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Juan Beccuti†

Abstract

This paper considers a multi-period setting where a monopolist, with short-term commitment, rents one unit of a durable good to a single consumer in every period. The consumer's valuation constitutes his private information and remains constant over time. By using a mechanism design approach, the paper shows that, when the monopolist and the consumer are sufficiently patient, the optimal renting strategy is to offer a simple price in every period. Although sophisticated mechanisms can make separation feasible when price-posting cannot achieve it, this happens precisely when separation is dominated by pooling. Moreover, the monopolist's choice of whether to discriminate or not depends on a simple and apparently myopic rule, reminiscent of its static equivalent.

Keywords: Durable good, renting, dynamic adverse selection, mechanism design, short-term commitment, price-posting.

JEL: D82, D86, D42.

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1. Introduction

Following the seminal contribution by Coase (1972), the durable goods literature offers valuable insights about the difficulty of exercising monopoly power in the absence of long-term commitment. While most results come under the assumption of simple price-posting, recent advances in dynamic mechanism design by Skreta (2006) and Doval & Skreta (2019*b*) show that this assumption is without loss of generality in a selling framework.

When a durable good is rented rather than sold, the optimality of price-posting remains an open issue.[1] Although the textbook analysis in Bolton & Dewatripont (2005) finds price-posting to be optimal in a two-period setting, Breig (2020) provides a four-period example for the sub-optimality of it.[2] The present paper uses a mechanism design approach with short-term commitment to investigate the optimality of price-posting in a setting with an arbitrary number of periods. The main contribution is to show that the monopolist cannot gain from offering a more sophisticated contract than a simple price in every period. However, this result depends intricately on the discounting of future payoffs. In particular, price-posting is optimal when the discount factor is either sufficiently high or sufficiently low. This positive finding complements Breig (2020) negative result because his example for the sub-optimality of price-posting relies on the discount factor taking intermediate values.

The paper considers a monopolist (*she*) renting a durable good to a single non-anonymous consumer (*he*) with unitary demand and private information about his valuation. The monopolist and the consumer face each other in a multi-period setting. While with two periods, the future information rents can never exceed the information rents of the current period, with more periods, the continuation values restrict how much separation the monopolist can achieve. Moreover, a multi-period environment provides a setting in which the monopolist can engage in a *gradual learning* strategy.

Hart & Tirole (1988) show that, when the number of periods is large, a monopolist renting a durable good is unable to price discriminate until the very end of the game. However, their setting restricts the monopolist to deterministic mechanisms: either the monopolist delivers the product after the consumer pays the required price or she does

[1] Renting differs from selling in that there exist new trade opportunities even after a positive transaction.

[2] Beccuti & Möller (2018) show that the optimality of price-posting in Bolton & Dewatripont (2005) is an artifact of the assumption that the monopolist and the consumer weigh future payoffs equally. In particular, price-posting turns to be suboptimal when the monopolist is more patient than the consumer. Bikhchandani & McCardle (2012) also consider the case with different levels of patience, but looks for the optimal prices when the monopolist has long-term commitment.

not deliver it at all.[3] Thus the monopolist's trade-off is simply between pooling (i.e., rent to any consumer at a low price) or price discrimination (i.e., rent to high types only at a high price).

The present paper extends the Hart & Tirole (1988) model by looking for the optimal menu of contracts for each period. In particular, the monopolist is not restricted to set a sequence of prices and, as a consequence, the monopolist may achieve separation through random mechanisms. While with deterministic mechanisms, the monopolist separates by refraining from trade with one type, with random mechanisms, the monopolist achieves separation by trading with both types with differing probabilities.

Related Literature. This paper connects to the literature on dynamic adverse selection, which shows that the *ratchet effect* harms the mechanism designer's market power due to her lack of long-term commitment. While the seminal papers (see, for instance, Freixas et al. (1985), Laffont & Tirole (1987), and Laffont & Tirole (1988)) restrict attention to a two-period framework, Hart & Tirole (1988) allow for longer horizons, but focus on price-posting. Devanur et al. (2019) study the dynamic pricing problem for a continuum of types, comparing the *partial* commitment with the short-term commitment case. This paper relaxes the assumption of price-posting in a framework with an arbitrary number of periods.

2. Model

A monopolist and a consumer interact repeatedly during a finite number of periods $T \geq 2$. In each period t, the monopolist rents one unit of a durable good produced at zero cost.[4] The consumer demands one unit of the good per period and is privately informed about his per-period valuation, denoted as his *type*. The consumer's type can be low ($\theta_L > 0$) or high ($\theta_H > \theta_L$) and is constant across periods. Define $\Delta\theta = \theta_H - \theta_L$ and let $\beta \in (0, 1)$ denote the monopolist's (prior) belief about the consumer's probability of having a high type. Both players discount the future with the same discount factor $\delta \in (0, 1)$.

The monopolist has short-term commitment, i.e., she can commit to a renting mechanism only for the current but not for future periods. In a setting in which there is direct communication between the consumer and the monopolist, as the one considered here, Bester & Strausz (2001) show that a modified revelation principle applies:[5] It allows the

[3] This restriction is present only in their short-term commitment case, while in the rest of the paper, they allow for general mechanisms.

[4] The environment considered in the present paper is equivalent to the repeated sale of a perishable good.

[5] See Doval & Skreta (2019*a*) for a setting with general *communication devices*.

monopolist to restrict to direct mechanisms and requires the consumer to reveal his true type only with strictly positive probability but not with certainty as in the static setting (Myerson (1981)).

Thus, at the beginning of every period, the monopolist offers a direct mechanism that specifies a payment from the consumer to the monopolist $w_m \in \Re$ and a likelihood of product-delivery $x_m \in [0,1]$ conditional on the consumer's message $m \in \{l, h\}$. Let $q_L \in [0,1)$ and $q_H \in (0,1]$ denote the probability with which the low- and the high-type consumer reports $m = h$ respectively. After observing a message the monopolist updates her belief about the consumer's type following Bayes' rule: $\beta_h \equiv \frac{\beta q_H}{Q}$ and $\beta_l \equiv \frac{\beta(1-q_H)}{1-Q}$, where $Q \equiv \beta q_H + (1-\beta) q_L$ is the ex-ante likelihood that the consumer reports a high type. Without loss of generality, the analysis focuses on the case where $q_H \geq q_L$ (equivalently $\beta_h \geq \beta \geq \beta_L$). When $q_H < q_L$, it is possible to rename messages and to interchange their roles.

When the consumer reports m his (instantaneous) surplus is given by $x_m\theta - w_m$ while the monopolist's (instantaneous) payoff is w_m. In the next period, the monopolist uses her updated belief β_m to propose a new mechanism. In what follows, $V_{t+1}(\beta_m)$ denotes the monopolist's continuation value for period t. Similarly, $U^H_{t+1}(\beta_m)$ and $U^L_{t+1}(\beta_m)$ denote the high- and the low-type consumer's continuation values respectively.

Using Perfect Bayesian Equilibrium as the solution concept, the monopolist's proposal at any period must be sequentially optimal given the reporting history up to that period. This is, the monopolist looks for the mechanism that maximizes her expected payoff taking into account the consumer's strategic behavior and her (potentially updated) belief about the consumer's type. Therefore, in period t, the monopolist solves

$$\max_{x_l, x_h, w_l, w_h, q_L<1, q_H>0} Q[w_h + \delta V_{t+1}(\beta_h)] + (1-Q)[w_l + \delta V_{t+1}(\beta_l)],$$

subject to

$$x_h\theta_H - w_h + \delta U^H_{t+1}(\beta_h) \geq x_l\theta_H - w_l + \delta U^H_{t+1}(\beta_l), \quad \text{(with equality when } q_H < 1) \quad (IC_H)$$

$$x_l\theta_L - w_l + \delta U^L_{t+1}(\beta_l) \geq x_h\theta_L - w_h + \delta U^L_{t+1}(\beta_h), \quad \text{(with equality when } q_L > 0) \quad (IC_L)$$

$$x_h\theta_H - w_h + \delta U^H_{t+1}(\beta_h) \geq 0, \quad (PC_H)$$

$$x_l\theta_L - w_l + \delta U^L_{t+1}(\beta_l) \geq 0. \quad (PC_L)$$

At any period, the consumer could choose not to participate and wait for the next period. In this case, the consumer not only gets zero instantaneous surplus, but it is also assumed that he gets zero continuation value.[6] Note also that, at some particular

[6]Since Bayes' rule does not apply after non-participation, it can be assumed that, in such a case, the

beliefs, the monopolist is indifferent among several mechanisms. To avoid more than one continuation equilibrium, the analysis assumes that the monopolist can credibly "promise" to offer the mechanism that reduces the consumer's reluctance to report his type.[7] For future references, a mechanism is a price-posting mechanism when there exist a $x_m \in \{0,1\}$ and a price $p \in \Re$ such that $w_m = px_m$ for all $m \in \{l,h\}$.

The monopolist's problem can be simplified using standard techniques.[8] In particular, in any period the monopolist chooses x_h, x_l, q_L, q_H to solve the *reduced program*:

$$\begin{aligned} \max_{x_h,x_l,q_L<1,q_H>0} \; & x_l\theta_L + Q\theta_H(x_h - x_l) + \\ & + (1-Q)\delta V_{t+1}(\beta_l) + Q\delta\{V_{t+1}(\beta_h) - [U^H_{t+1}(\beta_l) - U^H_{t+1}(\beta_h)]\}, \end{aligned} \tag{1}$$

subject to

$$x_h - \frac{\delta}{\Delta\theta}[U^H_{t+1}(\beta_l) - U^H_{t+1}(\beta_h)] \geq x_l, \quad \text{with equality if } q_L > 0. \quad (DMC)$$

The proof uses the fact that, as in the static case, the participation constraint of the high type (PC_H) is redundant, and the participation of the low type (PC_L) is binding at the optimum (implying $U^L_{t+1} = 0$ for any t). However, in contrast to the one-period setting, it is not clear which incentive constraint is binding at the optimum. Suppose (IC_L) is binding while (IC_H) is slack. Since (PC_L) is also binding, the price w_h is then equal to $x_h\theta_L$. Alternatively, if (IC_H) is the binding constraint, then it follows from $w_l = x_l\theta_L$ (i.e., from (PC_L) binding) that

$$\begin{aligned} w_h &= x_l\theta_L + (x_h - x_l)\theta_H - \delta[U^H_{t+1}(\beta_l) - U^H_{t+1}(\beta_h)], \\ &= x_l\theta_L + (x_h - x_l)\theta_H - \delta[U^H_{t+1}(\beta_l) - U^H_{t+1}(\beta_h)] + x_h\theta_L - x_h\theta_L, \\ &= (x_h - x_l)\Delta\theta - \delta[U^H_{t+1}(\beta_l) - U^H_{t+1}(\beta_h)] + x_h\theta_L, \\ &\geq x_h\theta_L, \end{aligned}$$

where the last inequality is due to the (DMC). Thus, by making (IC_H) binding, the monopolist cannot be worse off.

The Dynamic Monotonicity Constraint (DMC) results from substituting the (IC_H) into (IC_L). It not only requires the allocation to be increasing in the reported type but

monopolist assigns probability one to face a high-type consumer. Hence, the monopolist sets a high price in the next period and the consumer's continuation value is zero.

[7]For instance, in the last period T, the monopolist is indifferent between pooling and separating when $\beta = \theta_L/\theta_H$. Thus, when the monopolist's prior belief is higher than θ_L/θ_H in $T-1$, she promises to offer separation if her posterior belief becomes θ_L/θ_H after observing message l. Using the (IC_H), it can be seen that the incentives to truthfully report for the high-type consumer is larger when $U^H_T(\beta_l) = 0$ (due to future separation) than when $U^H_T(\beta_l) = \Delta\theta$ (due to future pooling). For the low-type, continuation values are zero under both mechanisms.

[8]A formal proof can be found in Beccuti & Möller (2019).

also imposes a wedge between the high-and the low-type's allocation. This wedge is a consequence of the *ratchet effect*: If the high-type consumer makes his type public, he will lose his future information rents (i.e., $U^H_{t+1}(\beta_l) - U^H_{t+1}(\beta_h)$). Therefore, to induce the consumer to reveal his type, the monopolist has to compensate him. In particular, when the monopolist does not want to refrain from trade with the low-type consumer, the (DMC) determines that the compensation comes in form of a reduction in x_l. Notice that $\delta = 0$ recovers the static case and its standard monotonicity constraint, when the ratchet effect does not play any role and there is no need for such compensation.

The (DMC) explains the difference with the Hart & Tirole (1988) approach. In particular, it restricts the degree of separation that the monopolist can achieve. Note, first, that an increment in x_h improves the monopolist's objective while it relaxes the constraint, implying $x^*_h = 1$. It follows that, separation by price-posting, i.e., with $x_l = 0$, may not be feasible if the number of periods remaining is sufficiently large. For instance, fully separating types via price-posting (with both types reporting truthfully) is not feasible when δ is sufficiently high since $\delta[U^H_{t+1}(0) - U^H_{t+1}(1)] = \delta[\sum_{i=0}^{T-(t+1)} \delta^i \Delta\theta] > \Delta\theta$. As an alternative, the monopolist may offer a semi-separating price-posting mechanism inducing $0 < \beta_l < \beta_h = 1$. Hart & Tirole (1988) show that, if the number of periods is large enough, even such semi-separating price-posting is not feasible (i.e., $\delta[U^H_{t+1}(\beta_l) - U^H_{t+1}(1)] > \Delta\theta$) and hence, when the monopolist is restricted to price-posting mechanism, she can only offer pooling.

However, for the same number of periods and monopolist's prior belief, the monopolist may still achieve some separation by offering a random delivery contract (i.e., $x_l \in (0, 1)$) and making (DMC) binding with $q_L > 0.$, i.e., she can choose another pair $q_L > 0, q_H < 1$ such that $1 - \frac{\delta}{\Delta\theta}[U^H_{t+1}(\beta_l) - U^H_{t+1}(\beta_h)] = x_l > 0$. The choice between pooling and this type of semi-separation is absent in Hart & Tirole (1988), due to their restriction to simple price-posting mechanisms.

3. Optimality of price-posting

The next proposition presents the main result of the paper.

Proposition 1. *Suppose there are $r > 2$ periods remaining and $\delta \in (\delta^*(r), 1)$, where $\delta^*(r)$ is the unique solution to $\delta^{r-2}(1 + \delta) = 1$ in $(0, 1)$. Then, the profit-maximizing renting mechanism is deterministic, $x_l, x_h \in \{0, 1\}$, i.e., it can be implemented by simple price-posting.*

All formal proofs can be found in the Appendix. In the following, we explain the intuition for the result of Proposition 1.

In any period, the monopolist has two alternatives. She may pool types (i.e., $x_l = x_h^* = 1$), giving up to the possibility of learning about the consumer's valuation. Alternatively, she may induce separation either by refraining from trade with the low type altogether or by offering him a random allocation. Note that the objective in the reduced program (1) is linear in x_l and decreases in x_l if $Q \geq \theta_L/\theta_H$. Suppose that the monopolist chooses to separate types by inducing $Q < \theta_L/\theta_H$. In this case, the monopolist's payoff increases with the likelihood of trading with the low type. Thus, the monopolist would like x_l to be different but as close as possible to x_h^*. However, the (DMC) imposes a restriction for the maximum value of x_l. This restriction arises as a cost of separation and, if the monopolist expects to receive a message l with high probability (i.e., Q is low), she is better off by not reducing the likelihood of trading with the low-type consumer, i.e., by pooling types. On the other hand, suppose she wants to separate by inducing $Q \geq \theta_L/\theta_H$ and reducing x_l. As in the two-period case, if it is feasible to induce a (potentially different) high Q with $q_L = 0$, then the monopolist will do so since the (DMC) does not bind and she can offer the separation price-posting $x_l = 0$. With more periods to go, this separation is feasible only if the prior β is high enough. For intermediate values of the prior, $q_L = 0$ is not feasible because, if δ is sufficiently large, the (DMC) does not hold even for $\beta_l = \beta$: $[U_{t+1}^H(\beta) - U_{t+1}^H(1)] > \frac{\Delta\theta}{\delta}$. Therefore, she can only separate by inducing $q_L > 0$ when the (DMC) determines the random allocation $x_l \in (0, 1)$ which decreases with δ. However, notice that for a large discount factor, and even when the monopolist restricts her learning to a small difference $[U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)]$, the likelihood of trading with the low type is small. Therefore, the monopolist faces a high cost of separation in exchange for low learning, and she is better off by pooling types.

The requirement that $\delta > \delta^*(r)$ ensures that x_l is low while the feasible learning is rather restricted. In particular, $\delta^{r-2}(1 + \delta) = 1$ implies the the high-type consumer can gain information rents in at most one period, i.e., $[U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] < \Delta\theta$.

As a result of the proposition, the monopolist has only to decide whether to pool or to separate types by incurring the cost of not trading with the low-type. As in the two-period setting, when the monopolist offers a separating price-posting, she does it by inducing truth telling from the low-type consumer. With such a mechanism, she is certain of facing a high-type after observing the message h. Therefore, for $q_L = 0$, it must hold that $1 - \frac{\delta}{\Delta\theta} U_{t+1}^H(\beta_l) > 0$ since the monopolist can only offer a mechanism that satisfies the (DMC). As $U_{t+1}^H(\beta)$ is decreasing in β, we can define $\hat{\beta}_l$ as the smallest β_l for which this inequality is still satisfied. In correspondence to $\hat{\beta}_l$, let $\hat{q}_H$ denote the likelihood with which the high-type is required to tell the truth to induce the posterior $\hat{\beta}_l$. With the help of $\hat{q}_H$, the next corollary states a rule governing the monopolist's choice between pooling and separation.

Corollary 1. *The monopolist finds it optimal to separate types when $\beta\hat{q}_H\theta_H \geq \theta_L$, and to pool them otherwise.*

Whether it is optimal to induce information revelation or not depends on a simple and apparently myopic rule. To choose between separation and pooling, in any given period the monopolist only needs to compare the payoffs of that period, disregarding all future payoffs. In particular, separation is optimal if the maximum feasible present period payoff from inducing the low-type to report truthfully is larger than the payoff from pooling both types, i.e., if $\beta \cdot \hat{q}_H \cdot \theta_H \geq \theta_L$.[9]

Continuation values matter only in that they determine how much separation the monopolist can possibly induce. This limitation goes unnoticed in a setting with two periods because, with only one period to go, future information rents can never exceed the information rents of the current period. Only by considering the case with more than two periods, it becomes clear that the degree of separation the monopolist can achieve can become restricted, making her choice between pooling and separation non-trivial. The following example serves to further illustrate these points.

Example. *The monopolist's ability to learn is limited according to*

$$\frac{\Delta\theta}{\delta} > U^H_{t+1}(\beta_l), \tag{2}$$

with $\beta_h = 1$ (i.e., $q_L = 0$), and $\beta_l = \frac{\beta(1-q_H)}{1-\beta q_H}$ from Bayes' rule.

In a two-period settings, the consumer's continuation values are (see, e.g., Bolton & Dewatripont (2005))

$$U^H_T(\beta) = \begin{cases} 0 & \text{if } \beta \geq \frac{\theta_L}{\theta_H} \\ \Delta\theta & \text{if } \beta < \frac{\theta_L}{\theta_H} \end{cases},$$

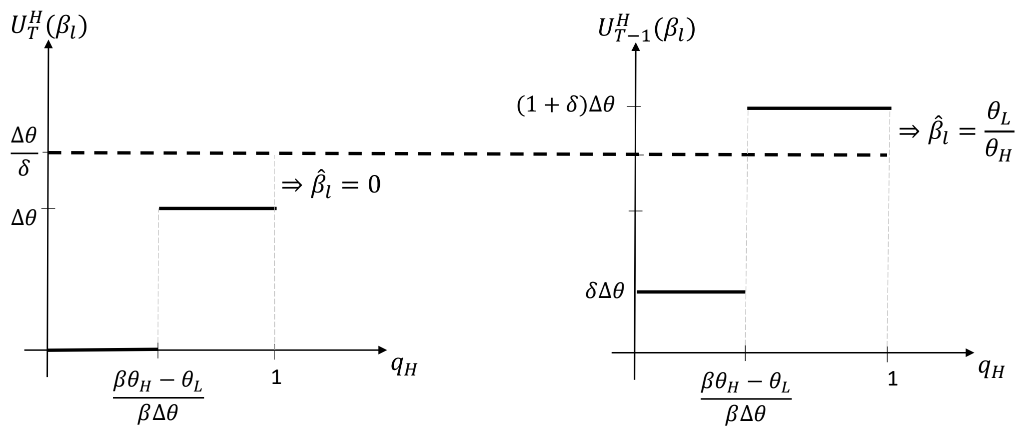
while in the three-period case,

$$U^H_{T-1}(\beta) = \begin{cases} 0 & \text{if } \beta > \frac{\theta_L}{\theta_H}\frac{\theta_H+\delta\Delta\theta}{\theta_L+\delta\Delta\theta} \\ \delta\Delta\theta & \text{if } \beta \in [\frac{\theta_L}{\theta_H}, \frac{\theta_L}{\theta_H}\frac{\theta_H+\delta\Delta\theta}{\theta_L+\delta\Delta\theta}] \\ (1+\delta)\Delta\theta & \text{if } \beta < \frac{\theta_L}{\theta_H} \end{cases}.$$

[9]Note, however, that the monopolist needs to take into account continuation values to decide among different semi-separating price-postings.

Note that, in the former case, condition (2) holds for any β_l, while in the latter case, the monopolist cannot induce $\beta_l < \theta_L/\theta_H$ when δ is sufficiently large.

The following picture illustrates the two-period setting (left hand panel), and the three-period one (right hand panel) when the monopolist's prior belief is $\beta \in [\frac{\theta_L}{\theta_H}, \frac{\theta_L}{\theta_H}\frac{\theta_H+\delta\Delta\theta}{\theta_L+\delta\Delta\theta}]$.



Let $\theta_L = 1$, $\theta_H = 2$, $\beta = 2/3$, and $\delta = 3/4$. In the two-period setting, there is no limit, $\beta \cdot 1 \cdot \theta_H = 4/3 > 1$, and the monopolist separates types. However, when $T = 3$, $\hat{\beta}_l = 0.5$ and, from Bayes' rule, $\hat{q}_H = \frac{\beta-\hat{\beta}_l}{\beta(1-\hat{\beta}_l)} = 0.5$. It follows that $\beta \cdot \hat{q}_H \cdot \theta_H = 2/3 < 1$ and the monopolist finds it optimal to pool types. Alternatively, if $\beta > 3/4$, then $\beta \cdot \hat{q}_H \cdot \theta_H > 1$ and the monopolist offers separation.

Proposition 1 and its Corollary show that the price-posting is optimal when δ is sufficiently high . In the opposite case, when δ is sufficiently low, it is straightforward to see that price-posting must also be optimal. In particular, for $\delta < 1/2$ it holds that $\delta[U_{t+1}^H(0) - U_{t+1}^H(1)] = \delta[\sum_{i=0}^{T-(t+1)} \delta^i \Delta\theta] < \Delta\theta$ for any number of periods and, hence, the (DMC) holds for any combination of posterior beliefs. As a consequence, $q_L = 0$ is feasible for any prior. Thus, when the prior is larger than θ_L/θ_H, the monopolist finds it optimal to separate with $x_l = 0$ by inducing $Q = \beta q_H \geq \theta_L/\theta_H$, while for low priors, she finds it optimal to pool with $Q < \theta_L/\theta_H$. On the other hand, for intermediate values of the discount factor, Breig (2020) shows that a monopolist restricted to price-postings may find it optimal to set a low price that the high-type consumer accepts while the low-type randomizes between acceptance and rejection. However, such price-posting makes the (IC_H) slack and can therefore be improved upon by the use of a random allocation. Hence, the optimality of price-posting must be non monotonic in the discount factor.

The next observation collects this implication. Since its proof is direct from previous discussion, it is omitted.

Observation. *The optimality of price-posting is not monotonic in the level of patience.*

4. Conclusion

The dynamic pricing problem when a monopolistic renter has short-term commitment has been broadly studied. This paper uses a mechanism design approach to derive the optimal renting strategy and shows that a restriction to price-posting comes without loss of generality when the discount factor is above a threshold. The choice between separation or pooling follows a simple and apparently myopic rule that only considers current payoffs. In particular, in a two consumer-types setting, separation is optimal when the monopolist can induce types to reveal themselves in a way that makes the likelihood of renting at a high price larger than the ratio of types.

Appendix A - Proofs

Proof of Proposition 1: The discussion following the reduced program at (1) already shows that for $x_h^* = 1$ in any period. It remains to show that $\delta > \delta^*(r)$ the optimal $x_l^* \in \{0, 1\}$.

The proof consists of two steps. Step 1 presents some useful properties of continuation values when the discount factor is large. Next, Step 2 proceeds by induction to show that $x_l^* \in \{0, 1\}$.

Step 1: Suppose any period $t \in \{1, ..., T\}$ and let r be the number of remaining periods at the beginning of the current one (i.e., any period $\{t, t+1, ..., T\}$ is equivalent to a period denoted with $\{r, r-1, ..., 1\}$). For the proof, and with the purpose of simplifying the notation in it, we refer to the remaining number of periods r.

Assume that $\delta \in (\delta^*(r), 1)$, where $\delta^*(r)$ is the unique solution in $(0, 1)$ to $\delta^{r-2}(1+\delta) = 1$ when $r \geq 2$.

The difference $U_{r-1}^H(\beta_l) - U_{r-1}^H(\beta_h)$ ($\equiv U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)$) is (at most) equal to the sum of the future information rents of the high-type consumer. Since the per-period information rent is $\Delta\theta$, the total future information rents in play is $\Delta\theta(1 + ... + \delta^{r-2})$. Hence, a difference in the continuation values larger than $\Delta\theta$ is, at least, equal to $\delta^{r-3}(1+\delta)\Delta\theta$ (i.e., the sum of the last two periods). However, in such a case, the (DMC) does not hold, since

$$1-\frac{\delta}{\Delta\theta}[U^H_{r-1}(\beta_l)-U^H_{r-1}(\beta_h)]\leq 1-\frac{\delta^{r-2}(1+\delta)\Delta\theta}{\Delta\theta}<0.$$

In other words,

$$U^H_{r-1}(\beta_l)-U^H_{r-1}(\beta_h)\in\{0,\delta^{r-2}\Delta\theta,...,\delta\Delta\theta,\Delta\theta\}, \tag{3}$$

and, by misreporting, the high-type gains informational rents in (at most) one period.

Step 2: From here, the proof proceeds by induction showing that the monopolist either pools by offering $x^*_l=1$ or separates with $x^*_l=0$. In particular, when ther are r remaining periods, there is a threshold $\hat{\beta}_r$ (with $\hat{\beta}_r\geq\hat{\beta}_{r-1}\geq..\geq\hat{\beta}_1$) such that, for any $\beta<\hat{\beta}_r$, the monopolist finds it optimal to pool types (i.e., $x_l=1$), while for any $\beta>\hat{\beta}_r$ she finds it optimal to separate them with price-posting (i.e., $x_l=0$). In the former case, w.l.g., $q^*_L=q^*_H$. In the latter, the low type does not randomize between contracts (i.e., $q^*_L=0$ and $\beta_h=1$), while the high type may randomize with $q^*_H\leq 1$ such that $U^H_{r-1}(\beta_l)\in\{0,\delta^{r-2}\Delta\theta\}$. At $\beta=\hat{\beta}_r$, the monopolist is indifferent between pooling or separating types, with $\hat{\beta}_r q^*_{H,r}(\hat{\beta}_r)\theta_H+\delta V_{r-1}(\hat{\beta}_r)=\theta_L+\delta V_{r-1}(\hat{\beta}_r)$, where $q^*_{H,r}(\beta)$ denotes the optimal q_H for the monopolist's belief β. Additionally, let $\hat{q}_{H,r}(\beta)\equiv\frac{\beta-\hat{\beta}_{r-1}}{\beta(1-\hat{\beta}_{r-1})}$ where $\hat{\beta}_{r-1}\equiv\inf\{\beta_l:U^H_{r-1}(\beta_l)\leq\delta^{r-2}\Delta\theta\}$. This is, for any $\beta\geq\hat{\beta}_{r-1}$, $\hat{q}_{H,r}(\beta)$ is the maximum feasible q_H when $q_L=0$. Additionally, $\hat{q}_{H,r}(\hat{\beta}_r)=q^*_{H,r}(\hat{\beta}_r)$ for every r.[10]

The last two periods are well known (see, e.g., Bolton & Dewatripont (2005), Chapter 9). In the last period T, when $r=1$, the monopolist offers $\{(x^*_l,x^*_h)\}=\{(1,1)\}$ with $q^*_L=q^*_H$ for $\beta\leq\theta_L/\theta_H$, and $\{(x^*_l,x^*_h),(q^*_L,q^*_H)\}=\{(0,1),(0,1)\}$ otherwise. The payoffs are

$$V_1(\beta)=\begin{cases}\beta q^*_H\theta_H\\ \theta_L\end{cases},\quad U^H_1(\beta)=\begin{cases}0 & \text{if } \beta\geq\frac{\theta_L}{\theta_H}\\ \Delta\theta & \text{if } \beta\leq\frac{\theta_L}{\theta_H}\end{cases}. \tag{4}$$

When $r=2$, the monopolist again offers pooling for low priors, while separates for large ones. In particular, she offers full separation $(q^*_L,q^*_H)=(0,1)$ for an intermediate value of the prior belief, while semi-separation $(q^*_L,q^*_H)=(0,\frac{\beta\theta_H-\theta_L}{\beta\Delta\theta})$ for large values. The payoffs are,

$$V_2(\beta)=\begin{cases}\beta q^*_H\theta_H+\delta\beta\theta_H\\ \beta q^*_H\theta_H+\delta\theta_L\\ \theta_L+\delta\theta_L\end{cases},\quad U^H_2(\beta)=\begin{cases}0 & \text{if } \beta\geq\frac{\theta_L}{\theta_H}\frac{\theta_H+\delta\Delta\theta}{\theta_L+\delta\Delta\theta}\\ \delta\Delta\theta & \text{if } \beta\in[\frac{\theta_L}{\theta_H},\frac{\theta_L}{\theta_H}\frac{\theta_H+\delta\Delta\theta}{\theta_L+\delta\Delta\theta}]\\ (1+\delta)\Delta\theta & \text{if } \beta\leq\frac{\theta_L}{\theta_H}\end{cases}. \tag{5}$$

[10] To simplify notation, in the main text, q_h and $\hat{q}_H$ denote $q_{H,r}(\beta)$ and $\hat{q}_{H,r}(\beta)$ respectively. For the purpose of the proof, the notation is now more specific.

Note that $\hat{\beta}_2 = \hat{\beta}_1 = \theta_L/\theta_H$, where the monopolist is indifferent between separating with $q^*_{H,i}(\hat{\beta}_i) = 1$ and pooling. Moreover, $\hat{q}_{H,i}(\hat{\beta}_i) = q^*_{H,i}(\hat{\beta}_i)$ in both $i = \{2, 1\}$.

Suppose now that in period $r-1$ the monopolist offers a price-posting to separate types when her belief is larger or equal than $\hat{\beta}_{r-1}$, or to pool them otherwise.

In any period the monopolist solves the reduced program (1). Because the objective is linear in x_l, then $x^*_l \in \{0, 1 - \frac{\delta}{\Delta\theta}[U^H_{r-1}(\beta_l) - U^H_{r-1}(\beta_h)], 1\}$, with the random allocation $x_l = 1 - \frac{\delta}{\Delta\theta}[U^H_{r-1}(\beta_l) - U^H_{r-1}(\beta_h)] \in (0, 1)$ as the only one that is not a price-posting. Notice that the case in (3) with $U^H_{r-1}(\beta_l) - U^H_{r-1}(\beta_h) = 0$ implies that the random mechanism allocates $x_l = 1$ as with the pooling price-posting.

When the monopolist screens types with $x_l = 1 - \frac{\delta}{\Delta\theta}[U^H_{r-1}(\beta_l) - U^H_{r-1}(\beta_h)]$, the reduced program at (1) becomes

$$\max_{q_L<1, q_H>0} \theta_L + \delta(1-Q)V_{r-1}(\beta_l) + \delta Q V_{r-1}(\beta_h) - \delta(1-Q)\frac{\theta_L}{\Delta\theta}[U^H_{r-1}(\beta_l) - U^H_{r-1}(\beta_h)]. \quad (6)$$

When $\beta_h > \hat{\beta}_i > \beta_l$ where $i \in \{r-1, ..., 1\}$, the monopolist is giving information rents to the high type in period i only after observing a message l. Thus, from (3), it must be $U^H_i(\beta_l) - U^H_i(\beta_h) = \delta^{r-i}\Delta\theta$. Because the monopolist cedes information rents in at most one period, from period $i-1$, the monopolist either pools or separates types under both posteriors. In the former case, she gives information rents no matter the messages reported and her expected continuation value for each of those periods is θ_L. In the latter case, she does not give information rents after any message, getting $(1-Q)\beta_l q^*_{H,j<i}(\beta_l)\theta_H + Q\beta_h q^*_{H,j<i}(\beta_h)\theta_H = \beta q^*_{H,j<i}(\beta)\theta_H$.[11] Otherwise, the "degree" of learning would be different and this would reflect in future $U^H_{(.)}(\beta_l) - U^H_{(.)}(\beta_h)$. Hence, $QV_{i-1}(\beta_h) + (1-Q)V_{i-1}(\beta_l) \equiv V_{i-1}(\beta)$.

We consider now two cases: **2.1)** when the prior is $\beta < \hat{\beta}_{r-1}$, and **2.2)** when $\beta \geq \hat{\beta}_{r-1}$.

2.1) Suppose the case $\beta < \hat{\beta}_{r-1}$. Since the monopolist is screening types with the random mechanism, then it must be that $\hat{\beta}_{i+1} \geq \beta_h > \hat{\beta}_i > \beta_l \geq \hat{\beta}_{i-1}$ for some $i \in \{r-1, ..., 1\}$. A larger difference between β_h and β_l, e.g., $\beta_l < \hat{\beta}_{i-1}$, implies that, while after message h the monopolist pools in period $i+1$, after message l she pools from period $i+1$ to $i-1$. This is, by misreporting, the high-type increases his future information rents in at least two periods, contradicting (3).

Therefore, the monopolist's payoff is

$$\theta_L \sum_{n=0}^{r-(i+1)} \delta^n + \delta^{r-i} Q\beta_h q^*_{H,i}(\beta_h)\theta_H + \delta^{r-(i-1)}V_{i-1}(\beta). \quad (7)$$

[11]Separation under both posteriors is only possible for those periods j in which $\beta_l \geq \hat{\beta}_j$.

Since $Q \equiv \frac{\beta-\beta_l}{\beta_h-\beta_l}$, then

$$\begin{aligned} Q\beta_h q^*_{H,i}(\beta_h) &\leq Q\beta_h \hat{q}_{H,i}(\beta_h) \\ &= \frac{\beta-\beta_l}{\beta_h-\beta_l}\beta_h \frac{\beta_h-\hat{\beta}_{i-1}}{\beta_h(1-\hat{\beta}_{i-1})} \\ &\leq \frac{\beta-\hat{\beta}_{i-1}}{\beta_h-\hat{\beta}_{i-1}}\beta_h \frac{\beta_h-\hat{\beta}_{i-1}}{\beta_h(1-\hat{\beta}_{i-1})} \\ &= \beta\frac{\beta-\hat{\beta}_{i-1}}{\beta(1-\hat{\beta}_{i-1})} \\ &= \beta\hat{q}_{H,i}(\beta), \end{aligned}$$

and the monopolist's payoff has the upper-bound

$$\theta_L \sum_{n=0}^{r-(i+1)} \delta^n + \delta^{r-i}\beta\hat{q}_{H,i}(\beta)\theta_H + \delta^{r-(i-1)}V_{i-1}(\beta).$$

When $\beta \in [\hat{\beta}_{i-1}, \hat{\beta}_i]$, then $\beta\hat{q}_{H,i}(\beta)\theta_H \leq \hat{\beta}_i q^*_{H,i}(\hat{\beta}_i)\theta_H = \theta_L$, and the monopolist is better off by pooling up to period $i-1$, achieving

$$\theta_L \sum_{n=0}^{r-i} \delta^n + \delta^{r-(i-1)}\beta q^*_{H,i-1}(\beta)\theta_H + \delta^{r-(i-2)}V_{i-2}(\beta).$$

When $\beta \in (\hat{\beta}_i, \hat{\beta}_{i+1}]$, then the monopolist pools types up to period $i+1$. In period i, and because also $\beta_h \in (\hat{\beta}_i, \hat{\beta}_{i+1}]$, she separates them with a price-posting that induces the same posterior $\beta_{l,i-1}$ than the random mechanisms, i.e., with $\beta q^*_{H,i}(\beta) = Q\beta_h q^*_{H,i}(\beta_h)$.

2.2) Consider now the case $\beta \geq \hat{\beta}_{r-1}$, i.e., when the random mechanism induces a separation such that $U^H_{r-1}(\beta_l) - U^H_{r-1}(\beta_h) = \delta^{r-2}\Delta\theta$.

When the posterior $\beta_l < \hat{\beta}_{r-1}$, it applies the previous analysis for $i = r-1$. This is, the monopolist achieves, with the random mechanism, at most the same payoff than with pooling.

On the other hand, when $\beta_l \geq \hat{\beta}_{r-1}$, with the random mechanism the monopolist separates types in $r-1$ under both posteriors, i.e.,

$$\begin{aligned} &(1-Q)V_{r-1}(\beta_l) + QV_{r-1}(\beta_h) = \\ &\qquad = \sum_{i=0}^{r-3} \delta^i [(1-Q)\beta_l q^*_{H,i-1}(\beta_l) + Q\beta_h q^*_{H,i-1}(\beta_h)]\theta_H + \delta^{r-2}[(1-Q)\theta_L + Q\beta_h\theta_H]. \end{aligned}$$

Since $\beta_l \geq \hat{\beta}_{r-1}$, it must be that $U^H_{r-1}(\beta_l) = \delta^{r-2}\Delta\theta$. Hence, the monopolist maximizes

$$\begin{aligned}
&\max_{q_L<1,q_H>0} \theta_L + \delta(1-Q)V_{r-1}(\beta_l) + \delta Q V_{r-1}(\beta_h) - \delta(1-Q)\frac{\theta_L}{\Delta\theta}U^H_{r-1}(\beta_l) \\
&\qquad = \max_{q_L<1,q_H>0} \theta_L + \sum_{i=1}^{r-2} \delta^i[(1-Q)\beta_l q^*_{H,i-1}(\beta_l) + Q\beta_h q^*_{H,i-1}(\beta_h)]\theta_H + \delta^{r-1}Q\beta_h\theta_H \\
&\qquad\quad = \max_{q_L<1,q_H>0} \theta_L + \sum_{i=1}^{r-2} \delta^i[\beta(1-q_H)q^*_{H,i-1}(\beta_l) + \beta q_H q^*_{H,i-1}(\beta_h)]\theta_H + \delta^{r-1}\beta q_H\theta_H.
\end{aligned}$$

The monopolist finds it optimal to induce the lowest feasible β_l (i.e., $\hat{\beta}_{r-1}$) and the highest feasible β_h (i.e., $\beta_h = 1$) such that $U^H_{r-1}(\beta_l) - U^H_{r-1}(\beta_h) = \delta^{r-2}\Delta\theta$ by choosing $q_L = 0$ and $q_H = \hat{q}_{H,r}$. To see this, suppose any other pair q_L, q_H such that $U^H_{r-1}(\beta_l) = \delta^{r-2}\Delta\theta$ and $U^H_{r-1}(\beta_h) = 0$. Then,

$$\begin{aligned}
&\beta(1-q_H)q^*_{H,i-1}(\beta_l) + \beta q_H q^*_{H,i-1}(\beta_h) \leq \beta(1-\hat{q}_{H,r}(\beta))q^*_{H,i-1}(\hat{\beta}_{r-1}) + \beta\hat{q}_{H,r}(\beta)q^*_{H,i-1}(1) \\
\Leftrightarrow \quad &(1-q_H)q^*_{H,i-1}(\beta_l) - (1-\hat{q}_{H,r}(\beta))q^*_{H,i-1}(\hat{\beta}_{r-1}) \leq \hat{q}_{H,r}(\beta) - q_H q^*_{H,i-1}(\beta_h).
\end{aligned}$$

Notice that both $q^*_{H,i-1}(\beta_l)$ and $q^*_{H,i-1}(\hat{\beta}_{r-1})$ must induce the same posterior $\tilde{\beta}_l$ if we want to have the same $U^H_{r-1}(\beta_l)$ in both cases. Thus, using $(1-q_H)q^*_{H,i-1}(\beta_l) = \frac{(1-Q)}{\beta}\frac{\beta_l-\tilde{\beta}_l}{1-\tilde{\beta}_l}$, it follows that

$$\begin{aligned}
(1-q_H)q^*_{H,i-1}(\beta_l) - (1-\hat{q}_{H,r}(\beta))q^*_{H,i-1}(\hat{\beta}_{r-1}) &= \frac{1}{\beta(1-\tilde{\beta}_l)}[(1-Q)(\beta_l-\tilde{\beta}_l) - (1-\hat{Q})(\hat{\beta}_{r-1}-\tilde{\beta}_l)] \\
&\leq \frac{1}{\beta(1-\tilde{\beta}_l)}[(1-Q)(1-\tilde{\beta}_l) - (1-\hat{Q})(1-\tilde{\beta}_l)] \\
&= \frac{1}{\beta}[\hat{Q} - Q] \\
&= \hat{q}_{H,r}(\beta) - \frac{Q}{\beta} \\
&\leq \hat{q}_{H,r}(\beta) - q_H q^*_{H,i-1}(\beta_h).
\end{aligned}$$

Therefore, by setting $q_H = \hat{q}_{H,r}$, the monopolist achieves:

$$\begin{aligned}
&\theta_L + \sum_{i=1}^{r-2} \delta^i[\beta(1-\hat{q}_{H,r}(\beta))q^*_{H,i-1}(\hat{\beta}_{r-1}) + \beta\hat{q}_{H,r}(\beta)]\theta_H + \delta^{r-1}\beta\hat{q}_{H,r}(\beta)\theta_H \\
&\qquad = \theta_L + \delta(1-\beta\hat{q}_{H,r}(\beta))V_{r-1}(\hat{\beta}_{r-1}) + \delta\beta\hat{q}_{H,r}(\beta)V_{r-1}(1) - \delta^{r-1}(1-\beta\hat{q}_{H,r}(\beta))\theta_L \qquad (8)
\end{aligned}$$

However, by offering a separating price-posting with $x_l = 0$, the monopolist can also induce $U^H_{r-1}(\beta_l) - U^H_{r-1}(\beta_h) = \delta^{r-2}\Delta\theta$, in which case, the (DMC) holds with strict inequality,

$$1 - \delta^{r-1} > 0,$$

implying $q_L = 0$ (i.e., $\beta_h = 1$). Thus, the monopolist can induce the same posteriors than with the random mechanism. After plugging into (1) the allocation $x_l = 0$, the monopolist gets

$$\beta \hat{q}_{H,r}(\beta)\theta_H + \delta(1 - \beta \hat{q}_{H,r}(\beta))V_{r-1}(\hat{\beta}_{r-1}) + \delta\beta\hat{q}_{H,r}(\beta)V_{r-1}(1) - \delta^{r-1}\beta\hat{q}_{H,r}(\beta)\Delta\theta. \quad (9)$$

Because $\beta\hat{q}_{H,r}(\beta)\theta_H \in [0, \theta_H]$ for $\beta \in [\hat{\beta}_{r-1}, 1]$, and is continuous and increasing in β, then there exists $\hat{\beta}_r \geq \hat{\beta}_{r-1}$ such that $\hat{\beta}_r\hat{q}_{H,r}(\hat{\beta}_r)\theta_H = \theta_L$. Hence, for any prior larger or equal to $\hat{\beta}_r$, the monopolist is better off by separating with the price-posting $x_L = 0$ than with the random allocation.

On the other hand, for a prior $\beta < \hat{\beta}_r$, the separation by random allocation dominates the one by price-posting. However, in such a case, the monopolist is better off by pooling. With pooling the monopolist achieves

$$\begin{aligned} V^{pool} &= \theta_L + \delta V_{r-1}(\beta) \\ &= \theta_L + \delta \max_{q_H \leq \hat{q}_{H,r-1}} \{\beta q_H \theta_H + \delta(1 - \beta q_H)V_{r-2}(\beta_{l,r-2}) + \delta\beta q_H[V_{r-2}(1) - U^H_{r-2}(\beta_{l,r-2})]\} \end{aligned} \quad (10)$$

On the other hand, with the random mechanism she gets (8) which is equivalent to:

$$\begin{aligned} = \theta_L + \delta\{(1 - \beta\hat{q}_{H,r}(\beta))[&\hat{\beta}_{r-1}q^*_{H,r-1}(\hat{\beta}_{r-1})\theta_H + \delta(1 - \hat{\beta}_{r-1}q^*_{H,r-1}(\hat{\beta}_{r-1}))V_{r-2}(\beta^*_{l,r-2}) + \\ &\delta\hat{\beta}_{r-1}q^*_{H,r-1}(\hat{\beta}_{r-1})V_{r-2}(1) - \delta\hat{\beta}_{r-1}q^*_{H,r-1}(\hat{\beta}_{r-1})U^H_{r-2}(\beta^*_{l,r-2})] + \\ &+ \beta\hat{q}_{H,r}(\beta)[\theta_H + \delta V_{r-2}(1)] - \delta^{r-2}\beta\hat{q}_{H,r}(\beta)\Delta\theta\}, \end{aligned}$$

with $\beta^*_{l,r-2} = \hat{\beta}_{r-2}$ because $q^*_{H,r-1}(\hat{\beta}_{r-1}) = \hat{q}_{H,r-1}(\hat{\beta}_{r-1})$. Using the identity $(1-\beta\hat{q}_{H,r}(\beta))(1-\hat{\beta}_{r-1}q^*_{H,r-1}(\hat{\beta}_{r-1})) = (1 - \beta q^*_{H,r-1}(\beta))$, the previous payoff is equal to

$$\begin{aligned} &\theta_L + \delta\{\beta q^*_{H,r-1}(\beta)\theta_H + \delta(1 - \beta q^*_{H,r-1}(\beta))V_{r-2}(\hat{\beta}_{r-2}) + \\ &\quad \delta\beta q^*_{H,r-1}(\beta)V_{r-2}(1) - \delta(1 - \beta\hat{q}_{H,r}(\beta))\hat{\beta}_{r-1}q^*_{H,r-1}(\hat{\beta}_{r-1})U^H_{r-2}(\hat{\beta}_{r-2}) - \delta^{r-2}\beta q^*_{H,r}(\beta)\Delta\theta\} \\ &\leq \theta_L + \delta\{\beta q^*_{H,r-1}(\beta)\theta_H + \delta(1 - \beta q^*_{H,r-1}(\beta))V_{r-2}(\hat{\beta}_{r-2}) + \\ &\qquad \delta\beta q^*_{H,r-1}(\beta)V_{r-2}(1) - \delta\beta q^*_{H,r-1}(\beta)U^H_{r-2}(\hat{\beta}_{r-2})\}, \end{aligned}$$

with equality for the prior $\hat{\beta}_r$. Note that the last expression is the solution of (10).

To show that $\hat{q}_{H,r}(\hat{\beta}_r) = q^*_{H,r}(\hat{\beta}_r)$, let suppose the contrary. Since $q^*_{H,r}(\hat{\beta}_r)$ must yield $U^H_{r-1}(\beta_l) = \{0, \delta^{r-2}\Delta\theta\}$, then, if it were the case with $U^H_{r-1}(\beta_l) = \delta^{r-2}\Delta\theta$, it should be that $\hat{q}_{H,r}(\hat{\beta}_r) = q^*_{H,r}(\hat{\beta}_r)$ by definition. On the other hand, if it were $U^H_{r-1}(\beta_l) = 0$, it should be $\hat{\beta}_r q^*_{H,r}(\hat{\beta}_r)\theta_H < \theta_L$. However, in this case, the monopolist, by offering the random mechanism gets

$$\max_{q_L<1, q_H>0} \theta_L + \delta(1 - Q)V_{r-1}(\beta_l) + \delta Q V_{r-1}(\beta_h),$$

and she is strictly better off than with the separating prices posting, yielding to a contradiction.

Finally, using the previous identity and algebraic manipulation, it is straightforward to show that (9) is equal to (10) when the prior is $\hat{\beta}_r$. Thus, $q^*_{H,r}(\hat{\beta}_r) = \theta_L/(\theta_H\hat{\beta}_r)$ is the optimal q_H for such a prior. Moreover, both payoffs are continuous and increasing in the prior, while $\partial(V^{(9)} - V^{(10)})/\partial\beta > 0$. Hence, the monopolist prefers the separation price-posting over pooling when $\beta > \hat{\beta}_r$. ∎

Proof of Corollary 1:

From previous proof, we know that, at the prior $\hat{\beta}_r$, the monopolist is indifferent between offering pooling or the separating price-posting, while she prefers the latter for larger priors. Note that, for $\beta \geq \hat{\beta}_r$, the monopolist can always induce $Q = \beta\hat{q}_{H,r}(\beta)\theta_H \geq \theta_L$ while for lower priors $\beta\hat{q}_{H,r}(\beta)\theta_H < \theta_L$. Thus, the monopolist knows that she has to offer the separating price-posting when the maximum feasible separation satisfies that conditions. ∎

References

Beccuti, J. & Möller, M. (2018), 'Dynamic adverse selection with a patient seller', *Journal of Economic Theory* **173**, 95–117.

Beccuti, J. & Möller, M. (2019), 'Screening by mode of trade', *Unpublished Manuscript* .

Bester, H. & Strausz, R. (2001), 'Contracting with imperfect commitment and the revelation principle: The single agent case', *Econometrica* **69**(4), 1077–1098.

Bikhchandani, S. & McCardle, K. (2012), 'Behavior-based price discrimination by a patient seller', *The BE Journal of Theoretical Economics* **12**(1).

Bolton, P. & Dewatripont, M. (2005), *Contract Theory*, MIT Press.

Breig, Z. (2020), 'Repeated contracting without commitment', *Unpublished Manuscript* .

Coase, R. (1972), 'Durability and monopoly', *Jourmal of Law and Economics* **15**(1), 143–149.

Devanur, N., Peres, Y. & Sivan, B. (2019), 'Perfect bayesian equilibria in repeated sales', *Games and Economic Behavior* .

Doval, L. & Skreta, V. (2019*a*), 'Mechanism design with limited commitment', *Unpublished Manuscript* .

Doval, L. & Skreta, V. (2019*b*), 'Optimal mechanism for the sale of a durable good', *Unpublished Manuscript* .

Freixas, X., Guesnerie, R. & Tirole, J. (1985), 'Planning under incomplete information and the ratchet effect', *Review of Economic Studies* **52**, 173–191.

Hart, O. & Tirole, J. (1988), 'Contract renegotiation and coasian dynamics', *Review of Economic Studies* **55**(4), 509–540.

Laffont, J. & Tirole, J. (1987), 'Comparative statics of the optimal dynamic incentive contract', *European Economic Review* **31**(4), 901–926.

Laffont, J. & Tirole, J. (1988), 'The dynamics of incentive contracts', *Econometrica* **56**, 1153–1175.

Myerson, R. (1981), 'Optimal auction design', *Mathematics of Operations Research* **6**(1), 58–73.

Skreta, V. (2006), 'Sequentially optimal mechanisms', *Review of Economic Studies* **73**(4), 1085–1111.