friedman rule by taxing old buyers is more efficient - it allows to achieve the same consumption levels at strictly lower hours worked.[13]

[13]We are assuming in this Friedman rule equilibrium that agents finance their CM consumption with capital, even

Proposition 1 shows that even though consumption is at the first-best level at the Friedman rule, there is still a welfare loss from hours worked, so it is not obvious that running the Friedman rule is welfare-maximizing - and as we will show, it turns out that the Friedman rule is only the welfare-maximizing policy under some conditions in this economy.

Next, we investigate the effects of inflation on total labor supply $H = h^b + h^s$ and total capital accumulation $K = k^b + k^s$ for both policy implementation schemes. As we will show, these effects depend on the absolute value of the elasticity of DM consumption, which we denote as $|\varepsilon_{q^m}|$, and on the coefficient of relative risk aversion, which we denote as $\eta(q) = -\frac{qu''(q)}{u'(q)}$.

Proposition 2. *With $\mathcal{I} = 0$ inflation affects total labor supply and total capital accumulation in the following way:*

1. *If $|\varepsilon_{q^m}| < 1$: $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} > 0$ and $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} < 0$: Reverse Mundell-Tobin effect.*
2. *If $|\varepsilon_{q^m}| > 1$: $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} < 0$ and $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$: Mundell-Tobin effect.*
3. *If $|\varepsilon_{q^m}| = 1$: $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} = 0$ and $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} = 0$: No Mundell-Tobin effect.*

where $|\varepsilon_{q^m}| = \frac{1}{\eta(q^m)}$. With $\mathcal{I} = 1$, $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} < 0$ and $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$ and thus the Mundell-Tobin effect holds for all values of $|\varepsilon_{q^m}|$.

The proof to this Proposition can be found in the appendix. As we have pointed out above, there are two channels through which capital accumulation is affected by the inflation rate when $\pi = 1$. Through the seller channel, inflation has a positive effect on capital accumulation. Sellers understand that buyers want to consume less DM goods at higher inflation rates, so they finance a larger share of their CM consumption through capital investment. Since their CM consumption is constant at x^*, financing a larger share of it through capital investment implies higher capital investment. The seller channel is independent of the monetary policy regime.

The transfer channel is only active for $\mathcal{I} = 0$. With fully illiquid capital, buyers accumulate capital only for CM consumption and expenditures when old. Since CM consumption is constant at x^*, the buyers' capital accumulation is independent of inflation if they don't pay taxes / receive transfers when old, which is the case when $\mathcal{I} = 1$. With $\mathcal{I} = 0$ however, the real value of the tax to be paid or transfers received when old varies with the inflation rate, and thereby affects the buyers' capital accumulation. For $\gamma < 1$ (deflation), buyers have to pay a tax when old, so they need to

though they are indifferent between money and capital on an individual level. For the economy as a whole, financing CM consumption with money would be much more inefficient. In the extreme case where all CM-consumption is financed with money, we have a pure monetary economy where agents don't invest into capital. In this case consumption in the DM and the CM would still be efficient at the Friedman rule. But total work would be excessive since buyers would have to provide the total real return on money of $1/\gamma = R$ by working.

accumulate more capital in order to provide for x^* and the tax payment. If inflation is positive instead, buyers receive a transfer when old, so they can partly finance x^* through the transfer and need to accumulate less capital. The effect of inflation on capital accumulation through the transfer channel has two components: On the one hand, higher inflation (less deflation) increases the nominal value of the transfer (decreases the nominal value of the tax), but on the other hand, higher inflation (less deflation) decreases the value of money. For $\gamma < 1$, less deflation decreases the nominal value of the tax and also the real value of money. Thus the real tax payment decreases and buyers hold less capital. For positive inflation rates, either effect can dominate, depending on the elasticity of DM consumption.[14]

Proposition 2 describes the effect on aggregate capital accumulation, which is given by the net effect of the two channels. Since the transfer channel is shut down for $\mathcal{I} = 1$, capital accumulation is always increasing in inflation when taxes are paid by the young, so there is a Mundell-Tobin effect. For $\mathcal{I} = 0$, the aggregate effect depends on the elasticity of DM consumption $|\varepsilon_{q^m}|$, as this governs both the sign of the transfer channel at higher inflation rates, and the relative strength of the two channels. With $\pi = 1$, the elasticity of DM consumption is fully determined by the coefficient of relative risk aversion $\eta(q^m)$. If $|\varepsilon_{q^m}| = 1$, the increase in capital accumulation with inflation through the seller channel is exactly offset by a decrease in capital accumulation coming from the transfer channel, and the aggregate capital investment remains constant. Thus, there is no Mundell-Tobin effect in this case. With $|\varepsilon_{q^m}| > 1$, the effect through the seller channel is strong, while there is only a weak negative effect on capital accumulation from the transfer channel at low inflation rates, so there is a Mundell-Tobin effect on aggregate. The reason that this happens for high values of elasticity is that in this case, buyers reduce their DM consumption by a lot if inflation increases, so in turn the sellers' capital accumulation is reacting strongly to changes in inflation. Strong changes in DM consumption also imply that the real value of the tax/transfer decreases rapidly as inflation increases, which weakens the negative effect on capital accumulation from the transfer channel. The contrary is true for $|\varepsilon_{q^m}| < 1$: DM consumption changes very little as inflation varies, implying that sellers' capital accumulation also varies very little with inflation. On the other hand, higher inflation rates leave the value of money almost unchanged, so the transfer channel has a strong negative effect on capital accumulation. On aggregate, the negative effect from the transfer channel dominates, such that there is a reverse Mundell-Tobin effect for aggregate capital accumulation.

Proposition 2 also shows that a Mundell-Tobin effect is correlated with a negative effect on aggregate labor supply. This may seem counterintuitive, as from equations (28) and (32) capital

[14] Specifically, if $|\varepsilon_{q^m}| > \frac{\gamma}{\gamma-1}$, the effect through the transfer channel of inflation on capital accumulation is positive, so a positive correlation is more likely for high DM elasticity and higher inflation rates.

seems to *increase* total labor in the CM. To understand the connection look at market clearing in the CM in the model with $\mathcal{I} = 1$ (without loss of generality). The total amount of goods consumed in the CM is $2x^*$. Out of these, $K^{\mathcal{I}=1}R$ are produced from capital investments in the previous period, and $\phi M + \tau = \phi_{+1}M$ come from intergenerational transfers - the amount which equals the money holdings of young buyers. To sum up:

$$2x^* = K^{\mathcal{I}=1}R + \phi_{+1}M. \tag{33}$$

Now suppose capital increases because inflation increases and there is a Mundell-Tobin effect. By (33), this decreases real money holdings, as $\phi_{+1}M = 2x^* - RK^{\mathcal{I}=1}$ and x^* is unaffected by inflation. By (28), the effect on total labor supply is then:

$$H^{\mathcal{I}=1} = K^{\mathcal{I}=1} + \phi_{+1}M = K^{\mathcal{I}=1} + 2x^* - K^{\mathcal{I}=1}R = 2x^* - K^{\mathcal{I}=1}(R-1).$$

Thus if capital increases, this *decreases* total labor because real money holdings decrease more than capital increases. The intuition is again that financing CM-consumption with capital instead of transfers is more efficient in this economy because $R > 1$, and therefore a shift in financing CM consumption from intergenerational transfers to capital accumulation decreases total labor - with more capital, the same amount of CM consumption can be achieved with less labor.

After having established the effects of inflation on capital and labor supply, we can derive the optimal monetary policy.

Proposition 3. *With $\pi = 1$, the optimal money growth rate under $\mathcal{I} = 0$ is $\gamma^* = 1/R$ for $|\varepsilon_{q^m}| \leq 1$, and $\gamma^* = \frac{|\varepsilon_{q^m}|}{|\varepsilon_{q^m}|+R-1} \in (1/R, 1)$ for $|\varepsilon_{q^m}| > 1$; $\gamma^* = 1$ is optimal under $\mathcal{I} = 1$. The optimal monetary policy regime is $\mathcal{I}^* = 0$, and the first-best allocation is not achievable with $\pi = 1$.*

The proof to this Proposition can be found in the appendix. The intuition behind the proof is as follows: We know from proposition 1 that the Friedman rule allows to achieve the first-best consumption level in the DM. Thus if aggregate labor supply is increasing in inflation or is independent from it ($\frac{\partial H}{\partial \gamma} \geq 0$, cases 1 and 3 from proposition 2) the Friedman rule must be the optimal monetary policy. Higher inflation would decrease consumption in the DM while increasing (or having no effect on) labor supply. If aggregate labor supply decreases in inflation ($\frac{\partial H}{\partial \gamma} < 0$), there is a policy tradeoff: Increasing the inflation rate reduces utility from DM consumption, but simultaneously reduces disutility from CM labor. This implies that the optimal inflation rate must lie above the Friedman rule. At the Friedman rule, the marginal costs of decreasing DM consumption are zero, but the benefits of decreasing the aggregate labor supply H are positive.

Thus it is optimal to increase inflation above the Friedman rule. This is what happens in case 2 and in the model with $\mathcal{I} = 1$. By how much inflation can be increased above the Friedman rule to further increase welfare then depends on the elasticity of DM consumption and on the tax regime. With $\mathcal{I} = 0$, implementing deflation and thus higher DM consumption is relatively cheaper, so γ^* is increasing in the elasticity of DM consumption and approaching 1 as DM consumption becomes infinitely elastic. With $\mathcal{I} = 1$ however, running any deflationary policy in order to increase DM consumption is too costly, so $\gamma^* = 1$ in this case.

The welfare results also imply an ordering of the two monetary policy regimes. We know that for $\gamma = 1$, both regimes are equivalent in terms of allocation and we also know that if monetary policy is implemented over young buyers, $\gamma^* = 1$ is the optimal inflation rate. But this allocation is always feasible, but not optimal, if monetary policy is implemented over old buyers. Thus, we can conclude that the optimal monetary policy is to always set $\mathcal{I}^* = 0$, and to set $\gamma^* = 1/R$ for $|\varepsilon_{q^m}| \leq 1$ and to set $\gamma^* =\in (1/R, 1)$ for $|\varepsilon_{q^m}| > 1$.

Note that $\gamma = 1$ means that the return on money equals the return on intergenerational transfers which the social planner faces; note also that financing old-age CM consumption with fiat money implies intergenerational transfers, as only young agents are willing to sell goods against money. With $\pi = 1$, buyers are only able to compensate sellers with money for DM production, so unless the DM is completely shut down, sellers inevitably end up with money when they enter the second CM of their life. Setting $\gamma = 1$ leads to the correct prices for the two ways of generating CM consumption, i.e., intergenerational transfers and capital accumulation. On the other hand, setting $\gamma = \frac{1}{R}$ is the only way to reach the first-best consumption level in the DM. This analysis points out the fundamental policy tradeoff in our model: Efficiency in the DM requires a different money growth rate than efficiency in the CM, and the optimal policy choice depends on how much buyers value DM consumption, or more precisely, how elastic DM consumption is.

Haslag and Martin (2007) have shown that a constant money stock is typically optimal in an OLG model, independent of the Mundell-Tobin effect. We can confirm that $\gamma = 1$ is the optimal monetary policy for $\mathcal{I} = 1$, independent of all other parameters. However, we have also shown that a Mundell-Tobin effect still exists in this case, but it is running through the seller channel only. More generally, we can show that implementing monetary policy in a less costly way, namely by taxing agents only once they are old, allows to shut down the Mundell-Tobin effect completely for certain parameters, and that in these cases, the Friedman rule is the optimal monetary policy.[15]

[15]While this works nicely in our model, it would not do the trick in pure OLG models. The difference is that relocation occurs during the final stage of an agent's life in models such as Smith (2002) or Haslag and Martin (2007). The reason that taxing the old is strictly cheaper in our model is that all agents know they have access to

The analysis so far shows that the first-best allocation is achieved automatically if capital is perfectly liquid, while the first-best consumption levels can be implemented at the Friedman rule in the case of perfectly illiquid capital, but at the cost of having agents work more in the first CM of their lives. Whether agents prefer this allocation over higher money growth rates in the case of perfectly illiquid capital depends on the way the Friedman rule is implemented and on the preferences of agents. But even if the Friedman rule is welfare maximizing in the case of perfectly illiquid capital, welfare is still strictly lower than in the case of perfectly liquid capital, i.e. below first best.

5 Equilibrium with partially liquid capital

We now turn to the case of partial relocation, which implies partially liquid capital. Uncertainty about relocation introduces a clear tradeoff between acquiring money or capital: Money provides insurance against the relocation shock, while capital offers a higher rate of return for $\phi_t/\phi_{t+1} > \frac{1}{R}$. At low inflation rates, acquiring money for insurance means only a small loss of return, thus making money relatively more attractive and depressing capital accumulation. At high inflation rates, acquiring money for insurance is really costly in terms of rate of return foregone, and thus capital accumulation becomes relatively more attractive. As we will see, this tradeoff adds a third channel through which capital accumulation is affected by the inflation rate. Since this channel only occurs when capital is partially liquid, we call it the *liquidity channel*.

For sellers, there is no uncertainty even with partially liquid capital. Thus, the results we found in equations (16) and (17) still hold. This implies two things: First, the seller's CM consumption is still unaffected by monetary policy and always at the first-best level; second, $p_t = \frac{R}{\phi_{t+1}}$ still holds.

Solving the buyers' lifetime problem (8) while making use of this result yields the following first-order conditions:

$$m_t : \quad \frac{\phi_t}{\phi_{t+1}} = \pi \frac{1}{R} u'\left(\frac{\phi_{t+1} m_t}{R}\right) + (1-\pi)\beta U'(\phi_{t+1} m_t + R(k_t^b - q_t^b) + (1-\mathcal{I})\tau^o) \tag{34}$$

$$k_t^b : \quad \frac{1}{\beta R} = \pi U'(R k_t^b) + (1-\pi) U'(\phi_{t+1} m_t + R(k_t^b - q_t^b) + (1-\mathcal{I})\tau^o) \tag{35}$$

$$q_t^b : \quad u'(q_t^b) = \beta R U'(\phi_{t+1} m_t + R(k_t^b - q_t^b) + (1-\mathcal{I})\tau^o). \tag{36}$$

We are now ready to define an equilibrium in the full model. Again we first derive the equilibrium when monetary policy is implemented over young buyers ($\mathcal{I} = 1$). Money market clearing

their capital when they have to pay the tax, and can thus fully pay the tax via capital investment. In pure OLG models, only non-relocated agents have access to their capital during the final stage of their life.

$m = M$ and stationarity $\phi/\phi_{+1} = \gamma$ are identical to before but market clearing in the DM is now:

$$\pi q^m + (1-\pi)q^b = q^s. \tag{37}$$

With (37), (21), (24) and (25) and the definitions and first-order conditions derived above we can define a stationary equilibrium with partially liquid capital as a list of eleven variables $\{q^m$, q^b, q^s, x^b, x^m, x^s, $\phi_{+1}M$, k^b, h^b, k^s, $h^s\}$ solving:

$$\pi u'(q^m) + (1-\pi)u'(q^b) = \gamma R \tag{38}$$

$$\pi U'(x^m) + (1-\pi)U'(x^b) = \frac{1}{\beta R} \tag{39}$$

$$u'(q^b) = \beta R U'(x^b) \tag{40}$$

$$x^m = x^b + R(q^b - q^m) \tag{41}$$

$$x^s = x^* \tag{42}$$

$$k^{b,\mathcal{I}=1} = \frac{x^m}{R} \tag{43}$$

$$h^{b,\mathcal{I}=1} = \phi_{+1}M + k^b. \tag{44}$$

$$\tag{45}$$

Aggregate labor supply and capital investments are given by:

$$K^{\mathcal{I}=1} = \frac{x^m}{R} + \frac{x^*}{R} - q^s \tag{46}$$

$$H^{\mathcal{I}=1} = \frac{x^m}{R} + q^m R + \frac{x^*}{R} - q^s. \tag{47}$$

With monetary policy implemented over old buyers ($\mathcal{I} = 0$) the only changes are in the labor supply and the capital investments of buyers and thus also in aggregate capital and labor supply.

$$h^{b,\mathcal{I}=0} = \gamma\phi_{+1}M + k^b \tag{48}$$

$$k^{b,\mathcal{I}=0} = \frac{x^m}{R} - q^m(\gamma - 1) \tag{49}$$

$$K^{\mathcal{I}=0} = \frac{x^m}{R} - q^m(\gamma - 1) + \frac{x^*}{R} - q^s \tag{50}$$

$$H^{\mathcal{I}=0} = \frac{x^m}{R} + \gamma q^m R - (\gamma - 1)q^m + \frac{x^*}{R} - q^s \tag{51}$$

Next, we are interpreting the equilibrium with a number of propositions. The proofs to all of them can be found in the appendix.

Proposition 4. *At the Friedman rule ($\gamma = \frac{1}{R}$), all DM trades are conducted with money, and the allocation is identical to an economy with $\pi = 1$. DM consumption is perfectly smoothed for relocated and non-relocated buyers and equal to the first-best level, i.e. $q^m = q^b = q^s = q^*$. CM* *consumption is also perfectly smoothed for relocated and non-relocated buyers and equal to first best* $x^m = x^b = x^s = x^*$.

The allocation under the Friedman rule achieves perfect insurance against the relocation shock and first best consumption in all markets. However, as we know from the last section, aggregate labor supply is above the first best in this equilibrium. All trades in the DM are made using money although capital would be accepted in some of them.

Proposition 5. *With inflation rates above the Friedman rule ($\gamma > \frac{1}{R}$), DM consumption is higher for non-relocated buyers and both consumption levels are below the first-best consumption level, i.e.* $q^m < q^s < q^b < q^*$. *CM consumption is higher for relocated buyers and above first-best consumption, while CM consumption for non-relocated buyers is below first best, i.e.* $x^m > x^* > x^b$. *Also, CM consumption for non-relocated buyers and DM consumption for all buyers decrease in inflation while CM consumption for relocated buyers increases in inflation, i.e.* $\frac{\partial q^m}{\partial \gamma}, \frac{\partial q^b}{\partial \gamma}, \frac{\partial q^s}{\partial \gamma}, \frac{\partial x^b}{\partial \gamma} < 0$ *and* $\frac{\partial x^m}{\partial \gamma} > 0$. *Total CM consumption* $X = x^* + \pi x^m + (1-\pi)x^b$ *increases in inflation.*

The proposition shows that deviations from the Friedman rule introduce consumption risk for the agents and that their consumption deviates from first best in all markets. For relocated buyers, DM consumption is lower than for nonrelocated buyers, as they can only use money to purchase special goods. Therefore, their CM consumption has to be higher, as they still have the capital they accumulated during the DM. Nonrelocated buyers smooth their consumption more. They consume less than first-best in both markets because the low return on money (which they accumulate due to the ex-ante uncertainty about relocation) makes them unwilling to accumulate enough assets to purchase first-best consumption levels. In contrast to the model with full relocation, where buyer consumption in the CM was x^* and independent of inflation, inflation now affects CM consumption of relocated buyers (and total CM-consumption) positively.

The following two propositions describe the effects of inflation on total capital and the labor supply, in the two monetary policy regimes and at or above the Friedman rule, respectively.

Proposition 6. *With* $\mathcal{I} = 1$, *there is a Mundell-Tobin effect* ($\frac{\partial K^{\mathcal{I}=1}}{\partial \gamma} > 0$) *for all parameters. At the Friedman rule, an increase in inflation reduces aggregate labor supply* ($\frac{\partial H^{\mathcal{I}=1}}{\partial \gamma} < 0$)*, but for sufficiently high inflation or liquidity of capital, a further increase in inflation increases aggregate labor supply.*

As in the model with fully illiquid capital, inflation always increases capital accumulation if monetary policy is implemented over young buyers. The effect is even stronger now, as the liquidity channel also makes buyers accumulate more capital instead of real balances if inflation increases in addition to the seller channel, which is still active. To see why the Mundell-Tobin effect and the effect on total labor supply don't always go in opposite directions anymore, we again look at market clearing in the CM in the model with taxes/transfers to young buyers (without loss of generality). Total CM-consumption is

$$X = x^* + \pi x^m + (1-\pi)x^b = K^{\mathcal{I}=1}R + \phi_{+1}M. \tag{52}$$

x^* is the sellers' CM consumption, and $\pi x^m + (1-\pi)x^b$ is the buyers' CM consumption. Total CM consumption is provided by capital investments $K^{\mathcal{I}=1}R$ and transfers $\phi_{+1}M$. Following the same steps as in the model with fully illiquid capital, total labor supply is $H^{\mathcal{I}=1} = X - K^{\mathcal{I}=1}(R-1)$. However, X is not independent of inflation anymore and thus the logic from before that an increase in inflation would always lead to a decrease in total labor is not valid anymore. From proposition 5 we know that X increases in inflation. Thus total labor supply still decreases over the Mundell-Tobin effect but increases over the effect of inflation on X. This is why there is no full correlation between the Mundell-Tobin effect and a negative effect of inflation on the labor supply anymore. In a way, total labor supply now has an intensive (inflation shifts the financing mix of CM consumption from on the spot production to capital) and an extensive margin (inflation increases total consumption). One can show that the positive effect of inflation on CM consumption is zero at the Friedman rule. This is why at the Friedman rule, the connection between the Mundell-Tobin effect and a negative effect of inflation on total labor still holds.

Proposition 7. *With $\mathcal{I} = 0$, inflation affects labor supply and capital investment at the Friedman rule in the following way:*

1. *If $|\varepsilon_{q^m}|_{FR} < 1$: $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} > 0$ and $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} < 0$: Reverse Mundell-Tobin effect.*
2. *If $|\varepsilon_{q^m}|_{FR} > 1$: $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} < 0$ and $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$: Mundell-Tobin effect.*
3. *If $|\varepsilon_{q^m}|_{FR} = 1$: $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} = 0$ and $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} = 0$: No Mundell-Tobin effect.*

where $|\varepsilon_{q^m}|_{FR} = \frac{1}{\eta(q^*)}\xi$ and $\xi \in (1, \infty)$ with $\xi = 1$ if $\pi = 1$, $\xi \to \infty$ as $\pi \to 0$, and ξ monotonically decreasing in π at the Friedman rule. This implies that with a decrease in π, it is more likely that a Mundell-Tobin effect occurs at the Friedman rule, and if there is already a Mundell-Tobin effect, it becomes more pronounced.

Away from the Friedman rule, the conditions for a Mundell-Tobin effect and a negative effect on aggregate labor supply are not identical. $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$ if $|\varepsilon_{q^m}| > \hat{\varepsilon}$ and $\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} < 0$ if $|\varepsilon_{q^m}| > \tilde{\varepsilon}$ where $\hat{\varepsilon} < 1 < \tilde{\varepsilon}$.

At the Friedman rule, the conditions for a Mundell-Tobin effect look identical to what we described in proposition 2 for $\pi = 1$ - there is a Mundell-Tobin effect for DM elasticity above 1, and a reverse Mundell-Tobin effect for DM elasticity below 1. However, note that the DM elasticity is now given by $\frac{1}{\eta(q^*)}\xi$ at the Friedman rule. Since $\xi = 1$ for $\pi = 1$ and ξ is decreasing in π, the proposition shows that with lower π, there are more values of $\eta(q^*)$ for which a Mundell-Tobin effect occurs at the Friedman rule; i.e., there are values for $\eta(q^*)$ for which a reverse Mundell-Tobin effect occurs for high values of π, but a Mundell-Tobin effect occurs for low values of π. If $\pi \to 0$, there is always a Mundell-Tobin effect at the Friedman rule. Further, if there is a Mundell-Tobin effect at the Friedman rule, the effect becomes stronger for lower π. The reason is the liquidity channel. At the Friedman rule, all DM trades are made using money and DM consumption is q^*. But if capital is relatively liquid (π is low) a lot of trades could be done using capital. The likelihood that buyers can use capital to pay in the DM is high. Thus if inflation is marginally above the Friedman rule and thus capital dominates money in terms of return, there is a strong incentive for buyers to substitute money for capital and massively increase capital holdings (and decrease labor as explained above). The lower π, the bigger this incentive, and the more likely that it dominates the agents' desire to smooth DM consumption which is captured by the risk-aversion coefficient. On the other hand if π is high, the incentive to increase capital is low at the Friedman rule if inflation marginally increases, because anyway most of the DM-trades are made using money, and the probability that buyers will be able to use capital to purchase goods in the DM is low.

Away from the Friedman rule, things are more complicated with partially liquid capital. In general, the Mundell-Tobin effect occurs when DM elasticity is above some threshold $\hat{\varepsilon} < 1$, but there is no simple representation of DM elasticity as a function of the coefficient of relative risk aversion in this case, so it is difficult to make general statements. Importantly, the threshold for whether inflation has a positive or a negative effect on aggregate labor supply is given by $\tilde{\varepsilon} > 1$, so away from the Friedman rule inflation does not generally have opposite effects on aggregate labor supply and aggregate capital accumulation. For $\hat{\varepsilon} < |\varepsilon_{q^m}| < \tilde{\varepsilon}$, a Mundell-Tobin effect coincides with a positive effect of inflation on aggregate labor supply.

Proposition 8. *For $\mathcal{I} = 1$, $\gamma^* = 1$ is the optimal money growth rate, independent of all other parameters. Furthermore, the Friedman rule is relatively more costly in this regime for low π.*

When monetary policy is implemented over young buyers, a constant money stock is optimal. This is not surprising, as we have shown in proposition 3 that a constant money stock is optimal in this monetary policy regime even if there is no uncertainty about relocation. With partially liquid capital, the Mundell-Tobin effect is generally stronger due to the liquidity channel, so there was no reason to expect a lower money growth rate to be optimal. Positive inflation rates are not optimal either, as for $\gamma > 1$, the additional distortions in consumption of relocated buyers are larger than benefits from increased capital accumulation. Further, the proposition also shows that running the Friedman rule is especially costly for low values of π. In fact, welfare at the Friedman rule is always the same independent of π, while welfare at $\gamma = 1$ is decreasing in π. The reason is that at the Friedman rule, all DM trades are made with money, which implies that a large share of the sellers' CM consumption is financed through intergenerational transfers, while at the first-best allocation, all CM consumption is financed by capital investment. Since for low levels of π, a large share of DM trades could be made using capital instead of money, the loss from running the Friedman rule is larger. For $\pi \to 0$, welfare at $\gamma = 1$ approaches the first-best.

Proposition 9. *For $\mathcal{I} = 0$ and $|\varepsilon_{q^m}|_{FR} \leq 1$, the optimal inflation rate is the Friedman rule, i.e. $\gamma^* = 1/R$. For $|\varepsilon_{q^m}|_{FR} > 1$, the optimal inflation rate is above the Friedman rule $\gamma^* = \frac{|\varepsilon_{q^m}|}{|\varepsilon_{q^m}|+R-1} \in (1/R, 1)$. In other words, for a given π, there is a threshold on risk aversion ξ, with the Friedman rule being the optimal monetary policy if and only if $\eta(q^*) > \xi$, and ξ is decreasing in π. When the optimal money growth rate is chosen, welfare is strictly higher with $\mathcal{I} = 0$.*

This proposition shows that higher liquidity of capital makes it less likely that the Friedman rule is the optimal monetary policy. In fact, when $\pi \to 0$, the Friedman rule is not optimal for any $\eta(q^*)$. This result directly follows from proposition 7. Due to the envelope theorem, an increase in inflation is welfare-increasing at the Friedman rule if it leads to a decrease in labor supply: such a decrease has a first-order effect on welfare, while the welfare losses from consumption in the DM and CM are only second order. Since a Mundell-Tobin effect coincides with a negative effect of inflation on labor supply at the Friedman rule, an increase in inflation is welfare-increasing if there is a Mundell-Tobin effect at the Friedman rule, which is the case for $|\varepsilon_{q^m}|_{FR} > 1$, with $|\varepsilon_{q^m}|_{FR} = \frac{1}{\eta(q^*)}\xi$. This shows that the liquidity channel makes it less likely that the Friedman rule is the optimal monetary policy. If the optimal monetary policy is not given by the Friedman rule, it is a function of the elasticity of DM consumption of relocated buyers, and it lies somewhere between the Friedman rule and a constant money stock. The result that monetary policy implementation over old buyers is better for welfare follows again from the fact that allocations coincide for $\gamma = 1$, so the optimal allocation for $\mathcal{I} = 1$ is always feasible with $\mathcal{I} = 0$, but generally not optimal.

As in the model with $\pi = 1$, the fundamental policy tradeoff is that setting $\gamma = \frac{1}{R}$ allows for efficiency in the DM, but misrepresents the cost of using money, and thus intergenerational transfers, to provide for CM consumption. While this is a relatively small issue if capital is illiquid and money is the only way to provide for DM consumption, the welfare loss from running the Friedman rule increases with the liquidity of capital, as most DM trades could be made with capital if it is relatively liquid.

Before we conclude, we want to note that proposition 9 shows that the premise behind Smith (2002) was correct: the Mundell-Tobin effect is enough to make deviations from the Friedman rule optimal. In our model, the Friedman rule is optimal if and only if there is no Mundell-Tobin effect at the Friedman rule, with higher liquidity of capital (i.e., a lower π) and lower risk aversion of buyers making it more likely that there is a Mundell-Tobin effect at the Friedman rule.

6 Conclusion

We have added a market which requires liquid assets to trade to an OLG model with relocation shocks, in order to study whether the Mundell-Tobin effect can make deviations from the Friedman rule optimal. We have shown that the Friedman rule is optimal if and only if there is no Mundell-Tobin effect at the Friedman rule, and that a Mundell-Tobin effect is more likely to occur if capital is relatively liquid, risk aversion of buyers is low, and if monetary policy is implemented by taxing the young. If the Friedman rule is not optimal, the optimal money growth rate lies somewhere between the Friedman rule and a constant money stock. While the Friedman rule allows for first-best consumption levels in the DM, it misrepresents the cost of using intergenerational transfers to provide for CM consumption during old age. These costs are correctly represented by a constant money stock. We have also shown that for any deflationary policy, taxing old agents is strictly better than taxing young agents when there is a productive investment opportunity in the economy.

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Appendix A

A.1 Proof of Proposition 2

Proof. We begin with the Mundell-Tobin effect in both models. When monetary policy is implemented over young buyers ($\mathcal{I} = 0$) total capital investment is given by:

$$K^{\mathcal{I}=1} = \frac{2x^*}{R} - q^m \tag{27}$$

DM-consumption shows up because it indirectly affects the sellers' capital accumlation. Since q^m is decreasing in inflation, we always get a Mundell-Tobin effect when monetary policy is implemented over young buyers:

$$\frac{\partial K^{\mathcal{I}=1}}{\partial \gamma} = -\frac{\partial q^m}{\partial \gamma} > 0 \tag{53}$$

When monetary policy is implemented over old buyers ($\mathcal{I} = 0$) total capital investment is:

$$K^{\mathcal{I}=0} = \frac{2x^*}{R} - q^m(\gamma - 1) - q^m = \frac{2x^*}{R} - q^m\gamma \tag{31}$$

with the first derivative:

$$\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} = -(\gamma\frac{\partial q^m}{\partial \gamma} + q^m) = \frac{u'(q^m)}{-u''(q^m)} - q^m = q^m\big(\frac{1}{\eta(q^m)} - 1\big). \tag{54}$$

Furthermore, $\frac{1}{\eta(q^m)} = |\varepsilon_{q^m}|$, since

$$|\varepsilon_{q^m}| = -\frac{dq^m/q^m}{d\gamma/\gamma} = -\frac{\gamma}{q^m}\frac{\partial q^m}{\partial \gamma} = -\frac{u'(q^m)}{q^m u''(q^m)}. \tag{55}$$

Thus $\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} > 0$ (we have a Mundell-Tobin effect) if $|\varepsilon_{q^m}| = \frac{1}{\eta(q^m)} > 1$.

Next we turn to the effects on total labor supply. In both monetary policy regimes, total labor supply is the sum of capital investments K and the work of buyers to acquire real balances. If monetary policy is instead implemented over the young buyers ($\mathcal{I} = 1$), total labor supply is:

$$H^{\mathcal{I}=1} = \gamma q^m R + \frac{x^*}{R} - q^m R(\gamma - 1) + \frac{x^*}{R} - q^m = \frac{2x^*}{R} + q^m(R - 1) \tag{28}$$

Buyers acquire real balances $\phi M = \gamma q^m R$ and get a transfer (if $\gamma > 1$) of $\tau = q^m R(\gamma - 1)$). So in this case the wealth effects of inflation on the holdings of real balances are canceled out and the effect of inflation on total labor supply must be negative:

$$\frac{\partial H^{\mathcal{I}=1}}{\partial \gamma} = \frac{\partial q^m}{\partial \gamma}(R - 1) < 0 \tag{56}$$

With monetary policy implemented over old buyers ($\mathcal{I}=0$) total labor supply in the CM is in equilibrium:

$$H^{\mathcal{I}=0}=\gamma q^m R+\frac{x^*}{R}-q^m(\gamma-1)+\frac{x^*}{R}-q^m=K^{\mathcal{I}=0}+\gamma q^m R=\frac{2x^*}{R}+(R-1)\gamma q^m \qquad (32)$$

Total labor supply is the sum of buyer real balances $\gamma q^m R$ and total capital investments. The effects of real balances on $K^{\mathcal{I}=0}$ and the real balance holdings $\gamma q^m R$ simplify to $(R-1)\gamma q^m$. Thus the sign of the derivative of γq^m, which is determined by $\eta(q^m)$, will also determine the sign of the derivative of total labor supply:

$$\frac{\partial H^{\mathcal{I}=0}}{\partial\gamma}=(R-1)(\frac{\partial q}{\partial\gamma}\gamma+q^m)=q^m(R-1)\big(1-\frac{1}{\eta(q^m)}\big) \qquad (57)$$

Thus we must have $\frac{\partial K^{\mathcal{I}=0}}{\partial\gamma}>0$, $\frac{\partial H^{\mathcal{I}=0}}{\partial\gamma}<0$ for $|\varepsilon_{q^m}|=\frac{1}{\eta(q^m)}>1$, and $\frac{\partial K^{\mathcal{I}=0}}{\partial\gamma}<0$, $\frac{\partial H^{\mathcal{I}=0}}{\partial\gamma}>0$ for $|\varepsilon_{q^m}|=\frac{1}{\eta(q^m)}<1$.

■

A.2 Proof of Proposition 3

Proof. Welfare of a representative generation with fully illiquid capital can be written as

$$V^g=-H+u(q^m)-q^m+2\beta U(x^*) \qquad (58)$$

Because CM consumption is independent of inflation for $\pi=1$, inflation affects welfare through DM consumption and aggregate labor supply:

$$\frac{\partial V^g}{\partial\gamma}=-\frac{\partial H}{\partial\gamma}+\frac{\partial q^m}{\partial\gamma}(u'(q^m)-1) \qquad (59)$$

In cases 1 and 3 from proposition 2, the aggregate labor supply rises in inflation or is independent from it ($\frac{\partial H}{\partial\gamma}\geq 0$). Thus, the Friedman rule must be the optimal monetary policy since this maximizes welfare in the DM ($q^m=q^*$) from (19). Higher inflation would decrease consumption in the DM while weakly increasing labor supply.[16]

In case 2 and with $\mathcal{I}=1$, aggregate labor supply decreases in inflation ($\frac{\partial H}{\partial\gamma}<0$). Thus, the optimal inflation rate must lie above the Friedman rule due to the envelope theorem.

In case 2 under $\mathcal{I}=0$, $\frac{\partial H}{\partial\gamma}$ given by (57) and optimal inflation rate $\gamma^*<1$ solves

[16] For inflation rates below the Friedman rule, our derivation of results is incorrect, because we assumed $\gamma\geq\frac{1}{R}$. It looks like further decreasing inflation is welfare-increasing if $|\varepsilon_{q^m}|<1$, but this is incorrect, as inflation below the Friedman rule leads to a regime switch where nobody accumulates capital. This clearly reduces aggregate welfare. Thus, $\gamma=\frac{1}{R}$ is a corner solution for $|\varepsilon_{q^m}|<1$.

$$-q^m(R-1) - \frac{\partial q^m}{\partial \gamma}\gamma^*(R{-}1) + \frac{\partial q^m}{\partial \gamma}(\gamma^* R - 1) = 0$$

$$\leftrightarrow$$

$$-\frac{\partial q^m}{\partial \gamma}\frac{\gamma^*}{q^m} = |\varepsilon_{q^m}| = \frac{(R-1)\gamma^*}{1-\gamma^*} \tag{60}$$

The interior solution solves

$$\gamma^* = \frac{|\varepsilon_{q^m}|}{|\varepsilon_{q^m}| + R - 1} \quad \in (1/R, 1) \tag{61}$$

If monetary policy is implemented over young buyers ($\mathcal{I} = 1$), $\frac{\partial H}{\partial \gamma}$ is given by (56) and optimal inflation rate γ^* solves

$$-\frac{\partial q}{\partial \gamma}(R-1) + \frac{\partial q}{\partial \gamma}(\gamma^* R - 1) = 0 \tag{62}$$

Thus $\gamma^* = 1$.

■

A.3 Proof of Proposition 4

Proof. We proof this by contradiction using the first four optimality conditions (38) to (41) which we repeat for convenience.

$$\pi u'(q^m) + (1-\pi)u'(q^b) = \gamma R \tag{38}$$

$$\pi U'(x^m) + (1-\pi)U'(x^b) = \frac{1}{\beta R} \tag{39}$$

$$u'(q^b) = \beta R U'(x^b) \tag{40}$$

$$x^m = x^b + R(q^b - q^m) \tag{41}$$

At the Friedman rule (38) becomes:

$$\pi u'(q^m) + (1-\pi)u'(q^b) = 1$$

Suppose $q^b > q^m$. From (41) this implies $x^m > x^b$. From the decreasing marginal utility of the utility functions and the weighted average formulation of the first and the second condition we must have $u'(q^m) > 1$ and $u'(q^b) < 1$ and $U'(x^m) > \frac{1}{\beta R}$ and $U'(x^b) < \frac{1}{\beta R}$. But this violates (40), the left-hand side would be < 1 and the right-hand side > 1. The opposite contradiction follows for assuming $q^b < q^m$. Thus we must have $q^b = q^m$ and $x^m = x^b$ and then $q^b = q^m = q^*$ and $x^m = x^b = x^*$ follow from (38) and (39). From $q^b = q^s$, it follows that all DM trades are made with money, and from that it follows that the allocation must be identical to an economy with $\pi = 1$. ■

A.4 Proof of Proposition 5

Proof. We first show that for $\gamma > 1/R$ we must have $q^* > q^b > q^m$ and $x^m > x^* > x^b$. The steps are the same as in the previous proof. Suppose that $q^b = q^m$ and $x^m = x^b$ from (41). This implies $u'(q) > 1$ from (38) and $\beta RU'(x) = 1$. Thus it violates (40). Similarly $q^b < q^m$ and $x^m < x^b$ imply $u'(q^b) > \gamma R > 1$ and $U'(x^b) < \frac{1}{\beta R}$ which also violates (40). Thus only $q^b > q^m$ and $x^m > x^b$ is possible, implying $u'(q^b) < \gamma R$ and $U'(x^b) > \frac{1}{\beta R}$. In this case (40) can hold if $u'(q^b) \in (1, \gamma R)$ which also implies $q^b < q^*$. Since $q^m < q^b$, q^m must also be below q^* and $x^m > x^*$ and $x^b < x^*$ follows from $U'(x^b) > \frac{1}{\beta R}$ and $U'(x^m) < \frac{1}{\beta R}$.
To find the effects of inflation on the consumption levels we differentiate (38) to (41) with respect to γ.

$$\pi u''(q^m)\frac{\partial q^m}{\partial \gamma} + (1-\pi)u''(q^b)\frac{\partial q^b}{\partial \gamma} = R$$
$$\pi U''(x^m)\frac{\partial x^m}{\partial \gamma} + (1-\pi)U''(x^b)\frac{\partial x^b}{\partial \gamma} = 0$$
$$u''(q^b)\frac{\partial q^b}{\partial \gamma} = \beta RU''(x^b)\frac{\partial x^b}{\partial \gamma}$$
$$\frac{\partial x^m}{\partial \gamma} = \frac{\partial x^b}{\partial \gamma} + R(\frac{\partial q^b}{\partial \gamma} - \frac{\partial q^m}{\partial \gamma}) \tag{63}$$

Solving this for the partial effects yields

$$\frac{\partial q^b}{\partial \gamma} = A\frac{\partial x^b}{\partial \gamma} < 0 \tag{64}$$
$$\frac{\partial x^m}{\partial \gamma} = -B\frac{\partial x^b}{\partial \gamma} > 0 \tag{65}$$
$$\frac{\partial q^m}{\partial \gamma} = \frac{(RA+1+B)}{R}\frac{\partial x^b}{\partial \gamma} < 0 \tag{66}$$
$$\frac{\partial x^b}{\partial \gamma} = \frac{R}{C} < 0, \tag{67}$$

where $A = \frac{\beta RU''(x^b)}{u''(q^b)}$ and $B = \frac{(1-\pi)U''(x^b)}{\pi U''(x^m)}$ are positive and $C = \pi u''(q^m)\frac{RA+1+B}{R} + (1-\pi)u''(q^b)A$ is negative because $u(\cdot)$ and $U(\cdot)$ are strictly concave. The effect on seller DM consumption q^s must also be negative because it is a weighted average of consumption of relocated and non-relocated buyers from (37).

The partial effect of inflation on aggregate CM-consumption $X = x^* + \pi x^m + (1-\pi)x^b$ is:

$$\frac{\partial X}{\partial \gamma} = \pi\frac{\partial x^m}{\partial \gamma} + (1-\pi)\frac{\partial x^b}{\partial \gamma} = -(\pi(1+B)-1)\frac{\partial x^b}{\partial \gamma} \tag{68}$$

which is positive since $\frac{\partial x^b}{\partial \gamma} < 0$ and $\pi(1+B) > 1$ for inflation rates above the Friedman rule as we show in the proof of proposition 6. ■

A.5 Proof of Proposition 6

Proof. With $\mathcal{I} = 1$, aggregate capital investment and labor supply are in equilibrium:

$$K^{\mathcal{I}=1} = \frac{x^m}{R} + \frac{x^*}{R} - q^s \tag{46}$$

$$H^{\mathcal{I}=1} = \frac{x^m}{R} + q^m R + \frac{x^*}{R} - q^s = K^{\mathcal{I}=1} + q^m R \tag{47}$$

Deriving $K^{\mathcal{I}=1}$ with respect to inflation γ yields

$$\frac{\partial K^{\mathcal{I}=1}}{\partial \gamma} = -\frac{\partial q^m}{\partial \gamma}\mathcal{X}_1 > 0, \tag{69}$$

with $\mathcal{X}_1 = \frac{B+RA+\pi(1+B)}{B+RA+1}$.

Already a visual inspection of (46) tells us that there must be a Mundell-Tobin effect. From the proof of proposition 5, x^m increases with inflation and q^m and q^b and thus also $q^s = \pi q^m + (1-\pi)q^b$ decrease with inflation. The derivative confirms this, as all terms in $\mathcal{X}_1$ are positive.

From (47), the effect of inflation on aggregate labor supply in the first CM is the sum of a positive Mundell-Tobin effect and a negative effect over the real money holdings of buyers $q^m R$. This can be written as:

$$\frac{\partial H^{\mathcal{I}=1}}{\partial \gamma} = -\frac{\partial q^m}{\partial \gamma}\mathcal{X}_1 + R\frac{\partial q^m}{\partial \gamma} = \frac{\partial q^m}{\partial \gamma}\big(R - \mathcal{X}_1\big), \tag{70}$$

so the effect of an increase in inflation on aggregate labor supply depends on $R \lesseqgtr \mathcal{X}_1$.

From the definition of B in the proof to proposition 5, we know

$$\pi(1+B) = \pi + (1-\pi)\frac{U''(x^b)}{U''(x^m)} \geq 1,$$

since $x^m \geq x^b$ and $U'''(x) > 0$.[17] This implies $\mathcal{X}_1 \geq 1$. $\mathcal{X}_1 = 1$ if either $\pi = 1$ (capital is fully illiquid) or if $x^m = x^b = x^*$ (at the Friedman rule), and strictly larger otherwise. From the partial derivatives, increasing inflation from the Friedman rule increases the spread between DM-consumption x^m/x^b. Therefore $\pi(1+B)$ and $\mathcal{X}_1$ must rise with γ, while they are decreasing in π. Thus, the effect of inflation on aggregate labor is always negative at the Friedman rule. Away from the Friedman rule, higher inflation and higher liquidity of capital make it more likely that there is a positive effect of inflation on aggregate labor supply.

■

[17] $U''(x^b) < U''(x^m)$, but since both second derivatives are negative $(U''(x^b), U''(x^m) < 0)$, $U''(x^b)/U''(x^m) > 1$.

A.6 Proof of Proposition 7

Proof. With $\mathcal{I} = 0$, aggregate capital investment and labor supply are in equilibrium:

$$K^{\mathcal{I}=0} = \frac{x^m}{R} + \frac{x^*}{R} - q^s - q^m(\gamma - 1) \tag{50}$$

$$H^{\mathcal{I}=0} = \frac{x^m}{R} + \gamma q^m R - (\gamma - 1)q^m + \frac{x^*}{R} - q^s = K^{\mathcal{I}=0} + \gamma q^m R \tag{51}$$

Differentiating aggregate capital investment with respect to inflation yields:

$$\frac{\partial K^{\mathcal{I}=0}}{\partial \gamma} = -\Big(\frac{\partial q^m}{\partial \gamma}\gamma \mathcal{X}_0 + q^m\Big), \tag{71}$$

with $\mathcal{X}_0 = \frac{\pi(1+B)-1}{\gamma(RA+1+B)} + 1$. $\mathcal{X}_0 = 1$ for $\pi = 1$ (since $B = 0$ for $\pi = 1$) and at the Friedman rule (since $B = \frac{1-\pi}{\pi}$ at the FR). The condition for a Mundell-Tobin effect is

$$-\frac{\partial q^m}{\partial \gamma} > \frac{q^m}{\gamma \mathcal{X}_0} \tag{72}$$

$$|\varepsilon_{q^m}| > \frac{1}{\mathcal{X}_0} = \hat{\varepsilon} \tag{73}$$

using the definition of the elasticity of DM-consumption with respect to inflation: $|\varepsilon_{q^m}| = -\frac{\gamma}{q^m}\frac{\partial q^m}{\partial \gamma}$.

At the Friedman rule $\mathcal{X}_0 = 1$, so there is a Mundell-Tobin effect if $|\varepsilon_{q^m}|_{FR} > 1$. Using (66) and (67) with $q^m = q^b = q^*$ and $x^m = x^b = x^*$, the derivative of q^m with respect to inflation and the elasticity are:

$$-\frac{\partial q^m}{\partial \gamma}|_{FR} = \frac{1}{u''(q^*)}\frac{R(\pi RA + 1)}{\pi(1 + RA)} \tag{74}$$

$$|\varepsilon_{q^m}|_{FR} = -\frac{u'(q^*)}{q^* u''(q^*)}\xi, \tag{75}$$

with $\xi = \frac{1+\pi RA}{\pi(1+RA)}$. Thus, there is a Mundell-Tobin effect at the Friedman rule if:

$$-\frac{u'(q^*)}{q^* u''(q^*)}\xi > 1$$

$$\Rightarrow -\frac{u'(q^*)}{q^* u''(q^*)} > \frac{1}{\xi}. \tag{76}$$

$\xi \to 1$ for $\pi \to 1$, and $\xi \to \infty$ for $\pi \to 0$, and it can easily be shown that ξ is monotonically decreasing in π at the Friedman rule. This implies that for a given risk aversion, a Mundell-Tobin effect is more likely to occur at the Friedman rule for lower π.

Combining (51) and (54) the derivative of the aggregate labor supply with respect to inflation is:

$$\frac{\partial H^{\mathcal{I}=0}}{\partial \gamma} = (R - 1)(\frac{\partial q^m}{\partial \gamma}\gamma + q^m) - \frac{\partial q^m}{\partial \gamma}(\mathcal{X}_0 - 1). \tag{77}$$

This is negative if

$$-\frac{\partial q^m}{\partial \gamma} > \frac{q^m(R-1)}{\gamma(R-1)-(\mathcal{X}_0-1)} \tag{78}$$

$$|\varepsilon_{q^m}| > \frac{\gamma(R-1)}{\gamma(R-1)-(\mathcal{X}_0-1)} = \tilde{\varepsilon}, \tag{79}$$

so there is a negative effect of inflation on aggregate labor supply for $\gamma(R-1) > \mathcal{X}_0 - 1$. Since $\mathcal{X}_0 = 1$ at the Friedman rule, $\hat{\varepsilon}|_{FR} = \tilde{\varepsilon}|_{FR} = 1$, so the conditions coincide, and a Mundell-Tobin effect (reverse Mundell-Tobin effect) always implies a negative (positive) effect of inflation on aggregate labor supply. Away from the Friedman rule, $\mathcal{X}_0 > 1$, so $\hat{\varepsilon} < 1$ while $\tilde{\varepsilon} > 1$, and thus there is a range of values for $|\varepsilon_{q^m}|$ for which a Mundell-Tobin effect comes along with a positive effect of inflation on aggregate labor supply.

■

A.7 Proof of Proposition 8

Proof. With monetary policy implemented over young buyers expected welfare of a representative generation is given by:

$$\begin{aligned} V^{\mathcal{I}=1} &= -H^{\mathcal{I}=1} + \pi(u(q^m) + \beta U(x^m)) + (1-\pi)(u(q^b) + \beta U(x^b)) - q^s + \beta U(x^*) \\ &= -\frac{x^m}{R} - q^m R - \frac{x^*}{R} + \pi(u(q^m) + \beta U(x^m)) + (1-\pi)(u(q^b) + \beta U(x^b)) + \beta U(x^*) \end{aligned} \tag{80}$$

Differentiating (80) with respect to inflation and replacing $u'(q^m), U'(x^m)$ and $u'(q^b)$ using the optimality conditions (38), (39) and (41) yields:

$$\frac{\partial V^{\mathcal{I}=1}}{\partial \gamma} = -R\frac{\partial q^m}{\partial \gamma} + \gamma R \frac{\partial q^m}{\partial \gamma} - (1-\pi)\beta R U'(x^b)\Big(\frac{\partial q^m}{\partial \gamma} - \frac{\partial q^b}{\partial \gamma} + \frac{\frac{\partial x^m}{\partial \gamma} - \frac{\partial x^b}{\partial \gamma}}{R}\Big)$$

The last bracket is equal to zero from (63), and we obtain the identical expression as in the model with fully illiquid capital, (62):

$$\frac{\partial V^{\mathcal{I}=1}}{\partial \gamma} = \frac{\partial q^m}{\partial \gamma} R(\gamma - 1) \tag{81}$$

At the Friedman rule this expression must be positive since $\frac{\partial q^m}{\partial \gamma} < 0$ from (66) (and from proposition 7 the marginal effect on the labor supply is negative). From there expected welfare increases in inflation for $\gamma < 1$ and decreases for $\gamma > 1$. Thus the unique optimum must be $\gamma^* = 1$ independent of π and the other parameters.

To show that the Friedman rule is relatively more costly at lower levels of π, we show that $V^{\mathcal{I}=1}(\gamma^*) - V^{\mathcal{I}=1}(\gamma^{FR})$, the difference in expected welfare under optimal monetary policy ($\gamma^* = 1$) and the Friedman rule ($\gamma = 1/R$), decreases in π. From proposition 4, welfare at the Friedman rule is given by:

$$V^{\mathcal{I}=1}(\gamma^{FR}) = -\frac{2x^*}{R} - q^* R + u(q^*) + 2\beta U(x^*) \tag{82}$$

which is independent of π. Thus to show that $V^{\mathcal{I}=1}(\gamma^*) - V^{\mathcal{I}=1}(\gamma^{FR})$ decreases in π it is sufficient to show that expected welfare under optimal monetary policy is decreasing in π. From (80) $V^{\mathcal{I}=1}(\gamma^*)$ is given by:

$$V^{\mathcal{I}=1}(\gamma^*) = -\frac{x^m}{R} - q^m R - \frac{x^*}{R} + \pi(u(q^m) + \beta U(x^m)) + (1-\pi)(u(q^b) + \beta U(x^b)) + \beta U(x^*) \quad (83)$$

where all optimality conditions (38) to (41) hold and (38) is evaluated at $\gamma^* = 1$. Since (83) is evaluated at the optimum we can ignore the indirect effects of π on the variables and directly take the partial derivative of (83) with respect to π.

$$\frac{\partial V^{\mathcal{I}=1}(\gamma^*)}{\partial \pi} = u(q^m) + \beta U(x^m) - (u(q^b) + \beta U(x^b)) < 0. \quad (84)$$

Changes in expected welfare at γ^* through π reflect differences in the utility of consumption as a relocated and a non-relocated buyer. If utility of consumption as a relocated buyer is higher, expected welfare would rise with π and vice versa. Since $q^b > q^m$ and $x^m > x^b$ it is not a priori clear where utility is higher. However, since for a non-relocated buyer the relocated allocation $\{q^m, x^m\}$ is also feasible but not chosen, it must be that utility of non-relocated buyers is higher or $u(q^b) + \beta U(x^b) > u(q^m) + \beta U(x^m)$. Thus $\frac{\partial V^{\mathcal{I}=1}(\gamma^*)}{\partial \pi} < 0$, implying that expected welfare at the optimal monetary policy is decreasing in π. In turn, this shows that the welfare loss of running the Friedman rule is decreasing in π.

■

A.8 Proof of Proposition 9

Proof. Expected welfare when monetary policy is implemented over old buyers is given by:

$$\begin{aligned} V^{\mathcal{I}=0} &= -H^{\mathcal{I}=0} + \pi(u(q^m) + \beta U(x^m)) + (1-\pi)(u(q^b) + \beta U(x^b)) - q^s + \beta U(x^*) \quad (85) \\ &= -\frac{x^m}{R} - q^m R\gamma + (\gamma - 1)q^m - \frac{x^*}{R} + \pi(u(q^m) + \beta U(x^m)) + (1-\pi)(u(q^b) + \beta U(x^b)) + \beta U(x^*) \end{aligned}$$

Differentiating (85) with respect to inflation and replacing $u'(q^m), U'(x^m)$ and $u'(q^b)$ using the optimality conditions (38), (39) and (41) yields:

$$\frac{\partial V^{\mathcal{I}=0}}{\partial \gamma} = \frac{\partial q^m}{\partial \gamma}(\gamma - 1) - (R-1)q^m - (1-\pi)\beta R U'(x^b)\Big(\frac{\partial q^m}{\partial \gamma} - \frac{\partial q^b}{\partial \gamma} + \frac{\frac{\partial x^m}{\partial \gamma} - \frac{\partial x^b}{\partial \gamma}}{R}\Big)$$

The last bracket is zero from (63) and we obtain the identical expression as in the model with fully illiquid capital, (60):

$$\frac{\partial V^{\mathcal{I}=0}}{\partial \gamma} = \frac{\partial q^m}{\partial \gamma}(\gamma - 1) - (R-1)q^m \quad (86)$$

Since $\frac{\partial q^m}{\partial \gamma} < 0$ an increase in inflation can only be welfare improving if there is deflation ($\gamma < 1$). When is a deviation from the Friedman rule welfare improving? Evaluating (86) at the Friedman rule:

$$\frac{\partial V^{\mathcal{I}=0}}{\partial \gamma}|_{FR} = q^*(R-1)\Big(|\varepsilon_{q^m}|_{FR} - 1\Big) \tag{87}$$

Thus an inflation rate above the Friedman rule, $\gamma^* > 1/R$ is optimal if $|\varepsilon_{q^m}|_{FR} - 1$ holds, which is the same condition as for a Mundell-Tobin effect and a negative effect on aggregate labor supply.

If $|\varepsilon_{q^m}|_{FR} - 1$ holds, the optimal inflation rate $\gamma^* > 1/R$ is given by (86) set to 0 which is exactly the same expression as for the case of fully illiquid capital (60)

$$(1-\gamma^*) - \frac{\partial q^m}{\partial \gamma} = (R-1)q^m \tag{88}$$

$$\leftrightarrow \tag{89}$$

$$-\frac{\partial q^m}{\partial \gamma}\frac{\gamma^*}{q^m} = |\varepsilon_{q^m}| = \frac{(R-1)\gamma^*}{1-\gamma^*}. \tag{90}$$

It can easily be seen that the right-hand-side of this expression equals 1 at the Friedman rule, ∞ for $\gamma = 1$, and is strictly increasing in γ $\forall \gamma \in \{\frac{1}{R}, 1\}$. From the proof to proposition 7, we know that $|\varepsilon_{q^m}|_{FR} = \frac{1}{\eta(q^*)}\xi$. Thus, the Friedman rule is optimal if and only if $\eta(q^*) > \xi$. Since ξ is monotonically decreasing in π and $\xi \to \infty$ for $\pi \to 0$, higher liquidity of capital (lower π) makes it less likely for the FR to be optimal, and the FR is never optimal when capital is perfectly liquid.

With partially liquid capital, the optimal inflation rate γ^* is still given by (61):

$$\gamma^* = \frac{|\varepsilon_{q^m}|}{|\varepsilon_{q^m}| + R - 1,} \quad \in (1/R, 1) \tag{61}$$

but with $|\varepsilon_{q^m}|$ now also being a function of π.

The result that welfare is higher with $\mathcal{I} = 0$, given that γ^* is chosen, follows once again from the fact that the optimal allocation under $\mathcal{I} = 1$, which is achieved by setting $\gamma^* = 1$, is feasible with $\mathcal{I} = 0$, but generally not optimal.

■