Screening by Mode of Trade

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Abstract

This paper proposes a mechanism design approach, capable of endogenizing a monopolist’s choice between selling and renting in a non-anonymous durable goods setting with short-term commitment. Allowing for mechanisms that determine the good’s allocation not only at the beginning but also at the end of a given period, we show that the profit-maximizing mechanism features screening by mode of trade. By selling to high types while renting to low types, the monopolist overcomes the obstacles encountered by intertemporal price discrimination and induces immediate separation of types for arbitrary low priors.

Keywords: Durable goods; Dynamic mechanism design; Coase problem; Ratchet effect; Screening
JEL: D82, D86, D42.

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1 Introduction

Coase’s (1972) seminal insight, that a seller’s tendency to cut prices for previously unsold units erodes his ability to screen customers, serves as foundation for the durable-goods literature. The idea that the Coase-problem might be overcome, and monopoly power be restored, by renting rather than selling (Bulow 1982) features prominently in this literature as it provides justification (retrospect) for several antitrust rulings against rent-only monopolists. In anonymous markets, renting mitigates a monopolist’s propensity to cut prices by eliminating the negative selection associated with a sale. However, with the rise of digital commerce and personalized pricing, markets have become increasingly non-anonymous and renting faces a problem similar to the one identified by Coase; the monopolist’s tendency to increase rentals tomorrow for customers renting today. With non-anonymous customers, this so called ratchet effect makes screening even harder to achieve under renting than under selling (Hart and Tirole 1988).

Recent advances in dynamic mechanism design by Bester and Strausz (2001, 2007) and Doval and Skreta (2019a) allow us to examine whether a monopolist’s ability to screen customers can be enhanced via the use of mechanisms more sophisticated than simple price-posting. While, in selling frameworks, general mechanisms turn out to have no bite (Skreta 2006; Doval and Skreta 2019b), in renting frameworks they improve screening via randomization (Beccuti and Möller 2018). However, in both settings the conclusion remains, that screening is prohibitively costly when the monopolist’s prior expectations about his customers’ willingness to pay are low.

In this article, we challenge this view by arguing that the monopolist’s inability to discriminate customers results from the restrictions on the monopolist’s mode of trade, imposed by the common definition of the set of feasible allocations. For this purpose, we extend the standard approach to mechanism design with short-term commitment by allowing for mechanisms that specify not only a likelihood of allocating the product to the consumer but also a likelihood of returning the product to the monopolist. Endogenizing the monopolist’s choice between selling and renting in this simple yet generic way, allows us to identify situations in which the profit-maximizing trading mechanism consists of

1Formalizations of Coase’s insight are provided by Stokey (1981), Gul et al. (1986), and Ausubel and Denecker (1989).

2United States v. United Shoe Mach. Corp. case 110 F. Supp. 295 (D. Mass. 1953) served as the basis for court orders against AT&T, IBM, and Xerox to offer products not only for rental but also for sale.

3Existing models assume the likelihood of return to be exogenous and equal to either one (renting frameworks, e.g. Hart and Tirole 1988; Beccuti and Möller 2018) or zero (selling frameworks, e.g. Fudenberg et al. 1983; Sobel and Takahashi 1983; Skreta 2006; Doval and Skreta 2019b).
screening by mode of trade. By selling to some consumer-types while renting to others the monopolist is able to reduce his costs of eliciting consumers’ private information to the extent that screening becomes optimal for arbitrarily low priors.

In Section 2 we introduce our model of a dynamic, non-anonymous durable goods market with short-term commitment. In each of two periods, a single, risk-neutral consumer has unit demand for a durable product. The consumer’s per-period valuation of the product can take two values, is constant across time, and constitutes the consumer’s private information, i.e. his type. The product is provided by a risk-neutral, monopolistic supplier with zero cost. In every period, the supplier can offer a mechanism. A mechanism elicits a message from the consumer, determining a monetary transfer and the probabilities of allocating the product to the consumer at the beginning of the period and of returning the product to the supplier at the end. The supplier’s lack of (long-term) commitment is captured by the assumption that mechanisms can specify transfers and (re)allocation-probabilities only for the current period and that, in every period, the supplier’s offer must be sequentially optimal.

We allow for the possibility that, with positive probability, the market breaks down in between periods, impeding further interaction between the two parties. Market breakdown is a key feature of our model as it makes (repeated) renting different from selling by jeopardizing potential future gains from trade. It may result from trade embargoes or the introduction of prohibitively high tariffs, or be simply due to a deterioration of the business relationship that obstructs further trade between the supplier and the consumer.

Following the formulation and discussion of the dynamic mechanism design problem in Section 4 we provide a characterization of the monopolist’s revenue maximizing trading mechanism in Section 5. Screening by mode of trade – selling to the high type and renting to the low type – turns out to be a feature of the optimal mechanism. Screening by mode of trade arises when the monopolist is patient, in that he discounts future payoffs less strongly than the consumer. Arguably, the case of a patient supplier is relevant,

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4Our model allows for the interpretation of a continuum of non-anonymous consumers. Importantly, the same (set of) consumer(s) is present in all periods. This distinguishes us from the literature studying the effects of consumers arriving over time (Conlisk et al. 1984; Board 2008; Deb and Said 2015; Garrett 2016).

5In practice, a mechanism can be implemented via a menu of contracts, specifying (re-)allocation probabilities and transfers, with contract-choice determined by the consumer’s message.

6Disputes between landlords and tenants are frequent in housing markets and there is evidence that the share of rentals increases in the efficiency of courts (Casas-Arce and Saiz 2010).

7While most of our analysis focuses on the (more interesting) case where the supplier’s prior is low, we complete our characterization of the optimal mechanism in Section 8. For high priors, the optimal mechanism posts either a rental or a sale price but never employs both modes of trade in combination.
because firms may have access to cheaper credit (Hirshleifer, 1958), and consumers might be present-biased (O’Donoghue and Rabin, 2013).

Our main result reveals an important qualitative difference between screening by mode of trade and the traditional forms of screening (i.e. intertemporal price discrimination) examined by the literature. In Section 6 we show that, while there exists a lower bound on the monopolist’s prior for which separation is optimal when trade is restricted to either selling or renting, immediate separation of types arises for arbitrarily low priors if the monopolist can screen by mode of trade. In other words, the common view, that a durable good monopolist’s ability to screen customers is limited, might have to be abandoned in settings where the choice between renting and selling can serve a screening purpose.

To understand why screening by mode of trade improves upon ordinary screening, first note that a monopolist with a low prior would find it optimal to serve both types of consumer if there was only one period of trade. In a dynamic context, heterogeneous discounting introduces a wedge between the monopolist’s future gain from learning the consumer’s type and the consumer’s future loss in information rent. As a consequence, screening can become optimal even for priors for which pooling is preferred from a static perspective. Because intertemporal price-discrimination (via pure selling or pure renting) requires the first-period exclusion of the low-type, it is subject to a trade-off between the monopolist’s static and dynamic objectives, making separation prohibitively costly when the monopolist’s prior is low. Screening by mode of trade overcomes this trade-off as it separates the high type from the low type while implementing first period trade with both types.

To see that screening by mode of trade can be optimal for arbitrarily low priors, note that, in the absence of asymmetric information, the monopolist would either rent or sell to both types of consumer. Although renting jeopardizes future gains from trade, a patient monopolist may prefer renting over selling because, from his viewpoint, the consumer’s discounted present value of his product is too small. As screening by mode of trade entails a sale to the high type and a rental to the low type, it comes at the cost of implementing the wrong mode of trade with one type of consumer. When the probability of market breakdown is low, the seller prefers renting over selling, so that not only the benefits but also the costs of screening arise from the monopolist’s interaction with the high type. As a consequence, screening by mode of trade can be optimal even when the likelihood of encountering a high type is (arbitrarily) low.

An important feature of our optimal mechanism is the use of randomization. In particular, the optimal mechanism may mix between renting and selling to the high type. Our
theory thus demonstrates the sub-optimality of deterministic mechanisms (price-posting) for a market where the mode of trade can be determined by the supplier. However, random mechanisms have been criticized for being more difficult to enforce than deterministic mechanisms (Laffont and Martimort, 2002, p.67). In Section 7 we therefore determine the supplier’s optimal deterministic mechanism. We show that the mere posting of a rental and a sale price is sufficient to obtain immediate separation of types for arbitrarily low priors. It is reassuring that our main result remains valid even in settings where random contracts are not feasible.

In summary, we can therefore conclude that in durable good markets, the mode of trade can constitute a powerful screening device. With non-anonymous consumers, the concurrence of selling and renting should not be seen as an indicator for the absence of monopoly power (Bulow, 1982) but rather as a potential feature of it; especially for markets where alternative motivations for renting, such as liquidity constraints or preference uncertainty, are less plausible.

Related literature. Although our theory is set in a durable goods framework, it relates, more generally, to the literature on dynamic adverse selection, initiated by Freixas et al. (1985) and Laffont and Tirole (1987, 1988). An important insight of this literature is that intertemporal screening by time might be substituted by intra-temporal screening by menu (Wang, 1998). The durable goods literature has employed a similar idea to show that in anonymous markets, the Coase-problem can be mitigated by giving consumers the choice between differentiated varieties (Kühn and Padilla (1996); Kühn (1998); Takeyama, 2002; Hahn, 2006; Inderst, 2008). Most closely related is Kühn and Padilla’s work on the simultaneous supply of a durable and a non-durable substitute. Our theory differs in its approach (mechanism design) and focus (non-anonymous markets) and demonstrates that the mere choice between buying and renting of a single product-variety can be sufficient to obtain immediate separation of consumer types.

The conditions under which immediate separation prevails in our model – heterogeneous time preferences and possibility of market breakdown – are ubiquitous features of real-world markets and constitute regular assumptions of the related literature on bargaining (e.g. Rubinstein and Wolinsky, 1985; Binmore et al., 1986; Fudenberg et al., 1983; Sobel and Takahashi, 1983). The significance of market breakdown for our results resonates well with the idea that enabling one of the parties to abandon the trade-relationship permanently can overcome the Coase problem in a selling framework (Board and Pycia, 2014) or mitigate the ratchet effect in a rental setting (Gerardi and Maestri, 2018).

While selling induces a time-invariant consumption pattern, renting entails the possi-
bility that consumption becomes “renegotiated” in the future. Laffont and Tirole (1990) consider a two-period renting framework with a divisible good, where a monopolist can offer long-term contracts that are subject to renegotiation. They show that the contract designed for the high type induces the same (efficient) consumption level in both periods while the contract designed for the low type becomes renegotiated. Although this pattern bears some similarity to screening by mode of trade, an important difference is that the low type’s consumption becomes renegotiated upwards whereas in our setting trade with the low type is first efficient and then moves downwards (market breakdown). It is the deferral of allocative efficiency into the future, which explains the power of the mode of trade as a screening device.

Finally, our result that combining selling with renting may reduce distortions arising from asymmetric information is reminiscent of the idea that leasing can mitigate the lemons problem in resale markets (Hendel and Lizzeri, 2002; Johnson and Waldman, 2003). Leasing contracts differ from renting contracts in that they entail the supplier’s commitment to a future (selling) price. When lessees obtain private information about their product’s (depreciated) quality, the associated option value is increasing in their valuation of quality. Hence, while both renting and leasing might serve as screening purposes in a durable goods setting, the conditions under which they emerge as an alternative to buying are markedly different.

2 Setup

Consider a monopolistic supplier of a non-divisible, durable product facing a single consumer during two periods. In each period, the consumer has unit demand. The consumer’s per-period valuation of the supplier’s product, $\theta$, is constant over time and constitutes the consumer’s private information. $\theta$ can take two values which we denote as the consumer’s type, $i \in \{L, H\}$. With probability $\beta \in (0, 1)$ the consumer’s valuation is high, $\theta = \theta^H > 0$, whereas with probability $1 - \beta$ the consumer’s valuation is low, $\theta = \theta^L \in (0, \theta^H)$. We call $\beta$ the supplier’s prior belief and abbreviate notation by defining $\Delta \theta \equiv \theta^H - \theta^L$. A discussion of the effects of extending our model to allow for more than two types and more than two periods is postponed until the Conclusion.

Our theory allows for the possibility that future trade-opportunities are not guaranteed but are subject to uncertainty. In particular, we make the following assumption:

\footnote{Maestri (2017) finds that in the limiting case of an infinite horizon and no discounting, the low type’s renegotiated contract becomes approximately efficient, thereby eradicating the monopolist’s ability to screen customers. See also Strulovici (2017).}
Assumption 1 (Market breakdown). In between periods, the market breaks down with probability $1 - \phi \in (0, 1)$, impeding further interaction between the two parties.

In case of market breakdown, the product’s allocation remains as determined previously. Assumption 1 therefore captures the idea that renting jeopardizes future gains from trade that could have been realized through a sale. While the relevance of market breakdown has been emphasized by the related literature on bargaining (e.g. Rubinstein and Wolinsky, 1985; Binmore et al., 1986), our model highlights its importance for identifying the mode of trade as a screening device in a durable goods framework.

Payoffs are as follows. If in any given period, the consumer makes a transfer $t \in \mathbb{R}$ to the supplier, then the consumer’s instantaneous payoff is $\theta - t$ if the product is allocated to him during that period. Otherwise the consumer’s payoff is $-t$. The supplier’s cost is normalized to zero, so that the supplier’s instantaneous payoff is given by his revenue $t$. The supplier and the consumer discount future payoffs with discount factors $\delta_S$ and $\delta_C$, respectively. We allow parties to differ with respect to their intertemporal preferences and make the following assumption:

Assumption 2 (Heterogeneous discounting). The supplier discounts future payoffs less strongly than the consumer, i.e. $0 \leq \delta_C < \delta_S \leq 1$.

It will become clear (see footnote 16) that the analysis of the opposite case where $\delta_C \geq \delta_S$ is trivial and it is therefore omitted. Although heterogeneous time preferences are commonly assumed in related models of bargaining with asymmetric information (Fudenberg et al., 1983; Sobel and Takahashi, 1983), the durable goods literature has largely focused on the case of homogeneous discounting in order to allow for a better comparison with the commitment case.

Mechanism design approach. We employ a mechanism design approach with short-term commitment to determine the supplier’s revenue maximizing trading mechanism. The supplier lacks long-term commitment in the sense that, a mechanism can specify monetary transfers and changes regarding the allocation of the supplier’s product only for the current period. We use Perfect Bayesian Equilibrium as our solution concept which requires that in every period the supplier’s offer must be optimal given his (potentially updated) belief about the consumer’s type. From a methodological viewpoint, the main

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Footnote 16: In models employing a dynamic mechanism design approach, heterogeneous discounting renders the commitment-benchmark ill-defined when parties are able to commit to intertemporal transfers (Krähmer and Strausz, 2015). Heterogeneous discounting is a feature of theories considering a monopolist’s choice of product durability (e.g. Barro, 1972; Bikhchandani and McCardle, 2012) assume a patient seller in their analysis of behavior-based price discrimination in a non-durable goods framework.
novelty of our analysis is that we allow for mechanisms that may change the allocation of the supplier’s product not only at the beginning but also at the end of the current period. This allows us to determine the optimal “mode of trade”, i.e. the supplier’s choice between renting and selling, as part of the mechanism design problem.

More specifically, in period 1 the supplier offers a (direct) mechanism which asks the consumer to report his type by issuing a message \( m \in \{L, H\} \). Conditional on the consumer’s message, the mechanism specifies: a transfer \( t_m \in \mathbb{R} \) from the consumer to the supplier; a likelihood \( d_m \in [0, 1] \) with which the product is delivered to the consumer at the beginning of period 1; and a likelihood \( r_m \in [0, 1] \) with which the product, conditional on having been delivered, is returned to the supplier at the end of period 1. If the product was delivered and not returned in period 1, the consumer enjoys the product in period 2. If the product was not delivered or delivered but returned, and if the market does not break down, then in period 2 the supplier posts a price. Restricting to price-posting in period 2 is without loss of generality because the supplier’s problem becomes equivalent to a static screening problem for which price-posting is known to be optimal.

There are two notable features of our approach. Firstly, our set of feasible mechanisms contains as special cases the selling mechanisms \( (r_L = r_H = 0) \) and renting mechanisms \( (r_L = r_H = 1) \) considered by the existing literature. Secondly, and most importantly, a generic mechanism in our model cannot be replicated via the mere re-definition of an “allocation”, neither as a selling nor as a renting mechanism. To see this, let \( \tilde{\beta}_m \) denote the supplier’s updated belief about the consumer’s type conditional on his message \( m \in \{L, H\} \) and let \( \tilde{U}^i(\tilde{\beta}_m) \) denote the consumer’s second period gains from trade when his type is \( i \in \{L, H\} \) (both to be determined in Section 5). Type \( i \)’s payoff from choosing message \( m \) under our general mechanism can then be written as

\[
U_m^i = d_m \theta_i - t_m + \delta_C [\phi \tilde{U}^i(\tilde{\beta}_m) + d_m (1 - r_m)(\theta_i - \phi \tilde{U}^i(\tilde{\beta}_m))] .
\]

Note from (1) that a selling mechanism \( (r_m = 0) \) and a renting mechanism \( (r_m = 1) \) differ in that only the former exhibits a direct, non-informational link between present allocation and future payoffs. More specifically, while under selling, future payoffs depend on \( d_m \) directly, under renting, the consumer’s choice between \( d_L \) and \( d_H \) influences his future payoffs only indirectly via its effect on the supplier’s updated belief \( \tilde{\beta}_m \). The reason for this difference is that under selling, trade is an “absorbing state” in the language of

\[\text{Bester and Strausz (2001) and Doval and Skreta (2019a) and our explanations in Section 4.}\]

\[\text{Note that we indicate messages by subscript to facilitate distinction from types which are denoted by superscript.}\]
Tirole (2016), whereas under renting trade is non-absorbing. In our approach, the return probabilities \( r_m \) can be used to fine-tune this non-informational link between present allocation and future payoffs. The supplier controls how absorbing trade is and can even make trade with one type more absorbing than with the other by choosing \( r_L \neq r_H \). It is in this sense that our mechanism design approach extends the existing renting or selling models.

3 Benchmark: Symmetric information

As a benchmark, consider the case of symmetric information, where the supplier can observe the consumer’s type \( \theta \). In this simple case, the supplier cannot do better than by either selling his product to the consumer in period 1 at a price of \((1 + \delta_C)\theta\) or by offering to rent in both periods at price \( \theta \). The supplier’s payoff from selling is larger than his (expected) payoff from renting if and only if \((1 + \delta_C)\theta \geq (1 + \phi\delta_S)\theta\). Note that, in the absence of private information, the supplier’s choice between selling and renting is determined by a simple trade-off. Renting jeopardizes future revenues in the event of market breakdown while selling requires an excessively discounted price due to the consumer’s impatience. Hence selling dominates renting if and only if the risk of market breakdown is sufficiently large. We record these insights in the following observation:

**Observation 1** (Symmetric information benchmark). Under symmetric information, the supplier implements the same mode of trade with both consumer-types, by selling if \( \phi < \frac{\delta_C}{\delta_S} \) and by renting if \( \phi > \frac{\delta_C}{\delta_S} \). From an ex ante perspective, the supplier’s expected payoff is given by

\[
V^* = [1 + \max\{\delta_C, \phi\delta_S\}][\beta\theta^H + (1 - \beta)\theta^L].
\]  

(2)

The payoff \( V^* \) constitutes an important benchmark for our subsequent analysis. In particular, the comparison of \( V^* \) with the supplier’s payoff under the optimal mechanism in the presence of asymmetric information, will allow us to assess the severity of dynamic adverse selection in our framework and to show how the problem is mitigated by the possibility of screening by mode of trade.

4 The supplier’s mechanism design problem

The first step towards the formulation of the supplier’s mechanism design problem is to determine the expected future gains from trade, conditional on the supplier’s updated belief about the consumer’s type, \( \tilde{\beta} \). As the second period is the last, the distinction
between renting and selling ceases to matter. It is well established that the optimal static mechanism consists of simple price-posting and it can be described as follows. The supplier will post a low price \( \theta_L \), accepted by both types, when his (updated) belief is such that \( \tilde{\beta} \leq \frac{\theta_L}{\alpha} \). When \( \tilde{\beta} > \frac{\theta_L}{\alpha} \), the supplier will post a high price \( \theta_H \), accepted only by the high type. The supplier’s expected future gains from trade are thus given by

\[
\tilde{V}(\tilde{\beta}) = \begin{cases} 
\tilde{\beta}\theta_H & \text{if } \tilde{\beta} \geq \frac{\theta_L}{\alpha} \\
\theta_L & \text{if } \tilde{\beta} \leq \frac{\theta_L}{\alpha} 
\end{cases}
\tag{3}
\]

The consumer’s future gains from trade are \( \tilde{U}_L(\tilde{\beta}) = 0 \) for the low type and

\[
\tilde{U}_H(\tilde{\beta}) = \begin{cases} 
0 & \text{if } \tilde{\beta} > \frac{\theta_L}{\alpha} \\
\Delta \theta & \text{if } \tilde{\beta} \leq \frac{\theta_L}{\alpha} 
\end{cases}
\tag{4}
\]

for the high type. The supplier’s updated belief \( \tilde{\beta} \) depends on the consumer’s (first period) message \( m \in \{L, H\} \) and therefore takes two values, \( \tilde{\beta}_L \) and \( \tilde{\beta}_H \), to be determined below. In order to abbreviate notation we will let \( \tilde{V}_m = \tilde{V}(\tilde{\beta}_m) \) and \( \tilde{U}^i_m = \tilde{U}^i(\tilde{\beta}_m) \).

We are now ready to formulate the mechanism design problem the supplier faces in period 1. Our analysis follows the machinery developed by the seminal article of Bester and Strausz [2001]. The main difficulty arising from the dynamic nature of our setting is that we cannot focus on mechanisms that induce true type revelation, making the evolution of the supplier’s beliefs become part of the mechanism design problem. To allow for the possibility that types can be misrepresented, let \( q^L < 1 \) and \( q^H > 0 \) denote the probabilities with which types \( L \) and \( H \) are induced to report message \( H \). Note that, without loss of generality, we can assume that \( q^H \geq q^L \), because if this was not the case, we could rename messages. Denote the ex-ante probability that messages \( H \) and \( L \) are issued by \( Q_H = \beta q^H + (1 - \beta)q^L \) and \( Q_L = 1 - Q_H \), respectively. Then Bayesian updating implies that the supplier’s posterior beliefs about the consumer’s type are given by

\[
\tilde{\beta}_L = \frac{\beta(1 - q^H)}{Q_L} \quad \text{and} \quad \tilde{\beta}_H = \frac{\beta q^H}{Q_H}
\tag{5}
\]

and it follows from \( q^H \geq q^L \) that \( \tilde{\beta}_L \leq \beta \leq \tilde{\beta}_H \). The supplier’s mechanism design problem is to choose a menu of contracts \( (t_L, d_L, r_L), (t_H, d_H, r_H) \in \mathbb{R} \times [0, 1]^2 \) and an information

\(^{12}\)It is possible to interpret \( q^i \) as the likelihood with which type \( i \) is induced to select the contract \( (t_H, d_H, r_H) \), “designed for the high type”, from the menu of contracts \( \{(t_L, d_L, r_L), (t_H, d_H, r_H)\} \). Given this rather intuitive interpretation, we chose to focus our analysis on the likelihoods \( q^i \). Alternatively, we could have formulated the mechanism design problem using the supplier’s corresponding posterior beliefs, as shown by the recent work of Doval and Skreta [2019a].
elicitation strategy, \( q^L \in [0, 1) \) and \( q^H \in (0, 1] \), to maximize his expected revenue

\[
V = \sum_{m \in \{L, H\}} Q_m [t_m + (1 - d_m + d_m r_m) \delta_S \tilde{V}_m]\tag{6}
\]

subject to the incentive and participation constraints

\[
\begin{align*}
U^H_H & \geq U^H_L \quad \text{with equality if } q^H < 1 \quad (IC^H) \\
U^L_L & \geq U^L_H \quad \text{with equality if } q^L > 0 \quad (IC^L) \\
U^H_H & \geq 0 \quad (PC^H) \\
U^L_L & \geq 0 \quad (PC^L)
\end{align*}
\]

with \( U^i_m \) given by (1). Note that the incentive constraints have to hold with equality whenever the information elicitation strategy allows the consumer to misrepresent his type.

We show in the Appendix that \((PC^H)\) is redundant and that at the optimum \((IC^H)\) and \((PC^L)\) must hold with equality\(^\text{13}\). Substitution of the transfers that make these constraints binding

\[
\begin{align*}
t^*_L &= d_L [1 + (1 - r_L) \delta_C] \theta^L \\
t^*_H &= d_H [1 + (1 - r_H) \delta_C] \theta^H - d_L [1 + (1 - r_L) \delta_C] \Delta \theta \\
&\quad - \delta_C \phi \{[1 - d_L (1 - r_L)] \tilde{U}^H_L - [1 - d_H (1 - r_H)] \tilde{U}^H_H\}
\end{align*}\tag{7, 8}
\]

simplifies the supplier’s mechanism design problem to the following reduced program:

\[
\begin{align*}
\max_{d_L, r_L, d_H, r_H, q^L, q^H} \quad & \sum_{i \in \{L, H\}} Q_i \{[d_i + d_i (1 - r_i) \delta_C] \theta^i + [1 - d_i (1 - r_i)] \phi [\delta_C \tilde{U}^i_i + \delta_S \tilde{V}_i]\} \\
& - Q_H \{[d_L + d_L (1 - r_L) \delta_C] \Delta \theta + [1 - d_L (1 - r_L)] \phi \delta_C \tilde{U}^H_L\} \\
\text{subject to } & \quad d_H [1 + (1 - r_H) \delta_C] - d_L [1 + (1 - r_L) \delta_C] \geq \\
& \quad \frac{\phi \delta_C}{\Delta \theta} \{[1 - d_L (1 - r_L)] \tilde{U}^H_L - [1 - d_H (1 - r_H)] \tilde{U}^H_H\}
\end{align*}\tag{9}
\]

subject to \( d_H [1 + (1 - r_H) \delta_C] - d_L [1 + (1 - r_L) \delta_C] \geq \)

\[
\frac{\phi \delta_C}{\Delta \theta} \{[1 - d_L (1 - r_L)] \tilde{U}^H_L - [1 - d_H (1 - r_H)] \tilde{U}^H_H\}
\]

with equality if \( q^L > 0 \).

The supplier’s program exhibits the familiar trade-off between maximization of surplus and minimization of information-rent left to the high type. The constraint is a dynamic version of the monotonicity constraint, requiring “trade” with the high type to be larger than “trade” with the low type.

\(^{13}\)Standard arguments from static mechanism design only allow us to conclude that at least one of the two incentive constraints must be binding.
A number of features of the supplier’s optimal mechanism follow directly from inspection of the reduced program. Note that because \( \tilde{U}_H + \tilde{V}_H \leq \theta^H \), an increase in \( d_H \) increases the supplier’s objective \((9)\) while relaxing the constraint \((DMC)\). Moreover, it is easy to see that the same holds for a decrease in \( r_H \) when \( \phi < \frac{\delta_C}{\delta_S} \). Hence we can make the following:

**Observation 2 (No distortion at the top).** The optimal mechanism sets \( d_{H*} = 1 \). Moreover, if the supplier prefers selling under symmetric information, i.e. if \( \phi < \frac{\delta_C}{\delta_S} \), then the optimal mechanism sells to the high type, i.e. \( r_{H*} = 0 \).

Observation 2 is reminiscent of the “no distortion at the top” result from static screening (e.g. Mussa and Rosen, 1978). Note however, that in our dynamic setting, allocative efficiency requires the product not only to be delivered \((d = 1)\) but also to never be returned \((r = 0)\). Because of the possibility of market breakdown, renting comes at the risk of a reduction in allocative efficiency. Observation 2 shows that when the supplier prefers selling over renting under symmetric information then he will indeed implement the efficient mode of trade with the high type. In other words, any inefficiency in the contract offered to the high type must be driven by an inherent preference for renting held by the supplier and can therefore be understood as a simple consequence of heterogeneous discounting. We will come back to this issue in our discussion of the optimal mechanism’s allocative efficiency at the end of Section 6.

5 **Screening by mode of trade**

In this section we characterize the supplier’s optimal mechanism for the case where the supplier’s prior belief is such that \( \beta < \frac{\theta_L}{\theta_H} \). The analysis of the remaining case, \( \beta \geq \frac{\theta_L}{\theta_H} \), is postponed until Section 8. We interpret the case where the supplier’s prior is low as a “poor market” and talk about a “good market” when the supplier’s prior is high. We analyze the two cases separately, because the supplier’s optimal mechanism exhibits strikingly different features in these two types of markets.

Solving the supplier’s reduced program in \((9)\) is a simple yet tedious application of linear programming whose details can be found in the Appendix. In order to report the
solution we define the following thresholds

\[
\phi(\beta) = \frac{\delta_C(\theta^L - \beta \theta^H)}{\delta_S(\theta^L - \beta \theta^H) + \beta(\delta_S - \delta_C)\Delta \theta}
\]  
(10)

\[
\bar{\phi}(\beta) = \min \left\{ \frac{\delta_S \theta^H - (\delta_S - \delta_C)\theta^L}{\delta_S \theta^H}, \frac{\delta_C \theta^L}{\delta_S \beta \theta^H} \right\}
\]  
(11)

\[
\beta = \frac{\delta_C \theta^L}{\delta_S \theta^H - (\delta_S - \delta_C)\theta^L} \in (0, \frac{\theta^L}{\theta^H})
\]  
(12)

where \( \phi \) and \( \bar{\phi} \) are decreasing and such that \( 0 < \phi(\beta) < \frac{\delta_C}{\delta_S} < \bar{\phi}(\beta) < 1 \) for all \( \beta \leq \frac{\theta^L}{\theta^H} \).

**Proposition 1 (Optimal mechanism).** For \( \beta < \frac{\theta^L}{\theta^H} \), the supplier’s optimal mechanism is characterized as follows:

- \( \phi \in (0, \phi) \): Sell only: Pooling \( \{(d_{L*}^*, r_{L*}^*), (d_{H*}^*, r_{H*}^*)\} = \{(1, 0), (1, 0)\} \).
- \( \phi \in [\phi, \bar{\phi}) \): Rent to low type, sell or mix to high type: Separation, \( \{(d_{L*}^*, r_{L*}^*), (d_{H*}^*, r_{H*}^*)\} = \{(1, 1), (1, r_{H*}^*)\} \) with \( r_{H*}^* = 0 \) for \( \phi \leq \frac{\delta_C}{\delta_S} \) and \( r_{H*}^* = 1 - \phi \) for \( \phi > \frac{\delta_C}{\delta_S} \).
- \( \phi \in [\bar{\phi}, 1) \): Rent only: Pooling, \( \{(d_{L*}^*, r_{L*}^*), (d_{H*}^*, r_{H*}^*)\} = \{(1, 1), (1, 1)\} \), if \( \beta < \beta \);
  Separation, \( \{(d_{L*}^*, r_{L*}^*), (d_{H*}^*, r_{H*}^*)\} = \{(1, 1), (1 - \phi \delta_C, 1)\} \), if \( \beta \geq \beta \).

**Posters are \( \tilde{\beta}_L = 0 \) and \( \tilde{\beta}_H = 1 \) under separation and \( \tilde{\beta}_L = \tilde{\beta}_H = \beta \) under pooling.**

Figure 1 provides a graphical representation of Proposition[1]. In the shaded areas, the optimal mechanism pools both types of consumer by offering only one contract; a selling contract \( (d_{L*}^*, r_{L*}^*) = (d_{H*}^*, r_{H*}^*) = (1, 0) \) when the likelihood of a future trade opportunity is low (\( \phi < \phi \)), and a renting contract \( (d_{L*}^*, r_{L*}^*) = (d_{H*}^*, r_{H*}^*) = (1, 1) \) when the likelihood of a future trade opportunity is high (\( \phi > \bar{\phi} \)). In the unshaded areas, the optimal mechanism induces full separation of consumer types. Note that the optimal mechanism screens the consumer in two mutually exclusive ways: (1) by decreasing the low type’s probability of delivery \( d_{L*}^* \) below \( d_{H*}^* \); or (2) by increasing the low type’s probability of return \( r_{L*}^* \) above \( r_{H*}^* \). Both, decreasing \( d_L \) or increasing \( r_L \) are possible means to achieve the monotonicity in “trade”,

\[
d_H[1 + (1 - r_H)\delta_C] - d_L[1 + (1 - r_L)\delta_C] \geq \delta_C[1 - d_L(1 - r_L)]\phi, \]  
(13)

necessary for separation[14]. The possibility to reduce “trade” with the low type by renting rather than selling is the novel feature of our approach, which becomes overlooked when the mode of trade is treated as exogenous.

[14]In a separating mechanism, the consumer’s second period payoffs are \( \bar{U}_L = \Delta \theta \) and \( \bar{U}_H = 0 \) and (13) follows from substitution of these payoffs into (DMC).
Figure 1: **Optimal Mechanism.** The supplier’s revenue-maximizing mechanism in dependence of his prior $\beta \in (0, \frac{\varphi_L}{\varphi_T})$ and the likelihood $\phi \in (0, 1)$ of a future trade opportunity. Unless noted otherwise, the optimal mechanism sets not only $d_H^* = 1$ but also $d_L^* = 1$. The thresholds $\phi$, $\bar{\phi}$, and $\beta$ are as defined in (10), (11), and (12), respectively.

In light of existing results, the most remarkable characteristic of our optimal mechanism is the emergence of an interval of moderate market breakdown probabilities $\phi \in [\frac{\Delta C}{\Delta S}, 1 - \frac{(\delta_S - \delta_C)\theta^L}{\Delta \theta^H}]$ for which the supplier screens the consumer (by mode of trade) even when he is approximately certain ($\beta \to 0$) that the consumer’s type is low. Screening by mode of trade overcomes the obstacles for discrimination posed by the Coase problem and the ratchet effect in that it induces immediate separation of types independently of the supplier’s expectations.

To understand this result, it is instructive to compare the pooling-payoffs from renting to both types, $V_{Pool}^R = (1 + \phi \delta_S)\theta^L$, with the payoffs from screening by mode of trade, which for the relevant range of parameters ($\phi > \frac{\Delta C}{\Delta S}$) can be written as

$$V_{Screen}^{MT} = V_{Pool}^R - \beta \phi (\phi \delta_S - \delta_C)\theta^H + \beta \phi (\delta_S - \delta_C)\Delta \theta.$$

(14)

Screening by mode of trade is costly because it entails the possibility of a sale (to the high type with probability $\phi$) although, as noted in Section 3, renting constitutes the

---

15 The payoffs from screening by mode of trade can be obtained from substitution of $d_L = d_H = r_L = 1$ and $r_H = 1 - \phi$ into (9).
supplier’s preferred mode of trade when $\phi > \frac{\delta_C}{\delta_S}$. The benefits from screening derive from the fact that, due to heterogeneous discounting, the supplier attaches a greater (future) value to the consumer’s private information than the consumer himself. There is scope for informational arbitrage and screening will occur when the corresponding benefits, $\beta\phi(\delta_S - \delta_C)\Delta \theta$, exceed the costs of selling rather than renting, $\beta\phi(\phi \delta_S - \delta_C)\theta^H$.

This happens when both modes of trade are rather similar in the eyes of the supplier, i.e. when $\phi \in \left[\frac{\delta_C}{\delta_S}, 1 - \frac{\delta_S - \delta_C}{\delta_S \theta^H}\right]$.

To understand why the choice between screening and pooling can become independent of the supplier’s prior note from (14) that under screening by mode of trade, both the benefits and the costs of screening are incurred with the high type, i.e. with probability $\beta$. This distinguishes screening by mode of trade from ordinary screening with a single mode of trade (see Section 6 for details). The costs of ordinary screening are incurred with the low type, i.e. with probability $1 - \beta$, in the form of a reduction in first period trade. As a consequence, separation becomes prohibitively costly when the supplier’s prior is low.

When the mode of trade can be used as a screening device, immediate separation can occur within a durable goods framework, independently of the supplier’s prior.

The power of screening by mode of trade becomes most apparent when we rewrite its payoffs in reference to the symmetric information benchmark $V^*$:

$$V^{MT}_{Screen} = V^* - \beta(1 + \phi \delta_C)\Delta \theta - \begin{cases} (1 - \beta)(\delta_C - \phi \delta_S)\theta^L & \text{if } \phi \leq \frac{\delta_C}{\delta_S} \\ \beta\phi(\phi \delta_S - \delta_C)\theta^H & \text{if } \phi \geq \frac{\delta_C}{\delta_S}. \end{cases}$$

The payoffs from screening by mode of trade are lower than in the symmetric information benchmark because of two effects: (1) the mechanism leaves information rents $\beta(1 + \phi \delta_C)\Delta \theta$ to the high type and (2) the mechanism creates a distortion with respect to the supplier’s preferred mode of trade. For $\phi < \frac{\delta_C}{\delta_S}$ the mechanism rents to the low type although selling is preferred in the benchmark. Similarly, for $\phi > \frac{\delta_C}{\delta_S}$ the mechanism mixes between renting and selling to the high type although renting is preferred in the benchmark. Note that for $\phi \rightarrow \frac{\delta_C}{\delta_S}$ both of these distortions become negligible and hence the payoff from screening by mode of trade becomes equal to $V^* - \beta(1 + \phi \delta_C)\Delta \theta$. For $\phi \rightarrow \frac{\delta_S}{\delta_S}$, screening comes at no cost beyond the standard information rent, i.e. screening becomes as powerful as it can possibly get.

Finally, a comment is in order regarding the optimal mechanism’s use of randomization. The optimal mechanism may randomize with respect to the product’s delivery ($d^*_L = 1 - \phi \delta_C$) as well as with respect to the product’s return ($r^*_H = 1 - \phi$). Interestingly, for an impatient supplier, i.e. for $\delta_C \geq \delta_S$ there would exist no informational arbitrage and hence the optimal mechanism would consist of simple pooling.
randomization is useful only for those parameters where renting constitutes the supplier’s preferred mode of trade \( (\phi \geq \frac{\delta_C}{\delta_S}) \). This result resonates well with the existing literature which has found that randomization can be optimal in a renting framework but not in a selling framework.\(^\text{17}\) Further discussion of this issue will follow our characterization of the optimal deterministic mechanism in Section 7.

6 Ordinary screening

While many products can be rented as well as sold, there are cases where the mode of trade cannot be freely determined by the supplier. For example, when a product requires complicated maintenance it may be practical to offer it for rental only, in combination with a service contract. On the contrary, when “proper” maintenance of a product is subject to the user’s moral hazard, a sale may be the only practical option.

In this section, we determine the supplier’s optimal mechanisms when the supplier is restricted to pure selling \( (r_L = r_H = 0) \) or pure renting \( (r_L = r_H = 1) \). Comparison with the results of the previous section will allow us to highlight the main advantages of screening by mode of trade over “ordinary” screening, with a single mode.

The characterization of the supplier’s optimal renting and selling mechanisms contained in the next proposition makes use of an additional threshold:

\[
\beta^S(\phi) \equiv \frac{\theta^L(1 + \delta_C - \phi \delta_S)}{\theta^H(1 + \delta_C) - \phi(\delta_S \theta^L + \delta_C \Delta \theta)} \in \left(\frac{\theta^L}{\theta^H}, \beta\right).
\]

(16)

**Proposition 2** (Exogenous mode of trade). When the supplier is restricted to a single mode of trade \( (r_L = r_H \in \{0, 1\}) \), he will refrain from screening unless his prior is sufficiently high. In particular:

- The optimal selling mechanism pools, \( \{(d^S_L, r^S_L), (d^S_H, r^S_H)\} = \{(1,0), (1,0)\} \), if \( \beta \leq \beta^S(\phi) \) and separates, \( \{(d^S_L, r^S_L), (d^S_H, r^S_H)\} = \{(0,0), (1,0)\} \), if \( \beta^S(\phi) \leq \beta < \frac{\theta^L}{\theta^H} \).

- The optimal renting mechanism pools, \( \{(d^R_L, r^R_L), (d^R_H, r^R_H)\} = \{(1,1), (1,1)\} \), if \( \beta \leq \beta^R \) and separates, \( \{(d^R_L, r^R_L), (d^R_H, r^R_H)\} = \{(1 - \phi \delta_C, 1), (1,1)\} \), if \( \beta \leq \beta < \frac{\theta^L}{\theta^H} \).

**Proposition 2** emphasizes the main effect of imposing the supplier to use either selling or renting as his sole mode of trade. In line with the findings of the existing literature, the scope for ordinary screening is rather restricted, no matter whether the supplier operates in a renting or in a selling framework. In both cases, the supplier decreases first period

\(^{17}\)Randomization can be optimal for a monopolist selling multiple varieties. See for example Thanassoulis (2004).
trade with the low type in order to reduce the information-rent left to the high type. This reduction comes at a loss of surplus and makes separation prohibitively costly when the supplier’s prior is low. Screening by mode of trade overcomes this problem as it induces separation while implementing first period trade with both types.

To improve our understanding of the difference between ordinary screening and screening by mode of trade, it is instructive to compare their payoffs in relation to the symmetric information benchmark, $V^*$. In our setting, allocative efficiency requires that the product is allocated to both types of consumer in every period. Due to the possibility of market breakdown, renting therefore constitutes an obstacle for efficiency. In order to focus on the efficiency losses caused by the presence of asymmetric information, consider the case where, in the benchmark, the supplier prefers selling over renting. In particular, let $\phi < \frac{\delta_C}{\delta_S}$ and compare the supplier’s payoff $V_{MT}^S$ from screening by mode of trade in (15) with the payoff from ordinary screening (via pure selling) given by

$$V_{Screen}^S = V^* - \beta(1 + \phi \delta_C)\Delta \theta - (1 - \beta)(\delta_C - \phi \delta_S)\theta_L - (\theta_L - \beta \theta_H). \quad (17)$$

The two screening mechanisms leave the same information-rent $\beta(1 + \phi \delta_C)\Delta \theta$ to the high-type consumer. Moreover, both mechanisms expose the low-type consumer to the risk of market breakdown in period two, resulting in the reduction of the supplier’s payoff by $(1 - \beta)(\delta_C - \phi \delta_S)\theta_L$. Ordinary screening and screening by mode of trade differ only in the term $\theta_L - \beta \theta_H$ which accounts for the fact that the former excludes the low type consumer from trade in the first period whereas the latter induces trade with both types.

Screening by mode of trade dominates ordinary screening by obviating first-period trade reductions, leading to the first-best allocation of the supplier’s product in period one. Hence, the increase in monopoly power that arises from screening by mode of trade does not necessarily come at the cost of allocative efficiency. Instead, it is possible that screening by mode of trade substitutes for ordinary screening, increasing not only the supplier’s revenue but, at the same time, benefiting allocative efficiency.

7 Price-posting

According to our analysis in the previous sections, the supplier’s optimal mechanism employs randomization, unless the mode of trade is restricted to pure selling. Both, the optimal (unrestricted) mechanism, as well as the optimal renting mechanism randomize

\[18\] Remember from Observation 1 that, even in the absence of asymmetric information, the supplier may choose renting over selling, thereby failing to maximize allocative efficiency, simply because he is more patient than the consumer.
the product’s allocation by either mixing between selling and renting, \( r_H \in (0, 1) \), or between delivery and exclusion, \( d_L \in (0, 1) \). In practice, such randomization may take the form of contracts that condition the consumer’s usage of the supplier’s product on exogenous random events. However, stochastic contracts are more difficult to verify and hence to enforce, and in some markets randomization might simply not be feasible. It is therefore important to understand how the supplier’s optimal mechanism is changed when contracts are restricted to be deterministic.

In this section we solve the supplier’s mechanism design problem under the additional constraint that allocation and re-allocation probabilities must be either one or zero, i.e. \( d_m, r_m \in \{0, 1\} \). This constraint is equivalent to requiring the supplier to set a rental and/or a sale price for his product in each period. Our subsequent analysis therefore relates our theory to the earlier price-posting models, with the difference that, in our setting, the supplier may choose to rent and sell. The following definitions are necessary to state our result:

\[
\beta^{PP}(\phi) \equiv \frac{\theta_L}{\theta_H + \phi(\delta_S - \delta_C)\Delta \theta}
\]

\[
\bar{\phi}^{PP}(\beta) \equiv \min \left( \frac{\delta_C\theta_H}{\delta_C\Delta \theta + \delta_S\theta_L}, \frac{\theta_L - \beta\theta_H(1 - \delta_C)}{\beta\delta_S\theta_H} \right).
\]

**Proposition 3** (Price-posting). Let \( \beta < \frac{\alpha_L}{\bar{\phi}^{PP}} \). When the supplier is restricted to deterministic mechanisms, i.e. \( d_m, r_m \in \{0, 1\} \), separation is less prevalent but screening by mode of trade continues to occur for arbitrarily low priors. In particular, \( \bar{\phi}^{PP} \in (\delta_S, \bar{\phi}) \), \( \beta^{PP} \in (\beta, \frac{\alpha_L}{\bar{\phi}^{PP}}) \), and the optimal price-posting mechanism is characterized as follows:

- \( \phi \in (0, \bar{\phi}) \): Sell at price \((1 + \delta_C)\theta_L\), accepted by both types (pooling).
- \( \phi \in [\bar{\phi}, \bar{\phi}^{PP}] \): Rent at price \( \theta_L \), accepted by the low type, sell at price \((1 + \delta_C)\theta_H - (1 + \delta_C\phi)\Delta \theta\), accepted by the high type (separation).
- \( \phi \in [\bar{\phi}^{PP}, 1) \): Rent at price \( \theta_L \), accepted by both types (pooling), if \( \beta \leq \beta^{PP} \); Rent at price \( \theta_H - \delta_C\phi\Delta \theta \), accepted by the high type only (separation) if \( \beta \geq \beta^{PP} \).

The characterization of the monopolist’s optimal price-posting mechanism is depicted in Figure 2. In terms of information-revelation, the optimal price-posting mechanism looks similar to the fully optimal mechanism depicted in Figure 1. The only difference is a left-shift of the threshold \( \bar{\phi} \) and an upward-shift of the threshold \( \beta \). The area of separation under price-posting is thus a strict subset of the area of separation under the fully optimal mechanism. This is intuitive, because a restriction to deterministic mechanisms affects
in (18) and (19), respectively. The dashed curves depict the corresponding thresholds, the supplier's payoff from separation but not his payoff from pooling, making separation trade induces separation for arbitrarily low priors in an intermediate interval of market-breakdown probabilities. Figure 2: Optimal Price-Posting Mechanism. The supplier's revenue-maximizing deterministic mechanism in dependence of his prior $\beta \in (0, \frac{\theta^L}{\theta^H})$ and the likelihood $\phi \in (0, 1)$ of a future trade opportunity. The thresholds $\beta^{PP}(\phi)$ and $\bar{\phi}^{PP}(\beta)$ are as defined in (18) and (19), respectively. The dashed curves depict the corresponding thresholds, $\beta$ and $\bar{\phi}$, under the (fully) optimal mechanism. Restricting to price-posting reduces the area where screening is optimal (unshaded) but leaves its qualitative features unchanged.

More importantly, even under the restriction to price-posting, screening by mode of trade induces separation for arbitrarily low priors in an intermediate interval of market-breakdown probabilities. Giving consumers the simple choice between a sale-price and a rental-price constitutes a powerful screening-device, capable of substituting inter-temporal discrimination by intra-period discrimination. It is reassuring to see that this insight extends to settings in which randomization is not feasible.

8 High expectations

Until now, our analysis has focused on the case where the supplier’s expectations about the consumer’s valuation were rather low, $\beta < \frac{\theta^L}{\theta^H}$. In this section, we complete our characterization of the supplier’s optimal mechanism, by considering the remaining case of high expectations, $\beta \geq \frac{\theta^L}{\theta^H}$. The main insight we obtain is that under these more
favorable market conditions, selling and renting are never employed in combination. Our theory thus predicts the concurrence of selling and renting for screening purposes to be a feature specific to poor markets, where expectations about consumer valuations are low.

For $\beta \geq \frac{\theta_L}{\theta_H}$, the description of the supplier’s optimal mechanism requires a final definition:

$$
\bar{\beta}(\phi) \equiv \begin{cases}
\left(1 + \frac{\phi \delta_C \Delta \theta^2}{\theta_H (\theta_H + \delta_C \theta_H - \phi \delta_S \theta_L)}\right)^{-1} & \text{if } \phi \leq \frac{\delta_C}{\delta_S} \\
\left(1 + \frac{\phi \delta_C \Delta \theta^2}{\theta_L (\theta_H + \delta_C \theta_H - \phi \delta_S \theta_L)}\right)^{-1} & \text{if } \phi \geq \frac{\delta_C}{\delta_S}.
\end{cases}
$$

(20)

Note that the threshold $\bar{\beta}$ is decreasing and such that $\bar{\beta}(\phi) \in \left(\theta_L, \theta_H, 1\right)$.

**Proposition 4** (High expectations). When the supplier’s prior is high, i.e. $\beta \geq \frac{\theta_L}{\theta_H}$, price-posting is optimal and the optimal mechanism implements either a rental or a sale, but never combines both modes of trade. It is characterized as follows:

- $\phi \in (0, \frac{\delta_C}{\delta_S})$: Sell at price $(1 + \delta_C)\theta_H - \delta_C \phi \Delta \theta$ accepted only by the high type (separation) if $\beta \leq \bar{\beta}(\phi)$; Sell at price $(1 + \delta_C)\theta_H$ accepted only by the high type with probability $q^H = \frac{\beta \theta_H - \theta_L}{\beta \Delta \theta}$ (semi-separation) if $\beta \geq \bar{\beta}(\phi)$.

- $\phi \in \left[\frac{\delta_C}{\delta_S}, 1\right)$: Rent at price $\theta_H - \delta_C \phi \Delta \theta$ accepted only by the high type (separation) if $\beta \leq \bar{\beta}(\phi)$; Rent at price $\theta_H$ accepted only by the high type with probability $q^H = \frac{\beta \theta_H - \theta_L}{\beta \Delta \theta}$ (semi-separation) if $\beta \geq \bar{\beta}(\phi)$.

Posterior values are $\bar{\beta}_L = 0$ and $\bar{\beta}_H = 1$ under separation, and $\bar{\beta}_L = \frac{\theta_L}{\theta_H}$ and $\bar{\beta}_H = 1$ under semi-separation.

A graphical representation of Proposition 4 can be seen in Figure 3. The supplier sells when the likelihood of future trade opportunities is low ($\phi \leq \frac{\delta_C}{\delta_S}$) but rents when their likelihood is high ($\phi \geq \frac{\delta_C}{\delta_S}$). Note that the supplier’s choice between renting and selling is the same as in the symmetric information benchmark, i.e. the mode of trade fails to be employed as a screening device.

With respect to the degree of revealed information, the optimal mechanism is reminiscent of Bolton and Dewatripont’s (2005) textbook analysis of the case where $\phi = 1$ and $\delta_C = \delta_S$. In particular, the optimal mechanism either separates or semi-separates types by inducing the high type to accept the supplier’s price either with certainty or with probability $q^H = \frac{\beta \theta_H - \theta_L}{\beta \Delta \theta}$. Semi-separation allows the supplier to maintain posterior beliefs sufficiently high to charge the price $\theta_H$ in the future, thereby reducing the high type’s information rent.
Figure 3: Optimal Mechanism (High Expectations). The supplier’s revenue-maximizing mechanism in dependence of his prior $\beta \in (\frac{\theta^L}{\theta^H}, 1)$ and the likelihood $\phi \in (0, 1)$ of a future trade opportunity. In the depicted range of parameters, the optimal mechanism sets $d^*_H = 1$ and $d^*_L = 0$ (making $r^*_L$ irrelevant), and induces truth-telling (rejection of the posted price) by the low type, $q^L = 0$. The threshold $\bar{\beta}(\phi)$ is as defined in (20).

To understand why, for high priors, screening by mode of trade fails to be employed, note that for $\beta \geq \frac{\theta^L}{\theta^H}$ the supplier would implement trade only with the high type in a static (one-period) context. Ordinary screening (with one mode of trade) achieves this objective by excluding the low type in period one. Ordinary screening is thus preferred over screening by mode of trade which induces trade with both types in period one. Intra-period screening via mode of trade substitutes for inter-temporal screening via time only when there exists a tension between static and dynamic objectives.

9 Conclusion

In this article, we have determined the optimal supply mechanism for a monopolistic, non-anonymous durable goods market. While we have made no restrictions on the set of feasible mechanisms, the analysis was simplified by our focus on a two-period, two-type framework. Before we summarize our main findings, we provide a brief discussion of the potential effects of relaxing these assumptions.

With a continuum of types, the mechanism design approach employed in this article is
no longer valid\footnote{Skreta (2006) and Doval and Skreta (2019a) provide techniques to deal with a continuum of types.}. Resorting to simple price-posting, we have determined the monopolist’s revenue-maximizing rental- and sale-prices for two periods of trade, when types are uniformly distributed on an interval. Screening by mode of trade, i.e. the simultaneous offer of a rental- and a sale-price in period one, turns out to be optimal for most parameter values. The market becomes segmented into three subsets of consumers, with high types buying, middle types renting, and low types refraining from trade. We take this finding as an indication that screening by mode of trade is not an artifact of our binary-type assumption but can be expected to occur more generally.

Similarly, extending our two-type model to allow for three periods of trade, we have been able to show that, under price-posting, screening by mode of trade continues to be optimal when the likelihood of market breakdown is taken from an intermediate range. Although the area of immediate separation is smaller than in the two-period case, we suspect that even for long horizons there always exists a range of breakdown-probabilities for which immediate separation via screening by mode of trade is optimal. The reason is that in the vicinity of the breakdown-probability for which the supplier is indifferent between selling and renting under symmetric information, screening by mode of trade comes at zero cost.

In summary, the main message of this article is that a durable good monopolist’s ability to screen customers is greatly enhanced when he can offer his products simultaneously for rental and for sale. Screening by mode of trade dominates ordinary, intertemporal screening because it substitutes the immediate exclusion of low valuation customers by their potential and postponed exclusion in the future. By making the mode of trade a variable of the monopolist’s mechanism design problem, we have shown that the common view, that a durable good monopolist’s ability to screen customers is rather limited, might have to be abandoned in situations where time preferences can be heterogeneous and future trade opportunities are not guaranteed.

Appendix: Proofs

Derivation of reduced program. As $\theta^H > \theta^L$ and $\bar{U}^H_L \geq \bar{U}^L_L = 0$, $(IC^H)$ and $(PC^L)$ together imply $(PC^H)$. Hence $(PC^H)$ is redundant. If $(PC^L)$ holds with strict inequality then raising both transfers $t_L$ and $t_H$ by a sufficiently small and identical amount increases the supplier’s objective while keeping all constraints satisfied. Hence the optimal mechanism must make $(PC^L)$ binding. Similarly, if both $(IC^L)$ and $(IC^H)$ hold with strict inequality then raising $t_H$ by a sufficiently small amount increases the objective while
maintaining both inequalities strict. Hence, at least one incentive constraint must hold with equality. In order to derive the reduced program, we assume that \((IC^H)\) is binding and then substitute the payments \((7)\) and \((8)\), that make \((PC^L)\) and \((IC^H)\) hold with equality, into the remaining constraint \((IC^L)\) and the objective \((3)\) to obtain \((DMC')\) and \((9)\), respectively. To see that this final step is without loss of generality, assume alternatively, that \((IC^L)\) is binding. Substitution of the payments that make \((PC^L)\) and \((IC^L)\) hold with equality,

\[
t_i = d_i[1 + (1 - r_i)\delta_C]\theta^i, \quad i \in \{L, H\}
\]

into the remaining constraint \((IC^H)\) and the objective \((3)\) leads to the following program:

\[
\max_{d_L, r_L, d_H, r_H, q^L, q^H} \sum_{i \in \{L,H\}} Q_i\{d_i + d_i(1 - r_i)\theta^i + [1 - d_i(1 - r_i)]\phi[\delta_S \tilde{V}_i + \delta_C \tilde{U}_i]\}
- Q_H\{d_L + d_L(1 - r_L)\delta_C\Delta\theta + [1 - d_L(1 - r_L)]\phi\delta_C \tilde{U}_L^H\}
- Q_H\Delta\theta\{d_H\{(1 + (1 - r_H)\delta_C) - d_L[(1 + (1 - r_L)\delta_C]\}
+ Q_H\phi\delta_C\{[1 - d_L(1 - r_L)]\tilde{U}_L^H - [1 - d_H(1 - r_H)]\tilde{U}_H^H\}
\text{subject to } d_H[(1 + (1 - r_H)\delta_C) - d_L[(1 + (1 - r_L)\delta_C] \geq \phi \delta_C\frac{\delta_S}{\Delta\theta}\{[1 - d_L(1 - r_L)]\tilde{U}_L^H - [1 - d_H(1 - r_H)]\tilde{U}_H^H\}
\text{with equality if } q^H < 1.
\]

Note that the objective \((22)\) is identical to the objective \((9)\) of the reduced program except for the last two lines and that the constraint \((DMC')\) is the same as \((DMC)\) except that it must hold with equality for \(q^H < 1\) rather than for \(q^L > 0\). Choosing \(q^H < 1\) makes \((DMC')\) binding and the last two lines of \((22)\) become zero, i.e. \((22)\) becomes identical to \((9)\). Alternatively, setting \(q^H = 1\) allows \((DMC')\) to be slack, but this makes the last two lines of \((22)\) become negative. In other words, for any mechanism that solves the above program, we can find a mechanism that solves the reduced program and leads to (at least weakly) larger payoff.

\(\square\)

Proof of Proposition 7. From \(\tilde{\beta}_L \leq \beta < \frac{\theta^L}{\theta^H}\) we have \(\tilde{U}_L^H = \Delta\theta\) and \(\tilde{V}_L = \theta^L\). We make use of Observation 2 by considering the cases \(\phi < \frac{\delta_C}{\delta_S}\) and \(\phi \geq \frac{\delta_C}{\delta_S}\) in separation.

Case \(\phi < \frac{\delta_C}{\delta_S}\): Substitution of \(d_L^* = 1\) and \(r_H^* = 0\) into the reduced program gives

\[
\max_{(d_L, r_L, q^L, q^H)} d_L[1 + (1 - r_L)\delta_C]\theta^L + (1 - d_L + d_L r_L)\phi\delta_S \theta^L
+ Q\{(1 - d_i)\theta^H + (1 - d_i + d_i r_i)[(\delta_C - \phi\delta_S)\theta^H + \phi(\delta_S - \delta_C)\Delta\theta]\}
\text{s.t. } 1 + (1 - \phi)\delta_C - d_L[(1 + (1 - r_L)(1 - \phi)\delta_C] \geq 0
\text{with equality if } q^L > 0.
\]
If the supplier sets \((d^*_L, r^*_L) = (1, 0)\) then he pools by selling to both types and his payoff is given by

\[ V^{SS} = (1 + \delta_C)\theta^L. \quad (24) \]

If the supplier chooses \((d_L, r_L) \neq (1, 0)\), then the constraint holds with strict inequality, and hence the low type must be induced to report the truth, i.e. \(q_L = 0\). Moreover, as the objective in (23) increases in \(Q\), while neither the objective nor the constraint depend any longer on \(\tilde{U}_H\) (because the mechanism sells to the high type), it must be optimal to induce truth-telling also from the high type, i.e. \(q_H = 1\). Substitution of \(q_L = 0\) and \(q_H = 1\) then leaves the following unconstrained program:

\[
\begin{align*}
\max_{(d_L, r_L)} & \quad d_L(\theta^L - \beta\theta^H) + (1 - r_L)d_L\{\delta_C(\theta^L - \beta\theta^H) - \phi[\delta_S(\theta^L - \beta\theta^H) + \beta(\delta_S - \delta_C)\Delta\theta]\} \\
& \quad + \beta[(1 + \delta_C)\theta^H - \phi\delta_C\Delta\theta] + (1 - \beta)\phi\delta_S\theta^L.
\end{align*}
\]

Consider the threshold \(\phi\) defined in (10) and note that \(\beta < \frac{\theta^L}{\delta^L} \) and \(\delta_C < \delta_S\) imply that \(\phi \in (0, \frac{\delta_C}{\delta_S})\). For \(\phi \leq \phi\) the objective in (25) is decreasing in \(r_L\) and increasing in \(d_L\). Hence for \(\phi \leq \phi\) the optimal mechanism sets \((d^*_L, r^*_L) = (1, 0)\), i.e. it pools by selling to both types, and the supplier’s maximized revenue is given by \(V^{SS}\). For \(\phi > \phi\) the objective in (25) is increasing in \(r_L\) and, after substitution of \(r^*_L = 1\), the remaining objective is clearly increasing in \(d_L\). Hence, for \(\phi \in (\phi, \frac{\delta_C}{\delta_S})\) it is optimal to set \((d^*_L, r^*_L) = (1, 1)\), i.e. the optimal mechanism rents to the low type but sells to the high type. The corresponding payoff is given by

\[ V^{RS} = \theta^L + \phi\delta_S\theta^L + \beta\{\delta_C\theta^H - \phi(\delta_C\Delta\theta + \delta_S\theta^L)\}. \quad (26) \]

This completes our characterization of the supplier’s optimal mechanism for the case where \(\phi < \frac{\delta_C}{\delta_S}\).

**Case \(\phi \geq \frac{\delta_C}{\delta_S}\):** In this case an increase in \(r_L\) increases the objective (9) while relaxing the constraint \((DMC)\). Substitution of \(d^*_H = 1\) and \(r^*_L = 1\) into the reduced program gives

\[
\begin{align*}
\max_{(d_L, r_H, q^L, q^H)} & \quad d_L\theta^L + (1 - Q)\phi\delta_S\theta^L + Q\{r_H\phi\delta_S\tilde{V}_H + [1 + (1 - r_H)\delta_C]\theta^H \}
& \quad + r_H\phi\delta_C\tilde{U}_H - d_L\theta^H - \phi\delta_C\Delta\theta \\
\text{s.t.} & \quad -d_L\Delta\theta - \phi\delta_C\Delta\theta + [1 + (1 - r_H)\delta_C]\Delta\theta + r_H\phi\delta_C\tilde{U}_H \geq 0 \\
& \quad \text{with equality if } q^L > 0.
\end{align*}
\]
Accounting for the piecewise definition of \( \tilde{V}_H \) and \( \tilde{U}_H^L \) in the following we consider in separation two possible types of mechanisms: Learning mechanisms which induce a posterior belief \( \beta_H > \frac{a_L}{a_H} \), and non-learning mechanisms which induce a posterior belief \( \beta_H \leq \frac{a_L}{a_H} \).

Non-learning mechanisms: If \((q^L,q^H)\) are such that \( \beta_H \leq \frac{a_L}{a_H} \), then \( \tilde{U}_H^H = \Delta \theta \) and \( \tilde{V}_H = \theta^L \), and the constraint in (27) can be written as \( d_L \leq 1 + (1 - r_H)(1 - \phi)\delta_C \). A non-learning mechanism must set \( r_H = 1 \), because for \( r_H < 1 \) the constraint could not be binding, and the low type would be induced to tell the truth, resulting in \( \beta_H = 1 \). Setting \( r_H^* = 1 \) the constraint is automatically satisfied and the fact that it must hold with equality when \( q^L > 0 \) is equivalent to the requirement that \( (1 - d_i)q^L = 0 \). The problem simplifies to the unconstrained program

\[
\max_{d_i,q^H} d_i\theta^L + \phi\delta_S\theta^L + \beta q^H (1 - d_i)\theta^H
\]

whose objective is increasing in \( d_L \). Setting \( d_L^* = 1 \), the corresponding mechanism \( (d_L^*,r_L^*),(d_H^*,r_H^*)\) = \{(1,1),(1,1)\} pools by renting to both types, and leads the payoff \( V^{RR} = (1 + \phi\delta_S)\theta^L \).

We have thus shown that for \( \phi \geq \frac{\delta_C}{\delta_S} \), \( V^{RR} \) is the highest payoff amongst all non-learning mechanisms.

Learning mechanisms: If \((q^L,q^H)\) are such that \( \beta_H > \frac{a_L}{a_H} \), then \( \tilde{U}_H^H = 0 \) and \( \tilde{V}_H = \frac{\partial U_H^H}{\partial H} \theta^H \), and the constraint in (27) becomes \( d_L \leq 1 + (1 - \phi)\delta_C - r_H\delta_C \). If \( d_L \) satisfies this constraint with strict inequality then \( q^L = 0 \) and the objective in (27) is increasing in \( d_L \), because \( \theta^L - Q_H\theta^H = \theta^L - \beta q^H\theta^H \geq \theta^L - \beta \theta^H > 0 \). Hence \( d_L^* = 1 \) is optimal if \( r_H \leq 1 - \phi \) and \( d_L^* = 1 + (1 - \phi)\delta_C - r_H\delta_C \) is optimal if \( r_H > 1 - \phi \). Consider these two alternatives in turn. If the supplier chooses \( r_H \leq 1 - \phi \), substitution of \( q^L = 0 \) and \( d_L = 1 \) into (27) gives the following unconstrained program:

\[
\max_{q^H>0,r_H \leq 1-\phi} \theta^L + (1 - \beta q^H)\phi\delta_S\theta^L + \beta q^H[r_H\phi\delta_S\theta^H + (1 - r_H)\delta_C\theta^H - \phi\delta_C\Delta \theta].
\]

From \( \phi \geq \frac{\delta_C}{\delta_S} \) it follows that it is optimal to set \( r_H^* = 1 - \phi \). For \( q^H \to 0 \) the remaining objective takes the value \( V^{RR} \) whereas for \( q^H = 1 \) the payoff becomes

\[
V^{RM} = (1 + \phi\delta_S)\theta^L + \beta \phi[(1 - \phi)\delta_S\theta^H - (\delta_S - \delta_C)\theta^L].
\]

The corresponding mechanism \( (d_L^*,r_L^*),(d_H^*,r_H^*)\) = \{(1,1),(1,1 - \phi)\} rents to the low type but mixes between renting and selling to the high type. Alternatively, if the supplier
chooses $r_H > 1 - \phi$ then substitution of $d^{**}_L = 1 + (1 - \phi)\delta_C - r_H\delta_C$ into (27) simplifies the program to

$$
\max_{q^L,q^H,r_H>1-\phi} \left[ 1 + (1 - \phi)\delta_C + \phi\delta_S - r_H\delta_C \right] \theta^L + \beta q^H r_H \phi\delta_S \theta^H \\
- \left[ \beta q^H + (1 - \beta)q^L \right] (\delta_S - \delta_C) \phi \theta^L.
$$

(32)

Note that the objective in (32) is decreasing in $q^L$ and supermodular in $(q^H, r_H)$. Hence it is optimal to set $q^L = 0$ and to choose either $q^H \to 0$ and $r_H = 1 - \phi$ or $q^H = 1$ and $r_H = 1$. The first possibility again results in the payoff $V^{RR}$. The second possibility gives the payoff

$$
V^{rR} = (1 + \phi \delta_S) \theta^L + \phi [\beta \delta_S \Delta \theta - (1 - \beta)\delta_C \theta^L]
$$

(33)

and corresponds to the mechanism $\{(d_L, r_L), (d_H, r_H)\} = \{(1 - \phi \delta_C, 1), (1, 1)\}$ which rents to both types but separates by delivering the product to the low type with a lower probability than to the high type.

A straightforward comparison of the payoff from the optimal non-learning mechanism, $V^{RR}$, with the payoffs $V^{RM}$ and $V^{rR}$ of the two candidates for the optimal learning mechanism completes our characterization of the supplier’s optimal mechanism for the case where $\phi \geq \frac{\delta_C}{\delta_S}$.

Proof of Proposition 2. To derive the optimal selling mechanism, substitute $r_L = r_H = 0$ into the reduced program to get:

$$
\max_{(d_L,q^H)} d_L [1 + \delta_C] \theta^L + (1 - d_L)\phi \delta_S \theta^L + Q_H (1 - d_L) \left[ [1 + \delta_C] \theta^H - \phi (\delta_C \Delta \theta + \delta_S \theta^L) \right]
$$

(34)

s.t. $1 - d_L \geq 0$ with equality if $q^L > 0$.

If $q^L > 0$, then $d_L = 1$ and the supplier obtains pooling payoff $(1 + \delta_C) \theta^L$. Alternatively, if $q^L = 0$, the program simplifies to

$$
\max_{(d_L,q^H)} d_L [1 + \delta_C] \theta^L + (1 - d_L)\phi \delta_S \theta^L + Q_H (1 - d_L) \left[ [1 + \delta_C] \theta^H - \phi (\delta_C \Delta \theta + \delta_S \theta^L) \right]
$$

(35)

Since the program is linear in $d_L$, it must be that $d_L \in \{0, 1\}$. Setting $d_L = 1$ leads to the pooling payoff. For $d_L = 0$ the remaining objective $\phi \delta_S \theta^L + Q_H (1 - d_L) \left[ [1 + \delta_C] \theta^H + \phi (\delta_C \Delta \theta + \delta_S \theta^L) \right]$ is increasing in $q^H$ and setting $q^H = 1$ gives the separating payoff

$$
V^S_{Screen} = \phi \delta_S \theta^L + \beta \theta^H + \delta_C \theta^H - \phi (\delta_S \theta^L + \delta_C \Delta \theta).
$$

(36)
This payoff is larger than the one from pooling if and only if \( \beta \geq \beta^S(\phi) \) with \( \beta^S(\phi) \) as defined in \([16]\).

To derive the optimal renting mechanism, substitute \( r_L = r_H = 1 \) into the reduced program to get:

\[
\max_{(d_L,q^L,q^H)} d_L\theta^L + \phi\delta_S\theta^L + Q_H\{(1-d_L)\theta^H + \phi(\delta_C(\tilde{U}^H_H - \Delta\theta) + \delta_S(\tilde{V}_H - \theta^L))\}
\]

s.t. \( (1-d_L)\Delta\theta + \phi\delta_C(\tilde{U}^H_H - \Delta\theta) \geq 0 \) with equality if \( q^L > 0 \).

For any \((q^L,q^H)\) such that \( \tilde{\beta}_H \leq \frac{\theta^H}{\Delta\theta} \) it must hold that \( q^L > 0 \), \( \tilde{U}^H_H = \Delta\theta \) and \( \tilde{V}_H = \theta^L \), i.e. the supplier gets the pooling payoff \((1 + \phi\delta_S)\theta^L\). Alternatively, if \((q^L,q^H)\) is such that \( \tilde{\beta}_H > \frac{\theta^H}{\Delta\theta} \), then \( \tilde{U}^H_H = 0 \) and \( \tilde{V}_H = \tilde{\beta}^H \theta^H \). The constraint must be binding, i.e. \( d_L = 1 - \phi\delta_C \), because for \( q^L = 0 \) the objective is increasing in \( d_L \). It then follows that \( q^L = 0 \) and \( q^H = 1 \) are optimal and the supplier’s maximized payoff from separation is given by

\[
V^R_{Screen} = \theta^L + (1-\beta)\phi(\delta_S - \delta_C)\theta^L + \beta\phi\delta_S\theta^H. \tag{38}
\]

This payoff is larger than the one from pooling if and only if \( \beta \geq \beta^H \) with \( \beta^H \) as defined in \([12]\).

**Proof of Proposition 3.** As for \( \phi < \frac{\delta_C}{\delta_S} \), the (fully) optimal mechanism is deterministic, we can restrict attention to the case where \( \phi \geq \frac{\delta_C}{\delta_S} \). Note first that the mechanism \( \{(d_L,r_L), (d_H,r_H)\} = \{(0,0),(1,0)\} \) which sells to \( H \) while excluding \( L \) is dominated by the mechanism \( \{(0,0),(1,1)\} \) which rents to \( H \) while excluding \( L \). Similarly, the mechanism \( \{(1,0),(1,0)\} \) which sells to both types is dominated by the mechanism \( \{(1,1),(1,1)\} \) which rents to both types. The simple reason is that, as in the symmetric information benchmark, for \( \phi \geq \frac{\delta_C}{\delta_S} \) the supplier prefers renting over selling. Further note that for the mechanism \( \{(1,0),(1,1)\} \) which rents to \( H \) while selling to \( L \), the constraint \((DMC)\) becomes \( \frac{\phi\tilde{U}^H_H}{\Delta\theta} \geq 1 \) and cannot be satisfied. In other words, the mechanism’s allocation is not implementable. Given that \( d^*_H = 1 \), the remaining candidates for the supplier’s optimal deterministic mechanism are: \( \{(1,1),(1,0)\} \) (selling to \( H \) while renting to \( L \) with payoff \( V^{RS} \)) given by \((26)\); \( \{(1,1),(1,1)\} \) (renting to both types) with payoff \( V^{RR} \) given by \((29)\); and finally \( \{(0,0),(1,1)\} \) (renting to \( H \) while excluding \( L \) which results in the payoff

\[
V^{ER} = \phi\delta_S\theta^L + \beta(\theta^H + \phi(\delta_S - \delta_C)\Delta\theta). \tag{39}
\]

Note that

\[
V^{RR} < V^{RS} \iff \phi < \frac{\delta_C\theta^H}{\delta_C\Delta\theta + \delta_S\theta^L} \quad \text{and} \quad V^{ER} < V^{RS} \iff \phi < \frac{\theta^L - \beta\theta^H(1-\delta_C)}{\beta\delta_S\theta^H}. \tag{40}
\]

27
Hence separation by mode of trade is optimal if and only if $\phi \in [\bar{\phi}, \tilde{\phi}^{PP}(\beta)]$ with $\tilde{\phi}^{PP}(\beta)$ given by (19). The threshold $\tilde{\phi}^{PP}(\beta)$ is decreasing in $\beta$ and it is straightforward to show that $\tilde{\phi}^{PP}(\beta) < \bar{\phi}$. Finally, for $\phi > \tilde{\phi}^{PP}(\beta)$ it remains to compare renting to both types with renting to only the high type:

$$V^{ER} < V^{RR} \iff \beta < \beta^{RPP}(\phi) \equiv \frac{\theta^L}{\theta^H + \phi(\delta_S - \delta_C)\Delta \theta}.$$

The threshold $\beta^{RPP}(\phi)$ is decreasing and converges to

$$\beta^{RPP}(1) = \frac{\theta^L}{\theta^H + (\delta_S - \delta_C)\Delta \theta} > \frac{\delta_C \theta^L}{\delta_C \theta^L + \delta_S \Delta \theta} = \beta.$$

The prices specified in Proposition 3 can be determined from (7) and (8) via substitution of the corresponding allocations $\{(d_L, r_L), (d_H, r_H)\}$.

**Proof of Proposition 4.** The proof consists of two steps. In the first step we determine the optimal separating mechanism inducing posteriors $\tilde{\beta_L}, \tilde{\beta_H}) = (0, 1)$. This allows us to show that for $\beta > \frac{\theta^L}{\theta^H}$ screening by mode of trade is dominated by ordinary, intertemporal screening. The second step compares the payoffs of the optimal separating mechanism with the payoffs from semi-separation $\tilde{\beta_L}, \tilde{\beta_H}) = (\frac{q^L}{q^{HR}}, 1)$ and pooling $\tilde{\beta_L}, \tilde{\beta_H}) = (\beta, \beta)$. As this comparison leads to results that are well known from the literature, it is omitted. Details are available on request.

Focusing on mechanisms that induce (full) separation $\tilde{\beta_L}, \tilde{\beta_H}) = (0, 1)$ allows us to set $q^L = 0$, $q^H = 1$, and $\bar{V}_L = \bar{V}_H = \theta^L$, $\bar{V}_H = 0$, $\bar{V}_L = \theta^H$. Substitution of these values together with $d^{**}_H = 1$ from Observation 2 into the reduced program leaves us with the following problem:

$$\max_{(d_L, r_L, r_H)} d_L \{\theta^L - \beta \theta^H - (1 - r_L)[\phi(1 - \beta)(\delta_S - \delta_C)\theta^L + (1 - \phi)\delta_C(\beta \theta^H - \theta^L)]\}$$

$$+ \phi \delta_S \theta^L + \beta\{[1 + \delta_C(1 - r_H)]\theta^H - \phi(\delta_C \Delta \theta + \delta_S \theta^L - r_H \delta_S \theta^H)\}$$

s.t. $1 + (1 - r_H)\delta_C - d_L[1 + (1 - r_L)\delta_C(1 - \phi)] \geq \phi \delta_C$.

As $\beta \theta^H > \theta^L$, a decrease in $d_L$ raises the objective while relaxing the constraint. Hence the optimal separating mechanism must set $d^{**}_L = 0$, i.e. it must exclude the low type. Screening by mode of trade $(d_L, r_L) = (1, 1)$ is dominated by ordinary screening because the supplier’s prior is such that serving only the high type would be optimal in a static setting.

$\square$
References


