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Monetary Policy Implementation and Pass-Through

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DISCUSSION PAPERS

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Monetary Policy Implementation and Pass-Through*

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Abstract: I provide a simple general equilibrium model of monetary policy implementation and pass-through for undergraduate and graduate teaching. Besides a household and a firm, the model features a continuum of commercial banks, a government, and a central bank. The household uses deposits and cash to transfer resources over time. Monetary policy is implemented with open market operations and interest on reserves policies. I show that open market operations affect the money market rate, the government bond yield, and the deposit rate through changes in the insurance yield on reserves. At the interest rate floor, the insurance yield is zero. Therefore, open market operations become ineffective when reserves are ample. By contrast, interest on reserves policies change interest rates even at the interest rate floor. In addition, I find that expansionary monetary policies decrease expected commercial bank profits. Also, they increase household cash holdings in a monotonic, but non-linear fashion.

JEL classification: E41, E43, E52, E58

Keywords: Monetary policy implementation, monetary policy pass-through, open market operations, interest on reserves, negative interest rate policies

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1 Introduction

In many advanced economies, monetary policy has changed considerably since the Great Financial Crisis of 2008/2009. Before the crisis, the banking sector operated with a structural liquidity deficit. Therefore, commercial banks faced the risk of of falling short of the minimum reserve requirement. The central bank adjusted the supply of reserves to meet the interest rate target. Expansions in the monetary base decreased interest rates, and vice versa. Today, commercial banks operate with abundant reserves. Adjusting the supply of reserves does no longer affect the short-term money market rate. Therefore, central banks started to manage the financial compensation of reserves. Despite these changes, monetary pass-through has been sustained. The channel through which interest rates react to monetary policy is, however, different from before the crisis.

Even though monetary policy has evolved significantly, teaching materials have changed only little.[1] This paper contributes a general equilibrium model of monetary policy implementation and pass-through suitable for undergraduate and graduate teaching. In particular, it uses a transparent model to discuss the pass-through of monetary policy to the money market rate, the government bond yield, and the deposit rate. A particular focus lies on the differences between open market operations and interest on reserves policies. In addition, the paper sheds light on the effects of monetary policy on expected commercial bank profits and household cash holdings.

I synthesize and simplify existing models from two streams of the literature. First, the partial equilibrium literature on monetary policy implementation. Pioneered by Poole (1968), this literature shows that monetary policy affects the money market rate through the commercial banks' risk of a liquidity shortage. Reserves serve as a buffer against the risk of a liquidity shortage. That is, reserves earn an insurance yield or liquidity premium.[2] The insurance

[1] Examples from course materials used at Swiss universities in 2019 include: "The central bank controls the level of the nominal interest rate by supplying the money that is demanded at that rate" and "To raise the interest rate, the central bank reduces the money supply."

[2] The insurance yield is an economic, rather than financial, benefit of reserves.

yield rises in the liquidity risk and falls in reserves. It is zero at the interest rate floor. I augment the model of Poole (1968) with the opportunity for central banks to conduct interest on reserves policies. In addition, I embed his model in a general equilibrium framework. Boutros and Witmer (2019) study monetary policy implementation in a negative interest rate environment. They find that below the effective lower bound, commercial banks rather hold cash than reserves. I expand their model by investigating the household decision between deposits and cash.[3] Canetg and Kaufmann (2019) establish the equivalence between central bank debt and interest on reserves. Analogous to their work, I identify a cost channel through which monetary policy affects the money market rate. In addition to their results, I explain how monetary policy changes the government bond yield and the deposit rate.

Second, my model relates to a literature analyzing monetary policy pass-through in general equilibrium models with a banking sector. Kashyap and Stein (2012) argue that it is optimal to use interest on reserves policies to manage the inflation-output trade-off and open market operations to support financial stability. Because my paper is concerned with monetary policy implementation and pass-through, I analyze the effectiveness of interest on reserves policies and open market operations from a monetary policy viewpoint only. Nonetheless, my approach is consistent with their work in that the central bank can use reserves independently of the interest rate target. Drechsler et al. (2018) study how liquidity premia on government bonds interact with monetary policy. In my model, commercial banks cannot liquidate government bonds to redeem deposit withdrawals. Rather, I impose that liquidity shortages have to be covered with discount window borrowing. Abstracting from emergency sales of governments bonds simplifies the pricing of government bonds. Assuming that government bonds cannot be liquidated after the realization of the liquidity shock captures imperfections in commercial banks' liquidity management. The modelling decision is justified by a large literature on monetary policy implementation, including Afonso and Lagos (2015) and Bech and Monnet (2016). Piazzesi and Schneider (2199)

[3]Following Coleman (1996), Feenstra (1986), and McCallum (1983), I motivate the desire to hold cash with a transaction cost argument. For more details, see Section 3.5

show that the provision of inside money is costly because higher leverage increases the commercial bank specific money market rate. To make the model more tractable, I assume that commercial banks trade at a common money market rate. In my model, issuing deposits is costly because it raises the minimum reserve requirement. Piazzesi et al. (2018) study the transmission of monetary policy to output and inflation in a New Keynesian model with a banking system. While sharing many characteristics in terms of modeling commercial banks, my model remains silent about monetary policy transmission. Motivated by the aspiration to develop a model suitable for teaching purposes, I can thereby isolate more transparently various forms and channels of monetary policy implementation and pass-through.

My findings resonates well with existing results. For example, the impotence of open market operations at the interest rate floor is described by many widely used textbooks such as Mankiw (2014), Blanchard and Johnson (2013), and Brunetti (2009). The effectiveness of interest on reserves policies at the interest rate floor is documented by Hendrickson (2017), Ireland (2014), and Goodfriend (2002). My results also agree with Berentsen et al. (2019) who find that expansionary interest on reserves policies decrease commercial bank profits. Furthermore, the response of household cash holdings are consistent with Assenmacher et al. (2019).

My model features five types of agents. A representative household, a firm, a continuum of homogeneous commercial banks, a government, and a central bank. There is only one period. The period is divided into a morning and an afternoon. In the morning, the household supplies labor to the firm. The household cannot consume in the morning. Therefore, it invests the labor income in deposits and cash. In the afternoon, the household consumes deposits and cash, including interest rate returns, and transfers. Commercial banks offer deposit accounts to the household. They optimize, in the morning, over money market borrowing and lending, government bond investments, and deposits. In doing so, they take into account that reserves serve as a buffer against potential liquidity outflows.

The liquidity shock materializes after the money market closes at noon. The government provides a pre-determined amount of government bonds as well as equity funding to commercial banks. The central bank implements monetary policy through open market operations and interest on reserves policies.

In equilibrium, the marginal benefit of reserves determines the money market rate, the government bond yield, and the deposit rate. The marginal benefit of reserves consists of two components. First, the insurance yield on reserves. Second, the interest on reserves. Open market operations affect interest rates because they change the insurance yield on reserves. By contrast, interest on reserves change the financial compensation of reserves.

The main results of the paper are summarized as follows. First, in response to an expansionary open market operation, the money market rate, the government bond yield, and the deposit rate fall. Open market operations work through the insurance yield on reserves. Therefore, they are ineffective at the interest rate floor. Likewise, in response to an expansionary interest on reserves policy, the money market rate, the government bond yield, and the deposit rate rise fall. Interest on reserves policies are effective even at the interest rate floor. Second, expected commercial bank profits decrease in response to an expansionary interest on reserves policy. Expansionary open market operations, too, depress expected commercial bank profits; however, only off the interest rate floor. Third, household cash holdings fall in response to an expansionary monetary policy operation. They do so in a monotonic, but non-linear fashion. The results are symmetric for contractionary monetary policy operations.

The remainder of this paper is structured as follows. Section 2 provides a short account of how monetary policy has evolved since the Great Financial Crisis. Section 3 contains a non-technical model summary, as well as the formal derivation of the model. Section 4 presents the main results. Section 5 concludes.

2 Monetary policy implementation in reality

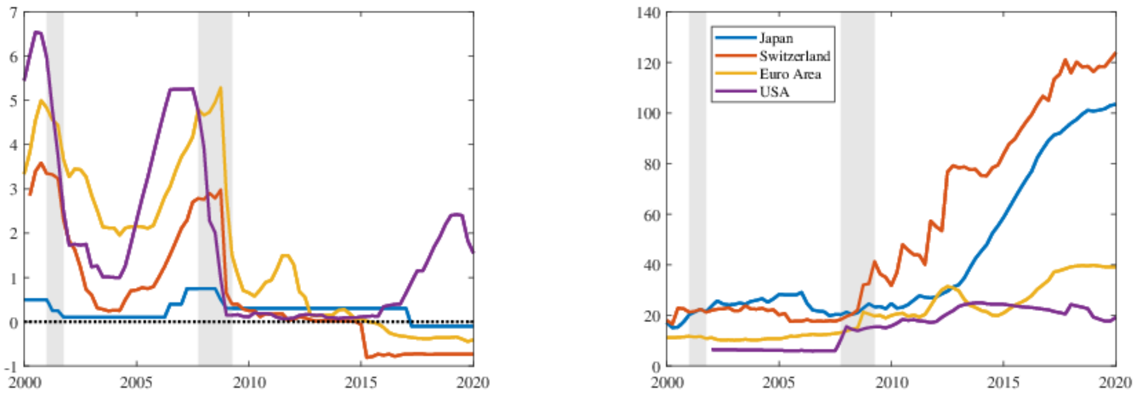
In many advanced economies before the Great Financial Crisis, central banks kept the monetary base in limited supply. A limited supply of base money maintained a structural liquidity deficit in the banking system. Therefore, to fulfill the minimum reserve requirement, commercial banks relied on the central bank. The Swiss National Bank (SNB), for example, used repurchase agreements (repos) to provide reserves to the banking sector. Reserves were auctioned in fixed rate tenders. That is, the SNB could lowered the repo rate to increase the amount of reserves in the banking sector. A greater amount of reserves in the banking sector depressed the insurance yield on reserves. In response, money market rates fell (Moser 2011). The Federal Reserve Bank (Fed) implemented monetary policy through open market operations. In particular, to decrease money market rates, the Fed purchased assets from financial intermediaries in exchange for reserves (Ihrig et al. 2015; Kroeger et al. 2018).

Since the Great Financial Crisis, monetary policy has changed considerably. Many central banks lowered their reference rates to zero. Some central banks even went negative, charging banks a negative interest on reserves (Figure 1a). At the same time, central banks injected massive amounts of reserves into the banking system (Figure 1b).

To illustrate, the size of the Swiss National Bank's balance sheet increased from 18% of GDP (CHF 150 billion) in 2007 to 120% of GDP (CHF 830 billion) in 2018. In 2019, commercial banks fulfilled their minimum reserve requirement by a factor 25 while in 2007, the banking system operated under a structural liquidity deficit (SNB 2019; Moser 2011). Currently (2019), the Swiss National Bank targets an interest rate of -0.75% (2007: 2.75%).

As reserves increased, the risk of a liquidity shortage became negligible. Therefore, the insurance yield on reserves fell to zero. In other words, the money market rate reached the interest rate floor. Managing the supply of reserves became ineffective. As a consequence, central banks began to use interest on reserves policies to implement monetary policy. The

Figure 1 — Negative Interest Rate Policies and Quantitative Easing



(a) Central Bank Reference Rates

(b) Central Bank Balance Sheets in % of GDP

Notes: The figure plots the discount rate for Japan, the 3-month LIBOR for Switzerland and the Euro Area, and the effective federal funds rate for the U.S. The gray shaded areas indicate the NBER recessions for the U.S. Data sources: National Bureau of Economic Research (NBER), Fred Economic Data (St. Louis Federal Reserve Bank), Staatssekretariat für Wirtschaft (Schweiz), and Swiss National Bank.

Federal Reserve Bank was granted the right to pay interest on reserves in fall 2008 in order to sustain positive interest rates amidst a structural liquidity surplus (Fed 2019). The Swiss National Bank first used interest on reserves when it announced a negative interest rate target in 2014 (SNB 2015).

3 Model setup

This section is organized as follows. First, I provide a non-technical model summary. Second, I derive the cashless model. In the cashless model, the household can only use deposits to transfer resources from the morning to the afternoon. Third, I discuss the model with cash. In the model with cash, the household can use deposits and cash to transfer resources from the morning to the afternoon. Forth, I provide the detailed timing of the model. Fifth, I justify the assumptions on the functional forms and the parameterization. Sixth, I present the model solution.

3.1 Non-technical model summary

There are five types of agents in the economy. A representative household, a firm, a continuum of homogeneous commercial banks, a government, and a central bank. There is only one period. The period is divided into a morning and an afternoon.

3.1.1 Household

In the morning, the household is endowed with a strictly positive, but numerically negligible endowment.[4] In addition, the household earns labor income. The household cannot consume in the morning. Therefore, it invests the endowment and labor income in deposits and cash. The household does not have any other means to transfer resources to the afternoon. In the afternoon, the household consumes deposit and cash, including interest rate returns, as well as firm profits, and the lump-sum transfers from the government and the central bank. The household optimizes over consumption, the extensive margin of its labor supply, deposits, and cash.[5] The household holds deposits and cash for two reasons. First, the household derives utility from deposits and cash. Second, either deposits or cash are needed to transfer resources from the morning to the afternoon.

3.1.2 Firm

The firm demands labor from the household in the morning. It produces the consumption good. The consumption good is supplied to the household in the afternoon. The firm profits are transferred to the household.

3.1.3 Commercial banks

As assets, commercial banks hold central bank reserves and government bonds. The amount of reserves is exogenous to commercial banks. Commercial banks are financed with

[4]The endowment ensures strictly positive consumption even if the household abstains from supplying labor. For more details, see Section 3.2.

[5]Appendix C shows that the qualitative results with regards to monetary policy implementation and pass-through do not change with if the household optimizes over the intensive margin of its labor supply.

deposits and equity. Deposits are provided by the household and equity is provided by the government.[6] Commercial banks exchange reserves among each other on the money market.

Commercial banks face a liquidity risk. In particular, the household randomly transfers a fraction of its deposits to another commercial bank. The liquidity shock materializes after the the money market closes at noon. A commercial bank is in a regulatory liquidity shortage if its reserves fall below the minimum reserve requirement. In a regulatory liquidity shortage, the commercial bank has to borrow at the central bank's discount window.

Commercial banks optimize over money market borrowing and lending, government bond investments, and deposits. They take interest on reserves and the discount rate as given. By contrast, the government bond yield, the money market rate, and the deposit rate are determined endogenously. Commercial bank profits are equal to the interest rate returns on reserves and government bonds, minus the interest rate payments on deposits, and – in case of a regulatory liquidity shortage – discount window borrowing. The profits are transferred to the government.[7]

3.1.4 Government

The government provides a pre-determined amount of government bonds. In addition, it provides equity funding to commercial banks. The government balances its budget through lump-sum transfers to/from the household.

3.1.5 Central bank

The central bank offers reserves to commercial banks and cash to the household. The asset side is made up of government bonds and claims from discount window lending. The central bank profits are equal to the interest rate returns on government bonds and discount window lending, minus the interest rate payments on reserves and cash. The yield on cash

[6] Appendix D shows that the results do not change if an entrepreneur provides equity.
[7] Appendix D shows that the results do not change with a different profit distribution.

is weakly negative. A negative return on cash may be interpreted as a loss of cash. Profits are transferred to the household. The central bank implements monetary policy through open market operations and interest on reserves policies.

3.1.6 *Equilibrium*

In equilibrium, the amount of government bonds available to commercial banks is fixed by exogenous monetary and fiscal policy decisions. Money market borrowing is zero because all commercial banks are alike prior to the closing of the money market. The equilibrium quantity of deposits is determined by the endowment, the labor supply of the household, and the wage.

The money market rate and the government bond yield ensure that the representative commercial bank is indifferent between asset classes (reserves, money market lendings, and government bonds). The deposit rate ensures that the representative commercial bank is indifferent between funding sources (deposits and money market borrowing). The deposit rate is lower than the money market rate because deposit funding increases the minimum reserve requirement. From the perspective of the household, the deposit rate ensures that deposits and cash are equally attractive.

3.1.7 *Preview of results*

Expansionary open market operations decrease the insurance yield on reserves. As a consequence, reserves become less attractive to commercial banks. The commercial banks' willingness to pay for reserves decreases. The money market rates falls. Because banks have to be indifferent between asset classes and funding sources, the government bond yield and the deposit rate fall, too. Similarly, lower interest on reserves make it less profitable for commercial banks to hold reserves. They become more inclined to lend funds on the money market. Thereby, they depress money market rates and, hence, the government bond yield and the deposit rate.

Open market operations affect interest rates through changes in the insurance yield on reserves. At the interest rate floor, the insurance yield is zero. Therefore, open market operations become ineffective when reserves are ample. By contrast, interest on reserves policies change interest rates even at the interest rate floor.

3.2 The cashless model

In this setup, the household does not have access to cash.

3.2.1 Household

The representative household solves a utility maximization problem subject to two budget constraints. The utility function is additively separable in consumption $C > 0$, deposits $D > 0$, and hours worked $N \geq 0$.

The budget constraint of the morning is associated to the Lagrange multiplier λ_1.[8] In the morning, the household is endowed with $F > 0$, allowing for an extensive labor supply decision $N \in \{0, 1\}$ as in Chetty (2005).[9] Furthermore, the household invests in deposits. The household does not have any other means to transfer resources to the afternoon. In particular, it does not have access to government bonds and commercial bank equity. The nominal wage is W.

The budget constraint of the afternoon is associated to the Lagrange multiplier λ_2. In the afternoon, the household consumes deposit, including interest rate returns, as well as firm profits Π^F, and the lump-sum transfers from the central bank $T^{cb,HH}$ and the government $T^{g,HH}$. The net deposit rate i^d is determined endogenously.

The household holds deposits for two reasons. First, it derives utility from deposits. Second, deposits are needed to transfer resources from the morning to the afternoon. Formally, the

[8] The timing of the model is outlined in Section 3.4.

[9] I impose $F > 0$ to ensure strictly positive consumption at $N = 0$. Appendix C presents a model in which the household optimizes over the intensive margin of its labor supply. Setting up the model with an intensive labor supply decision constraints the parameter space for which I can compute an analytical solution. Moreover, it complicates the model without providing additional insights. The qualitative results with regards to monetary policy implementation and pass-through remain unaffected.

second motivation is sufficient to ensure strictly positive deposits in equilibrium. However, I find it is useful to additionally include deposits in the utility function to later determine the interior partitioning between deposits and cash.[10]

The household solves the following optimization problem.

$$\mathcal{L} = u(C) + u(N) + u(D) + \lambda_1(F + WN - D) + \lambda_2(T^{cb,HH} + T^{g,HH} + \Pi^F + (1 + i^d)D - C)$$

Household optimality requires

$$u'(D) + (1 + i^d)u'(C) = \lambda_1 \ ,$$

where $u'(C) > 0$ and $u''(C) < 0$, $u'(D) > 0$ and $u''(D) < 0$, and

$$N = \begin{cases} 1, & \text{if } \lambda_1 W \geq u'(N) \quad \text{(1a)} \\ 0, & \text{otherwise,} \quad \text{(1b)} \end{cases}$$

where $u'(N) < 0$ and $u''(N) > 0$.

Earning an additional unit of labor income has two benefits. First, labor income can be invested in deposits. Deposits yield a direct utility. Second, higher labor income allows for higher consumption in the afternoon. In particular, earning an additional labor income unit in the morning increases consumption by $(1 + i^d)$ units in the afternoon. The cost of earning labor income is positive because the household derives utility from leisure. Optimal behaviour requires that the household works if the benefits of supplying labor outsize the costs.

3.2.2 *Firm*

The firm solves a profit maximization problem subject to the production function $Y = N_d^\alpha$, where $\alpha \in (0, 1]$. $N_d \geq 0$ denotes the quantity of labor demanded.

$$\mathcal{L} = (N_d^\alpha - WN_d) + \varepsilon_{N_d} N_d$$

[10] For details, see Section 3.3. The same argument applies to cash in the model with cash.

Firm optimality under $N_d > 0$ requires

$$\alpha \frac{Y}{N_d} = W \ .$$

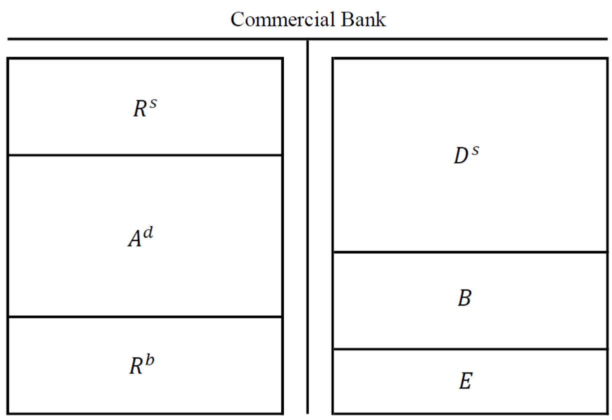
The benefit of hiring an additional worker is the worker's contribution to output; that is, the marginal product of labor. The costs of hiring a worker is the wage. The firm increases (decreases) the quantity of labor demanded if the benefit of an additional worker is greater (smaller) than the cost. Optimal behavior requires that the marginal product of labor equals the wage.

3.2.3 Commercial banks

In the morning, the central bank offers non-borrowed reserves $R^s > 0$ to commercial banks. In addition, commercial banks demand government bonds $A^d \geq 0$. The assets are financed with deposits $D^s \geq 0$ and equity $E \geq 0$.

Commercial banks engage in money market trading. In particular, commercial banks exchange non-borrowed reserves among each other at the net money market rate i^m. B is money market borrowing and R^b are reserves acquired through money market borrowing. Money market borrowing may be negative. By construction, reserves acquired through money market borrowing equal money market borrowing ($R^b \equiv B$).

Figure 2 — Commercial bank balance sheet



Notes: Commercial banks hold non-borrowed reserves R^s, government bonds A^d, and reserves acquired through money market borrowing R^b. The assets are financed with deposits D^s, money market borrowing B, and equity E. Money market borrowing B, and $R^b \equiv B$ may be negative.

After the money market closes at noon, commercial banks face a zero-mean liquidity shock $Z \leq D^s$, where $Z > 0$ is a liquidity outflow. In particular, the household randomly transfers a fraction of its deposit to another commercial bank. A commercial bank is short of liquidity if reserves after the shock fall short of the minimum reserve requirement $K \equiv \delta D^s$, where $\delta \in [0, 1]$. Formally, the regulatory liquidity shortage is $X \equiv Z - (R^s + R^b) + K$. In a regulatory liquidity shortage, the commercial bank has to borrow at the central bank's discount window. Commercial banks cannot lend funds to the central bank through the discount window. Therefore, borrowed reserves are weakly positive $R^x \equiv \max(0, X)$. After potential discount window borrowing, the balance sheet of the each commercial bank satisfies

$$
\begin{aligned}
&R^s + A^d + R^b + R^x = D^s + E + B + X\mathbb{I}_{X\geq 0} \\
&R^b \equiv B \\
&R^x \equiv \max(0, X) \; ,
\end{aligned}
$$

where $\mathbb{I}_{X\geq 0} = 1$ if $X \geq 0$ and $\mathbb{I}_{X\geq 0} = 0$ otherwise. Non-borrowed reserves R^s and reserves acquired through money market borrowing R^b pay the exogenous net interest rate i^r (interest on reserves). Interest on reserves may be negative. The government bond yield is i^a. Borrowed reserves do not pay interest. The funding costs are i^d for deposits, i^m for money market borrowing, and $i^x \equiv \max(0, \gamma i^r + g)$ for discount window borrowing. $g \geq 0$ is the width of the interest rate corridor in a Poole (1968) model and $\gamma \in [0,1]$ allows to calibrate the sensitivity of the discount rate with respect to the interest on reserves. To ensure that regulatory liquidity shortages are costly, I restrict the discount window rate to be weakly positive.

Each commercial bank optimizes over money market borrowing, government bond investments, and deposits. That is, in equilibrium, the money market rate, the government bond yield, and the deposit rate must be such that the commercial bank is indifferent between each pair of assets (reserves, government bonds, money market lendings) prior to potential discount window borrowing. Likewise, the commercial bank has to be indifferent between its funding sources (deposits, money market borrowing) prior to potential discount window borrowing.

The commercial bank's optimization problem is constrained by three non-negativity conditions. First, I impose a non-negativity constraints on $(R^s + B)$. The non-negativity constraint on reserves captures the commercial bank's inability to lend more reserves on the money market than it has available. Second, the commercial bank cannot hold negative amounts of government bonds ($A^d \geq 0$). Third, the commercial bank cannot offer negative deposits $D^s \geq 0$. The profit maximization problem takes the following form.

$$\begin{aligned}\mathcal{L} = i^r(R^s + R^b) + i^a A^d - i^m B - i^d \mathbb{E}(D^s - Z) - i^x \int_{\hat{z}}^{\infty} X f(Z) dZ \\ + \varepsilon_R (R^s + B) + \varepsilon_A A^d + \varepsilon_D D^s \, ,\end{aligned}$$

where $\mathbb{E}$ denotes the commercial bank's expectation conditional on an information set prior to the closing of the money market and $\hat{z} \equiv (R^s + R^b) - K$ is the realization of the liquidity shock for which $X = 0$.

Using the Leibniz rule and $X(\hat{z}) = 0$, the Karush-Kuhn-Tucker conditions require

$$\begin{aligned}
&B\big(i^r + i^x P(X > 0) - i^m + \varepsilon_R\big) = 0 \\
&A^d\big(i^a - (i^r + i^x P(X > 0)) - \varepsilon_R\big) = 0 \\
&D^s\big(i^r + i^x P(X > 0)(1-\delta) - i^d + \varepsilon_R\big) = 0 \\
&\varepsilon_R(R^s + B) = 0 \\
&\varepsilon_A A^d = 0 \\
&\varepsilon_D D^s = 0 \,,
\end{aligned}$$

with $R^s + B \geq 0$, $A^d \geq 0$, $D^s \geq 0$, $\varepsilon_R \geq 0$, $\varepsilon_A \geq 0$, and $\varepsilon_D \geq 0$, as in Wallace (2004).

Let us first analyze the money market. The marginal cost (benefit) of borrowing (lending) one unit of reserves on the money market is the money market rate i^m. The marginal benefit of an additional unit of reserves is the interest on reserves i^r plus the insurance yield on reserves $i^x P(X > 0)$. If the marginal cost of borrowing reserves on the money market is strictly greater (smaller) than the marginal benefit of reserves, the commercial bank decreases (increases) money market borrowing. Money market borrowing may become negative. $B < 0$ is money market lending. If the marginal benefit of lending reserves on the money market is strictly greater than the marginal benefit of reserves for each potential $B > -R^s$, the commercial bank is lower bound constrained in money market lending, reflected in $\varepsilon_R \geq 0$. If the marginal benefit (cost) of lending (borrowing) reserves on the money market equals the marginal benefit of an additional unit of reserves, the commercial bank is indifferent between increasing and decreasing money market lending and borrowing at $B \in (-R^s, 0]$ and $B > 0$, respectively. The marginal cost of the lower bound constraint on $R^s + B$ is then zero ($\varepsilon_R = 0$).

Second, the market for government bonds. Commercial banks buy government bonds with reserves.[11] The marginal return of an additional unit of government bonds is the government bond yield i^a. The marginal benefit of an additional unit of reserves is the

[11] Commercial banks can only buy government bonds from the central bank or from other commercial banks. Therefore, if a commercial bank increases its government bond investments, it simultaneously decreases its reserves.

interest on reserves i^r plus the insurance yield on reserves $i^x P(X > 0)$. If the marginal return on government bonds is strictly smaller (greater) than the marginal benefit of reserves, the commercial bank decreases (increases) its government bond investments. If the marginal return on government bonds is strictly smaller than the marginal benefit of reserves for each potential $A^d > 0$, the commercial bank abstains from investing in government bonds ($A^d = 0$), reflected in $\varepsilon_A \geq 0$. On the contrary, if the marginal return on government bonds is strictly greater than the marginal benefit of reserves for each potential $A^d > 0$, the commercial bank is lower bound constrained in reserves, reflected in $\varepsilon_R \geq 0$. If the marginal return on government bonds equals the marginal benefit of reserves, the commercial bank is indifferent between increasing and decreasing government bonds at $A^d > 0$. The marginal cost of the lower bound constraints on A^d and $(R^s + B)$ are then zero ($\varepsilon_A = \varepsilon_R = 0$).

Third, the deposit market. Attracting deposits from another commercial bank increases reserves.[12] The marginal cost of offering an additional unit of deposits is i^d. The marginal benefit of an additional unit of reserves is the interest on reserves i^r plus the insurance yield on reserves acquired through deposits $i^x P(X > 0)(1 - \delta)$. The insurance yield on reserves acquired through deposits is lower than the insurance yield on reserves acquired through either money market borrowing or sales of government bonds. The reason is that an additional unit of reserves acquired through deposits increases the minimum reserve requirement by δ. If the marginal cost of deposits is strictly greater (smaller) than the marginal benefit of reserves acquired through deposits, the commercial bank decreases (increases) deposits. If the marginal cost of deposits is strictly greater than the marginal benefit of reserves acquired through deposits for each potential $D^s > 0$, the commercial bank is either lower bound constrained in reserves, reflected in $\varepsilon_R > 0$, or lower bound constrained in deposits, reflected in $\varepsilon_D > 0$ (or both). If the marginal cost of deposits is equal to the marginal benefit of an additional unit of reserves acquired through deposits, the commercial bank is indifferent between increasing and decreasing deposits at $D^s > 0$. The

[12]Commercial banks cannot create deposits in the process of granting a loan. Rather, they attract deposits from other commercial banks. Therefore, if they increase deposits, reserves increase, too.

marginal cost of the lower bound constraints on $(R^s + B)$ and D^s are then zero ($\underline{\varepsilon}_R = \underline{\varepsilon}_D = 0$). Commercial bank profits are

$$\Pi^{com} = i^r R^s + i^a A^d - i^d D^s - i^x X \mathbb{I}_{X \geq 0} \, .$$

There are two sources of costs from operating a commercial bank. First, holding reserves is costly because the interest on reserves is weakly smaller than the deposit rate ($i^r - i^d \leq 0$). Second, regulatory liquidity shortages are costly if they materialize ($i^x X \geq 0$). Meanwhile, commercial banks incur a gain from being in business. In particular, the government bond yield is weakly greater than the deposit rate ($i^a - i^d \geq 0$). Therefore, depending on the composition of the asset side of the balance sheet, commercial bank profits may be positive or negative. I assume that in equilibrium, A^d is large enough to cover the commercial bank's operating costs.[13]

3.2.4 Government

In the morning, the government provides a pre-determined amount of government bonds $A^s > 0$ to the central bank. In addition, it provides equity funding to commercial banks.[14]

The government retains commercial bank profits Π^{com}. At the same time, it spends (earns) resources on interest rate payments if the endogenous government bond yield is positive (negative). The budget is balanced through lump-sum transfers to/from the household.

$$T^{g,HH} = \Pi^{com} - i^a A^s$$

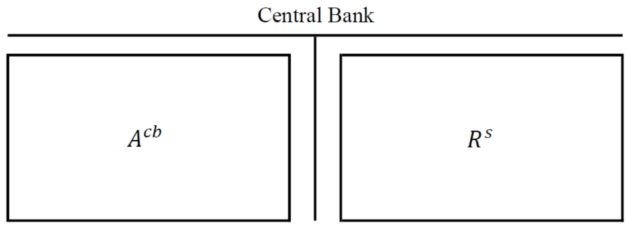
[13] More explicitly, I impose that the central bank abstains from implementing policies which imply strictly negative expected commercial bank profits. Appendix A.2 derives a condition under which expected commercial bank profits are weakly positive.

[14] Appendix D presents a model in which an entrepreneur provides equity. Setting up the model with an entrepreneur constraints the parameter space for which I can compute an analytical solution. Moreover, it complicates the model without providing additional insights. The results from the cashless model do not change.

3.2.5 Central bank

On the liability side, the central bank offers reserves R^s to commercial banks. The monetary base is $M^b \equiv R^s \in (0, A^s)$. On the asset side, the central bank holds government bonds $A^{cb} = M^b$. The remaining amount of government bonds $A^{com} \equiv A^s - A^{cb}$ is supplied to commercial banks.

Figure 3 — Central bank balance sheet



Notes: The central bank holds government bonds A^{cb} to cover reserves R^s.

If necessary, the central bank increases the reserves of commercial banks through discount window lending. The balance sheet of the central bank satisfies

$$A^{cb} + X\mathbb{I}_{X\geq 0} = R^s + R^x \ .$$

Central bank profits are equal to the interest rate returns on government bonds and discount window lending, minus the interest rate payments on reserves. Profits are transferred to the household.

$$T^{cb,HH} = i^a A^{cb} + i^x X\mathbb{I}_{X\geq 0} - i^r R^s$$

The central bank has two monetary policy tools. First, the monetary base. Second, the interest on reserves.

3.3 The model with cash

This section discusses a model extension in which the household uses deposits and cash to transfer resources from the morning to the afternoon.

3.3.1 Household

The household utility function is augmented with $u(M)$. The marginal utility of cash is positive. Therefore, in addition to deposits, the household invest an endogenous $M \in (0, M^b]$ into cash. Cash holdings are upper bound constrained by the size of the monetary base. The exogenous net cash yield is $i^{mon} \leq 0$. A negative return on cash may be interpreted as a loss of cash.

The household problem takes the following form.

$$\begin{aligned}\mathcal{L} = {} & u(C) + u(N) + u(D) + u(M) \\ & + \lambda_1(F + WN - D - M) \\ & + \lambda_2(T^{cb,HH} + T^{g,HH} + \Pi^F + (1+i^d)D + (1+i^{mon})M - C) \\ & + \bar{\varepsilon}_M(M^b - M)\end{aligned}$$

The additional first order conditions with respect to M require

$$\begin{aligned}& u'(M) + (1+i^{mon})u'(C) - \bar{\varepsilon}_M = u'(D) + (1+i^d)u'(C) \\ & \bar{\varepsilon}_M(M^b - M) = 0\ .\end{aligned}$$

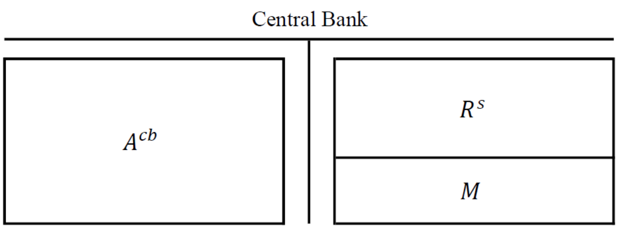
Investing in deposits has two benefits. First, deposits yield a direct utility. Second, deposits allow for higher consumption. In particular, an additional unit of deposits increases consumption by $(1 + i^d)$ units in the afternoon. Likewise, cash delivers a direct utility. An additional unit of cash increases consumption by $(1 + i^{mon}) \leq 1$ units in the afternoon. Optimal household behaviour requires that the household is indifferent between the two assets. For sake of exposition, assume that neither deposits nor cash deliver direct utility. In this case, the household only holds the asset with the higher return.[15] To avoid such corner solutions, I assume a functional form for the utility function that features an infinite marginal utility of deposits and cash at zero (see Section 3.5). Under this assumption, the household holds deposits and cash simultaneously. The share of deposits is higher, the higher the deposit rate and the greater the relative weight of deposits in the utility function.

[15] The household may be upper bound constraint in cash, reflected in $\bar{\varepsilon}_M \geq 0$.

3.3.2 Central bank

In the model with cash, the central bank offers reserves R^s to commercial banks and cash M to the household. The monetary base is $M^b \equiv R^s + M$.

Figure 4 — Central bank balance sheet



Notes: The central bank holds government bonds A^{cb} to cover reserves R^s and cash M.

The balance sheet of the central bank satisfies

$$A^{cb} + X\mathbb{I}_{X\geq 0} = R^s + M + R^x \ .$$

Compared to the cashless model, central bank profits are augmented with the seigniorage gains from issuing cash. Because the yield on cash is negative, the central bank makes a flow profit from issuing cash.

$$T^{cb,HH} = i^a A^{cb} + i^x X\mathbb{I}_{X\geq 0} - i^r R^s - i^{mon} M$$

3.4 Timing

$Morning$	$Noon$	$Afternoon$
the household is endowed with F	nature draws a liquidity shock Z	commercial banks borrow X at the central bank's discount window
the household supplies N to the firm		commercial banks pay $i^x X$ to the central bank
the firm demands N^d from the household		commercial banks pay $i^d D^s$ to the household
the firm pays W to the household		commercial banks transfer Π^{com} to the government
the firm produces Y		the central bank pay $i^r R^s$ to commercial banks
the government offers government bonds A^s to the central bank		the central bank transfers $T^{cb,HH}$ to the household
the government provides equity funding E to commercial banks		the government pays $i^a A^{com}$ to commercial banks
the central bank decides on M^b		the government pays $i^a A^{cb}$ to the central bank
the central bank offers reserves R^s to commercial banks		the government transfers $T^{g,HH}$ to the household
the central bank offers cash M to the household *(model with cash only)*		the firm transfers Π^F to the household
the central bank supplies government bonds A^{com} to commercial banks		the household pays $i^{mon} M$ to the central bank *(model with cash only)*
commercial banks demand A^d from the central bank		the household consumes C
commercial banks offers deposits D^s to the household		
the household invests in M *(model with cash only)*		
the household invests in D		

3.5 Functional forms and parameterization

For the functional form of the household utility function, I assume two log-specifications $u(C) = ln(C)$ and $u(D) = \omega_D ln(D)$. In addition, I assume $u(N) = -\frac{\phi N^{1+\varphi}}{1+\varphi}$, where $\varphi \geq 0$ is the inverse of the Frisch labor supply elasticity and ϕ the marginal disutility of labor at $N = 1$. Following Feenstra (1986), I interpret the utility gains from cash as lower transaction costs. That is, I replace $u(M)$ in the utility function with a transaction cost function $\phi(C, M)$ in the afternoon budget constraint of the household. For the functional form of the transaction cost function, I assume $\phi(C, M) = \kappa \frac{C}{M}$ with $\kappa > 0$.[16]

I parameterize the model with the aim of illustrating the qualitative features of the model. In particular, I set $\omega_D = 0.02$, implying that one unit of deposits yields a marginal utility of approximately 2% of consumption in the cashless model. Furthermore, I assume $\kappa = 0.01$, implying transaction costs of about 2.5% of consumption in the model with cash. For the yield on cash, I consider $i^{mon} = -0.05$ at which the household invest about 30% of its labor income in cash. Moreover, I assume a linear production function ($\alpha = 1$) and a strictly positive, but numerically irrelevant $F = 0.001$.

With regard to the exogenous policy variables, I assume a rather high amount of government bonds $A^s = 5$ (about 500% of consumption in the cashless model). Acknowledging the fact that in the model, government bonds capture any form of commercial bank assets different from reserves, the number seems, however, justifiable. In order to sensibly visualize the difference between the government bond yield and the deposit rate, I set the minimum reserve requirement to $\delta = 0.5$. For the discount rate, I assume a one-for-one correlation with the interest on reserves ($\gamma = 1$). Furthermore, I set the width of the interest rate corridor to one percentage point ($g = 0.01$). Finally, I assume a uniform distribution of the liquidity shock in $Z \in [-0.5, 0.5]$. The rather wide range allows for a meaningful visual representation of the model.

[16] It is possible to determine the equilibrium with $\omega_D = 0$ and $\kappa = 0$. In this case, however, the household does not hold deposits and cash simultaneously. As discussed in Section 3.2, I find it useful to assume $\omega_D > 0$ and $\kappa > 0$ to determine the interior partitioning between deposits and cash.

With respect to monetary policy, I assume values in $M^b \in [0.5, 1.5]$ for the monetary base, implying a monetary base of about $50-150\%$ of consumption in the cashless model. Finally, I analyze interest on reserves within the range of $i^r \in [-0.02, 0.02]$.

3.6 Model solution

Appendix A.1 derives the analytical solution for the cashless model. The money market rate, the government bond yield, the deposit rate, and the quantity of deposits are

$$i^m = i^r + i^x \left(\frac{\bar{Z} - M^b + \delta(F+\alpha)}{\bar{Z} - \underline{Z}}\right) \mathbb{I}_{\bar{X}\geq 0} \tag{2}$$

$$i^a = i^r + i^x \left(\frac{\bar{Z} - M^b + \delta(F+\alpha)}{\bar{Z} - \underline{Z}}\right) \mathbb{I}_{\bar{X}\geq 0} \tag{3}$$

$$i^d = i^r + i^x \left(\frac{\bar{Z} - M^b + \delta(F+\alpha)}{\bar{Z} - \underline{Z}}\right) \mathbb{I}_{\bar{X}\geq 0}(1-\delta) \tag{4}$$

$$D^s = (F+\alpha)\,, \tag{5}$$

respectively, where $\mathbb{I}_{\bar{X}\geq 0} = 1$ if the maximum regulatory liquidity shortage is strictly positive (formally: $\bar{X}(\bar{Z}) > 0$) and $\mathbb{I}_{\bar{X}\geq 0} = 0$ otherwise.

The model with cash does not have an analytical solution. Therefore, I use a numerical procedure to solve the model. The algorithm is outlined in Appendix B.1.

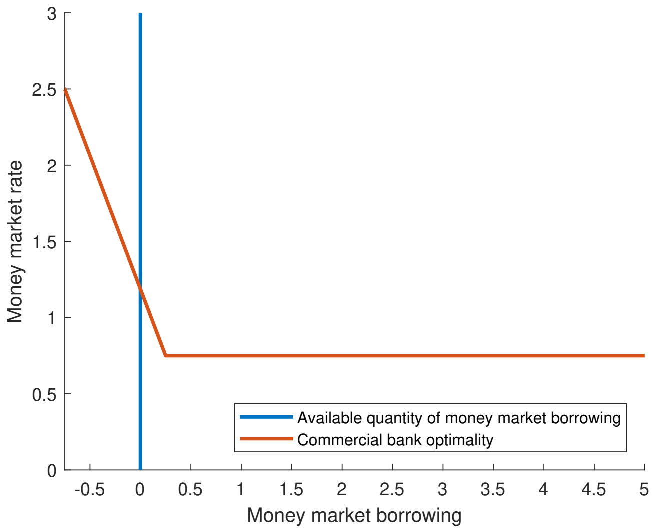
4 Results

The results are organized as follows. First, I discuss individually the money market, the market for government bonds, and the deposit market. Second, I analyze the general equilibrium effects of monetary policy. Third, I elaborate on the effects of monetary policy on expected commercial bank profits. Forth, I present the additional insight from the model with cash.

4.1 Market by market

Figure 5 shows the money market. The money market rate is the price at which the commercial bank borrows reserves on the money market. Reserves are valuable to the commercial bank because they yield a financial return – the interest on reserves – and an economic return, the insurance yield on reserves. The commercial bank borrows up to the point at which the cost of borrowing reserves equals the commercial bank's valuation of reserves (equation 2; depicted in red).

Figure 5 — The money market



Notes: The x-axis is money market borrowing B. The y-axis is the money market rate i^m in percent. The blue line is the available quantity of money market borrowing. The red line is the first order condition of the commercial bank in the cashless model. The figure assumes a monetary base $M^b = 0.75$ and an interest on reserves $i^r = 0.75\%$.

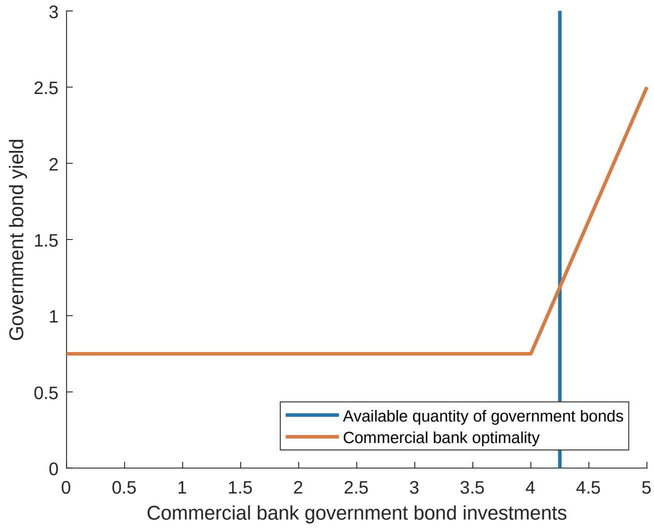
The money market rate falls with money market borrowing. The reason is that money market borrowing increases reserves. Higher reserves reduce the risk of a regulatory liquidity shortage. Therefore, the insurance yield on reserves falls. As the insurance yield falls, reserves become less attractive to the commercial bank. The commercial bank's willingness to pay for reserves decreases. The money market rate falls.

If money market borrowing is sufficiently high, the probability of a regulatory liquidity shortage is zero. A further increase in money market borrowing does no longer affect the insurance yield on reserves. The money market rate is therefore constant in money market borrowing and equal to the interest on reserves.

Commercial banks are alike prior to the closing of the money market. Therefore, by the market clearing condition, money market borrowing is zero (depicted in blue).

Figure 6 displays the market for government bonds. The financial return on government bonds is the government bond yield. The commercial bank buys government bonds with reserves. If the financial return on government bonds is greater (lower) than the commercial bank's valuation of reserves, the commercial bank increases (decreases) its government bond investments. It does so until it is indifferent between reserves and government bonds (equation 3, depicted in red).

Figure 6 — The government bond market



Notes: The x-axis is commercial bank government bond investments A^d. The y-axis is the government bond yield i^a in percent. The blue line is the available quantity of government bonds. The red line is the first order condition of the commercial bank in the cashless model. The figure assumes a monetary base $M^b = 0.75$ and an interest on reserves $i^r = 0.75\%$.

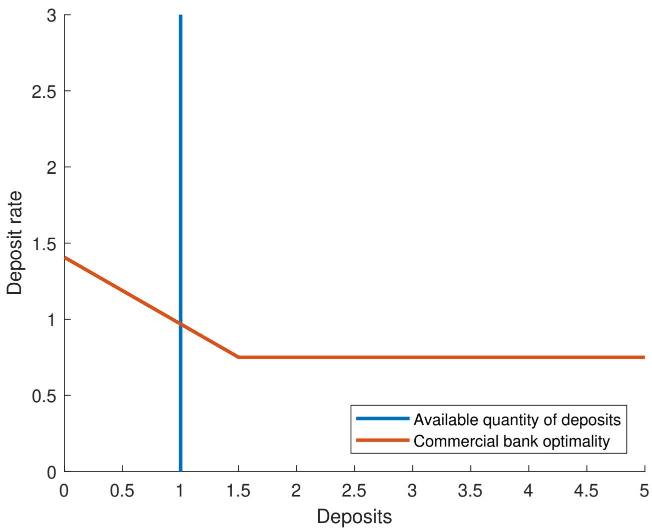
The government bond yield rises in government bond investments. The reason is that government bond investments decrease reserves. Lower reserves increase the risk of a regulatory liquidity shortage. Therefore, the insurance yield on reserves rises and reserves become more attractive to the commercial bank. To maintain the indifference between reserves and government bonds, the commercial bank demands a higher yield on government bonds.

If government bond holdings are sufficiently low, reserves are high enough to ensure that the probability of a regulatory liquidity shortage is zero. A further decrease in government bond investments does no longer affect the insurance yield on reserves. From the indifference requirement between reserves and government bonds, it follows that the government bond yield is constant in government bond investments, too, and equal to the interest on reserves.

The total amount of government bonds is pre-determined by the government. The central bank decides on the amount of government bonds available to commercial banks (depicted in blue).

Figure 7 presents the deposit market. Attracting deposits from another commercial bank increases reserves. Alternatively, the commercial bank can raise reserves through money market borrowing. Optimal commercial bank behavior requires the commercial bank to be indifferent between deposits and money market funding (equation 4, depicted in red).

Figure 7 — The deposit market



Notes: The x-axis is deposits D^s. The y-axis is the deposit rate i^d in percent. The blue line is the available quantity of deposits. The red line is the first order condition of the commercial bank in the cashless model. The figure assumes a monetary base $M^b = 0.75$ and an interest on reserves $i^r = 0.75\%$.

The deposit rate falls in deposits. The reason is that deposits, like money market borrowing, increase reserves. Higher reserves reduce the risk of a regulatory liquidity shortage. Therefore, the insurance yield on reserves falls. Reserves become less attractive and the commercial bank's willingness to pay for deposits falls. The deposit rate decreases. In contrast to money market funding, higher deposits increases the minimum reserve requirement. That is, raising reserves with deposits decreases the risk of a regulatory liquidity shortage by less than raising reserves through money market borrowing. The deposit rate is, for that reason, lower than the money market rate.

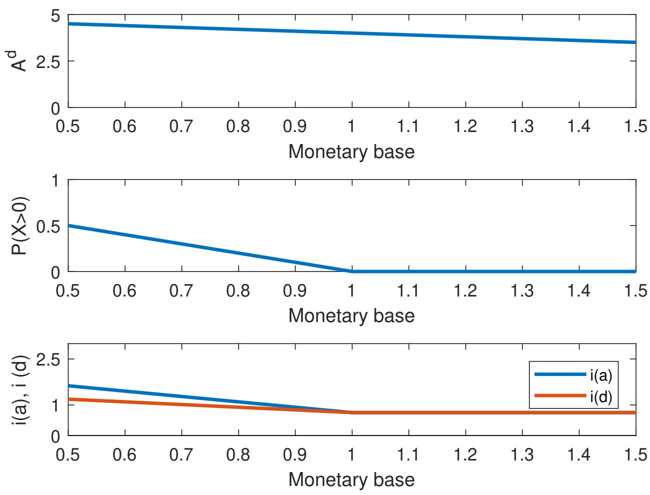
If deposits are sufficiently high, the probability of a regulatory liquidity shortage is zero. As a consequence, the insurance yield on reserves is zero, too. Raising additional reserves through deposits or money market borrowing yields the interest on reserves only. The deposit rate is therefore constant in deposits and equal to the money market rate and the interest on reserves.

The equilibrium quantity of deposits is determined by the endowment and labor income of the household (equation 5, depicted in blue).

4.2 Monetary policy effects on interest rates

The central bank implements monetary policy through open market operations and interest on reserves policies.

Figure 8 — Open market operations



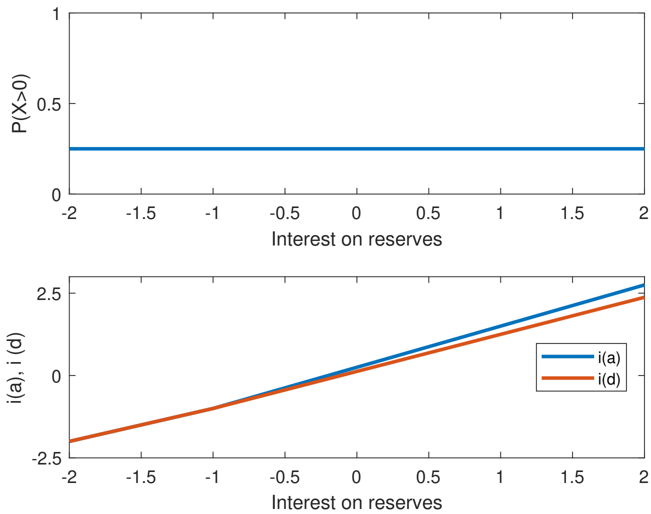
Notes: The figure plots equilibrium characteristics of the cashless model conditional on the monetary base $M^b \in [0.5, 1.5]$. The first panel plots commercial bank government bond investments, the second panel the probability of a regulatory liquidity shortage, and the third panel the government bond yield (in blue) and the deposit rate (in red) in percent. The figure assumes an interest on reserves $i^r = 0.75\%$.

First, I analyze the effects of an expansionary open market operation. An expansionary open market operation is characterized by higher reserves and a lower amount of government bonds available to commercial banks (Figure 8, panel A). Higher reserves imply a lower probability of a regulatory liquidity shortage for the commercial bank (Figure 8, panel B).

The lower probability of a regulatory liquidity shortage is reflected in a lower insurance yield on reserves. As a consequence, the commercial bank reduces its willingness to pay for money market funding. The money market rate falls. Optimal behavior requires them to be indifferent between funding sources. Therefore, the deposit rate falls, too (Figure 8, panel C). Finally, indifference between asset classes dictates a reduction in the willingness to pay for government bonds. Amid the decrease in the (perfectly inelastic) amount of government bonds available to commercial banks, the government bond yield falls (Figure 8, panel C). The results are symmetric. That is, a contractionary open market operation has the opposite effects of an expansionary open market operation.

Open market operations are only effective off the interest rate floor. At the interest rate floor, the insurance yield of reserves is zero; that is, independent of reserves. Because open market operations work through changes in the insurance yield, the money market rate, the deposit rate, and the government bond yield do not react to open market operations at the interest rate floor. They are equal to the interest on reserves.

Figure 9 — Interest on reserves policies off the interest rate floor

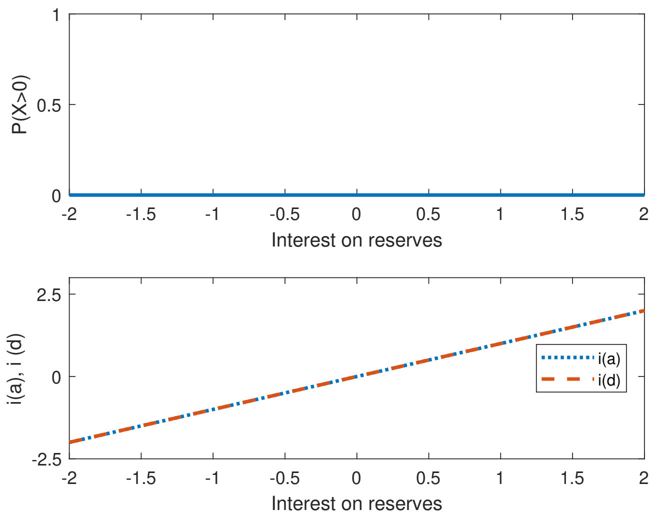


Notes: The figure plots equilibrium characteristics of the cashless model conditional on the interest on reserves $i^r \in [-2\%, 2\%]$. The first panel plots the probability of a regulatory liquidity shortage and the second panel the government bond yield (in blue) and the deposit rate (in red) in percent. The figure assumes a monetary base $M^b = 0.75$.

Second, I analyse an expansionary interest on reserves policy off the interest rate floor (Figure 9, panel A). An expansionary interest on reserves policy is characterized by lower interest on reserves. Lower interest on reserves decrease the commercial bank's willingness to pay for deposits and money market funding. The money market rate and the deposit rate fall (Figure 9, panel B). Likewise, the commercial bank decreases its willingness to pay for government bonds. The government bond yield falls (Figure 9, panel B). All results are symmetric; that is, a contractionary interest on reserves policy has the opposite effects of an expansionary interest on reserves policy.

The interest rate differential between the government bond yield and the deposit rate shirks as interest on reserves decrease. The reason is that changes in the interest on reserves affect the discount rate, too. If the interest on reserves becomes sufficiently negative, the deposit rate equals the government bond yield (Figure 9, panel B) even if the probability of a liquidity shortage is strictly positive. This is because the discount rate hits its zero lower bound if $\gamma i^r + g \leq 0$.

Figure 10 — Interest on reserves policies at the interest rate floor



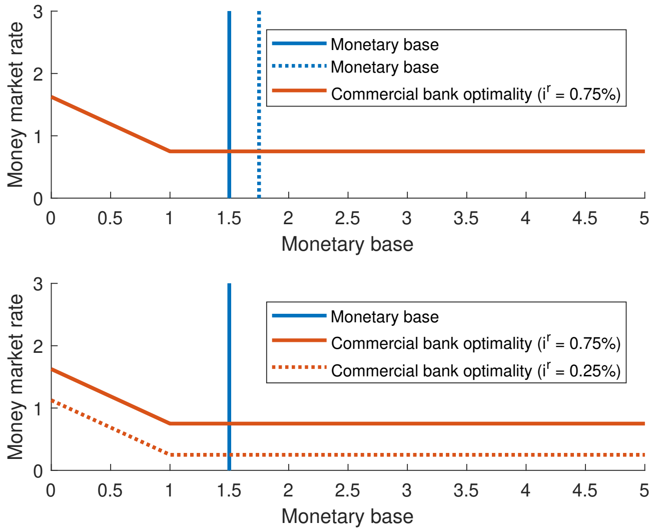
Notes: The figure plots equilibrium characteristics of the cashless model conditional on the interest on reserves $i^r \in [-2\%, 2\%]$. The first panel plots the probability of a regulatory liquidity shortage and the second panel the government bond yield (in blue) and the deposit rate (in red) in percent. The figure assumes a monetary base $M^b = 2$.

Third, I analyze an interest on reserves policy at the interest rate floor.[17] At the interest rate floor, the probability of a regulatory liquidity shortage is zero (Figure 10, panel A). In contrast to open market operations, interest on reserves policies affect interest rate even at

[17]For brevity, I abstain from discussing the limitations of open market operations at the interest rate ceiling.

the interest rate floor (Figure 10, panel B).

Figure 11 — Monetary policy and the money market



Notes: The figure plots equilibrium characteristics of the cashless model conditional on the monetary base $M^b \in [0, 5]$. The x-axis is the monetary base M^d. The y-axis is the money market rate i^m in percent. The blue line is the money supply. The red line is the first order condition of the commercial bank in the cashless model (the money demand). Panel A shows that an increase in the monetary base from $M^b = 1.5$ to $M^b = 1.75$ does not change the money market rate (0.75%). By contrast, panel B shows that a decrease in the interest on reserves from $i^r = 0.75\%$ to $i^r = 0.25\%$ decreases the money market rate from $i^m = 0.75\%$ to $i^m = 0.25\%$.

Figure 11 highlights the difference between open market operations and interest on reserves policies at the interest rate floor in an (i^m, M^b) space. The (i^m, M^b) space is frequently used to illustrate monetary policy pass-through in undergraduate teaching. Usually, these illustrations assume that reserves do not pay interest. My exposition is conditional on an interest on reserves of 0.75%. That is, in my example, the interest rate floor is strictly positive.

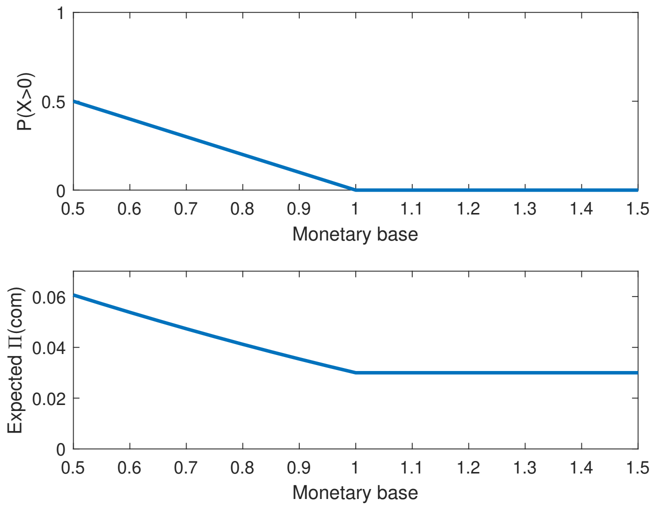
Panel A shows that an expansionary open market operations – an increase in the monetary base from $M^b = 1.5$ to $M^b = 1.75$ – is ineffective at the interest rate floor. By contrast, panel B illustrates that an expansionary interest on reserves policy – a decrease in the interest on reserves from $i^r = 0.75\%$ to $i^r = 0.25\%$ – affects the money market rate even at the interest rate floor.[18] The reason is that interest on reserves policies work through changes in the money demand, rather than through changes in the money supply.

[18]For exposition, I assume that the discount rate is independent of the interest on reserves; that is, $\gamma = 0$. With $\gamma > 0$, not only the intercept, but also the slope of the money demand function change in the interest on reserves.

4.3 Monetary policy effects on expected commercial bank profits

In this section, I discuss the effects of monetary policy on expected commercial bank profits.

Figure 12 — Expected commercial bank profits



Notes: The figure plots equilibrium characteristics of the cashless model conditional on the monetary base $M^b \in [0.5, 1.5]$. The first panel plots the probability of a regulatory liquidity shortage and the second panel expected commercial bank profits. The figure assumes an interest on reserves $i^r = 0.75\%$.

Figure 12 plots expected commercial bank profits as a function of the monetary base. The economy operates at the interest rate floor if the monetary base is sufficiently high. At the interest rate floor, the probability of a regulatory liquidity shortage is zero (panel A). Therefore, expected costs of regulatory liquidity shortages are zero. Commercial bank assets (reserves and government bonds) yield the interest on reserves. Likewise, deposits cost the interest on reserves. The expected interest rate differential between assets and liabilities is $i^r E$. In other words, since commercial bank equity is positive, expected commercial bank profits are positive (negative) if the interest on reserves is positive (negative). At the interest rate floor, monetary policy affects expected commercial bank profits through interest on reserves policies only.

Off the interest rate floor, the commercial bank earns a strictly positive interest rate differential between government bonds and deposits. At the same time, being off the interest rate floor is costly. First, the interest rate differential between reserves and deposits is negative. Second, the expected costs of a liquidity shortage are positive. In sum, expected commercial bank profits are weakly positive off the interest rate floor if the share of government bonds in commercial bank assets is sufficiently high.[19]

Monetary policy affects expected commercial bank profits off the interest rate floor through open market operations and interest on reserves policies. Expansionary open market operations increase the share of reserves in commercial bank assets. Higher reserves depress expected commercial bank profits because reserves yield a lower financial return than government bonds. Expansionary interest on reserves policies decrease expected commercial bank profits for two reasons. First, they decrease by an equal amount the financial return on commercial bank assets and liabilities. Because commercial bank equity is positive, the overall effect on expected commercial bank profit is negative. Second, lower interest on reserves compress the interest rate differential between commercial bank assets and deposits.[20]

4.4 Insights from the model with cash

Introducing cash to the model does not change the qualitative results of the model. Nonetheless, it provides an additional insight. Namely, expansionary monetary policy operations increase household cash holdings.

The household chooses deposits and cash such that both assets yield the same economic benefit to the household. Since the marginal benefit of deposits (cash) goes to infinity as deposits (cash) go to zero, the household holds a strictly positive amount of both assets. The economic value of deposits depends positively on the deposit rate. As a consequence, monetary policy affects the optimal partitioning between deposits and cash. In particular, the household decreases deposits in response to expansionary monetary policies. The

[19] Appendix A.2 provides a formal condition for weakly positive expected commercial bank profits off the interest rate floor.

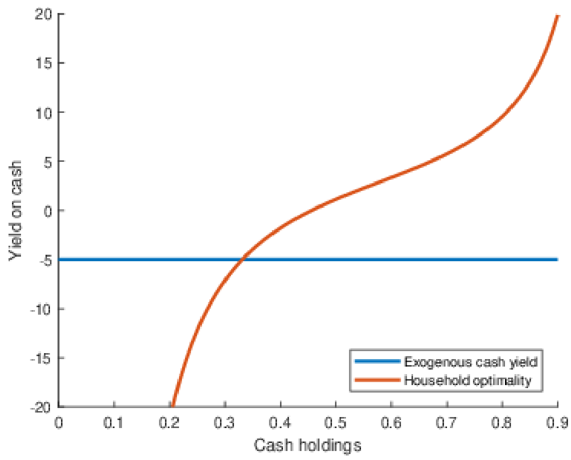
[20] For more details, see the discussion of Figure 9 in Section 4.2.

household optimality conditions can be expressed as follows.[21]

$$i^{mon} = i^d + \omega_D \frac{1}{\alpha - M} + \kappa \frac{1}{M^2} \; .$$

Figure 13 depicts the left hand side of the above equation in blue. The right hand side is shown in red. The graph illustrates that changes in the deposit rate – which shift the red curve – have monotonic, but non-linear effects on the optimal quantity of cash (M) and deposits ($\alpha - M$).

Figure 13 — The cash market



Notes: The x-axis is household cash holdings M. The y-axis is the yield on cash i^{mon} in percent. The blue line is the exogenous yield on cash. The red line is the first order condition of the household in the model with cash. The figure assumes a monetary base $M^b = 0.75$ and an interest on reserves $i^r = 0.75\%$.

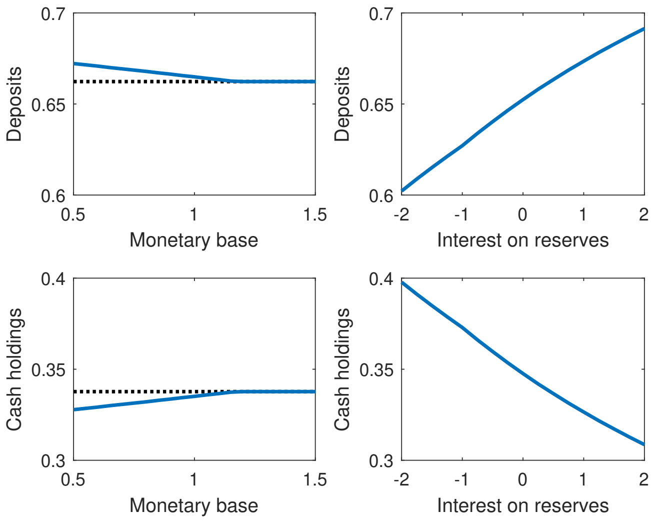
Figure 14, column 1 shows the response of deposits and cash to variations in the monetary base. Off the interest rate floor, expansionary open market operations decrease the deposit rate.[22] As a consequence, deposits become less attractive for the household. The household substitutes in a monotonic, but non-linear fashion from deposits to cash. At the interest rate floor, open market operations do not affect the deposit rate. Thus, the partitioning between deposits and cash remains unaffected by open market operations at the interest rate floor.

[21]See Appendix B for the derivation.

[22]Suppose the household is upper bound constrained in its cash holdings. Then, a marginal expansion of the monetary base is absorbed, in full, by higher household cash holdings. As a consequence, reserves would remain constant. Expansionary open market operations would, therefore, not affect the marginal benefit of reserves, and thus the deposit rate, even off the interest rate floor. For brevity, I abstract from analyzing this case.

Figure 14, column 2 presents the response of deposits and cash to variations in the interest on reserves. Interest on reserves policies affect the deposit rate even at the interest rate floor. In response to an expansionary interest on reserves policy, deposits fall and cash increases. Interest on reserves policies have non-linear effects of cash and deposits for two reasons. First, the relationship between the deposit rate and cash in the household optimality condition is non linear. Second, if $\gamma i^r + g \leq 0$, the discount rate hits its zero lower bound. That is, for sufficiently low interest on reserves, the insurance yield is zero even if the probability of a regulatory liquidity shortage is positive.[23]

Figure 14 — Monetary policy operations in the model with cash



Notes: The figure plots equilibrium characteristics of the model with cash. The left column depicts deposits and cash conditional on the monetary base $M^b \in [0.5, 1.5]$, assuming an interest on reserves $i^r = 0.75\%$. The right column depicts deposits and cash conditional on the interest on reserves $i^r \in [-2\%, 2\%]$, assuming a monetary base $M^b = 0.75$. The black dotted lines depict deposit and household cash holdings at the interest rate floor.

5 Concluding remarks

Since the Great Financial Crisis, the implementation of monetary policy in many jurisdictions has evolved from managing the supply of reserves to administrating interest on reserves. Open market operations affect the money market rate, the government bond yield, and the deposit rate through changes in the insurance yield on reserves. Interest

[23] In other words, at $\gamma i^r + g \leq 0$, further decreases in the interest on reserves do no longer translate into a lower insurance yield on reserves.

on reserves policies change interest rates more directly, namely through variation in the financial compensation of reserves. The change in monetary policy implementation has ensured monetary policy pass-through amidst the massive expansion of reserves.

My paper develops an accessible general equilibrium model suitable to analyze monetary policy implementation and pass-through. It demonstrates the impotence of open market operations at the interest rate floor. Furthermore, it shows that central banks can use interest on reserve policies to implement monetary policy independently of the level of reserves. Expansionary monetary policy operations have adverse effects on commercial bank profits. Also, they increase household cash holdings in a monotonic, but non-linear fashion.

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A The cashless model

In equilibrium, the following equations have to be satisfied under the following assumptions. Government policy: $A^s > 0$, allowing for $E \geq 0$ (that is, $A^s > D$). Monetary policy: $M^b \in (0, A^s)$, satisfying the conditions on weakly positive expected commercial bank profits as derived in Appendix A.2. Exogenous policy parameters: $\gamma \in [0,1]$, $g \geq 0$, $\delta \in [0,1]$. Utility function: $\omega_D \geq 0$, $C > 0$, $D > 0$, and $N \geq 0$. Endowment: $F > 0$. Production function: $\alpha \in (0,1]$.

Source	Equation	Endogenous variable
Household optimality	$\frac{1}{C} = \lambda_2$	$[C, \lambda_2]$
	$\omega_D \frac{1}{D} + (1+i^d)\lambda_2 = \lambda_1$	$[D, i^d, \lambda_1]$
	equation 1b	$[N]$
Household budget constraints	$F + WN = D$	$[W]$
	$T^{cb,HH} + T^{g,HH} + \Pi^F + (1+i^d)D = C$	$[T^{cb,HH}, T^{g,HH}, \Pi^F]$
Firm optimality	$\alpha \frac{Y}{N_d} = W$	$[N_d, Y]$
Production function	$Y = N_d^\alpha$	
Firm profits	$\Pi^F = Y - WN_d$	
Commercial bank optimality	$B(i^r + i^x P(X > 0) - i^m + \varepsilon_R) = 0$	$[B, i^x, X, i^m, \varepsilon_R]$
	$A^d(i^a - (i^r + i^x P(X > 0)) + \varepsilon_A - \varepsilon_R) = 0$	$[A^d, i^a, \varepsilon_A]$
	$D^s(i^r + i^x P(X > 0)(1-\delta) - i^d + \varepsilon_D + \varepsilon_R) = 0$	$[D^s, \varepsilon_D]$
	$\varepsilon_R(R^s + B) = 0$	$[R^s]$
	$\varepsilon_A A^d = 0$	
	$\varepsilon_D D^s = 0$	
Commercial bank balance sheet	$R^s + A^d + R^b + R^x = D^s + E + B + X\mathbb{I}_{X\geq 0}$	$[R^b, R^x, E]$
Commercial bank profits	$\Pi^{com} = i^r R^s + i^a A^d - i^d D^s - i^x X\mathbb{I}_{X\geq 0}$	$[\Pi^{com}]$
Government	$T^{g,HH} = \Pi^{com} - i^a A^s$	
Central bank balance sheet	$A^{cb} + X\mathbb{I}_{X\geq 0} = R^s + R^x$	$[A^{cb}]$
Central bank profits	$T^{cb,HH} = i^a A^{cb} + i^x X\mathbb{I}_{X\geq 0} - i^r R^s$	
Definitions	$M^b \equiv R^s$	
	$X \equiv Z - (R^s + R^b) + K$	$[K]$
	$R^x \equiv \max(0, X)$	
	$R^b \equiv B$	
	$A^s \equiv A^{com} + A^{cb}$	$[A^{com}]$
	$i^x \equiv \max(0, \gamma i^r + g)$	
	$K \equiv \delta D^s$	
Market clearing conditions	$C = Y + F$	
	$N^d = N$	
	$D = D^s$	
	$B = 0$	
	$A^d = A^{com}$	

A.1 Derivation of the solution

In the following, I use the market clearing conditions and the assumptions outlined in the previous subsection to simplify the system of equations. Furthermore, I assume that the disutility of labor is sufficiently small to ensure $N = 1$.[24] In an interior solution, the following equations have to be satisfied.

$$\frac{1}{C} = \lambda_2 \tag{6}$$

$$\omega_D \frac{1}{D^s} + (1 + i^d)\lambda_2 = \lambda_1 \tag{7}$$

$$F + W = D^s \tag{8}$$

$$T^{cb,HH} + T^{g,HH} + \Pi^F + (1 + i^d)D^s = C \tag{9}$$

$$\alpha = W \tag{10}$$

$$C = 1 + F \tag{11}$$

$$\Pi^F = C - F - W \tag{12}$$

$$i^a = i^r + i^x P(X > 0) \tag{13}$$

$$i^d = i^r + i^x P(X > 0)(1 - \delta) \tag{14}$$

$$R^s + A^{com} + R^x = D^s + E + X\mathbb{I}_{X\geq 0} \tag{15}$$

$$\Pi^{com} = i^r R^s + i^a A^{com} - i^d D^s - i^x X\mathbb{I}_{X\geq 0} \tag{16}$$

$$T^{g,HH} = \Pi^{com} - i^a A^s \tag{17}$$

$$A^{cb} + X\mathbb{I}_{X\geq 0} = R^s + R^x \tag{18}$$

$$T^{cb,HH} = i^a A^{cb} + i^x X\mathbb{I}_{X\geq 0} - i^r R^s \tag{19}$$

$$M^b \equiv R^s \tag{20}$$

$$X \equiv Z - R^s + K \tag{21}$$

$$R^x \equiv \max(0, X) \tag{22}$$

$$A^s \equiv A^{com} + A^{cb} \tag{23}$$

$$i^x \equiv \max(0, \gamma i^r + g) \tag{24}$$

$$K \equiv \delta D^s \tag{25}$$

Use equation 11 ($C = 1 + F$) and equation 10 ($W = \alpha$) to determine deposits and firm profits by equation 8 and equation 12, respectively.

$$D^s = F + \alpha \tag{26}$$

$$\Pi^F = 1 - \alpha \tag{27}$$

[24]With $N = 0$, deposits equal endowment. The qualitative results with regards to monetary policy implementation and pass-through remain unaffected. For brevity, I abstain from presenting the explicit solution under $N = 0$.

Next, use equation 13 and equation 14 to determine the government bond yield and the deposit rate, respectively. Make use of equation 20 ($M^b \equiv R^s$), equation 21, and equation 25.

$$i^a = i^r + i^x P(X > 0) \tag{28}$$

$$i^a = i^r + i^x (1 - P(X < 0)) \tag{29}$$

$$i^a = i^r + i^x (1 - P(Z < R^s - K)) \tag{30}$$

$$i^a = i^r + i^x \left(1 - P(Z < M^b - \delta D^s)\right) \tag{31}$$

$$i^a = i^r + i^x \left(1 - P(Z < M^b - \delta(F + \alpha))\right) \tag{32}$$

$$i^a = i^r + i^x \left(1 - \frac{M^b - \delta(F+\alpha) - \underline{Z}}{\bar{Z} - \underline{Z}}\right) \mathbb{I}_{\bar{X} \geq 0} \tag{33}$$

$$i^a = i^r + i^x \left(\frac{\bar{Z} - M^b + \delta(F+\alpha)}{\bar{Z} - \underline{Z}}\right) \mathbb{I}_{\bar{X} \geq 0} \tag{34}$$

and

$$i^d = i^r + i^x \left(\frac{\bar{Z} - M^b + \delta(F+\alpha)}{\bar{Z} - \underline{Z}}\right) \mathbb{I}_{\bar{X} \geq 0}(1 - \delta) \ , \tag{35}$$

where $i^x \equiv \max(0, \gamma i^r + g)$ from equation 24.[25] In terms of quantities, use equation 18, equation 23, equation 21, equation 22, equation 25, and equation 15 to determine

$$A^{cb} = M^b \tag{36}$$

$$A^{com} = A^s - M^b \tag{37}$$

$$X = Z - M^b + \delta(F + \alpha) \tag{38}$$

$$R^x = \max(0, Z - M^b + \delta(F + \alpha)) \tag{39}$$

$$K = \delta(F + \alpha) \tag{40}$$

$$E = A^s - (F + \alpha) \ , \tag{41}$$

respectively. By equation 16, commercial bank profits are

$$\Pi^{com} = i^r M^b + i^a(A^s - M^b) - i^d(F + \alpha) - i^x(Z - M^b + \delta(F+\alpha))\mathbb{I}_{X \geq 0} \tag{42}$$

$$\Pi^{com} = (i^r - i^a)M^b + i^a A^s - i^d(F + \alpha) - i^x(Z - M^b + \delta(F+\alpha))\mathbb{I}_{X \geq 0} \ . \tag{43}$$

By equation 17, government transfers are

$$T^{g,HH} = (i^r - i^a)M^b - i^d(F + \alpha) - i^x(Z - M^b + \delta(F+\alpha))\mathbb{I}_{X \geq 0} \ . \tag{44}$$

By equation 19, central bank transfers are

$$T^{cb,HH} = i^a M^b + i^x(Z - M^b + \delta(F+\alpha))\mathbb{I}_{X \geq 0} - i^r M^b \tag{45}$$

$$T^{cb,HH} = (i^a - i^r)M^b + i^x(Z - M^b + \delta(F+\alpha))\mathbb{I}_{X \geq 0} \ . \tag{46}$$

[25] I assume $\underline{Z} \leq -\delta(F + \alpha)$ to ensure that $\left(\frac{\bar{Z} - M^b + \delta(F+\alpha)}{\bar{Z} - \underline{Z}}\right) \leq 1$.

Finally, equation 6 and equation 7 determine

$$\lambda_1 = \frac{\omega_D}{(F+\alpha)} + (1+i^d)\frac{1}{1+F} \tag{47}$$

$$\lambda_2 = \frac{1}{1+F} \; . \tag{48}$$

As expected, using the model solution in equation 9 yields a true statement.

A.2 Expected commercial bank profits

Making use of equation 21, equation 25, and the model solution for D^s (equation 26), I can express the expected regulatory liquidity shortage as

$$\tilde{X} \equiv \mathbb{E}(X|X>0) \tag{49}$$

$$\tilde{X} \equiv \frac{\bar{Z} - R^s + \delta(F+\alpha)}{2} \; , \tag{50}$$

where I use that $\mathbb{E}(X|X>c) = \frac{b+c}{2}$ for $X \sim U(-b,b)$, according to Ouyang (1993). Expected commercial bank profits are

$$\mathbb{E}\Pi^{com} = i^r R^s + i^a A^{com} - i^d D^s - i^x \tilde{X} \; . \tag{51}$$

Next, I replace i^a and i^d with equation 13 and equation 14, respectively. Moreover, I use equation 20 and equation 37 to replace A^{com} with $(A^s - R^s)$. Expected commercial bank profits are weakly positive if

$$i^r R^s + \Big(i^r + i^x P(X>0)\Big)A^{com} - \Big(i^r + i^x P(X>0)(1-\delta)\Big)D^s - i^x\tilde{X} \geq 0 \tag{52}$$

$$i^r(R^s + A^{com} - D^s) + i^x P(X>0)\left[A^{com} - (1-\delta)D^s\right] - i^x\tilde{X} \geq 0 \tag{53}$$

$$i^r(A^s - D^s) + i^x P(X>0)\left[A^s - R^s - (1-\delta)D^s\right] - i^x\tilde{X} \geq 0 \tag{54}$$

$$i^r(A^s - (F+\alpha)) + i^x P(X>0)\left[A^s - M^b - (1-\delta)(F+\alpha) - \frac{\bar{Z} - \underline{Z}}{2}\right] \geq 0 \; . \tag{55}$$

At the interest rate floor, the probability of a regulatory liquidity shortage is zero. Since $A^s > (F+\alpha)$, expected commercial bank profits are positive (negative) if the interest on reserves is positive (negative). Off the interest rate floor, assuming $i^r \geq 0$,

$$A^s - (1-\delta)(F+\alpha) - \frac{\bar{Z} - \underline{Z}}{2} \geq M^b \tag{56}$$

is sufficient to ensure weakly positive expected commercial bank profits. Off the interest rate floor, assuming $i^r < 0$,

$$(A^s - (F+\alpha))\left(\frac{i^r}{i^x}\right)\left(\frac{\bar{Z} - R^s + \delta(F+\alpha)}{\bar{Z} - \underline{Z}}\right)^{-1} + \left[A^s - (1-\delta)(F+\alpha) - \frac{\bar{Z} - \underline{Z}}{2}\right] \geq M^b \tag{57}$$

is sufficient to ensure weakly positive expected commercial bank profits.

B The model with cash

In equilibrium, the following equations have to be satisfied under the following (additional) assumptions. Utility function: $M \in (0, M^b]$. Exogenous yield on cash: $i^{mon} \leq 0$. Transaction cost function $\kappa > 0$.

Source	Equation	Endogenous variable
Household optimality	$\frac{1}{C} = \lambda_2 \left(1 + \kappa \frac{1}{M}\right)$	$[C, \lambda_2, M]$
	$\omega_D \frac{1}{D} + (1 + i^d)\lambda_2 = \lambda_1$	$[D, i^d, \lambda_1]$
	equation 1b	$[N]$
	$\left((1 + i^{mon}) + \kappa \frac{C}{M^2}\right) \lambda_2 - \bar{\varepsilon}_M = \lambda_1$	$[\bar{\varepsilon}_M]$
	$\bar{\varepsilon}_M (M^b - M) = 0$	
Household budget constraints	$F + WN = D + M$	$[W]$
	$T^{cb,HH} + T^{g,HH} + \Pi^F + (1 + i^d)D + (1 + i^{mon})M = C + \phi(C, M)$	$[T^{cb,HH}, T^{g,HH}, \Pi^F, \phi(C, M)]$
Firm optimality	$\alpha \frac{Y}{N_d} = W$	$[N_d, Y]$
Production function	$Y = N_d^\alpha$	
Firm profits	$\Pi^F = Y - WN_d$	
Commercial bank optimality	$B(i^r + i^x P(X > 0) - i^m + \varepsilon_R) = 0$	$[B, i^x, X, i^m, \varepsilon_R]$
	$A^d(i^a - (i^r + i^x P(X > 0)) + \varepsilon_A - \varepsilon_R) = 0$	$[A^d, i^a, \varepsilon_A]$
	$D^s(i^r + i^x P(X > 0)(1 - \delta) - i^d + \varepsilon_D + \varepsilon_R) = 0$	$[D^s, \varepsilon_D]$
	$\varepsilon_R(R^s + B) = 0$	$[R^s]$
	$\varepsilon_A A^d = 0$	
	$\varepsilon_D D^s = 0$	
Commercial bank balance sheet	$R^s + A^d + R^b + R^x = D^s + E + B + X\mathbb{I}_{X \geq 0}$	$[R^b, R^x, E]$
Commercial bank profits	$\Pi^{com} = i^r R^s + i^a A^d - i^d D^s - i^x X\mathbb{I}_{X \geq 0}$	$[\Pi^{com}]$
Government	$T^{g,HH} = \Pi^{com} - i^a A^s$	
Central bank balance sheet	$A^{cb} + X\mathbb{I}_{X \geq 0} = R^s + M + R^x$	$[A^{cb}]$
Central bank profits	$T^{cb,HH} = i^a A^{cb} + i^x X\mathbb{I}_{X \geq 0} - i^r R^s - i^{mon} M$	
Definitions	$\phi(C, M) \equiv \kappa \frac{C}{M}$	
	$M^b \equiv R^s + M$	
	$X \equiv Z - (R^s + R^b) + K$	$[K]$
	$R^x \equiv \max(0, X)$	
	$R^b \equiv B$	
	$A^s \equiv A^{com} + A^{cb}$	$[A^{com}]$
	$i^x \equiv \max(0, \gamma i^r + g)$	
	$K \equiv \delta D^s$	
Market clearing conditions	$C = Y + F - \phi(C, M)$	
	$N^d = N$	
	$D = D^s$	
	$B = 0$	
	$A^d = A^{com}$	

B.1 Derivation of the solution

In the following, I use the market clearing conditions and the assumptions outlined in the previous subsection to simplify the system of equations. Furthermore, I assume that the disutility of labor is sufficiently small to ensure $N = 1$, as in Appendix A.1. In an interior solution, the following equations have to be satisfied.

$$\frac{1}{C} = \lambda_2(1 + \kappa\frac{1}{M}) \tag{58}$$

$$\omega_D\frac{1}{D^s} + (1 + i^d)\lambda_2 = \lambda_1 \tag{59}$$

$$\left((1 + i^{mon}) + \kappa\frac{1}{M^2}\right)\lambda_2 = \lambda_1 \tag{60}$$

$$F + W = D^s + M \tag{61}$$

$$T^{cb,HH} + T^{g,HH} + \Pi^F + (1 + i^d)D^s + (1 + i^{mon})M = C + \phi(C, M) \tag{62}$$

$$\alpha = W \tag{63}$$

$$C - F + \phi(C, M) = 1 \tag{64}$$

$$\Pi^F = C - F + \phi(C, M) - W \tag{65}$$

$$i^a = i^r + i^x P(X > 0) \tag{66}$$

$$i^d = i^r + i^x P(X > 0)(1 - \delta) \tag{67}$$

$$R^s + A^{com} + R^x = D^s + E + X\mathbb{I}_{X\geq 0} \tag{68}$$

$$\Pi^{com} = i^r R^s + i^a A^{com} - i^d D^s - i^x X\mathbb{I}_{X\geq 0} \tag{69}$$

$$T^{g,HH} = \Pi^{com} - i^a A^s \tag{70}$$

$$A^{cb} + X\mathbb{I}_{X\geq 0} = R^s + M + R^x \tag{71}$$

$$T^{cb,HH} = i^a A^{cb} + i^x X\mathbb{I}_{X\geq 0} - i^r R^s - i^{mon} M \tag{72}$$

$$\phi(C, M) \equiv \kappa\frac{C}{M} \tag{73}$$

$$M^b \equiv R^s + M \tag{74}$$

$$X \equiv Z - R^s + K \tag{75}$$

$$R^x \equiv \max(0, X) \tag{76}$$

$$A^s \equiv A^{com} + A^{cb} \tag{77}$$

$$i^x \equiv \max(0, \gamma i^r + g) \tag{78}$$

$$K \equiv \delta D^s \tag{79}$$

Combine equation 64 and equation 73 ($\phi(C, M) \equiv \kappa\frac{C}{M}$).

$$C = \frac{1 + F}{1 + \kappa\frac{1}{M}} \tag{80}$$

Next, combine equation 58 with the previous result (equation 80) to get $\lambda_2 = \frac{1}{1+F}$. Use this result as well as equation 61 and equation 63 ($W = \alpha$) in equation 59.

$$\lambda_1 = \omega_D \frac{1}{D^s} + (1+i^d)\frac{1}{1+F} \tag{81}$$

$$\lambda_1 = \omega_D \frac{1}{F+\alpha-M} + (1+i^d)\frac{1}{1+F} \tag{82}$$

Make use of $\lambda_2 = \frac{1}{1+F}$ in equation 60.

$$\lambda_1 = \left((1+i^{mon}) + \kappa\frac{1}{M^2}\right)\frac{1}{1+F} \tag{83}$$

To solve the model numerically, I apply the following procedure. First, I consider various M on a grid $M \in (0, F+\alpha)$. Second, I compute $\lambda_1^{(1)}$ from equation 82 and $\lambda_1^{(2)}$ from equation 83. Third, I select the M with the smallest absolute difference in $\lambda_1^{(1)} - \lambda_1^{(2)}$. The remaining variables are computed as in Appendix A.1.

$$D^s = F + \alpha - M \tag{84}$$

$$\Pi^F = 1 - \alpha \tag{85}$$

$$R^s = M^b - M \tag{86}$$

$$i^a = i^r + i^x\left(\frac{\bar{Z} - M^b + M(1-\delta) + \delta(F+\alpha)}{\bar{Z} - \underline{Z}}\right)\mathbb{I}_{\bar{X}\geq 0} \tag{87}$$

$$i^d = i^r + i^x\left(\frac{\bar{Z} - M^b + M(1-\delta) + \delta(F+\alpha)}{\bar{Z} - \underline{Z}}\right)\mathbb{I}_{\bar{X}\geq 0}(1-\delta) \tag{88}$$

$$A^{cb} = M^b \tag{89}$$

$$A^{com} = A^s - M^b \tag{90}$$

$$X = Z - M^b + M(1-\delta) + \delta(F+\alpha) \tag{91}$$

$$R^x = \max(0, Z - M^b + M(1-\delta) + \delta(F+\alpha)) \tag{92}$$

$$K = \delta((F+\alpha) - M) \tag{93}$$

$$E = A^s - (F+\alpha) \tag{94}$$

$$\Pi^{com} = (i^r - i^a)M^b + i^a A^s - i^d(F+\alpha) + (i^d - i^r)M - i^x(Z - M^b + M(1-\delta) + \delta(F+\alpha))\mathbb{I}_{X\geq 0} \tag{95}$$

$$T^{g,HH} = (i^r - i^a)M^b - i^d(F+\alpha) + (i^d - i^r)M - i^x(Z - M^b + M(1-\delta) + \delta(F+\alpha))\mathbb{I}_{X\geq 0} \tag{96}$$

$$T^{cb,HH} = (i^a - i^r)M^b + i^x(Z - M^b + M(1-\delta) + \delta(F+\alpha))\mathbb{I}_{X\geq 0} + (i^r - i^{mon})M \tag{97}$$

$$\lambda_2 = \frac{1}{1+F}\ , \tag{98}$$

where $i^x \equiv \max(0, \gamma i^r + g)$ from equation 78.[26] As expected, using the model solution in equation 62 yields a true statement.

[26] I assume $\underline{Z} \leq -\delta(F+\alpha)$ to ensure that $\left(\frac{\bar{Z} - R^s + \delta(F+\alpha-M)}{\bar{Z} - \underline{Z}}\right) \leq 1$.

C The cashless model with an intensive labor decision

In this variation, instead of letting the household decide on the extensive margin of $N \in \{0, 1\}$, I let the household optimize over the intensive margin of its labor supply. Because the origin of the assumption of a strictly positive endowment is not relevant if the household optimizes over the intensive margin of its labor supply, I set $F = 0$ for tractability. The qualitative results with regards to monetary policy implementation and pass-through from the cashless model with an extensive labor decision remain unaffected.

For $N > 0$, the first order conditions with respect to labor is

$$\lambda_1 W = \phi N^{\varphi} \ . \tag{99}$$

The household balances the marginal utility of receiving W units of additional income in the morning ($\lambda_1 W$) with the marginal utility of leisure ϕN^{φ}. The marginal utility of an additional unit of income in the morning (λ_1) is positive because it allows the household to invest in deposits. Deposits yield utility. In addition, they are needed to buy consumption goods in the afternoon. If the marginal utility of receiving labor income is strictly smaller (greater) than the marginal utility of leisure, the household supplies less (more) labor. If the marginal utility of of receiving labor income equals the marginal utility of leisure, the household is indifferent between increasing and decreasing labor supply at $N > 0$.

C.1 Derivation of the solution

In an interior solution, the following equations have to be satisfied.

$$\frac{1}{C} = \lambda_2 \tag{100}$$

$$\lambda_1 W = \phi N^{\varphi} \tag{101}$$

$$\omega_D \frac{1}{D^s} + (1 + i^d)\lambda_2 = \lambda_1 \tag{102}$$

$$WN = D^s \tag{103}$$

$$T^{cb,HH} + T^{g,HH} + \Pi^F + (1 + i^d)D^s = C \tag{104}$$

$$\alpha \frac{C}{N} = W \tag{105}$$

$$C = N^{\alpha} \tag{106}$$

$$\Pi^F = C - WN \tag{107}$$

$$i^a = i^r + i^x P(X > 0) \tag{108}$$

$$i^d = i^r + i^x P(X > 0)(1 - \delta) \tag{109}$$

$$R^s + A^{com} + R^x = D^s + E + X\mathbb{I}_{X \geq 0} \tag{110}$$

$$\Pi^{com} = i^r R^s + i^a A^{com} - i^d D^s - i^x X\mathbb{I}_{X \geq 0} \tag{111}$$

$$T^{g,HH} = \Pi^{com} - i^a A^s \tag{112}$$

$$A^{cb} + X\mathbb{I}_{X \geq 0} = R^s + R^x \tag{113}$$

$$T^{cb,HH} = i^a A^{cb} + i^x X\mathbb{I}_{X \geq 0} - i^r R^s \tag{114}$$

$$M^b \equiv R^s \tag{115}$$

$$X \equiv Z - R^s + K \tag{116}$$

$$R^x \equiv \max(0, X) \tag{117}$$

$$A^s \equiv A^{com} + A^{cb} \tag{118}$$

$$i^x \equiv \max(0, \gamma i^r + g) \tag{119}$$

$$K \equiv \delta D^s \tag{120}$$

Solve equation 102 and equation 109 for i^d and equate

$$\frac{1}{\lambda_2}\left(\lambda_1 - \lambda_2 - \omega_D \frac{1}{D^s}\right) = i^r + i^x P(X > 0)(1 - \delta)\,. \tag{121}$$

Use equation 100, equation 101, equation 105, and equation 106 to replace, sequentially, λ_2, λ_1, W, and C.

$$C\left(\frac{\phi N^{\varphi}}{W}-\frac{1}{C}-\omega_D\frac{1}{D^s}\right)=i^r+i^xP(X>0)(1-\delta) \tag{122}$$

$$C\left(\frac{\phi}{\alpha}N^{1-\alpha+\varphi}-\frac{1}{C}-\omega_D\frac{1}{D^s}\right)=i^r+i^xP(X>0)(1-\delta) \tag{123}$$

$$N^{\alpha}\left(\frac{\phi}{\alpha}N^{1-\alpha+\varphi}-\frac{1}{N^{\alpha}}-\omega_D\frac{1}{D^s}\right)=i^r+i^xP(X>0)(1-\delta) \tag{124}$$

$$\left(\frac{\phi}{\alpha}N^{1+\varphi}-1\right)-N^{\alpha}\left(\omega_D\frac{1}{D^s}\right)=i^r+i^xP(X>0)(1-\delta) \tag{125}$$

Equation 103, equation 105, and equation 106 imply $\alpha N^{\alpha}=D^s$.

$$\left(\frac{\phi}{\alpha}\left(\frac{D^s}{\alpha}\right)^{\frac{1+\varphi}{\alpha}}-1\right)-\frac{\omega_D}{\alpha}=i^r+i^xP(X>0)(1-\delta) \tag{126}$$

Next, use equation 116 and equation 120 to replace X and K, respectively. Make use of equation 115 ($M^b\equiv R^s$).

$$\frac{\phi}{\alpha}\left(\frac{D^s}{\alpha}\right)^{\frac{1+\varphi}{\alpha}}-\frac{\omega_D}{\alpha}=(1+i^r)+i^xP(Z-R^s+K>0)(1-\delta) \tag{127}$$

$$\frac{\phi}{\alpha}\left(\frac{D^s}{\alpha}\right)^{\frac{1+\varphi}{\alpha}}-\frac{\omega_D}{\alpha}=(1+i^r)+i^xP(Z>R^s-K)(1-\delta) \tag{128}$$

$$\frac{\phi}{\alpha}\left(\frac{D^s}{\alpha}\right)^{\frac{1+\varphi}{\alpha}}-\frac{\omega_D}{\alpha}=(1+i^r)+i^x\left[1-P(Z<R^s-K)\right](1-\delta) \tag{129}$$

$$\frac{\phi}{\alpha}\left(\frac{D^s}{\alpha}\right)^{\frac{1+\varphi}{\alpha}}-\frac{\omega_D}{\alpha}=(1+i^r)+i^x\left[1-\frac{(R^s-K)-\underline{Z}}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0}(1-\delta) \tag{130}$$

$$\frac{\phi}{\alpha}\left(\frac{D^s}{\alpha}\right)^{\frac{1+\varphi}{\alpha}}-\frac{\omega_D}{\alpha}=(1+i^r)+i^x\left[\frac{\bar{Z}-(R^s-\delta D^s)}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0}(1-\delta) \tag{131}$$

$$\frac{\phi}{\alpha}\left(\frac{D^s}{\alpha}\right)^{\frac{1+\varphi}{\alpha}}-\frac{\omega_D}{\alpha}=(1+i^r)+i^x\left[\frac{\bar{Z}-(M^b-\delta D^s)}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0}(1-\delta) \tag{132}$$

To simplify, assume $\alpha=1$ and $\varphi=0$. The solution for deposits is

$$\phi D^s-\omega_D=(1+i^r)+i^x\left[\frac{\bar{Z}-(M^b-\delta D^s)}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0}(1-\delta) \tag{133}$$

$$\phi D^s-\left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}D^s=(1+i^r)+i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0}+\omega_D \tag{134}$$

$$D^s=\frac{(1+i^r)+i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0}+\omega_D}{\phi-\left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}}\,, \tag{135}$$

where $i^x \equiv \max(0, \gamma i^r + g)$ by equation 119. Combining equation 108 and with the results from equation 135 yields the solution for the government bond yield.

$$i^a = i^r + i^x P(X > 0) \tag{136}$$

$$i^a = i^r + i^x \left[\frac{\bar{Z} - (M^b - \delta D^s)}{\bar{Z} - \underline{Z}}\right] \mathbb{I}_{\bar{X}>0} \tag{137}$$

$$i^a = i^r + i^x \left[\frac{\bar{Z} - M^b}{\bar{Z} - \underline{Z}}\right] \mathbb{I}_{\bar{X}>0} + \frac{\delta i^x}{\bar{Z} - \underline{Z}} \left(\frac{(1 + i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}}\right) \mathbb{I}_{\bar{X}>0} \tag{138}$$

Combining equation 109 and with the results from equation 135 yields the solution for the deposit rate.

$$i^d = i^r + (1-\delta) i^x \left[\frac{\bar{Z} - M^b}{\bar{Z} - \underline{Z}}\right] \mathbb{I}_{\bar{X}>0} + \frac{\delta(1-\delta) i^x}{\bar{Z} - \underline{Z}} \left(\frac{(1 + i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}}\right) \mathbb{I}_{\bar{X}>0} \tag{139}$$

Next, use equation 113, equation 115, equation 117, and equation 118 to determine the partitioning of government bonds between the central bank and commercial banks.

$$A^{cb} = M^b \tag{140}$$

$$A^{com} = A^s - M^b \tag{141}$$

Combine the results from equation 135 and equation 141 with equation 110 and equation 117 to pin down equity.

$$E = R^s + A^{com} - D^s \tag{142}$$

$$E = A^s - D^s \tag{143}$$

$$E = A^s - \frac{(1 + i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}} \tag{144}$$

Labor market outcomes (hours worked and the wage) and firm profits are determined by equation 106, equation 105, and equation 107, respectively.

$$N = \frac{(1 + i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}} \tag{145}$$

$$W = 1 \tag{146}$$

$$\Pi^F = 0 \tag{147}$$

For the household, use equation 106, equation 100, and equation 101 to pin down consumption and the two Lagrange multipliers.

$$C = \frac{(1+i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}} \tag{148}$$

$$\lambda_1 = \phi \tag{149}$$

$$\lambda_2 = \left(\frac{(1+i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}}\right)^{-1} \tag{150}$$

Equation 116 (together with equation 120), equation 117, and equation 120 determine the regulatory liquidity shortage, the discount window borrowing, and the minimum reserve requirement, respectively.

$$X \equiv Z - M^b + \delta\left(\frac{(1+i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}}\right) \tag{151}$$

$$R^x \equiv \max\left(0, Z - M^b + \delta\left(\frac{(1+i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}}\right)\right) \tag{152}$$

$$K \equiv \delta\left(\frac{(1+i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}}\right) \tag{153}$$

Equation 111 determines, together with equation 113, equation 115, equation 117, and equation 118, commercial bank profits.

$$\Pi^{com} = i^r R^s + i^a A^{com} - i^d D^s - i^x X\mathbb{I}_{X\geq 0} \tag{154}$$

$$\Pi^{com} = i^r R^s + i^a(A^s - R^s) - i^d D^s - i^x X\mathbb{I}_{X\geq 0} \tag{155}$$

$$\Pi^{com} = (i^r - i^a)R^s + i^a A^s - i^d D^s - i^x X\mathbb{I}_{X\geq 0} \tag{156}$$

$$\Pi^{com} = (i^r - i^a)M^b + i^a A^s - i^d\left(\frac{(1+i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}}\right) - i^x X\mathbb{I}_{X\geq 0}\,, \tag{157}$$

where the government bond yield, the deposit rate, and the regulatory liquidity shortage are given by equation 138, equation 139, and equation 151, respectively.

Equation 114, together with equation 115, determines central bank transfers.

$$T^{cb,HH} = (i^a - i^r)M^b + i^x X\mathbb{I}_{X\geq 0} \tag{158}$$

Finally, using the result for commercial bank profits (equation 157) in equation 112 pins down government transfers.

$$T^{g,HH} = (i^r - i^a)M^b - i^d \left(\frac{(1+i^r) + i^x(1-\delta)\left[\frac{\bar{Z}-M^b}{\bar{Z}-\underline{Z}}\right]\mathbb{I}_{\bar{X}>0} + \omega_D}{\phi - \left(\frac{\delta(1-\delta)i^x}{\bar{Z}-\underline{Z}}\right)\mathbb{I}_{\bar{X}>0}} \right) - i^x X \mathbb{I}_{X\geq 0} \tag{159}$$

As expected, using the model solution in equation 104 yields a true statement.

D The cashless model with an entrepreneur

In this variation, I assume that instead of the government, an entrepreneur provides equity to commercial banks. The main results from the cashless model under $N = 1$ do not change. For exposition, I assume $F = 0$.

The entrepreneur solves a linear utility maximization problem subject to two budget constraints. In addition, the entrepreneur faces two non-negativity constraint on consumption $C^e \geq 0$ and hours worked $N^e \geq 0$ (associated to the multipliers $\underline{\varepsilon}_{C^e}$ and $\underline{\varepsilon}_{N^e}$, respectively).

In the morning, the entrepreneur supplies labor. He/she transfers labor income to the afternoon by means of commercial bank equity. The budget constraint of the morning is associated to λ_1^e. In the afternoon, the entrepreneur consumes equity and commercial bank profits. The budget constraint of the afternoon is associated to λ_2^e. The entrepreneur solves the following optimization problem.

$$\mathcal{L} = C^e - \gamma^e N^e + \lambda_1^e(WN^e - E) + \lambda_2^e(E + \Pi^{com} - C^e) + \underline{\varepsilon}_{N^e} N^e + \underline{\varepsilon}_{C^e} C^e \ , \tag{160}$$

where $\gamma^e \in [0, \alpha)$. Using the budget constraints in the utility function, the optimization problem of the entrepreneur becomes

$$\mathcal{L} = (WN^e - \Pi^{com}) - \gamma^e N^e + \underline{\varepsilon}_{N^e} N^e \ . \tag{161}$$

The entrepreneur balances the marginal utility of labor (which is the wage W) with the marginal utility of leisure γ^e. If the wage is strictly smaller than the marginal utility of leisure, the entrepreneur abstains from supplying labor ($N^e = 0$), reflected in $\underline{\varepsilon}_{N^e} \geq 0$. If the wage is weakly greater than the marginal utility of leisure, the entrepreneur is willing to supply any quantity $N^e > 0$. The marginal cost of the lower bound constraint on N^e is then zero ($\underline{\varepsilon}_{N^e} = 0$).

D.1 Derivation of the solution

Compared to the cashless model of Appendix A, the production function, the government transfers, the goods market clearing condition, and the labor market clearing condition change.

$$Y = (N + N^e)^\alpha \tag{162}$$

$$T^{g,HH} = -i^a A^s \tag{163}$$

$$Y = C + C^e \tag{164}$$

$$N^d = N + N^e \tag{165}$$

Moreover, there are two additional equations to determine N^e and C^e.

$$WN^e = E \tag{166}$$

$$C^e = E + \Pi^{com} \tag{167}$$

As in Appendix A.1, I combine the first order condition of the firm with the production function to determine W, assuming $\alpha = 1$ and $F = 0$.[27] In the course of the derivation, I use $E = A^s - D^s$ and $D^s = WN$ with $N = 1$ from Appendix A.1.

$$W = \alpha \frac{Y}{N + N^e} \tag{168}$$

$$W = \alpha \left(N + N^e\right)^{\alpha-1} \tag{169}$$

$$W = \alpha \left(N + \frac{E}{W}\right)^{\alpha-1} \tag{170}$$

$$W = \alpha \left(N + \frac{A^s - WN}{W}\right)^{\alpha-1} \tag{171}$$

$$W = \alpha \left(\frac{A^s}{W}\right)^{\alpha-1} \tag{172}$$

$$W = 1 \tag{173}$$

By equation 166 and equation 167, the solutions for N^e and C^e

$$N^e = A^s - 1 \tag{174}$$

$$C^e = (A^s - 1) + \Pi^{com} , \tag{175}$$

where Π^{com} is unchanged from Appendix A.1. From here, I proceed as in Appendix A.1. The solutions for output, total labor supply, government transfers, and household consumption

[27] The model does not have an analytical solution for $\alpha \in (0, 1)$. $F = 0$ is assumed for simplicity.

are

$$Y = A^s \tag{176}$$

$$N^d = A^s \tag{177}$$

$$T^{g,HH} = -i^a A^s \tag{178}$$

$$C = 1 - \Pi^{com} \, , \tag{179}$$

respectively, where i^a is unchanged from Appendix A.1. The other variables are unaffected by the introduction of an entrepreneur.