Shocking Interest Rate Floors

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DISCUSSION PAPERS

Shocking Interest Rate Floors[*]

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Abstract: We identify the dynamic causal effects of interest rate floor shocks, exploiting regular auctions of Swiss central bank debt securities (SNB Bills). A theoretical model shows that variation in the volume of, and yield on, central bank debt changes the interest rate floor. In addition, the model establishes the equivalence between central bank debt and interest-bearing reserves when reserves are ample. Based on these insights, the empirical analysis identifies an interest rate floor shock in a dynamic event study of SNB Bill auctions. A restrictive interest rate floor shock causes an increase in the money market rate, a persistent appreciation of the Swiss franc, a decline in long-term interest rates, and a decline in stock prices. We then perform policy experiments under various identifying assumptions in which the central bank raises the interest rate floor from 0% to 0.25%. Such a policy change causes a 3-6% appreciation of the Swiss franc and a 5-20% decline in stock prices.

JEL classification: E41, E43, E44, E52, E58, C32

Keywords: Exit strategies, interest rate floors, central bank debt securities, interest on reserves, monetary policy shocks, identification through heteroscedasticity

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1 Introduction

Central banks massively expanded the monetary base in the wake of the financial and sovereign debt crises. How to raise money market rates in such an environment has since become a central question. The Federal Reserve Bank, for example, decided to pay interest on reserves to control the Federal Funds Rate in a floor system. Empirical evidence on the impact of interest rate floors is scarce, however, because changes in interest on reserves are rare and endogenous to the state of the economy.[1] This paper proposes to identify random variation in the interest rate floor using auctions of interest-bearing central bank debt securities.[2]

We use a money market model in the spirit of Poole (1968) and Boutros and Witmer (forthcoming) to show that central bank debt securities may generate variation in the interest rate floor. We find that the volume of central bank debt affects the money market rate for two reasons. First, issuing debt securities absorbs reserves and increases the likelihood of a costly liquidity shortage (reserve-absorbing channel). Second, a greater supply of debt securities reduces the cost of holding reserves because a greater share of non-interest bearing reserves can be invested in interest-bearing central bank debt (cost channel). With ample reserves, the probability of a liquidity shortage is close to zero and the reserve-absorbing channel is negligible. In this case, the volume of, and yield on, debt securities jointly determine the interest rate floor. Another way to implement an interest rate floor is paying interest on reserves. If appropriately scaled, interest on reserve policies and debt security programs yield the same interest rate floor. Therefore, analyzing random variation in the main parameters of a central bank debt security program provides evidence on the effects of random variation in interest on reserves.

The empirical analysis exploits a novel daily data set of SNB Bill auctions to identify the dynamic causal effects of an interest rate floor shock. From 2008 to 2011, the Swiss National Bank (SNB) auctioned debt securities on a pre-determined weekly schedule to soak up reserves created in emergency liquidity provisions and foreign exchange interventions. We propose to estimate dynamic causal effects combining identification through heteroscedasticity (Rigobon 2003) with local projections or vector autoregressions (Jordà 2005; Stock and Watson 2018; Lütkepohl 2012; Lütkepohl et al. 2018). A restrictive interest rate floor shock causes an increase in the money market rate, a persistent appreciation of the Swiss franc, a decline in stock prices, and a decline in long-term interest rates. One interpretation of the decline in long-term interest rates is that markets expected money market rates to remain persistently low because of the restrictive impact of the appreciation. In addition, we perform policy experiments in which the central bank raises the interest rate floor from

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[1]To this day, the Federal Reserve raised the interest on reserves only nine times.

[2]Central bank debt securities are widely used. The IMF survey on Information System for Instruments of Monetary Policy (ISIMP) covers 125 countries. Of these countries, about 35% issue central bank debt securities (Gray and Pongsaparn 2015). About 75% of the countries have legal provisions to emit central bank debt securities.

0% to 0.25%. The point estimates of two alternative identification schemes suggest an appreciation between 3% and 6% and a decline in stock prices between 5% and 20%.[3]

The paper contributes to research analyzing exit strategies with money market models.[4] Most studies focus on interest on reserve policies. Berentsen et al. (2018) is one exception investigating the welfare implications of central bank debt.[5] In line with our findings they show that debt securities determine an interest rate floor if reserves are ample. Our paper uses a different model to discuss the equivalence between debt securities and interest-bearing reserves. Debt securities raise the money market rate because the volume of, and yield on, debt securities reduce the cost of holding reserves. Therefore, we identify a cost channel, which is also present when paying interest on reserves (see Ireland 2014).[6]

In addition, the paper is related to a literature using event studies to estimate the causal effects of monetary policy.[7] We propose an approach tailored to the characteristics of the SNB Bills program. We lack information on the exact timing of the SNB Bill auctions and the exact timing of the release of the auction results. Therefore, we cannot use changes in interest rate futures in a narrow time window to identify the surprise component of an event as in Kuttner (2001), Gürkaynak et al. (2005), Gertler and Karadi (2015), and Nakamura and Steinsson (2018a). Instead, we follow Rigobon (2003) and exploit changes in the variance of economic data during deterministic regime changes (see Rigobon and Sack 2004; Gürkaynak et al. 2018; Nakamura and Steinsson 2018a; Cieslak and Schrimpf 2019, for monetary policy applications).[8] We measure the effect of an interest rate floor shock by comparing the variance of financial market variables on an auction day to the variance on a day without an auction. Because we do not want to restrict the response of the money market rate, our strategy imposes that a restrictive interest rate floor shock causes a decline in stock market prices (similar to

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[3]These findings are in line with Bäurle and Kaufmann (2018) who estimate impulse responses to a contractionary risk premium shock in Switzerland before the financial crisis as well as from January 2010 to July 2011. They find that risk premium shocks have a more persistent impact on the exchange rate and prices in the second period during which the effective lower bound was binding. Our paper shows that the SNB Bills program influenced this effective lower bound.

[4]Methodologically, our model closely follows Boutros and Witmer (forthcoming). They add an option for commercial banks to exchange central bank reserves for cash to analyze the implementation of policy rates below the yield on cash. Bech and Keister (2017) propose a related model to study liquidity cover ratios.

[5]Other papers on central bank debt securities focus on conceptual and operational issues (see e.g. Dziobek and Dalton 2005; Nyawata 2012; Gray and Pongsaparn 2015). These papers discuss eligibility of central bank debt as collateral in repurchase operations, treatment for capital requirements, liquidity of marketable central bank debt, coordination of debt management and monetary operations, possible threats to balance sheets of central banks (losses and recapitalization), and coordination of emissions with the treasury.

[6]Therefore, many insights from the literature on interest on reserves resonate well with our findings (e.g. Ennis 2018; Bech and Klee 2011; Keister et al. 2008; Goodfriend 2002). There are some differences, however. Keister et al. (2008) establish that paying interest on reserves "divorces" money from monetary policy in that reserve-absorbing or -providing operations have no impact on money market rates if reserves are ample. In contrast, issuing reserve-absorbing debt securities mechanically affects the costs of holding reserves. Thus, with debt securities, the divorce between monetary policy and money is incomplete.

[7]See Gürkaynak and Wright (2013) for an overview on event studies. More generally, our paper is related to a literature identifying the causal impact of monetary policy (see Nakamura and Steinsson 2018b, for a survey).

[8]As Nakamura and Steinsson (2018a) show this comes at the cost of a less efficient estimator because financial market variables are influenced by substantial background noise unrelated to the monetary policy decision.

Jarociński and Karadi 2018).[9]

The remainder of this paper is structured as follows. Section 2 discusses the purpose and main characteristics of the SNB's debt security program. Section 3 incorporates central bank debt in a money market model to study its impact on interest rates. Section 4 presents the estimation and identification strategy before we discuss the empirical findings in Section 5. Section 6 concludes.

2   The SNB Bills program

From October 2008 to July 2011, the SNB used debt securities to absorb reserves from the banking system. The SNB issued 232 Bills on 167 auction days.[10] Most SNB Bills had a short time to maturity from 7 to 85 days; some of them had a term to maturity up to one year.[11] They were denominated in Swiss francs.

The auctions usually took place on a specific day of the week from 2pm to 2:30pm and the results were published in the afternoon. Several days before an auction, the SNB announced the auction day, a price range for valid bids, the term of the Bills, and the payment date.[12] Bills were then allotted in American auctions with a variable rate tender. Each participant submitted the amount of Bills he or she was willing to accept jointly with a price. The SNB then chose the marginal price above which participants' offers were satisfied. Thereafter, participants paid the price they offered in their bids. The transactions were usually settled two working days after the auction.

SNB Bills were bought by a broad range of counterparties. Buyers either had reserve accounts at the SNB, access to the CH Repo Market, or access to the OTC Spot Market of the SIX Exchange (see SNB 2016).[13] The SNB has a relatively broad access policy to its facilities, including domestic securities brokers/dealers, insurance companies, asset managers of collective investment schemes, as well as foreign banks.[14] Participants in the CH Repo Market largely coincide with banks having access to the SNB's facilities and reserve accounts.[15] SNB Bills therefore reduced the volume of reserves and

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[9]Most studies measuring the impact of monetary policy shocks on the economy implicitly identify the sign of the response assuming that a restrictive monetary policy shock is associated with an increase in a money market rate. Because we aim to test whether an SNB Bill auction raises interest rates, we avoid this assumption.

[10]Our source is publicly accessible information on SNB Bill auctions (see Appendix A).

[11]Sometimes, the SNB auctioned two Bills with different term to maturity on the same day. The term was determined in such a way that the Bills matured on a day with a new auction.

[12]This information stems from a small number of announcement forms stored on a web archive. We cannot establish, however, whether the content and release date of these announcement forms changed over time.

[13]SNB Bills were transferable and traded on the SIX Exchange. Parties without access to SNB's facilities could therefore purchase SNB Bills too, but only through a counterparty eligible to participate in an SNB Bill auction. In addition, given the large denomination of the Bills of CHF 1 million, SNB Bill holders were likely large firms or institutional investors.

[14]Kraenzlin and Nellen (2015) report that in December 2010, 62 of 170 participants in the CH Repo Market were located abroad, most of them in Austria (24), Germany (16), and in the UK (6). See https://www.six-group.com/repo/dam/downloads/publications/repo-participant-list.pdf for a list of participants.

[15]According to Kraenzlin and Nellen (2015): "As a rule of thumb, banks that have access to the Swiss repo system are also eligible to participate in the SNB's open market operations and have access to the SNB's standing facilities."

the monetary base.[16]

Figure 1 — Liabilities of the SNB

(a) Weekly reserves and SNB Bills   (b) All monthly liabilities
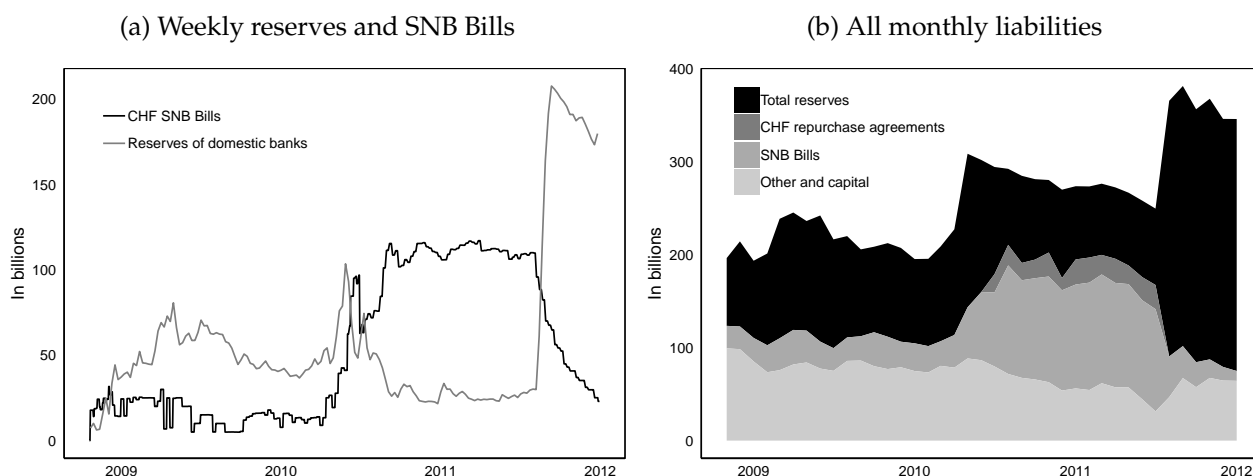
Notes: Panel (a) shows the daily volume of outstanding CHF SNB Bills and average weekly reserves of domestic banks (not accounting for repurchases of SNB Bills). Panel (b) shows monthly liabilities of the SNB (accounting for repurchases of SNB Bills). Sight liabilities comprise reserves of domestic, as well as foreign banks, and the Swiss government. SNB Bills comprise denominations in USD and CHF.

Figure 1 panel (a) shows that SNB Bills indeed absorbed reserves from the banking system. In early 2010 reserves rose quickly because of foreign exchange interventions.[17] Thereafter, the SNB stepped up its SNB Bills program, which led to a strong decline in domestic bank reserves. Conversely, when the SNB decided to expand reserves in August 2011, the amount of outstanding Bills fell strongly. SNB Bills were the main tool to drain reserves. Panel (b) shows monthly values of total reserves, including accounts of domestic and foreign counterparties, as well as the Swiss government. SNB Bills accounted for more than 40% of total SNB liabilities in 2011, making up more than 17% of Swiss nominal GDP. The SNB also drained reserves with reverse repurchase agreements (reverse repo). These were less important than SNB Bills, however (10% of total liabilities).

Why did the SNB issue debt securities?[18] Our interpretation is that the SNB aimed to keep the 3M Libor from falling to zero amid its liquidity-providing operations during the financial and sovereign

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[16]Conversely, the program enabled the SNB to quickly increase reserves. It could either repurchase outstanding SNB Bills in open market operations or stop renewing SNB Bills that fell due. According to Moser (2012) no repurchases occurred before the SNB stopped issuing debt securities in August 2011.

[17]Foreign currency investments rose from CHF 95 billion in December 2009 to CHF 225 billion in June 2010. See https://data.snb.ch/en/topics/snb#!/cube/snbbipo.

[18]The literature stresses two main purposes to issue central bank debt in advanced economies. First, central banks finance particular operations with debt securities (see e.g. Rule 2011; SNB 2009b). For example, the Bank of England emits debt securities in foreign currency to finance its foreign exchange reserves. The central banks of Malaysia and Switzerland issued foreign currency debt to finance subsidiaries supporting troubled commercial banks during the Asian crisis and the 2008 financial crisis, respectively (see Rule 2011; SNB 2009b). Second, industrialized countries have started to consider and employ central bank debt securities to drain reserves (Nyawata 2012). Speeches by members of the governing board in 2009 suggest that the SNB may use debt securities to tighten monetary policy in the future. For example, according to Jordan (2009): "SNB Bills will play an important role in the transition from the current zero interest rate environment towards a traditional interest rate policy." (our translation). Developing economies frequently use central bank debt securities to sterilize foreign exchange interventions.

debt crises. As long as the 3M Libor remained close to target, policy makers could communicate that the emergency liquidity provisions and foreign exchange interventions had no impact on the monetary policy stance. In fact, the SNB announced on 15 October 2008 that SNB Bills "[. . .] will serve to absorb liquidity, thereby neutralizing the monetary policy impact of measures to inject liquidity." (SNB 2008b).[19]

Figure 2 panel (a) presents the evolution of the 3M Libor, the 3M SAR, and the marginal SNB Bill rate. The 3M SAR is a secured money market rate based on actual repo transactions and binding quotes. We construct the marginal rate by averaging the marginal yield of SNB Bills weighted by the volume outstanding.[20] In 2008 the SNB engaged in emergency liquidity provisions after the bankruptcy of Lehman Brothers when "tensions in international money markets [led] to upward pressure on short-term Swiss franc money market rates" (SNB 2008a). While the SNB was willing to reduce risk premia on money market rates it wanted to avoid a decline of the 3M Libor below target. Indeed, the marginal SNB Bill yield remained close to the 3M SAR. This is suggestive evidence that SNB Bills helped to keep riskless money market rates and the 3M Libor from falling below target.

Figure 2 — Marginal rate and maturity of SNB Bills

(a) Marginal rate and money market rates

(b) Outstanding volume and term to maturity
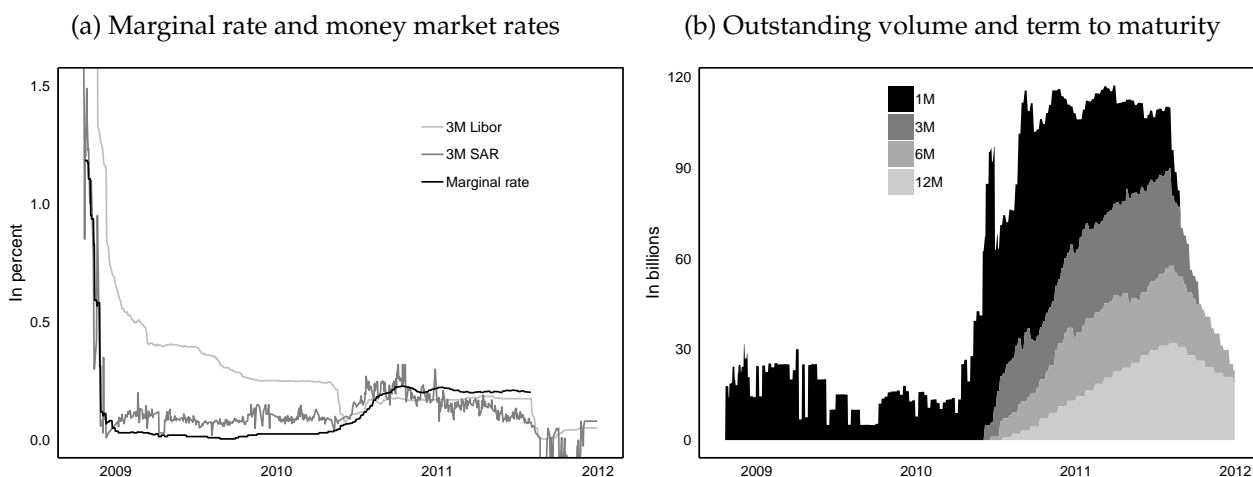
Notes: Panel (a) shows the marginal rate, calculated by averaging the marginal yield weighted by the volume outstanding. The 3M Libor is the SNB's operational target. The 3M SAR (Swiss Average Rate) is a three-month interest rate based on actual repo transactions and binding quotes. Panel (b) shows the daily volume of outstanding CHF SNB Bills according to term to maturity of approximately one month and less, 3 months, 6 months, and 12 months. Both figures do not account for repurchases of SNB Bills. Therefore, we do not show the marginal SNB Bill yield after August 2011 in panel (a).

We observe a similar co-movement of the 3M Libor, the 3M SAR, and the marginal SNB Bill rate in 2010, after the SNB conducted substantial exchange rate interventions. Arguably, without any further action, the 3M Libor would have fallen to zero because of the increase in reserves. To keep

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[19]Policy makers viewed the program as irrelevant for the stance of monetary policy as long as commercial banks held substantial reserves. Moser (2011): "From a monetary policy perspective, liquidity draining operations are negligible, as long as commercial banks hold excess liquidity and interest rates are close to zero." (our translation).

[20]The marginal rate does not account for repurchases of SNB Bills. Therefore, we do not show the marginal rate after August 2011.

the 3M Libor close to its point target at 0.25% the SNB increased the supply of SNB Bills. In addition, the SNB started to issue Bills with a higher marginal yield and a longer term to maturity (see panels a and b). The rise in all three interest rates coincides with the change in the SNB Bills program.[21] Indeed, policy makers suggested that they managed to bring the 3M Libor up to 0.2%. To move it closer to 0.25%, however, they would have had to absorb even more reserves (see Moser 2011).

3 Central bank debt in a money market model

We develop a money market model closely related to Poole (1968) and Boutros and Witmer (forthcoming) to show that central bank debt securities determine an interest rate floor for the money market rate.[22]

3.1 Agents

We assume a continuum of commercial banks on the interval $i \in [0, 1]$. Each commercial bank holds an exogenous amount of pre-determined reserves $R_i > 0$, manages an exogenous amount of deposits $D \geq R_i$, and experiences a zero-mean liquidity shock $Z \leq D$, where $Z > 0$ is a liquidity outflow.[23] Assets A_i are residually determined by R_i and D, assuming commercial banks' capital to be zero (see Figure 3). A commercial bank borrows additional reserves B_i from the money market at the net money market rate i_m.[24] Reserves bear no interest, deposits cost i_d, and assets earn i_a.

Commercial bank i invests a fraction π_i of its pre-auction reserves ($F_i \equiv B_i + R_i$) in central bank debt securities.[25] After the debt security settlement, commercial bank i holds $(1 - \pi_i)F_i$ in post-auction reserves. Thus, the commercial bank has a buffer of $E_i \equiv (1 - \pi_i)F_i - K$ to absorb potential liquidity outflows, where K is the exogenous minimum reserve requirement. If a commercial bank faces a liquidity shortage, it has to borrow at the central bank's discount window. The amount of reserves that a commercial bank borrows at the discount window is $X_i \equiv \max(0, Z_i - E_i)$.

The central bank provides reserves ($R = \int_0^1 R_i \mathrm{d}i$) and runs a discount window facility at which commercial banks can borrow an unlimited amount of additional reserves at the exogenous discount

Figure 3 — Stylized commercial bank balance sheet

Commercial Bank i

Notes: F_i: pre-auction reserves; $(1-\pi_i)F_i$: post-auction reserves; $\pi_i F_i$: commercial bank i's investment in central bank debt securities; R_i: pre-determined reserves; A_i: other assets; D: deposits. The commercial bank borrows additional reserves on the money market in this example ($B_i > 0$). In equilibrium, aggregated money market borrowing must be zero.

window rate i_x. Beyond that, the central bank has two levers to affect the money market rate. First, the central bank decides over the maximum fraction of pre-auction reserves that each commercial bank can invest in debt securities ($\pi^{cb} \in [0,1]$). Second, the central bank determines the yield on debt securities ($0 \leq i_b \leq i_x$).[26]

3.2 Timing

We assume a standard timing convention (see e.g. Boutros and Witmer forthcoming). First, the central bank determines the volume of, and yield on, debt securities (π^{cb} and i_b). Second, commercial banks decide on the optimal amount of money market borrowing (B_i) and the optimal amount of debt securities ($\pi_i \in [0, \pi^{cb}]$). These two decisions pin down the commercial banks' money demand functions. Third, commercial banks engage in the money market. At this stage, we impose the market clearing condition ($\int_i B_i \mathrm{d}i = 0$). Fourth, central bank debt securities are settled and each commercial bank holds post-auction reserves ($(1 - \pi_i)F_i$) and debt securities ($\pi_i F_i$). Fifth, the liquidity shock materializes.[27] Sixth, a commercial bank borrows at the central bank's discount window if it faces a liquidity shortage (i.e. if $X_i > 0$).

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[26]The Swiss National Bank issued debt securities in auctions. Therefore, the marginal yield depended on the bids and vice versa. For tractability, we analyze the role of each parameter independently.

[27]The fact that the liquidity shock materializes after the money market closes captures imperfections on the money market that prevent commercial banks from exactly targeting their required reserves before the closing of the money market. The work of Bech and Monnet (2016), Afonso and Lagos (2015), and Ennis and Weinberg (2013) justifies the simplification.

3.3 Commercial bank problem

The commercial bank maximizes expected profits $\mathbb{E}(\Pi_i)$ over the optimal amount of money market borrowing B_i and the fraction of pre-auction reserves invested in debt securities π_i.[28] The expected profit of commercial bank i is:

$$\mathbb{E}(\Pi_i) = i_a A_i + i_b \pi_i (R_i + B_i) - i_m B_i - i_d \mathbb{E}(D - Z) - i_x \int_{\hat{z}_i}^{\infty} X_i f(Z) dZ \ , \tag{1}$$

where $\hat{z}_i$ is the realization of the liquidity shock that is associated to zero borrowing at the discount window. Reserve requirements K do not enter the commercial bank's problem as a constraint but only through the probability of a liquidity shortage. K is not a constraint to the expected profit maximization problem because commercial banks can borrow at the discount window before having to comply with reserve requirements.

Commercial banks have an incentive to hold reserves for two reasons. First, a higher stock of reserves lowers the probability of a liquidity shortage. A lower probability of a liquidity shortage is beneficial because, in equilibrium, the discount rate is higher than the money market rate. Second, borrowing reserves on the money market increases the interest rate payments that commercial banks receive from investing in debt securities.

Commercial banks trade-off the marginal benefits of borrowing on the money market (a lower probability of a liquidity shortage and a higher interest rate income from investing in debt securities) against the marginal costs of borrowing (the money market rate). Optimal commercial bank behavior requires that these are equal.[29]

3.4 Equilibrium

In equilibrium, a money market rate i_m prevails at which it is beneficial for commercial banks with a relatively scarce (ample) amount of pre-determined reserves R_i to borrow (lend) on the money market. After the closing of the money market, each commercial bank holds the same amount of pre-auction reserves ($F_i = F \ \forall i$) and thus faces the same probability of a liquidity shortage ($P(X_i > 0) = P(X > 0) \ \forall i$). Therefore, each bank invests the same share of pre-auction reserves in debt securities ($\pi_i = \pi \ \forall i$).

Proposition 1 *If each commercial bank maximizes expected profits with respect to B_i and π_i, the money market rate equals $i_m = i_b \pi + (1 - \pi) i_x P(X > 0)$, where $\pi = \min(\pi^{cb}, \bar{\pi})$.*

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[28]The commercial bank takes interest rates, reserve balances R_i, deposits D, asset holdings A_i, the distribution of Z, the minimum reserve requirement K, and the supply of debt securities π^{cb} as given.

[29]Optimal commercial bank behavior also implies that lending on the money market is, on the margin, equally attractive as holding a portfolio of size F_i, partly invested in central bank debt securities and partly invested in non-interest-bearing reserves.

Proof. See Appendix B, Section B.1.

Proposition 1 establishes the money market equilibrium that prevails if commercial banks can invest in central bank debt securities. If the central bank does not supply debt securities, the money market rate is strictly downward-sloping in reserves as in Poole (1968). If reserves go to infinity, the probability of a liquidity shortage goes to zero. The money market rate then approaches the interest rate floor. In Poole (1968) the interest rate floor is zero because, in the aggregate, non-interest bearing reserves cannot be invested in interest-bearing assets.

In our model, the interest rate floor is strictly positive. The reason is that commercial banks can exchange non-interest bearing reserves for interest-bearing debt securities. In addition, our model differs from Poole (1968) in that debt securities drain reserves from the interbank market.

Commercial banks are willing to invest in debt securities up to some threshold $\bar{\pi}$. Accepting an additional unit of debt securities beyond the threshold decreases expected profits through the increased probability of a liquidity shortage. If reserves are ample, the probability of a liquidity shortage is low. Therefore, it is beneficial for commercial banks to exchange a large share of reserves for debt securities. In contrast, if reserves are scarce, commercial banks are not willing to absorb a lot of debt securities. Increasing the supply of debt securities is therefore less effective if reserves are scarce.[30]

Corollary 1.1 *With ample reserves the floor on the money market rate is $\underline{i}_m \equiv \lim_{F \to \infty} i_m = \pi i_b$.*

Proof. Because $\lim_{F \to \infty} X = -\infty$, $\lim_{F \to \infty} P(X > 0) = 0$. The result then follows directly from proposition 1.

Corollary 1.1 establishes that the volume of, and yield on, debt securities jointly determine a floor on the money market rate i_m.[31] Even with ample reserves central bank debt securities therefore allow to raise the money market rate.

Corollary 1.2 *The response of the money market rate i_m to an increase in the yield on debt securities i_b is $\frac{di_m}{di_b} = \pi \geq 0$.*

Proof. Follows immediately from proposition 1.

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[30]See Section 3.6 for an illustration and Appendix C, Section C.1 for a technical discussion.

[31]The upper limit binds for sufficiently low reserves. The money market rate is constrained by an upper bound because commercial banks would not borrow at a money market rate above the discount window rate. In fact, at sufficiently low F, commercial banks abstain from holding any debt securities ($\bar{\pi} = 0$) such that the upper limit is equal to the discount rate i_x. For more details, see Appendix C, Sections C.1 and C.2.

The money market rate strictly increases in the yield on debt securities if commercial banks are willing to hold these securities. A higher yield lowers the cost of holding pre-auction reserves without affecting the amount of post-auction reserves available as a liquidity buffer.[32] Variation in the yield on debt securities therefore affect the money market rate only through a cost channel.

Corollary 1.3 *The response of the money market rate to an increase in the supply of debt securities is* $\frac{di_m}{d\pi^{cb}} = i_b - i_x P(X > 0) + (1 - \pi) i_x \frac{\partial P(X>0)}{\partial \pi} > 0$ *if* $\pi < \bar{\pi}$.

Proof. Follows from proposition 1 and Appendix C, Section C.1 and C.2.

The money market rate rises in the supply of debt securities as long as commercial banks' expected profits rise in debt security investments, that is, as long as commercial banks are non-satiated with debt securities ($\pi < \bar{\pi}$). Commercial banks are willing to invest a larger fraction of pre-auction reserves in debt securities if pre-auction reserves are high.[33]

In sum, a higher supply of debt securities increases the money market rate through two channels. First, the cost of holding pre-auction reserves falls because reserves can, in part, be invested in interest-bearing debt securities. This investment opportunity translates a greater supply of debt securities into a higher demand for pre-auction reserves (cost channel). Second, debt security investments increase the probability of a liquidity shortage because debt securities drain reserves (reserve-absorbing channel). The reserve-absorbing channel is negligible, however, if reserves are ample.

3.5 Central bank debt securities and interest on reserves

To establish the equivalence between interest on reserve policies and central bank debt security programs, we derive the money market rate i_m^{ior} that prevails if commercial banks earn positive interest on reserves $i_{ior} > 0$ instead of having access to central bank debt securities.

Proposition 2 *Suppose that commercial banks earn $i_{ior} > 0$ on reserves. If each commercial bank maximizes expected profits with respect to B_i and π_i, the money market rate equals $i_m^{ior} = i_{ior} + i_x P(X^{ior} > 0)$, where $X^{ior} = Z - (F - K)$.*

Proof. See Appendix B, Section B.2.

Proposition 2 establishes the money market equilibrium that prevails if commercial banks earn a

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[32]Equivalently, a higher yield on debt securities increases the return on a portfolio of size F, partly invested in central bank debt securities and partly invested in non-interest-bearing reserves.

[33]The money market rate does not react to an increase in the supply of debt securities if commercial banks are satiated with debt securities. For more details, see Appendix C, Section C.1.

strictly positive interest on reserves.[34] In contrast to issuing debt securities, which affects the money market rate through the cost channel and the reserve-absorbing channel, interest on reserve policies affect the money market rate only through the cost channel.

Corollary 2.1 *With ample reserves the floor on the money market rate is $\underline{i}_m^{ior} \equiv \lim_{F\to\infty} i_m^{ior} = i_{ior}$.*

Proof. Because $\lim_{F\to\infty} X^{ior} = -\infty$, $\lim_{F\to\infty} P(X^{ior} > 0) = 0$. The result then follows directly from proposition 2.

From Corollary 1.1 and 2.1, it follows that a central bank can implement an interest rate floor either with an interest on reserves policy or with a debt security program. The interest rate floor in the two regimes is equal if $i_{ior} = i_b \pi$. Therefore, analyzing random variation in the main parameters of a central bank debt security program provides evidence on the money market effects of variation in interest on reserves.

3.6 Illustration

To illustrate the qualitative features of the money market model, we calibrate its parameters to match the situation in Switzerland in August 2010. At the time, Swiss commercial banks held approximately CHF 10 billion as minimum reserves ($K = 10$). The minimum reserve requirement was 2.5%, implying $D = 400$.[35] The discount rate was 0.55% (SNB 2010a). Focusing on the qualitative predictions of the model, we assume a rather arbitrary distribution of the liquidity shock $Z \sim N(0, 4)$. The marginal SNB Bill yield ranged from 0.05% to 1.25%, exceeding 0.55% only before the 3M Libor target range fell to [0% 0.75%] in March 2009.[36] At the peak of the program, SNB Bills accounted for 80% of pre-auction reserves. Consequently, we vary i_b and π^{cb} between $[0, i_x]$ and $[0, 1]$, respectively. Pre-auction reserves reached CHF 140 billion in August 2010. For pre-auction reserves, we thus consider values between 0 and 150 billion.[37]

We first discuss the impact of changing the volume of debt securities on the money market rate. The left panel of Figure 4 shows the money market rate as a function of pre-auction reserves for various π^{cb} conditional on $i_b = 0.1\%$. The money demand curves are strictly downward sloping in pre-auction reserves, unless commercial banks are satiated with a positive amount of debt securities ($\pi = \bar{\pi} > 0$). The money market rate falls in pre-auction reserves because a greater liquidity buffer decreases the likelihood of a liquidity shortage (the liquidity effect).

If commercial banks are satiated with a positive amount of debt securities, the money demand curves

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[34]Many others have established a similar result, e.g. Goodfriend (2002).

[35]SNB (2010b) and Bundesrat (2004)

[36]The SNB targeted the lower end of the range at approximately 0.25% (see SNB 2009a).

[37]The value of i_a is irrelevant for the money market rate because, in the aggregate, commercial banks cannot exchange reserves for private assets.

are flat in pre-auction reserves because commercial banks increase their debt security investments if pre-auction reserves rise. Higher investments in debt securities, in turn, increase the return on pre-auction reserves. The resulting upward pressure on the money market rate exactly offsets the liquidity effect.[38]

Figure 4 — Money demand curves
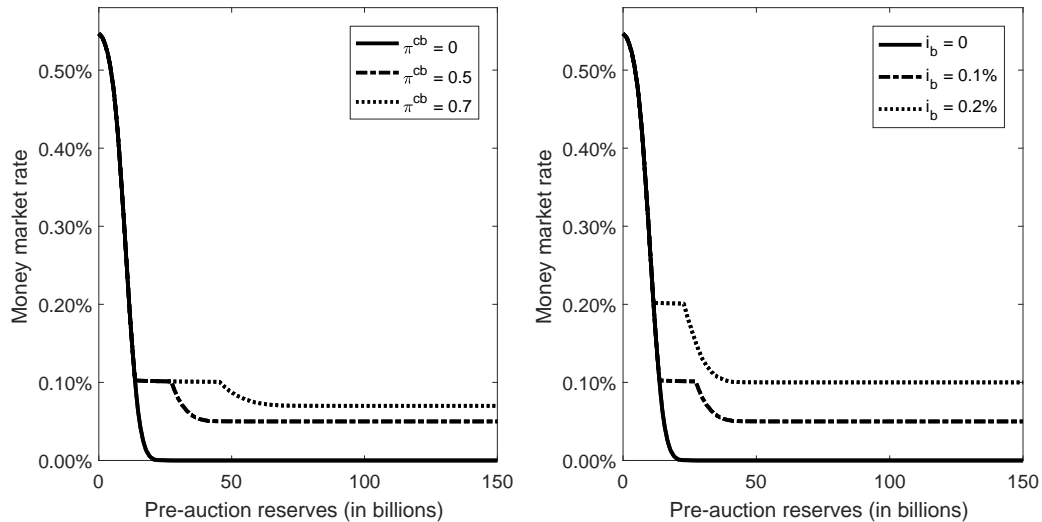
Notes: Conditional on $i_b = 0.10\%$ (left panel) and $\pi^{cb} = 0.5$ (right panel).

If pre-auction reserves are abundant, the money market rate hits the interest rate floor. For example, the interest rate floor equals 0.05% if the central bank allows commercial banks to invest up to 50% of their pre-auction reserves in debt securities and the yield on debt securities amounts to 0.1%. If the central bank supplies debt securities with a positive yield, the floor is strictly positive.

An increase in π^{cb} raises the money market rate if $\pi < \bar{\pi}$ through the reserve-absorbing channel and the cost channel. Increasing the supply of debt securities decreases post-auction reserves available to absorb liquidity shocks. Additionally, debt securities provide an attractive investment opportunity to commercial banks, which lowers the costs of holding pre-auction reserves.

A larger supply of debt securities is ineffective in raising the money market rate if commercial banks are satiated with debt securities ($\pi = \bar{\pi}$). In this case, the additional supply is not absorbed. For example, at $F = 40$, commercial banks optimally invest 65.9% of their pre-auction reserves in debt securities. Thus, increasing π^{cb} from 0.7 to 0.8 does not affect the money market rate. In contrast, increasing π^{cb} from 0.5 to 0.7 raises the money market rate in this situation.

Next, we examine the impact of changing the yield on debt securities. The right panel of Figure 4 depicts the money market rate as a function of pre-auction reserves for various yields on central bank debt securities conditional on $\pi^{cb} = 0.5$. Similar to the previous analysis, money demand is downward sloping if commercial banks are non-satiated with debt securities or if commercial banks

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[38]See Appendix B, Section B.1 and Appendix C, Section C.2, for a discussion.

do not invest in debt securities at all ($\pi < \bar{\pi}$ or $\pi = 0$). In contrast, if commercial banks are satiated with a positive amount of debt securities, money demand is flat in pre-auction reserves. In addition, if the yield on debt securities is strictly positive, the floor on the money market rate is strictly positive.

Finally, if commercial banks invest in debt securities, increasing the yield on debt securities raises the money market rate through the cost channel independently of the level of pre-auction reserves.

4 Empirical strategy

We identify the effects of a surprise change in the interest rate floor through random variation in SNB Bill auctions. Our theoretical model shows that the interest rate floor rises if the central bank increases the volume of, or marginal yield on, central bank debt securities. In addition, the central bank may reveal information about its intentions to change interest rates in the future. For example, issuing debt securities with longer terms to maturity absorb reserves for longer and may thus reveal that the central bank intends to keep the money market rate higher for longer.

We estimate the causal effect of an SNB Bill auction through heteroscedasticity (see Rigobon 2003; Rigobon and Sack 2004; Nakamura and Steinsson 2018a). As Gürkaynak et al. (2018) emphasize, this strategy captures the impact of all information revealed during an event and not only the "headline" news.[39] To estimate dynamic causal effects we combine identification through heteroscedasticity with vector autoregressions (VAR) and local projections. The two approaches have advantages and disadvantages. Local projections are more robust with respect to model misspecification (see Jordà 2005) and vector autoregressions are more efficient (see Stock and Watson 2018).

4.1 Model

To fix ideas, assume that the data stem from a VAR of order 1 including a stock price index (s_t) and the money market rate (r_t):[40]

$$y_t = \Phi y_{t-1} + \Psi e_t \,,$$

where $y_t = [s_t \quad r_t]'$, $e_t = [e_{1t} \quad e_{2t}]'$ is a vector of i.i.d. structural shocks with $V[e_{1t}] = V[e_{2t}] = 1$, Φ is a (2×2) matrix of autoregressive coefficients, and Ψ is a (2×2) matrix measuring the immediate impact of the two structural shocks.[41]

Without loss of generality, assume that shock 1 occurs only in specific periods ("events") whereas

---

[39]Event studies that control for expectations using surveys or interest rate futures may only capture part of the surprise component (the "headline" news). Suppose we identify the surprise component of labor market releases comparing survey expectations for the unemployment rate with the actual outcome published in the labor market report. If the report contains additional surprising information, for example on the participation rate, on the employment rate, or on methodological changes, we only measure part of the surprise component that affects financial variables (see Gürkaynak et al. 2018).

[40]Appendix D discusses cases that are more general.

[41]The unit normalization of the variance of the structural shocks is standard in structural VAR analysis.

shock 2 occurs in all periods. Then, the variance-covariance matrix of the one-step-ahead forecast error ($\varepsilon_{t|t-1} = \Psi e_t$) changes during an event. Let $\Omega_{t \in E}$ ($\Omega_{t \notin E}$) denote the variance-covariance matrix of $\varepsilon_{t|t-1}$ during events (no events), and ψ_{ij} the impact of variable i to shock j. Then, we can show that:

$$
\Omega_{t \in E} = \begin{bmatrix} \psi_{s1}^2 + \psi_{s2}^2 & \psi_{s1}\psi_{r1} + \psi_{s2}\psi_{r2} \\ \cdot & \psi_{r1}^2 + \psi_{r2}^2 \end{bmatrix}
\tag{2}
$$

$$
\Omega_{t \notin E} = \begin{bmatrix} \psi_{s2}^2 & \psi_{s2}\psi_{r2} \\ \cdot & \psi_{r2}^2 \end{bmatrix}
$$

$$
\tilde{\Omega} = \Omega_{t \in E} - \Omega_{t \notin E} = \begin{bmatrix} \psi_{s1}^2 & \psi_{s1}\psi_{r1} \\ \cdot & \psi_{r1}^2 \end{bmatrix} .
$$

The diagonal of $\tilde{\Omega}$ contains the squared impact coefficients of shock 1 on the stock price index and the money market rate. Intuitively, we can pin down the (absolute) impact of the surprise component of an event comparing the variance of financial market variables during event days to a counterfactual variance that prevails on non-event days.

In Appendix D and the next section, we discuss all assumptions for identification in detail. At this stage, we highlight two of them. First, we need to assume that an event indeed had an impact on one of the variables (e.g. $|\psi_{s1}| > 0$). Second, we need to impose a sign restriction on this variable (e.g. $\psi_{s1} < 0$). If we impose that a restrictive SNB Bill shock causes a decline in stock prices we can recursively compute all the responses by:

$$
\psi_{s1} = -\sqrt{\tilde{\Omega}_{11}}
\tag{3}
$$

$$
\psi_{r1} = \tilde{\Omega}_{12}/\psi_{s1} ,
$$

where $\tilde{\Omega}_{ij}$ denotes the row i, column j element of $\tilde{\Omega}$.[42] This resembles the estimator used by Rigobon (2003) and Nakamura and Steinsson (2018a).[43] The main difference is that we do not associate the response with a particular policy variable for which the response is normalized to unity. Instead, we impose a sign restriction, which allows us to estimate the impulse responses to a one standard deviation structural shock for all variables in the system (see Appendix D for a discussion).

Most event studies only measure the immediate impact. To estimate the dynamic causal effect we propose a VAR and a local projection approach. If the data stem from a VAR of order 1 we can

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[42]In our empirical application a restrictive SNB Bill shock will lead to lower stock prices, similar to Jarociński and Karadi (2018). Nakamura and Steinsson (2018a) assume that a restrictive monetary policy shock leads to higher interest rates.

[43]As Nakamura and Steinsson (2018a), we do not exploit all available information in the estimation. In fact, the variance of the residual associated with variable 1 allows to independently compute ψ_{r1}. Therefore, it would be possible to derive a more efficient GMM estimator and construct a test of the overidentifying restrictions (see Rigobon and Sack 2004).

compute the impulse response at various horizons h as:[44]

$$
\begin{bmatrix} \psi_{h,s1} \\ \psi_{h,r1} \end{bmatrix} = \Phi^h \begin{bmatrix} \psi_{s1} \\ \psi_{r1} \end{bmatrix} , \text{ for } h = 1, \ldots, H .
$$

Alternatively, we can estimate the impulse responses with local projections. A local projection is a direct forecast of y_t. Suppose we observe how an econometrician revises its h-step-ahead forecast when one additional observation becomes available. We can estimate the impulse response after h periods by comparing the variance of these forecast revisions for periods with an event, to the variance for periods without an event. Intuitively, if the econometrician systematically revises its forecast more strongly when an event occurs, these revisions reveal information about the causal impact of the event. In what follows, we show how to estimate the impulse responses.

To fix ideas, assume that the data stem from a VAR of order 1 and we estimate the impulse response for horizon $h = 1$. Iterating the VAR backwards and taking conditional expectations we can show that the one- and two-step-ahead local projections are:

$$
\begin{aligned}
\mathbb{E}[y_t | y_{t-1}] &= \Phi y_{t-1} \\
\mathbb{E}[y_t | y_{t-2}] &= \Phi^2 y_{t-2} .
\end{aligned}
$$

It follows that the one- and two-step-ahead forecast errors amount to:

$$
\begin{aligned}
y_t - \mathbb{E}[y_t | y_{t-1}] &= \varepsilon_{t|t-1} = \Psi e_t \\
y_t - \mathbb{E}[y_t | y_{t-2}] &= \varepsilon_{t|t-2} = \Psi e_t + \Phi \Psi e_{t-1} .
\end{aligned}
$$

The difference between the two- and one-step-ahead forecast errors depends only on the structural shocks in $t - 1$ and the impulse response after one period ($\Psi_1 \equiv \Phi \Psi$):

$$
\varepsilon_{t|t-2} - \varepsilon_{t|t-1} = \Psi_1 e_{t-1} .
$$

Therefore, we can use equations (2) and (3), with the variance-covariance matrices of $\varepsilon_{t|t-h-1} - \varepsilon_{t|t-h}$, for periods with and without an event in $t - h$ to estimate the impulse response for horizon h.

4.2 Identification

To identify the impulse responses to an SNB Bill shock we impose five assumptions (see also Appendix D): (i) All parameters except the residual variances of the VAR and local projections are constant; (ii) the variances of the structural shocks are unity; (iii) SNB Bill auctions affect stock prices

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[44]Appendix D gives the formulas for a VAR of order p.

($\tilde{\Omega}_{11} > 0$); (iv) the stock price index declines after a restrictive SNB Bill auction; (v) SNB Bill shocks only occur on deterministic auction days. For the local projection approach, we additionally impose that the decline of the stock price index is permanent.

(i) and (ii) are standard in identification schemes through heteroscedasticity and structural VAR analysis, respectively. We need (iii) to identify the impulse responses for variables other than the stock price index (see equation 3). To provide a measure of how likely this assumption holds, we report a one-sided test for $\tilde{\Omega}_{11} = 0$, against the alternative that $\tilde{\Omega}_{11} > 0$.[45] (iv) is a relatively innocuous normalization defining a "restrictive" SNB Bill auction. (v) allows to construct a counterfactual variance-covariance matrix to control for random shocks that affect financial markets every day.

Figure 5 — Timing of auctions

(a) Allotted volume CHF SNB Bills by weekday

(b) Share of all auctions by weekday
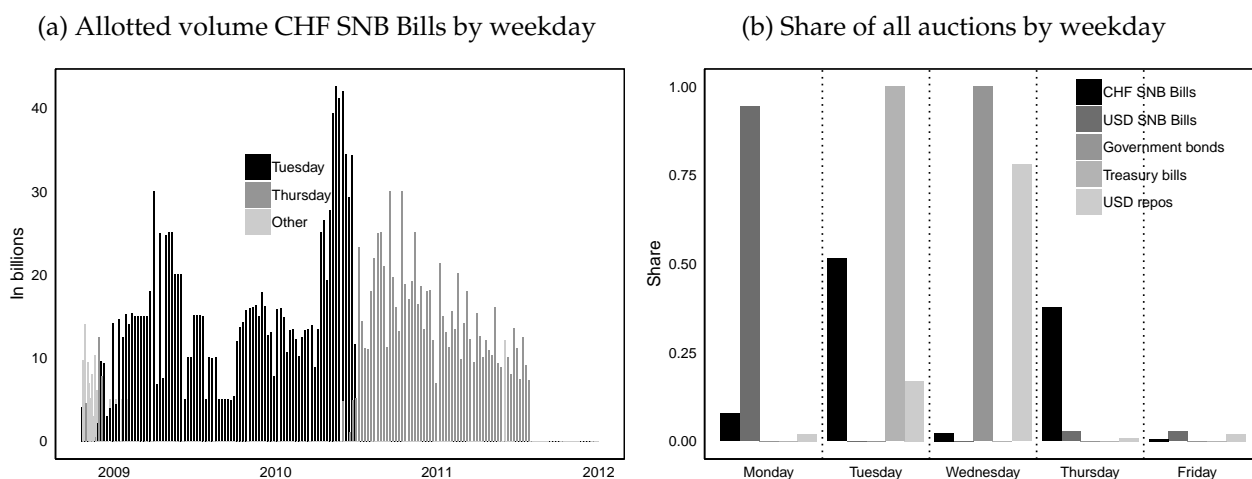
The last assumption implies that the SNB auction schedule was pre-determined and no events occurred according to the same deterministic schedule. Indeed, the auctions were announced in advance.[46] In addition, one auction took place almost every week.[47] Finally, most of the auctions took place according to a regular and predictable schedule. In the first part of the sample period, most auctions occurred on Tuesday (see panel a, Figure 5). In the second part of the sample, the auction day was changed to Thursday. One motivation to change the auction day may have been to highlight the difference between treasury bill auctions and SNB Bill auctions, which were both held on Tuesday in the first half of the sample (see panel b).[48] Indeed, the auction day changed in July

[45]See Appendix D for a Monte Carlo simulation.

[46]We have limited information from a web archive that the announcement sheet was published three working days prior to the auction. The date, term, a minimum and maximum price, as well as the auction type were public knowledge before the auction. Appendix A gives the link to the current auction announcement sheet.

[47]Only in the last week of the years 2009 and 2010 no auction took place. In the beginning of the program, the SNB sometimes held two auctions per week.

[48]Rule (2011) discusses general guidelines for issuing central bank debt securities suggesting that "[. . .] clear and precise announcements mean that market participants have a clear knowledge as to the size and regularity of central bank operations which they can separate from the governments issuance."

2010, when the SNB started to regularly issue SNB Bills with a similar maturity as treasury bills.[49]

In addition, we require that no other deterministic events occurred on SNB Bill auction days or on the days we use to estimate the counterfactual variance-covariance matrix. Panel (b) shows that the SNB also held auctions for repurchase agreements (in CHF and USD), USD SNB Bills, government bonds, and treasury bills.[50] USD SNB Bill auctions occurred usually on Monday. Government bond auctions took place on Wednesday, usually in the second week of the month. USD repo auctions took place predominantly on Wednesday but only on the height of the financial crisis.[51] Treasury bills were auctioned each Tuesday. Almost no auctions occurred on Friday. The figure does not report the SNB's CHF repo operations. To the best of our knowledge these operations took place each business day between 9:00 and 9:10 and therefore do not affect our identification scheme.

To identify the dynamic causal effect with local projections, we have to restrict the sign for one variable at every horizon. Because we estimate the response of financial market variables, we prefer to restrict the sign of the cumulative response. The reason is that financial market variables tend to follow a random walk. If true, the response of the first differences is zero for $h > 0$. The cumulative response, however, is non-zero even in the long run. Appendix D shows that the local projection approach readily extends to estimating and restricting the sign of the cumulative response.[52]

4.3 Implementation

Our main specification uses log-changes of a Swiss nominal effective exchange rate index, a Swiss stock price index (Swiss Market Index, SMI), as well as the change of a one-week money market rate. We express the exchange rate as one unit of a basket of foreign currencies in terms of Swiss francs. Therefore, a decline in the exchange rate is an effective appreciation of the Swiss franc. The SMI comprises 20 companies with the largest market capitalization on the SIX Exchange.[53] The one-week money market rate is a zero coupon yield inferred from interest rate swaps based on SARON, an overnight rate calculated from actual repo transactions and binding quotes.[54] We then augment the model gradually with other variables.[55]

To deal with missing values we remove all weekends. Therefore, our results are measured in

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[49]Treasury bills have a maturity of three months and more. All SNB Bill auctions with a maturity of three months and more indeed occurred on Thursday or Friday.

[50]Treasury bill and government bond auctions are conducted on behalf of the Swiss Confederation.

[51]Although the facility remained in place, the outstanding volume fell to zero in early 2009. We therefore ignore all auction days with a zero volume.

[52]In addition, we discuss how to incorporate exogenous variables and provide some simulation results to illustrate identification issues when restricting the sign of the response.

[53]See www.six-group.com/exchanges/knowhow/products/shares/indices_de.html.

[54]See www.six-group.com/exchanges/indices/data_centre/swiss_reference_rates/reference_rates_en.html. Although there are Swiss Average Rates available for maturities of one week and more, the quotes are often missing for periods in which the number of transactions and/or the volume of the transactions were too low. We therefore prefer to use interest rate derivatives in the empirical analysis.

[55]See Appendix A for all data sources.

weekdays. In addition, there are missing values in stock price data when markets are closed on holidays. We replace these missing values with the last available observation.

To control for other deterministic events that coincide with SNB Bill auctions we estimate the variance-covariance matrix for auction days, excluding observations on which one of the following events occurred: treasury bill auctions, government bond auctions, CHF SNB Bill payment days, USD SNB Bill auctions, and USD repo auctions. In addition, we exclude these events when computing the counterfactual variance-covariance matrix.

The estimation sample ranges from 20 October 2008 to 28 July 2011. We include one lag of the endogenous variables and a constant in all models. All models are estimated by OLS. For inference, we use 5,000 samples from a moving block bootstrap as described in ch. 8 by Efron and Tibshirani (1993). We set the block size to 15 business days assuming that changes in financial market variables are not very persistent.

5 Empirical results

What is the impact of a surprisingly restrictive SNB Bill auction? Figure 6 shows the impulse responses for the exchange rate (in percent), the one-week money market rate (in percentage points), and the response of the stock market index SMI (in percent). The negative sign of the SMI response reflects our identifying assumption.

According to the VAR approach, a restrictive SNB Bill shock causes a 0.4% appreciation of the Swiss franc and a 1% reduction in the SMI. In line with the theoretical model, the money market rate increases by 0.02 percentage points. All three responses are highly persistent. The local projection approach paints a more nuanced picture. Panel (b) confirms that the impact on the exchange rate is relatively persistent. In addition, we observe a decline in stock prices by about 1%. The response of the money market rate is particularly large after one business day and approaches zero thereafter. The temporary impact on the money market rate is consistent with the fact that most SNB Bills had a relatively short term to maturity of 7 to 28 days.

When computing the point estimates and confidence intervals, we only use those bootstrap samples for which we can compute the impulse response. If the difference between the variance-covariance matrix on auction days and the counterfactual variance-covariance matrix is negative, we have to disregard the bootstrap sample. A negative difference occurs more often if, in population, the identifying assumption does not hold. This suggests that we can test whether the identifying assumption holds by calculating the share of bootstrap samples we use for inference. Up to five business days, the impulse responses are based on 93% of all bootstrap samples. After 10 business

Figure 6 — Impulse responses to a restrictive SNB Bill auction
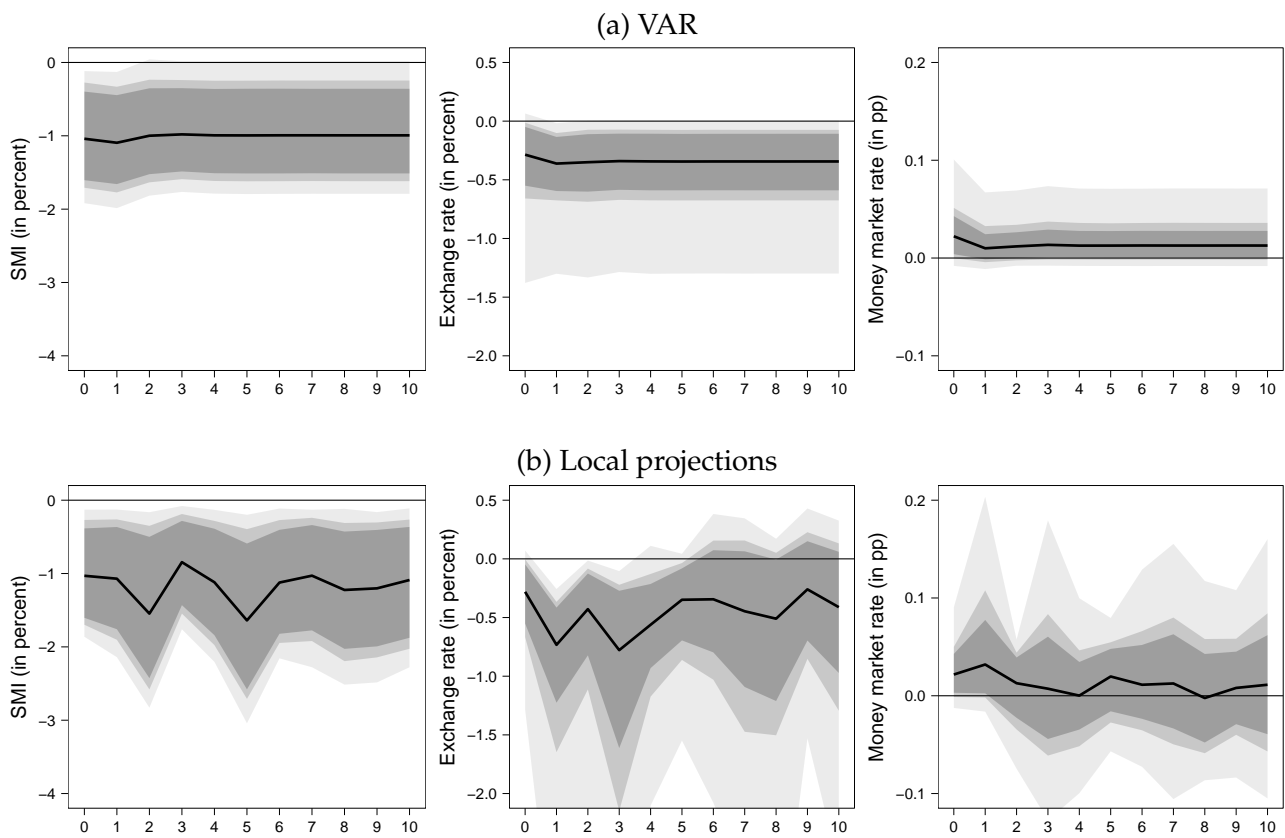
(a) VAR

(b) Local projections

Notes: Estimated response for up to 10 business days to a 1 std. restrictive SNB Bill shock in black. The response of stock market prices and the exchange rate are measured in percent, the response of interest rates in percentage points. The shaded areas give 99%, 95%, and 90% confidence intervals based on a block bootstrap algorithm.

days, the results are less reliable; the impulse responses are only based on 83% of all bootstrap samples.[56]

Figure 7 — Impulse responses of short- and long-term interest rates
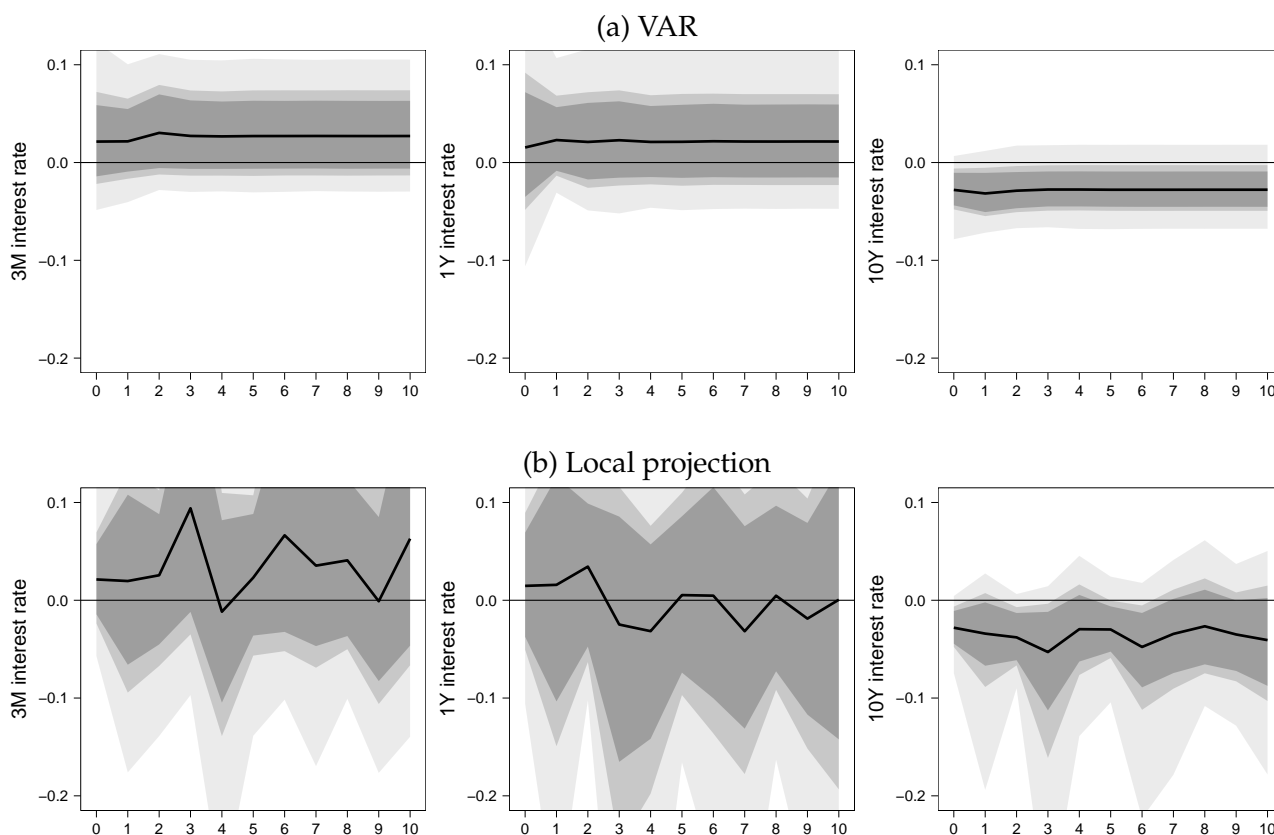
(a) VAR

(b) Local projection

Notes: Estimated response for up to 10 business days to a 1 std. restrictive SNB Bill shock in black. The response of interest rates are measured in percentage points. The shaded areas give 99%, 95%, and 90% confidence intervals based on a block bootstrap algorithm.

Next, we ask how a restrictive SNB Bill shock affects interest rates of various maturities. We add changes of zero coupon yields based on interest rate swaps for maturities of three months, one year, and 10 years (see Figure 7).[57] Panel (a) shows that three-month and one-year rates increase slightly but the responses are much less precisely estimated and indistinguishable from zero. By contrast, the ten-year rate declines. Although the local projection impulse responses are even more noisy they confirm the results from the VAR approach qualitatively. In particular, ten-year rates decline after a restrictive SNB Bill auction. One explanation for the decline of long-term interest rates is that an SNB Bill auction appreciates the Swiss franc. Because of the restrictive impact on the exchange rate, markets lower their short-term interest rate expectations. Therefore, long-term interest rates decline.

Long-term interest rates could decline because of lower expected real rates or lower inflation expectations. Figure 8 provides evidence that inflation expectations were not affected. We show the

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[56]In what follows we discuss this statistic only for cases where results may not be as reliable.

[57]We also experimented with zero coupon government bond yields. The results are qualitatively identical.

Figure 8 — Impulse responses of corporate bonds denominated in Swiss francs
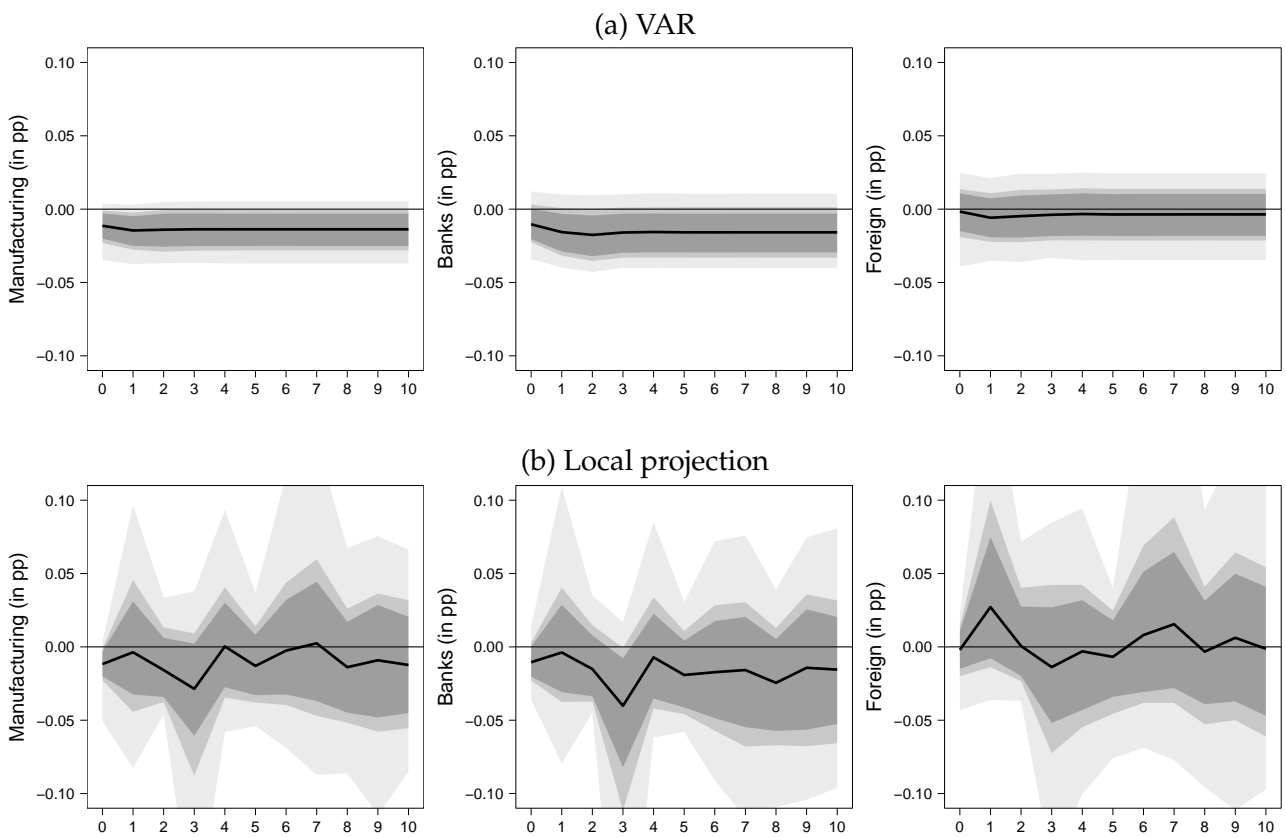
(a) VAR

(b) Local projection

Notes: Estimated response for up to 10 business days to a 1 std. restrictive SNB Bill shock in black. The response of interest rates are measured in percentage points. The shaded areas give 99%, 95%, and 90% confidence intervals based on a block bootstrap algorithm.

results from a model including corporate bonds of domestic companies and foreign companies that issue corporate bonds in Swiss francs.[58] Because both, domestic and foreign bonds, are denominated in Swiss francs they should be equally affected by changes in inflation expectations. Therefore, the difference between the responses reflects changes in domestic real interest rates relative to abroad. We find a decline in yields for domestic corporate bonds but not for foreign corporate bonds. Therefore, future expected real interest rates declined in Switzerland relative to real interest rates abroad.[59] The local projection estimates are imprecise and confirm the result only on impact.

Does the term of an SNB Bill matter? On the one hand, short-term Bills may have little effect because they soak up reserves only for a brief period. On the other hand, the impact may be large if financial market participants expect that the SNB systematically rolls over newly created Bills when they mature. Figure 9 shows the responses to SNB Bill auctions with a short term to maturity of 6 months or less. We find similar impulse responses as in the main specification when we limit the sample to short-term SNB Bill auctions. Unfortunately, estimates for longer maturities are not reliable because we find an overwhelming share of responses that do not fulfill our identifying assumption. We therefore do not report these results.

5.1 Policy experiments

We use our estimates to perform policy experiments. In particular, we ask how an increase of the interest rate floor from 0% to 0.25% affects stock prices and the exchange rate. We assume that the central bank implements a debt security program that persistently raises the one-week money market rate to 0.25%.

First, we simulate such a policy by scaling the variance of the structural shock to obtain the desired response of the money market rate. Second, we perform the policy experiment using an alternative identification scheme imposing a normalization on the initial response of the money market rate.[60] We choose the scale of the shock and the initial response, respectively, to generate a money market rate response of 0.25% after five weekdays.

According to the first identification scheme such a policy has relevant effects on stock prices and the exchange rate (see Figure 10, panel a). At the upper end of the 90% confidence interval, the stock price index falls by 7% and the exchange rate appreciates by 3%. The point estimates point to a rather strong decline in stock prices by 20% and a 6% appreciation.

According to the second identification scheme, the results are qualitatively similar (see panel b). However, the responses are smaller in absolute value. The stock price index falls by 5% and the

---

[58] All yields have a term to maturity of 8 years. Foreign bonds are issued by corporations with an excellent credit rating.

[59] Unfortunately, we cannot assess whether relative real interest rates decline because of a decline in risk premia.

[60] Appendix D shows that this is a scaled version of the estimator used by Nakamura and Steinsson (2018a).

Figure 9 — Impulse responses to a restrictive short-term SNB Bill auction
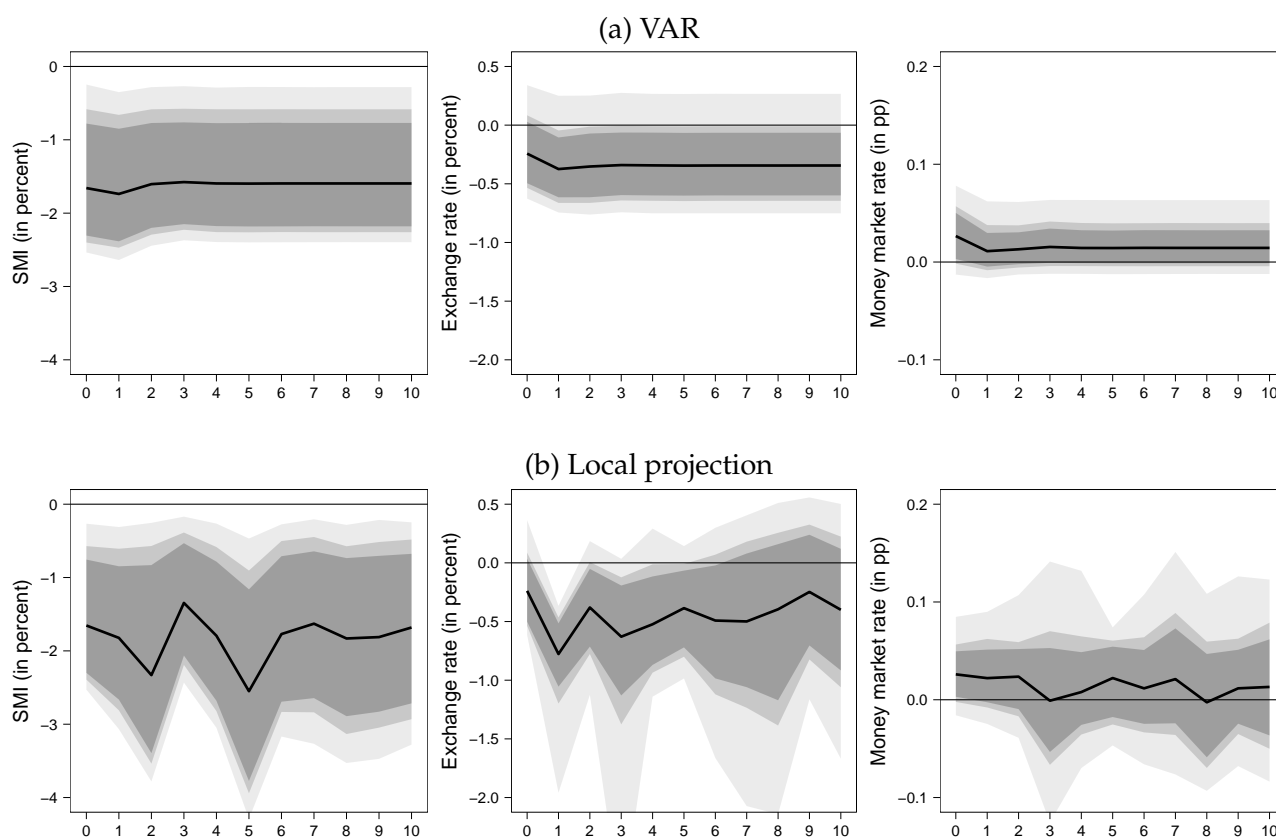
(a) VAR

(b) Local projection

Notes: Estimated response for up to 10 business days to a 1 std. restrictive SNB Bill shock in black. The response of stock market prices and the exchange rate are measured in percent, the response of interest rates in percentage points. The shaded areas give 99%, 95%, and 90% confidence intervals based on a block bootstrap algorithm.

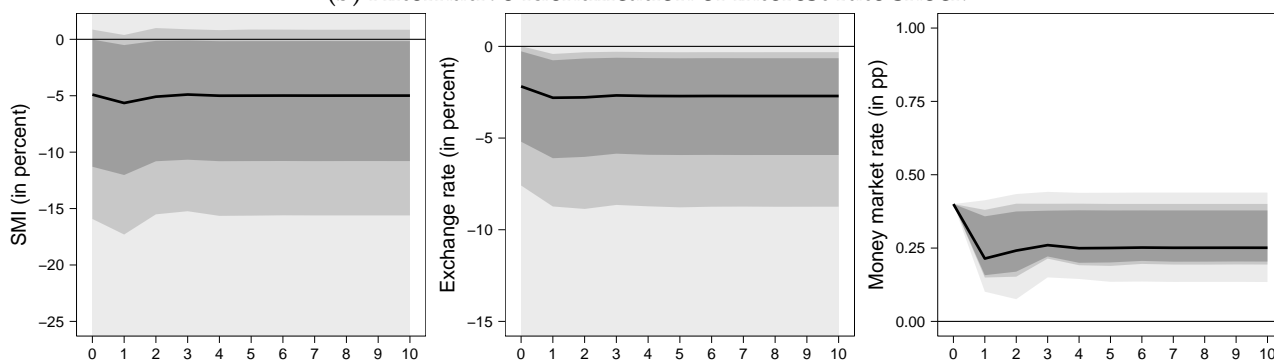## Figure 10 — Effect of a 0.25% interest rate floor

(a) Main specification with scaled shock
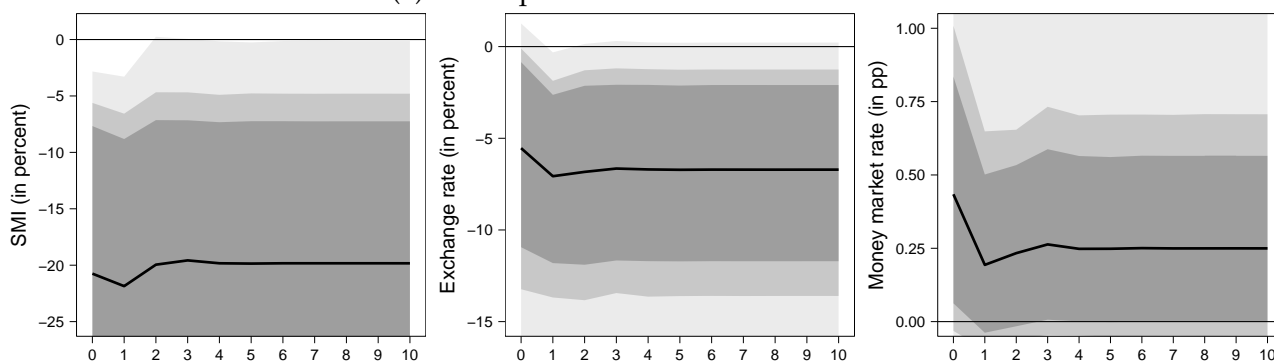
(b) Alternative identification of interest rate shock

Notes: Estimated response for up to 10 business days to a restrictive SNB Bill shock leading to a 0.25% increase in the money market rate (panel a). The alternative identification scheme imposes an initial response of the money market rate leading to a 0.25% money market rate after five weekdays (panel b). The response of stock market prices and the exchange rate are measured in percent, the response of interest rates in percentage points. The shaded areas give 99%, 95%, and 90% confidence intervals based on a block bootstrap algorithm.

exchange rate appreciates by 3%. Note that the alternative identification scheme captures the impact of an SNB Bill auction only to the extent that it affects the one-week money market rate. The smaller response may therefore reflect that SNB Bill auctions not only worked through the short-term money market rate but also through expectations about future policy actions. The first identification scheme may better capture these changes in expectations.

5.2 Placebo tests

As placebo tests we estimate the impulse responses using dummy variables, for which we expect the impact on financial market variables to be zero. Our placebo tests confirm that actual SNB Bill auction days systematically differ from other days without an auction.

First, we estimate the model on a sample from January 2005 to January 2008. During this period, no SNB Bill auctions took place. Instead, we generate a placebo dummy variable with 170 random auctions occurring on either Tuesday or Thursday. We then estimate the immediate impulse responses on 200 different placebo samples. Based on these placebo estimates, we can compute various statistics. The average impact indicates whether we find the same sign and magnitude of the effect as in the actual sample. Then, we compute the share of placebo samples in which we reject the null hypothesis of no response at the 10% level. We test the null against the one-sided alternative of a drop in the SMI, an appreciation of the exchange rate, and an increase in the interest rate. The share of rejections indicates how often we would falsely draw the same conclusion as in the main specification. Finally, we compute share of placebo samples where at least 90% of the bootstrapped impulse responses are consistent with our identifying assumption. This share indicates how likely we would wrongly conclude that our identifying assumption is satisfied.

Panel (a) of Table 1 shows the results. The summary statistics suggest that there was no impact of the placebo auctions on financial market variables before the SNB Bills program was in place. The average coefficient on the exchange rate and the money market rate are small and of the opposite sign compared to our actual estimates. In addition, rejection rates for the money market rate and the exchange rate are small. The negative coefficient on the SMI and the high rejection rate reflect our identifying assumption. Additionally, we can investigate how many of the bootstrap samples are consistent with the imposed identifying assumption. The last row shows that this is almost never the case.

Second, we perform a placebo test using the original sample during which the SNB Bills program was in place. We randomly select placebo days on which no SNB Bill auction occurred. Panel (b) shows that there is no evidence that the Swiss franc appreciates or the interest rate increases during these random auction days, although we condition on a decline in stock prices. In fact, the average coefficient carries the opposite sign relative to our main specification. Moreover, the identifying

Table 1 — Placebo tests

(a) Random auction day before SNB Bill program

	SMI	Exchange rate	Interest rate
Average impact	-0.28	0.10	-0.02
Share rejected	1.00	0.02	0.05
Share identified	0.01	0.01	0.01

(b) Random auction day during SNB Bill program (excluding actual auctions)

	SMI	Exchange rate	Interest rate
Average impact	-0.47	0.53	-0.02
Share rejected	1.00	0.00	0.00
Share identified	0.00	0.00	0.00

Notes: Results of two placebo simulations using 170 auction dummies generated on random days. Panel (a) uses a sample from 2005 to 2008, when no SNB Bills were issued, with random auctions occurring on Tuesdays or Wednesdays. Panel (b) uses the actual sample, when SNB Bills were issued, with random auctions occurring on days without an SNB Bill auction. We generate 200 placebo samples. The first row reports the average of the immediate impact (averaged over all bootstrap replications and placebo samples). The second row reports the share of placebo samples rejecting the null hypothesis of no response at the 10% level. We test the null against the one-sided alternative of a drop in the SMI, an appreciation of the exchange rate, and an increase in the interest rate. The last row shows the share of placebo samples where at least 90% of the bootstrapped impulse responses exist.

assumption almost never holds.

Third, we examine whether other events had an impact on financial market variables (the payment day, auctions of SNB Bills in USD, and treasury bill auctions). The payment day should not matter if all information about the auction was already revealed on the auction day. SNB Bill auctions in USD should not affect the Swiss money market rate because they do not drain CHF reserves. Finally, we expect that treasury bill auctions have no impact because the SNB issues these bills on behalf of the Swiss Confederation, that is, without the intention of draining reserves. The results are shown in Appendix E for brevity. The responses of the exchange rate and the interest rates are close to zero even though we condition on a drop in stock market prices. Only for treasury bill auctions we find an appreciation. However, the share of bootstrap replications where the identifying assumption is fulfilled amounts only to 37% suggesting that these results are less reliable.

5.3 Robustness tests

We perform a range of robustness tests (Appendix E shows the results). Adding exogenous variables, increasing the number of lags, or increasing the block size in the bootstrap algorithm to 40, does not change the results.[61] We also use different stock market indexes to identify the impulse responses (i.e. the MSCI Switzerland and the SPI, both including a broader range of Swiss companies). In both cases, the results remain the same. Then, we estimate the model with an overnight money market

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[61]Specifically we added the EUR/USD exchange rate, a European stock market index, and a European stock market volatility index.

rate. While we still find an appreciation of the currency, there is no effect on the overnight rate. Finally, we estimate a model including bilateral exchange rates (CHF/EUR, CHF/USD, CHF/JPY). SNB Bills cause an appreciation against the EUR, an imprecisely estimated appreciation against the USD, and no appreciation against the JPY. Finally, we control only for those treasury bill auction days on which no SNB Bill auction took place. This increases the number of auction days in the first half of the sample when both auctions usually occurred on Tuesday. The results remain qualitatively similar, although the response of the money market rate is less precisely estimated and the absolute impact is generally smaller. This is in line with the idea that treasury bill auctions had no or a smaller impact on financial market variables than SNB Bill auctions.

6 Concluding remarks

Central banks created large amounts of reserves in the wake of the financial and sovereign debt crises. To raise the money market rate in such an environment, central banks can sell assets, pay interest on reserves, or issue interest-bearing debt securities (see e.g. Berentsen et al. 2018). Our paper exploits the Swiss National Bank's debt security program to estimate the causal effects of such exit strategies on financial market variables.

We use a theoretical model of the money market to show how central banks affect the interest rate floor with debt securities. The money market rate increases in the volume of, and yield on, debt securities through a cost channel. Similar to interest on reserves, central bank debt is therefore able to implement an interest rate floor. In addition, central bank debt affects the money market rate through a reserve-absorbing channel. The reserve-absorbing channel is particularly relevant if reserves are scarce and negligible if reserves are ample. These theoretical considerations suggest that we can estimate the impact of variation in the interest rate floor by unexpected changes in the SNB's debt security program.

Our empirical analysis shows that SNB Bill auctions indeed affect financial market variables. A restrictive SNB Bill auction raises the money market rate, appreciates the Swiss franc, reduces long-term interest rates, and pushes down stock prices. Our estimates suggest relevant effects on stock prices and the exchange rate even for a small change in the interest rate floor from 0% to 0.25%. Raising the money market rate through an interest rate floor therefore affects asset prices and exchange rates even if reserves are ample.

References

Afonso, Gara and Ricardo Lagos (2015) "Trade dynamics in the market for federal funds," *Econometrica*, Vol. 83, pp. 263–313.

Bäurle, Gregor and Daniel Kaufmann (2018) "Measuring exchange rate, price, and output dynamics at the effective lower bound," *Oxford Bulletin of Economics and Statistics*, Vol. 80, pp. 1243–1266.

Bech, Morten and Todd Keister (2017) "Liquidity regulation and the implementation of monetary policy," *Journal of Monetary Economics*, Vol. 92, pp. 64–77.

Bech, Morten and Elizabeth Klee (2011) "The mechanics of a graceful exit: Interest on reserves and segmentation in the federal funds market," *Journal of Monetary Economics*, Vol. 58, pp. 415–431.

Bech, Morten and Cyril Monnet (2016) "A search-based model of the interbank money market and monetary policy implementation," *Journal of Economic Theory*, Vol. 164, pp. 32–67.

Berentsen, Aleksander, Sébastien Kraenzlin, and Benjamin Müller (2018) "Exit strategies for monetary policy," *Journal of Monetary Economics*, Vol. 99, pp. 20–40.

Boutros, Michael and Jonathan Witmer (forthcoming) "Monetary policy implementation in a negative rate environment," *Journal of Money, Credit and Banking*, URL: https://onlinelibrary.wiley.com/doi/full/10.1111/jmcb.12605.

Bundesrat (2004) "Verordnung zum Bundesgesetz über die Schweizerische Nationalbank vom 18. März 2004, SR 951.131.," Verordnung, Der Schweizerische Bundesrat.

Christensen, Jens H.E. and Signe Krogstrup (forthcoming) "Transmission of quantitative easing: The role of central bank reserves," *The Economic Journal*, URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/ecoj.12600.

Cieslak, Anna and Andreas Schrimpf (2019) "Non-monetary news in central bank communication," *Journal of International Economics*, Vol. 118, pp. 293–315.

Dziobek, Claudia H. and John W. Dalton (2005) "Central bank losses and experiences in selected countries," IMF Working Papers 05/72, International Monetary Fund.

Efron, Bradley and Robert J. Tibshirani (1993) *An Introduction to the Bootstrap*, New York: Chapman & Hall/CRC.

Ennis, Huberto M. (2018) "A simple general equilibrium model of large excess reserves," *Journal of Monetary Economics*, Vol. 98, pp. 50 – 65.

Ennis, Huberto M. and John A. Weinberg (2013) "Over-the-counter loans, adverse selection, and stigma in the interbank market," *Review of Economic Dynamics*, Vol. 16, pp. 601–616.

Gagnon, Joseph E. and Brian Sack (2014) "Monetary policy with abundant liquidity: a new operating framework for the Federal Reserve," Policy Brief 14-4, Peterson Institute for International Economics.

Gertler, Mark and Peter Karadi (2015) "Monetary policy surprises, credit costs, and economic activity," *American Economic Journal: Macroeconomics*, Vol. 7, pp. 44–76.

Goodfriend, Marvin (2002) "Interest on reserves and monetary policy," *Economic Policy Review*, Vol. 8, pp. 77–84.

Gray, Simon T. and Runchana Pongsaparn (2015) "Issuance of central bank securities; International experiences and guidelines," IMF Working Papers 15/106, International Monetary Fund.

Gürkaynak, Refet S., Burçin Kisacikoğlu, and Jonathan H. Wright (2018) "Missing events in event studies: Identifying the effects of partially-measured news surprises," NBER Working Papers 25016, National Bureau of Economic Research.

Gürkaynak, Refet S., Brian Sack, and Eric Swanson (2005) "Do actions speak louder than words? The response of asset prices to monetary policy actions and statements," *International Journal of Central Banking*, Vol. 1, pp. 55–93.

Gürkaynak, Refet S. and Jonathan H. Wright (2013) "Identification and inference using event studies," *The Manchester School*, Vol. 81, pp. 48–65, URL: https://doi.org/10.1111/manc.12020.

Ireland, Peter N. (2014) "The macroeconomic effects of interest on reserves," *Macroeconomic Dynamics*, Vol. 18, pp. 1271–1312.

Jarociński, Marek and Peter Karadi (2018) "Deconstructing monetary policy surprises - The role of information shocks," CEPR Discussion Papers 12765, Center for Economic Policy Research.

Jordà, Òscar (2005) "Estimation and inference of impulse responses by local projections," *American Economic Review*, Vol. 95, pp. 161–182.

Jordan, Thomas J. (2009) "10 Jahre Franken-Repomarkt - eine Erfolgsgeschichte (2 July 2009)," speech, Swiss National Bank.

Keister, Todd, Antoine Martin, and James J. McAndrews (2008) "Divorcing money from monetary policy," *Economic Policy Review*, Vol. 14, pp. 41–56.

Kraenzlin, Sébastien and Thomas Nellen (2015) "Access policy and money market segmentation," *Journal of Monetary Economics*, Vol. 71, pp. 1–12.

Kuttner, Kenneth N. (2001) "Monetary policy surprises and interest rates: Evidence from the Fed funds futures market," *Journal of Monetary Economics*, Vol. 47, pp. 523–544.

Lütkepohl, Helmut (2012) "Identifying structural vector autoregressions via changes in volatility," DIW Berlin Discussion Papers 1259, DIW Berlin, German Institute for Economic Research.

Lütkepohl, Helmut, Mika Meitz, Aleksei Netunajev, and Pentti Saikkonen (2018) "Testing identification via heteroskedasticity in structural vector autoregressive models," DIW Berlin Discussion Papers 1764, DIW Berlin, German Institute for Economic Research.

Moser, Dewet (2011) "Die neue Umsetzung der Geldpolitik und ihre Auswirkungen auf den Geldmarkt (Zusammenfassung, 24 March 2011)," speech, Swiss National Bank.

———— (2012) "Umsetzung der Geldpolitik im Wandel der Zeit (22 March 2012)," speech, Swiss National Bank.

Nakamura, Emi and Jón Steinsson (2018a) "High frequency identification of monetary non-neutrality: The information effect," *Quarterly Journal of Economics*, Vol. 133, pp. 1283–1330.

Nakamura, Emi and Jón Steinsson (2018b) "Identification in macroeconomics," *Journal of Economic Perspectives*, Vol. 32, pp. 59–86.

Nyawata, Obert (2012) "Treasury bills and/or central bank bills for absorbing surplus liquidity; the main considerations," IMF Working Papers 12/40, International Monetary Fund.

Poole, William (1968) "Commercial bank reserve management in a stochastic model: implications for monetary policy," *The Journal of Finance*, Vol. 23, pp. 769–791.

Rigobon, Roberto (2003) "Identification through heteroskedasticity," *The Review of Economics and Statistics*, Vol. 85, pp. 777–792.

Rigobon, Roberto and Brian Sack (2004) "The impact of monetary policy on asset prices," *Journal of Monetary Economics*, Vol. 51, pp. 1553–1575.

Rule, Garreth (2011) "Issuing central bank securities," Centre for Central Banking Studies Handbook 30, Bank of England.

SNB (2008a) "Swiss National Bank and European Central Bank cooperate to provide Swiss franc liquidity (15 October 2008)," press release, Swiss National Bank.

———— (2008b) "Swiss National Bank issuing SNB Bills to steer money market liquidity (15 October 2008)," press release, Swiss National Bank.

———— (2009a) "Swiss National Bank takes decisive action to forcefully relax monetary conditions (12 March 2009)," press release, Swiss National Bank.

———— (2009b) "Swiss National Bank to issue SNB Bills in US dollars (2 February 2009)," press release, Swiss National Bank.

———— (2010a) "Geldpolitisch wichtige Daten für die Woche zum 27 August 2010," press release, Swiss National Bank.

———— (2010b) "Geldpolitisch wichtige Daten für die Woche zum 29. Oktober 2010," press release, Swiss National Bank.

———— (2011a) "Swiss National Bank expands measures against strong Swiss franc (10 August 2011)," press release, Swiss National Bank.

———— (2011b) "Swiss National Bank intensifies measures against strong Swiss franc (17 August 2011)," press release, Swiss National Bank.

———— (2011c) "Swiss National Bank takes measures against strong Swiss franc (3 August 2011)," press release, Swiss National Bank.

———— (2016) "SNB Bills – Issue Conditions (1 March 2016)," press release, Swiss National Bank.

Stock, James H. and Mark W. Watson (2018) "Identification and estimation of dynamic causal effects in macroeconomics using external instruments," *The Economic Journal*, Vol. 128, pp. 917–948.

Online Appendix

A Data

Table A.1 — Data and sources

Name	Time stamp	Source	Links
CHF SNB Bills		SNB	`www.snb.ch/en/mmr/reference/snb_bills_res/source/snb_bills_res.en.pdf` and `www.snb.ch/en/mmr/reference/snb_bills_new/source/snb_bills_new.en.pdf`
USD SNB Bills		SNB	`www.snb.ch/de/mmr/reference/snb_usd_repo_results_20181121/source/snb_usd_repo_results_20181121.de.pdf`
CHF repo auctions		SNB	`www.snb.ch/en/mmr/reference/repo_mb23/source/repo_mb23.en.pdf`
USD repo auctions		SNB	`www.snb.ch/en/mmr/reference/snb_usd_repo_results_20190123/source/snb_usd_repo_results_20190123.en.pdf`
Government bond auctions		Federal Finance Administration	`www.admin.ch/gov/de/start/dokumentation/medienmitteilungen.msg-id-16132.html`
Treasury bill auctions		Federal Finance Administration	`www.newsd.admin.ch/newsd/message/attachments/54842.pdf`
Weekly reserves		SNB	`www.snb.ch/de/iabout/stat/statrep/id/statpub_impdata`
Monthly liabilities		SNB	`data.snb.ch/en/topics/snb#!/cube/snbbipo`
Stock market data (SMI, SPI, MSCI, VSTOXX, EURO STOXX 50)	Trade close	Datastream	
Effective exchange rate	11:00 am	SNB	`data.snb.ch/en/topics/ziredev#!/cube/devwkieffid` and `www.snb.ch/n/mmr/reference/economic_studies_2017_11/source/economic_studies_2017_11.n.pdf`
Bilateral exchange rates	2:15 am	ECB	`www.ecb.europa.eu/stats/policy_and_exchange_rates/euro_reference_exchange_rates/html/index.en.html`
Swiss Average Rates (SAR)	4:00 pm	SNB	`data.snb.ch/de/topics/ziredev#!/cube/zirepo`
Zero coupon yields based on IRS on SARON	4:00 pm	Reuters/Datastream	
Government bond yields	Morning	SNB	`data.snb.ch/en/topics/ziredev#!/cube/rendoblid`
Corporate bond yields	Morning	SNB	`data.snb.ch/en/topics/ziredev#!/cube/rendoblid`
Libor interest rates	12:00 am	Federal Reserve Bank of St. Louis	`fred.stlouisfed.org/categories/33003`

B Proofs

B.1 Proof of proposition 1

Optimization with respect to $\pi_i \in [0, \pi^{cb}]$ of

$$\mathbb{L} = i_a A_i + i_b \pi_i (R_i + B_i) - i_m B_i - i_d \mathbb{E}(D - Z) - i_x \int_{\hat{z}_i}^{\infty} X_i f(Z) dZ - \lambda_{1,i}(\pi_i - \pi^{cb}) + \lambda_{2,i} \pi_i$$

yields the following first order condition

$$F_i \left(i_b - i_x \int_{\hat{z}_i}^{\infty} f(Z) dZ \right) - \lambda_{1,i} + \lambda_{2,i} = 0$$

$$F_i \left(i_b - i_x (1 - F(Z < (1 - \pi_i) F_i - K)) \right) - \lambda_{1,i} + \lambda_{2,i} = 0$$

$$F_i \left(i_b - i_x P(X_i > 0) \right) - \lambda_{1,i} + \lambda_{2,i} = 0$$

together with the complementary slackness conditions

$$\lambda_{1,i}(\pi_i - \pi^{cb}) = 0$$

$$\lambda_{2,i} \pi_i = 0 .$$

Therefore, $\lambda_{1,i} > 0$ if $\pi_i = \pi^{cb}$ and $\lambda_{2,i} > 0$ if $\pi_i = 0$. Thus, a commercial bank optimally holds $\pi_i = \pi^{cb}$ if $i_b - i_x P(X_i > 0) > 0$, $\pi_i = 0$ if $i_b - i_x P(X_i > 0) < 0$, and $\pi_i \in (0, \pi^{cb})$ if $i_b - i_x P(X_i > 0) = 0$.[62] If expected profits do not change in π_i (i.e. if $i_b - i_x P(X_i > 0) = 0$), commercial bank i is indifferent between holding an additional unit of debt securities and an additional unit of reserves. Therefore, $i_b - i_x P(X_i > 0) = 0$ defines the indifference fraction $\bar{\pi}_i$.

$$i_b = i_x P(X_i(\bar{\pi}_i) > 0)$$

$$\frac{i_b}{i_x} = P(Z > (1 - \bar{\pi}_i) F_i - K)$$

$$\bar{\pi}_i = 1 - \frac{P^{-1}\left(1 - \frac{i_b}{i_x}\right) + K}{F_i}$$

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[62] Expected commercial bank profits strictly increase in π_i if $i_b - i_x P(X_i > 0) > 0$. A commercial bank is willing to hold any π^{cb} up to the point at which a marginal increase in π_i does no longer increase expected profits. Formally,

$$\frac{d\mathbb{E}(\Pi_i)}{d\pi_i} = i_b F_i - i_x \left[\int_{\hat{z}_i}^{\infty} \frac{\partial X_i}{\partial \pi_i} f(Z) dZ + X_i(\hat{z}_i) f(\hat{z}_i) \frac{\partial(\hat{z}_i)}{\partial \pi_i} \right]$$

$$\frac{d\mathbb{E}(\Pi_i)}{d\pi_i} = F_i \left(i_b - i_x P(X_i > 0) \right) .$$

where $P^{-1}(\cdot)$ is the inverse cumulative distribution function of X_i. Because we assume that the central bank is the only institution that offers central bank debt securities, we impose $\min(\bar{\pi}_i) = 0$.

$$\bar{\pi}_i = \min\left(0, 1 - \frac{P^{-1}\left(1 - \frac{i_b}{i_x}\right) + K}{F_i}\right)$$

In sum, $\pi_i = \min(\pi^{cb}, \bar{\pi}_i)$. Optimization with respect to $B_i \geq -R_i$ of

$$\mathbb{L} = i_a A_i + i_b \pi_i (R_i + B_i) - i_m B_i - i_d \mathbb{E}(D - Z) - i_x \int_{\hat{z}_i}^{\infty} X_i f(Z) dZ + \mu_i (B_i + R_i)$$

yields the following first order condition

$$i_b \min(\pi^{cb}, \bar{\pi}_i) + i_b \frac{\partial \min(\pi^{cb}, \bar{\pi}_i)}{\partial B_i} - i_m - i_x \left[\int_{\hat{z}_i}^{\infty} \frac{\partial X_i}{\partial B_i} f(Z) dZ + X_i(\hat{z}_i) f(\hat{z}_i) \frac{\partial(\hat{z}_i)}{\partial B_i} \right] + \mu_i = 0$$

$$i_m = i_b \left(\min(\pi^{cb}, \bar{\pi}_i) + \frac{\partial \min(\pi^{cb}, \bar{\pi}_i)}{\partial B_i} \right) - i_x \int_{\hat{z}_i}^{\infty} \frac{\partial X_i}{\partial B_i} f(Z) dZ + \mu_i,$$

where $X_i = Z - (1 - \pi_i)(B_i + R_i) + K$, $X_i(\hat{z}_i) = 0$, and

$$\frac{\partial X_i}{\partial B_i} = -(1 - \min(\pi^{cb}, \bar{\pi}_i)) + \frac{\partial \min(\pi^{cb}, \bar{\pi}_i)}{\partial B_i} \ .$$

Using that $\hat{z}_i = (1 - \min(\pi^{cb}, \bar{\pi}_i))(B_i + R_i) - K$ it follows that

$$i_m = i_b \left(\min(\pi^{cb}, \bar{\pi}_i) + \frac{\partial \min(\pi^{cb}, \bar{\pi}_i)}{\partial B_i} \right) + \left((1 - \min(\pi^{cb}, \bar{\pi}_i)) - \frac{\partial \min(\pi^{cb}, \bar{\pi}_i)}{\partial B_i} \right) i_x P(X_i > 0) + \mu_i \ .$$

More transparently

$$i_m = \begin{cases} i_b \pi_i + (1 - \pi_i) i_x P(X_i > 0) + \mu_i, & \text{if } \pi^{cb} < \bar{\pi}_i \\ i_b \pi_i + (1 - \pi_i) i_x P(X_i > 0) + \frac{\partial \bar{\pi}_i}{\partial B_i}(i_b - i_x P(X_i > 0)) + \mu_i, & \text{if } \pi^{cb} \geq \bar{\pi}_i \end{cases}$$

Use that if $\pi^{cb} > \bar{\pi}_i$, we have $\pi_i = \bar{\pi}_i$ which, by construction, implies $i_b - i_x P(X_i > 0) = 0$. Furthermore, the complementary slackness condition requires

$$\mu_i(B_i + R_i) = 0 \ .$$

Hence, if $B_i > -R_i$ (which is true in equilibrium)

$$i_m = i_b \pi_i + (1 - \pi_i) i_x P(X_i > 0) \ .$$

Guess the solution to be $B_i + R_i = \zeta_1 \ \forall i$ and $\pi_i = \pi \ \forall i$. Then

$$i_m = i_b \pi + (1 - \pi) i_x P(X(\zeta_1) > 0) .$$

To check if the guess is compatible with the market clearing condition, combine the (integrated) guess $\int_0^1 (B_i + R_i) \mathrm{d}i = \zeta_1$ with the market clearing condition $\int_0^1 B_i \mathrm{d}i = 0$ to obtain

$$\int_0^1 R_i \mathrm{d}i = \zeta_1 . \tag{4}$$

Furthermore, by definition, $\int_0^1 (B_i + R_i) \mathrm{d}i = \int_0^1 F_i \mathrm{d}i$. In combination with the market clearing condition $\int_0^1 B_i \mathrm{d}i = 0$, it follows that

$$\int_0^1 R_i \mathrm{d}i = F . \tag{5}$$

Combining equations 4 and 5 yields $\zeta_1 = F$. Thus, in equilibrium, the money market rate is equal to

$$i_m = i_b \pi_i + (1 - \pi) i_x P(X > 0) ,$$

where $X = Z - (1 - \pi) F + K$.

B.2   Proof of proposition 2

Optimization with respect to $B_i \geq -R_i$ of

$$\mathbb{L} = i_a A_i + i_{ior}(B_i + R_i) - i_m^{ior} B_i - i_d \mathbb{E}(D - Z) - i_x \int_{\hat{z}_i^{ior}}^{\infty} X_i^{ior} f(Z) dZ + \mu_i (B_i + R_i)$$

yields the following first order condition

$$i_{ior} - i_m^{ior} - i_x \left[\int_{\hat{z}_i^{ior}}^{\infty} \frac{\partial X_i^{ior}}{\partial B_i} f(Z) dZ + X_i^{ior}(\hat{z}_i^{ior}) f(\hat{z}_i^{ior}) \frac{\partial(\hat{z}_i^{ior})}{\partial B_i} \right] + \mu_i = 0$$

$$i_m^{ior} = i_{ior} - i_x \int_{\hat{z}_i^{ior}}^{\infty} \frac{\partial X_i^{ior}}{\partial B_i} f(Z) dZ + \mu_i ,$$

where $X_i^{ior} = Z - (B_i + R_i) + K$, and

$$\frac{\partial X_i^{ior}}{\partial B_i} = -1 .$$

Using that $\hat{z}_i^{ior} = (B_i + R_i) - K$ it follows that

$$i_m^{ior} = i_{ior} + i_x \int_{\hat{z}_i^{ior}}^{\infty} f(Z)dZ + \mu_i$$

$$i_m^{ior} = i_{ior} + i_x P(X_i^{ior} > 0) + \mu_i \ .$$

Furthermore, the complementary slackness condition requires

$$\mu_i(B_i + R_i) = 0 \ .$$

Hence, if $B_i > -R_i$ (which is true in equilibrium)

$$i_m^{ior} = i_{ior} + i_x P(X_i^{ior} > 0) \ .$$

Guess the solution to be $B_i + R_i = \zeta_2 \ \forall i$ and $\pi_i = \pi \ \forall i$. Then

$$i_m^{ior} = i_{ior} + i_x P(X^{ior}(\zeta_2) > 0) \ .$$

To check if the guess is compatible with the market clearing condition, combine the (integrated) guess $\int_0^1 (B_i + R_i)di = \zeta_2$ with the market clearing condition $\int_0^1 B_i di = 0$ to obtain

$$\int_0^1 R_i di = \zeta_2 \ . \tag{6}$$

Furthermore, by definition, $\int_0^1 (B_i + R_i)di = \int_0^1 F_i di$. In combination with the market clearing condition $\int_0^1 B_i di = 0$, it follows that

$$\int_0^1 R_i di = F \ . \tag{7}$$

Combining equation 6 and 7 yields $\zeta_2 = F$. Thus, in equilibrium, the money market rate is equal to

$$i_m^{ior} = i_{ior} + i_x P(X^{ior} > 0) \ ,$$

where $X^{ior} = Z - F + K$.

C Further insights from the money market model

C.1 The maximum fraction of F invested in debt securities

By construction, $\bar{\pi}_i$ is the fraction of pre-auction reserves that makes a commercial bank indifferent between holding an additional unit of debt securities and an additional unit of reserves.

Proposition 3 *The maximum fraction of F_i invested in debt securities is* $\bar{\pi}_i = \max\left(0, 1 - \frac{P^{-1}\left(1 - \frac{i_b}{i_x}\right) + K}{F_i}\right)$.

Proof. Solve $\frac{d\mathbb{E}(\Pi_i)}{d\pi_i} = 0 \Leftrightarrow i_b - i_x P(X_i > 0) = 0$ for $\bar{\pi}_i$ and assume that the central bank is the only institution that offers central bank debt securities (i.e. impose $\min(\bar{\pi}_i) = 0$).

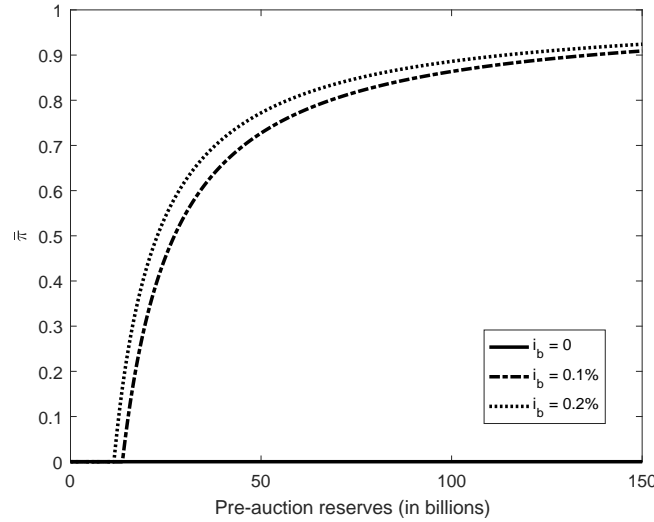Figure C.1 — The maximum fraction of F_i invested in debt securities ($\bar{\pi}_i$)

Figure C.1 presents $\bar{\pi} = \bar{\pi}_i \ \forall i$ conditional on i_b and F. A tighter money market (a lower F) leads to lower desired debt security holdings (lower $\bar{\pi}$) for two reasons. First, the probability of a liquidity shortage rises in debt security investments. Second, liquidity shortages are costly if the discount rate is higher than the yield on debt securities ($\frac{i_b}{i_x} < 1$). At the margin it does not pay off for commercial banks to hold an extra unit of debt securities if this triggers, in expectation, a liquidity shortage costing $i_x > i_b$.

If pre-auction reserves F become sufficiently scarce, commercial banks abstain from investing in debt securities ($\lim_{F_i \to 0} \bar{\pi}_i = 0$). In contrast, if pre-auction reserves go to infinity, commercial banks are willing to hold any fraction of reserves in debt securities ($\lim_{F_i \to \infty} \bar{\pi}_i = 1$).

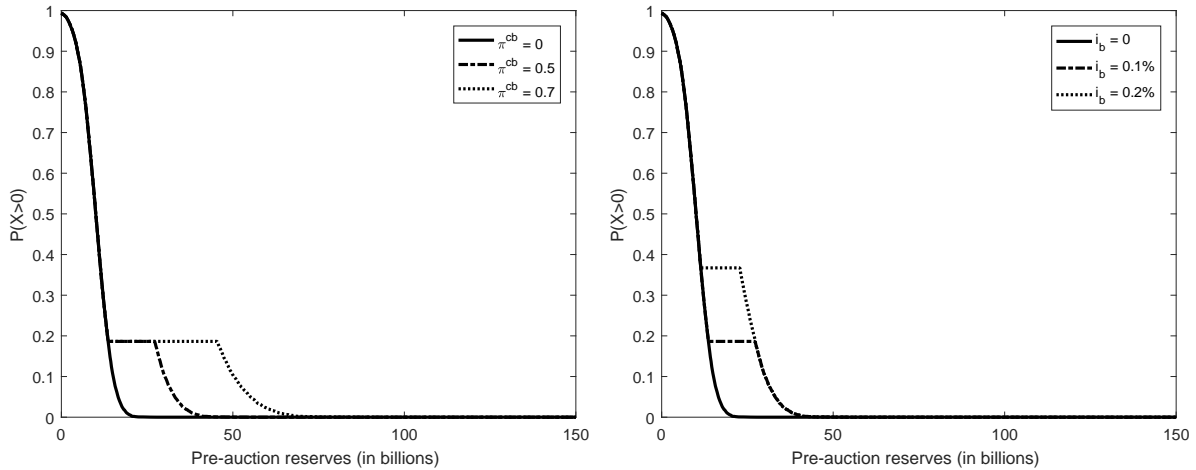## C.2 The probability of a liquidity shortage

Proposition 4 *The probability of a liquidity shortage weakly decreases in pre-auction reserves. Formally,*
$\frac{\partial P(X_i > 0)}{\partial F_i} = \mathbb{I}_{\pi_i < \bar{\pi}_i}\left(-\phi\left((1 - \pi_i)F_i - K\right)\left[1 - \pi_i\right]\right) < 0$, *where $\phi((1 - \pi_i)F_i - K) > 0$ is the probability*

density function of Z evaluated at $(1 - \pi_i)F_i - K$ and $\mathbb{I}_{\pi_i < \bar{\pi}_i}$ is an indicator function taking on the value 1 if $\pi_i < \bar{\pi}_i$ and zero otherwise.

Proof. Take the partial derivative of $P(X_i > 0) = 1 - F_i(Z < \hat{z}_i)$ with respect to F_i and use $\hat{z}_i = (1 - \pi_i)F_i - K$.

Figure C.2 — The probability of a liquidity shortage

Notes: Conditional on $i_b = 0.10\%$ (left panel) and $\pi^{cb} = 0.5$ (right panel).

The left panel in Figure C.2 displays the probability of a liquidity shortage, $P(X_i > 0)$, as a function of pre-auction reserves, F_i, conditional on $i_b = 0.10\%$. $P(X_i > 0)$ is constant in F_i if the commercial bank holds its desired amount of post-auction reserves L_i (because $\bar{\pi}_i$ adjusts to changes in F_i to ensure $(1 - \bar{\pi}_i)F_i = L_i$). In contrast, if $(1 - \pi_i)F_i > L_i$, a decrease in F_i increases the probability of a liquidity shortage.

The right panel in Figure C.2 plots $P(X_i > 0)$ as a function of pre-auction reserves conditional on $\pi = 0.50$. The probability of a liquidity shortage increases in the yield of the debt securities because in relative terms, liquidity shortages become less costly in i_b.

D   Identification of dynamic causal effects through heteroscedasticity

This section derives a dynamic event study estimator. The estimator is closely related to the heteroscedasticity-based identification schemes by Rigobon (2003), Rigobon and Sack (2004), Lütkepohl (2012), Lütkepohl et al. (2018), and Nakamura and Steinsson (2018a).

D.1 Identification through heteroscedasticity

Suppose that the data are generated by an N-variable structural VAR(1):[63]

$$By_t = \Gamma y_{t-1} + e_t, \quad E[e_t e_t'] = D,$$

where e_t denotes an N-dimensional vector of i.i.d. structural shocks. The reduced-form representation of the VAR reads:

$$y_t = \Phi y_{t-1} + \Psi e_t .$$

where $\Phi = B^{-1}\Gamma$ is a matrix of autoregressive coefficients and $\Psi \equiv B^{-1}$ measures the immediate impact of the i.i.d. structural shocks e_t on y_t. The identification problem manifests itself in the fact that we cannot estimate Ψ from the reduced-form representation of the VAR. However, we can estimate Φ and the one-step-ahead forecast errors (residuals):

$$\varepsilon_{t|t-1} = \Psi e_t .$$

Heteroscedasiticity-based identification schemes assume that the variance of some of the shocks increases only in some periods, while other parameters of the model remain unchanged. In our setting we assume:

Assumption 1 *Structural shock 1 (e_{1t}) affects y_t only in particular periods ("events"). The timing is deterministic and known to the econometrician.*

Assumption 2 *All other parameters are constant.*

Under these assumptions, the residuals are a linear combination of structural shocks, where shock e_{1t} appears only if an event occurs

$$\varepsilon_{t|t-1} = \begin{bmatrix} \mathbb{I}_{(t \in E)}\psi_{11}e_{1t} + \psi_{12}e_{2t} + \cdots + \psi_{1N}e_{Nt} \\ \mathbb{I}_{(t \in E)}\psi_{21}e_{1t} + \psi_{22}e_{2t} + \cdots + \psi_{2N}e_{Nt} \\ \cdots \\ \mathbb{I}_{(t \in E)}\psi_{N1}e_{1t} + \psi_{N2}e_{2t} + \cdots + \psi_{NN}e_{Nt} \end{bmatrix},$$

where we define $\mathbb{I}_{(t \in E)}$ as an indicator function that equals one if the event occurs in period t and zero otherwise. Thus, the variance of the residuals differs for periods with an event and without an

---

[63]We ignore deterministic terms for simplicity.

event. Let the variance-covariance matrix of the residuals if no event occurs be:

$$
\Omega_{t \notin E} = \begin{bmatrix} \omega_{11} & \omega_{12} & \dots & \omega_{1N} \\ & \ddots & & \vdots \\ & & & \omega_{NN} \end{bmatrix}.
$$

Then, the variance-covariance matrix of the residuals if an event occurs then amounts to:

$$
\Omega_{t \in E} = \begin{bmatrix} \psi_{11}^2 \sigma_1^2 + \omega_{11} & \psi_{11}\psi_{21}\sigma_1^2 + \omega_{12} & \dots & \psi_{11}\psi_{N1}\sigma_1^2 + \omega_{1N} \\ & \ddots & & \vdots \\ & & & \psi_{N1}^2 \sigma_1^2 + \omega_{NN} \end{bmatrix}.
$$

It follows that the difference between the two covariance matrices depends only on the impulse responses to shock 1, and the variance of the structural shock.

$$
\tilde{\Omega} = \Omega_{t \in E} - \Omega_{t \notin E} = \begin{bmatrix} \psi_{11}^2 \sigma_1^2 & \psi_{11}\psi_{21}\sigma_1^2 & \dots & \psi_{11}\psi_{N1}\sigma_1^2 \\ & \ddots & & \vdots \\ & & & \psi_{N1}^2 \sigma_1^2 \end{bmatrix}
$$

At this stage, it is useful to discuss whether we can identify the parameters. First, we cannot identify the sign of the responses without further assumptions. If we multiply all ψ_{ij} by -1 we obtain the same expression for $\tilde{\Omega}$. Second, we cannot identify the scale of the impulse responses. If we multiply all impulse responses by a factor γ and divide the standard deviation of the structural shocks by γ we obtain the same expression for $\tilde{\Omega}$. Therefore, we need to impose additional identifying assumptions.

Assumption 3 *The variance of the structural shocks is unity.*

This is a standard assumption in structural VAR analysis and reflects that we cannot distinguish the response to the structural shock from the standard deviation of the structural shock.

Assumption 4 *The sign of one particular response is known.*

Without loss of generality, let us assume that we know that the response of variable 1 is positive. We can estimate the responses from the first row of $\tilde{\Omega}$ as follows:[64]

$$
\begin{aligned}
\psi_{11} &= \sqrt{\tilde{\Omega}_{11}} \\
\psi_{N1} &= \tilde{\Omega}_{N1}/\psi_{11},
\end{aligned}
$$

where $\tilde{\Omega}_{ij}$ denotes the row i, column j element of $\tilde{\Omega}$. From these expressions we see that we can

---

[64]If it is negative, we would multiply the response by -1.

compute the responses of all other variables only if $\tilde{\Omega}_{11} > 0$. Otherwise, the responses do not exist. Therefore, we formally need another identifying assumption:

Assumption 5 *Structural shock 1 (e_{1t}) affects y_t ($\tilde{\Omega}_{11} > 0$).*

Three remarks are in order. First, with our additional identifying assumptions, the model is overidentified and we could set up a more efficient GMM estimator and test the overidentifying restrictions (see Rigobon and Sack 2004). Second, we see that we may run into a "weak instrument" problem. If ψ_{11} is close to zero we overestimate ψ_{N1}. This is related to the identifying assumption that there is an impact of structural shock 1 on variable 1 when the event occurs. If $\tilde{\Omega}_{11}$ is zero ψ_{N1} we cannot compute ψ_{N1} because the identifying assumption is violated. Third, this resembles the estimator used by Rigobon (2003) and Nakamura and Steinsson (2018a). The main difference is that we do not associate the response with a particular policy variable for which the response is normalized to unity. Instead, we impose a sign restriction on one of the variables which allows us to estimate the impulse response to a one standard deviation structural shock for all variables in the system.

The estimator by Nakamura and Steinsson (2018a) is particularly useful when we want to restrict the response of one variable to a specific value. For example, we may be interested in the impact of an increase in the money market rate by 0.25 percentage points. Let us choose the variance of the structural shock so that the response of variable 1 corresponds to the desired value, say c. Then,

$$\psi_{11} = \sqrt{\frac{\tilde{\Omega}_{11}}{\sigma_1^2}} = c .$$

This expression determines the variance of the structural shock needed to obtain the desired response. In addition, note that

$$\psi_{21} = \frac{\tilde{\Omega}_{12}}{\psi_{11}\sigma_1^2} .$$

Replacing the response of variable 1 by c and the variance of the structural shock by $\sigma_1^2 = \tilde{\Omega}_{11}/c^2$ yields

$$\psi_{21} = \frac{\tilde{\Omega}_{12}}{\tilde{\Omega}_{11}}c ,$$

which is exactly the estimator by Nakamura and Steinsson (2018a) when setting $c = 1$.

D.2 Impulse responses using a VAR

It is straightforward to recursively compute the impulse responses using the VAR parameters. Let $\Psi_0 = [\psi_{11} \ \psi_{21} \ \ldots \ \psi_{N1}]'$ be the impact of structural shock 1 on all variables in the system. For a

VAR(1) we can compute the dynamic impact after h periods as:

$$\Psi_h = \Phi^h \Psi_0 \, .$$

To estimate the dynamic impact for a VAR(p), we can use the recursion:

$$
\begin{aligned}
\Psi_1 &= \Phi_1 \Psi_0 \\
\Psi_2 &= \Phi_1 \Psi_1 + \Phi_2 \Psi_0 \\
&\vdots \\
\Psi_h &= \Phi_1 \Psi_{h-1} + \Phi_2 \Psi_{h-2} + \dots \Phi_p \Psi_{h-p} \quad \text{for } h \geq p \, .
\end{aligned}
$$

D.3   Impulse responses using local projections

Estimating impulse responses using a VAR is more efficient relative to local projections (Stock and Watson 2018). Local projections are more robust, however, if the model is misspecified (Jordà 2005). We therefore propose to alternatively estimate the impulse responses using local projections. Besides larger standard errors, the local projection approach comes at the cost that we have to restrict the sign of one response at all horizons.

To keep the discussion as simple as possible, we still assume that the data are generated by a VAR(1) but we want to estimate the impulse response using local projections. Let Ψ_h denote the impulse response after h periods. We can iterate the VAR backwards one period to obtain:

$$
\begin{aligned}
y_t &= \Phi(\Phi y_{t-2} + \Psi_0 e_{t-1}) + \Psi_0 e_t \\
y_t &= \Phi^2 y_{t-2} + \Psi_1 e_{t-1} + \Psi_0 e_t \, ,
\end{aligned}
$$

where $\Psi_1 \equiv \Phi \Psi_0$ denotes the impulse response after one period. We exploit that the change in the direct forecast (local projection) of an econometrician when one additional observation becomes available differs between periods with an without an event.

$$
\begin{aligned}
\mathbb{E}[y_t|y_{t-1}] - \mathbb{E}[y_t|y_{t-2}] &= \mathbb{E}[y_t|y_{t-1}] - y_t - \mathbb{E}[y_t|y_{t-2}] + y_t \\
&= \varepsilon_{t|t-2} - \varepsilon_{t|t-1} \, .
\end{aligned}
$$

We exploit that the difference between the two-step-ahead and one-step-ahead forecast errors equals the period t impact of the structural shocks from period $t-1$:

$$\varepsilon_{t|t-2} - \varepsilon_{t|t-1} = \Psi_1 e_{t-1} \, .$$

Let $\Omega_1 \equiv V[\varepsilon_{t|t-2} - \varepsilon_{t|t-1}]$. We can base our estimator of the dynamic response on the following covariance matrices:

$$\tilde{\Omega}_1 = \Omega_{1,t-1\in E} - \Omega_{1,t-1\notin E} \,,$$

and estimate the dynamic response by:

$$\begin{aligned}
\psi_{1,11} &= \sqrt{\tilde{\Omega}_{1,11}} \\
\psi_{1,N1} &= \tilde{\Omega}_{1,N1}/\psi_{1,11} \,,
\end{aligned}$$

where $\tilde{\Omega}_{1,ij}$ is the row i column j element of $\tilde{\Omega}_1$.

More generally, to estimate the response after h periods we can iterate the VAR backwards to obtain:

$$y_t = \Phi^{h+1}y_{t-h-1} + \Psi_0 e_t + \Psi_1 e_{t-1} + \Psi_2 e_{t-2} + \cdots + \Psi_h e_{t-h} \,,$$

and

$$\varepsilon_{t|t-h-1} - \varepsilon_{t|t-h} = \Psi_h e_{t-h} \,.$$

Then we can estimate the impulse response using

$$\tilde{\Omega}_h = \Omega_{h,t-h\in E} - \Omega_{h,t-h\notin E} \,,$$

and

$$\begin{aligned}
\psi_{h,11} &= \sqrt{\tilde{\Omega}_{h,11}} \\
\psi_{h,N1} &= \tilde{\Omega}_{h,N1}/\psi_{h,11} \,.
\end{aligned}$$

We estimate the response after h periods comparing the variance of the forecast revision in period $t - h$ if an event occurred h periods ago with the variance if no event occurred h periods ago. However, we require a sign restriction at every horizon. For many macroeconomic and financial market variables, restricting the sign of the response at every horizon does not seem sensible. It may be more reasonable to restrict the sign of the cumulative response if we expect the shock to have a large immediate impact which fades away slowly. For example, changes in asset prices may respond immediately but the response may be zero at longer horizons. Therefore, it may be reasonable to restrict the cumulative response of asset price returns.

To identify the cumulative response, we can add lagged y_t to the left-hand-side of the local projection (similar as Stock and Watson 2018). We restrict the discussion to $h = 1$, but for longer horizons the

derivations are similar. First, note that the cumulative response amounts to:

$$\Psi_0 + \Psi_1 = (I + \Phi)\Psi_0 \, ,$$

where I is a conformable identity matrix. If we add y_{t-1} on both sides of the equation of the VAR iterated one period backwards we obtain:

$$y_t + y_{t-1} = \Phi^2 y_{t-2} + \Psi_1 e_{t-1} + \Psi_0 e_t + y_{t-1} \, .$$

Replacing y_{t-1} with the $\Phi y_{t-2} + \Psi_0 e_{t-1}$ yields

$$
\begin{aligned}
y_t + y_{t-1} &= \Phi^2 y_{t-2} + \Psi_1 e_{t-1} + \Psi_0 e_t + \Phi y_{t-2} + \Psi_0 e_{t-1} \\
&= (\Phi + \Phi^2) y_{t-2} + (\Psi_0 + \Psi_1) e_{t-1} + \Psi_0 e_t \, .
\end{aligned}
$$

The two-step-ahead forecast error comprises a linear combination of past (period $t-1$) and current (period t) shocks. We can remove period t shocks by subtracting the one-step-ahead forecast error.

$$\varepsilon_{t|t-2} - \varepsilon_{t|t-1} = (\Psi_0 + \Psi_1) e_{t-1}$$

and proceed as before to estimate the response.

More generally, let $\varepsilon_{t|t-h-1}$ be the $h-1$ step-ahead forecast error in

$$\sum_{j=0}^{h} y_{t-j} = \Phi^{h+1} y_{t-h-1} + \varepsilon_{t|t-h-1} \, .$$

It follows that

$$\varepsilon_{t|t-h-1} - \varepsilon_{t|t-h} = \left(\sum_{j=0}^{h} \Psi_h \right) e_{t-h} \, .$$

Let $\Omega_h = V[\varepsilon_{t|t-h-1} - \varepsilon_{t|t-h}]$. We can then estimate the cumulative response using the same estimator as before.

D.4 Exogenous variables

It is straightforward to include exogenous variables. If the data are generated by a VARX(1) we have

$$y_t = \Theta_0 x_t + \Theta_1 x_{t-1} + \Phi y_{t-1} + \Psi e_t \, .$$

For estimating the immediate impact, we only have to include the concurrent and lagged exogenous variables when estimating the residuals and proceed as before. Estimating the dynamic impact using

the VAR approach then immediately follows. It turns out that we can also use the same approach when estimating the responses with local projections by including exogenous variables up to lag $h-1$ when estimating $\varepsilon_{t|t-h}$. The reason is that the impact of overlapping exogenous variables cancels out when calculating the difference $\varepsilon_{t|t-h-1} - \varepsilon_{t|t-h}$.

D.5 Simulations

To show under which circumstances the identification scheme works and illustrate potential pitfalls, we first simulate 250,000 observations from a VAR(2):

$$y_t = B^{-1}\Gamma_1 y_{t-1} + B^{-1}\Gamma_2 y_{t-2} + B^{-1} e_t ,$$

with

$$B^{-1} = \begin{bmatrix} 0.7 & 0.5 & 0.3 \\ 0.8 & -0.9 & 0.1 \\ 0.5 & 0.4 & 0.2 \end{bmatrix} \quad \Gamma_1 = \begin{bmatrix} 0.3 & 0.7 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.4 & 0.5 & -0.4 \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} 0.1 & 0.2 & 0.05 \\ 0.2 & 0.3 & 0.1 \\ 0.2 & 0.3 & 0.1 \end{bmatrix} \quad E[e_t e_t'] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

When simulating the data we impose that shock 1 happens every 5th period while the other shocks occur every period. To estimate the impact of shock 1 we restrict the response of the first variable to be positive.
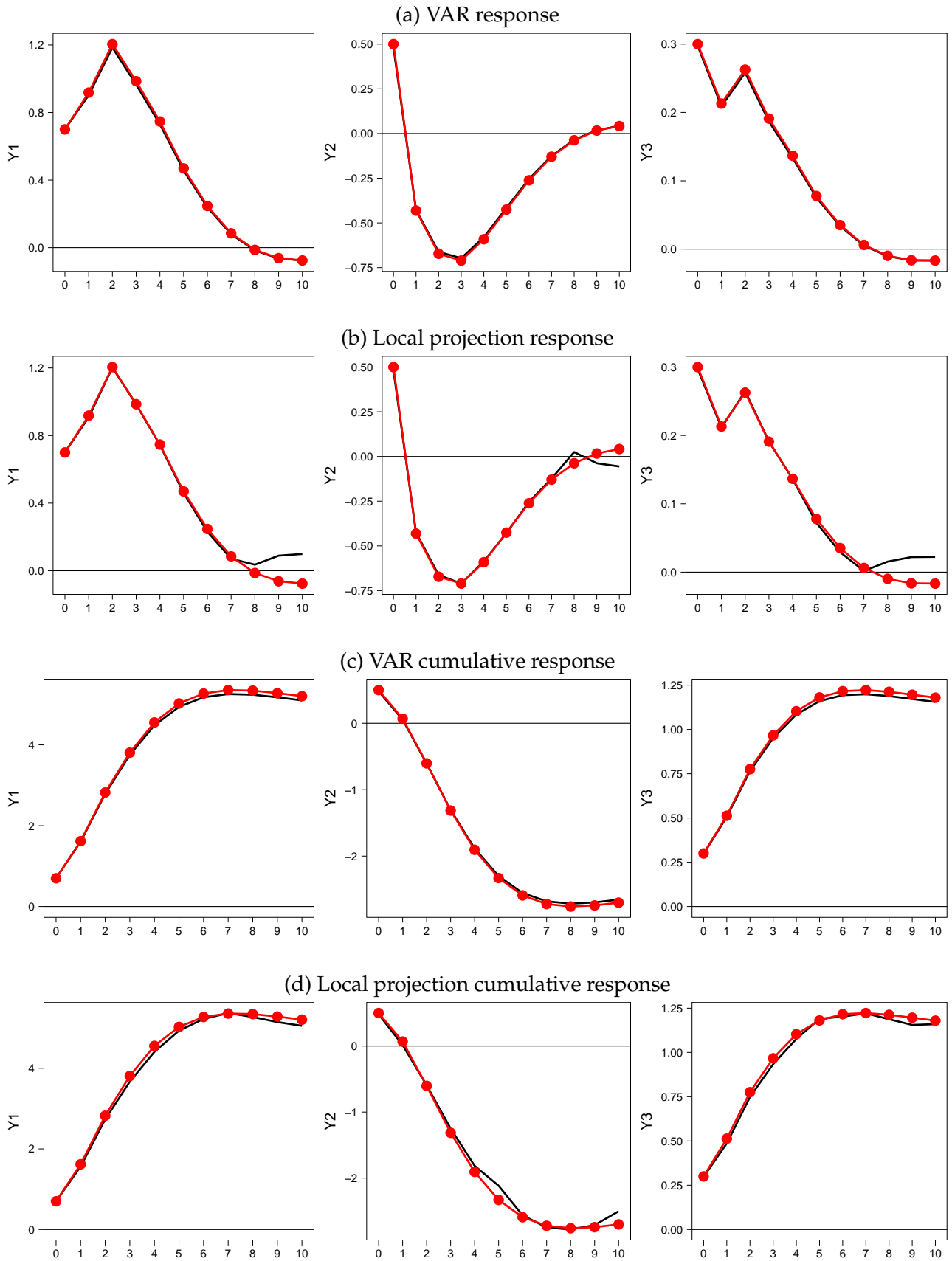
The black lines in Figure D.1 show the estimates of the impulse response functions.[65] The red lines are the actual impulse response functions. Not surprisingly, the VAR approach identifies the responses (panels a and c).

Panels (b) and (d) show the estimates using local projections. In panel (b) the identifying assumption requires that the response of variable 1 is always positive. We have an identification problem when the red line crosses the zero line. Because of our identification scheme, the response of variable 1 has to remain positive, leading to a wrong sign in the responses of all other variables. Panel (d) shows that that we avoid this problem when identifying the response by restricting the cumulative response. Because the cumulative response of variable 1 is always positive, we also identify the responses of all other variables.

Next, we illustrate the empirical implementation of our two approaches by simulating a smaller number of observations (2,000) and estimate confidence intervals using a moving block bootstrap approach (see Efron and Tibshirani 1993, section 8.6). We set the block size to 80 and draw 5,000 bootstrap samples.
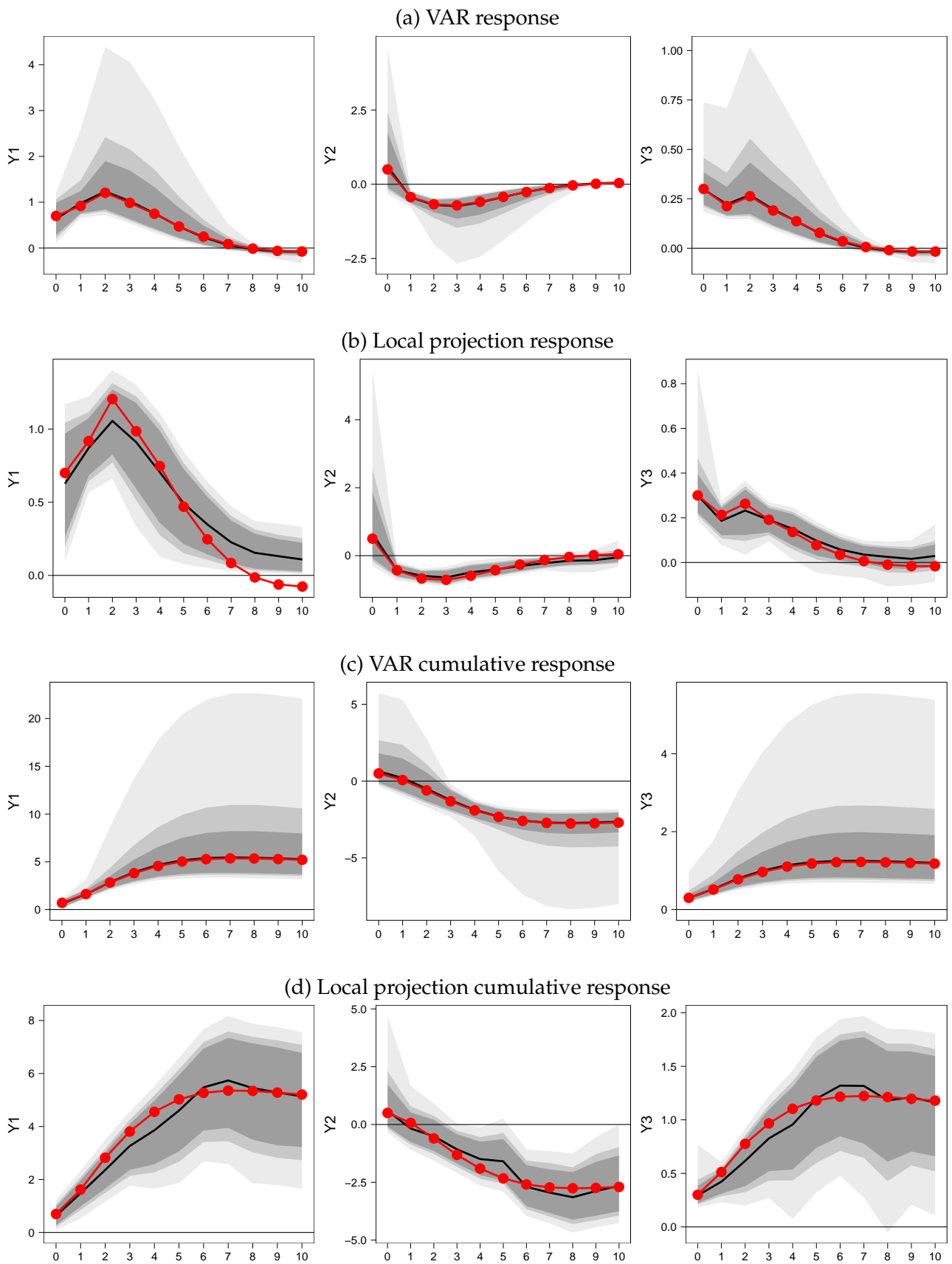
---

[65]We scaled the impulse response using the inverse standard deviation of structural shock 1 so that it matches the actual impulse response. In practice, as noted before, we are not able to separately identify the scale of the impulse response and the standard deviation of the structural shock.

Figure D.1 — Estimates based on 250,000 observations

(a) VAR response

(b) Local projection response

(c) VAR cumulative response

(d) Local projection cumulative response

Notes: Actual response (red line with dots) and estimated response (black line).

Figure D.2 — Estimates based on 2,000 observations

(a) VAR response

(b) Local projection response

(c) VAR cumulative response

(d) Local projection cumulative response

Notes: Actual response (red line with dots) and estimated response (black line). The shaded areas give 99%, 95%, and 90% confidence intervals based on a block bootstrap algorithm.

Figure D.2 shows that the VAR approach is more efficient and identifies the (cumulative) responses. The local projection approach does not identify the non-cumulative responses. Moreover, the local projection estimator yields imprecise cumulative responses. Therefore, we face the following trade-off in practice. Either we choose a particular model to estimate the dynamic effects. These estimates will be precise but biased if the VAR is misspecified. Or, we use local projections which are more robust with respect to model misspecification. This comes at the cost, however, that these responses are less precisely estimated and we have to impose a plausible sign restriction at every horizon.

We also perform a Monte Carlo exercise to examine the properties of a test for one of the identifying assumptions ($\tilde{\Omega}_{11} > 0$). We simulate 500 samples with 2,000 observations from the VAR and estimate the impulse responses in each sample with the block bootstrap algorithm. Then, we test in each sample whether $\tilde{\Omega}_{11} = 0$ against the one-sided alternative that $\tilde{\Omega}_{11} > 0$ at the 10% level. Finally, we compute the share of Monte Carlo samples where we reject the null hypothesis. We would expect to reject the null hypothesis in 90% of all cases because we know that the identifying assumption holds in population. Indeed, we find a rejection rate of 91%.
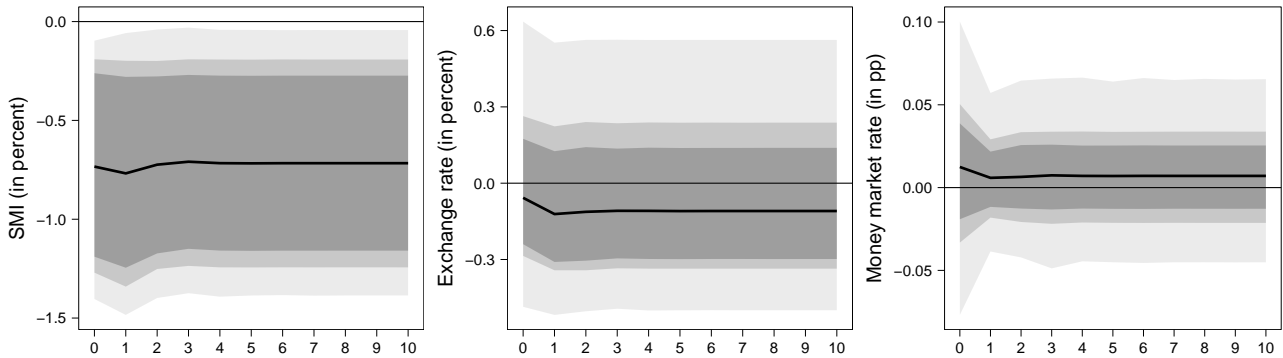
E Robustness and placebo tests

Figure E.1 — Placebo tests
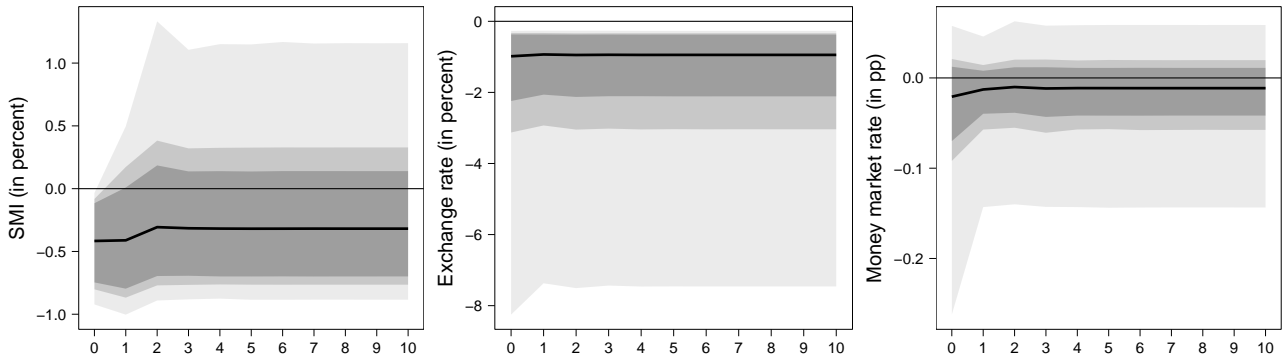
(a) VAR for payment day
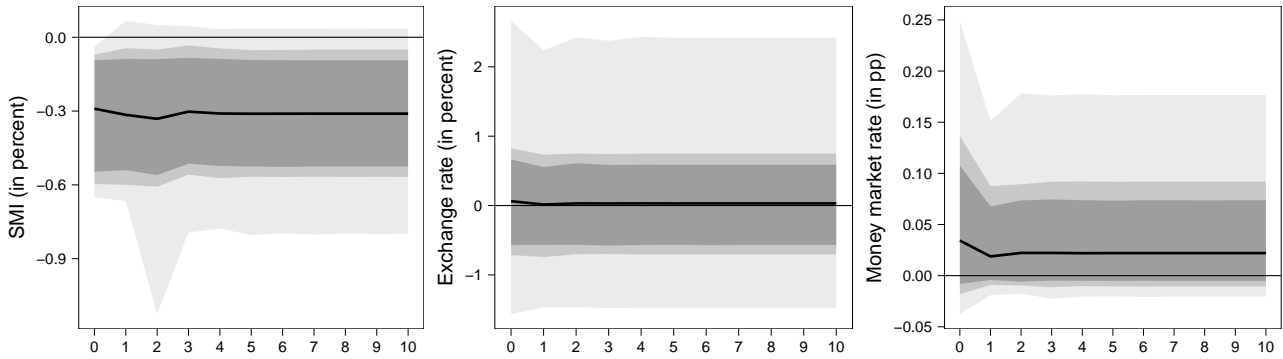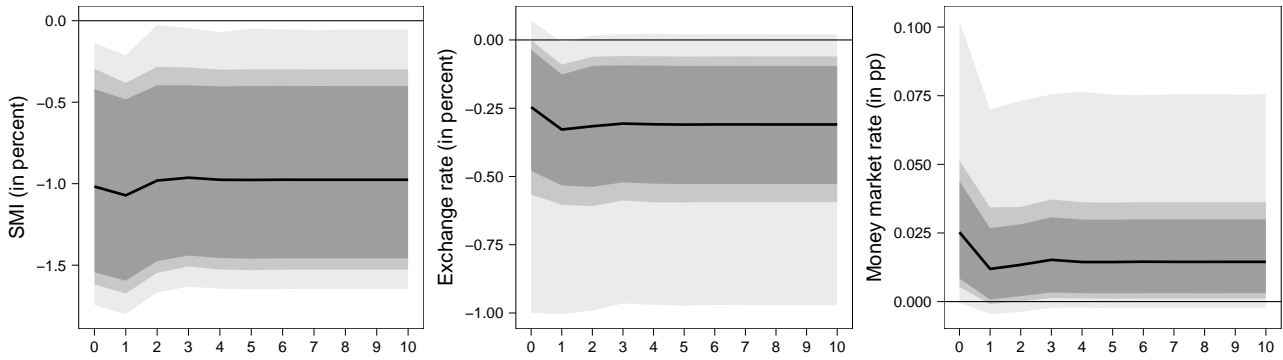

(b) VAR for USD Bill auctions
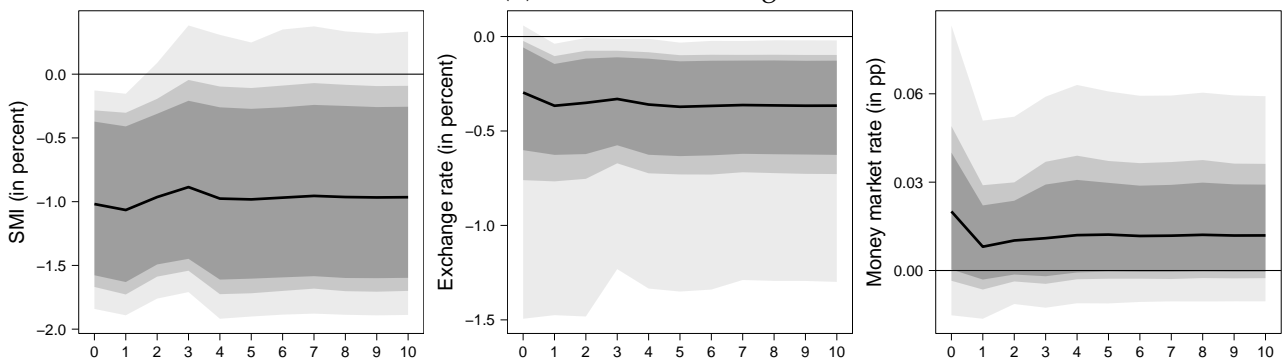

(c) VAR for treasury bill auctions

Notes: Estimated response for up to 10 business days to a 1 std. restrictive SNB Bill shock in black. The response of stock market prices and the exchange rate are measured in percent, the response of interest rates in percentage points. The shaded areas give 99%, 95%, and 90% confidence intervals based on a block bootstrap algorithm.
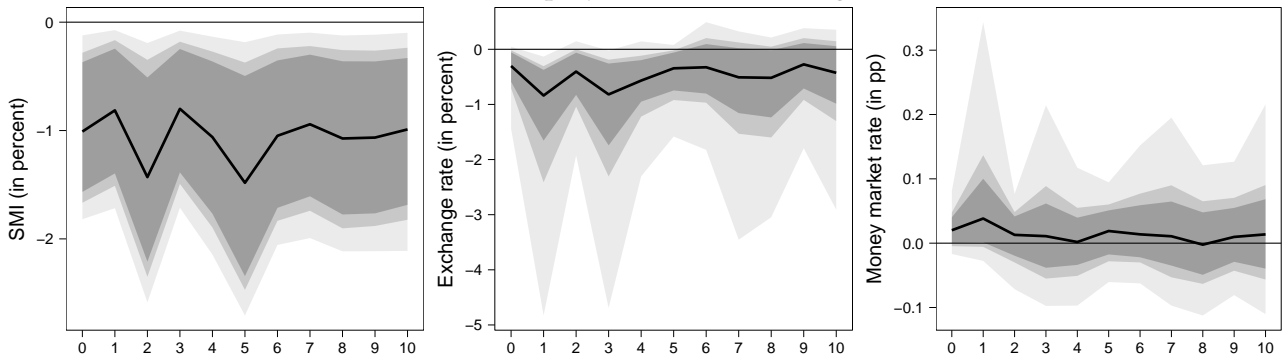
Figure E.2 — Robustness tests
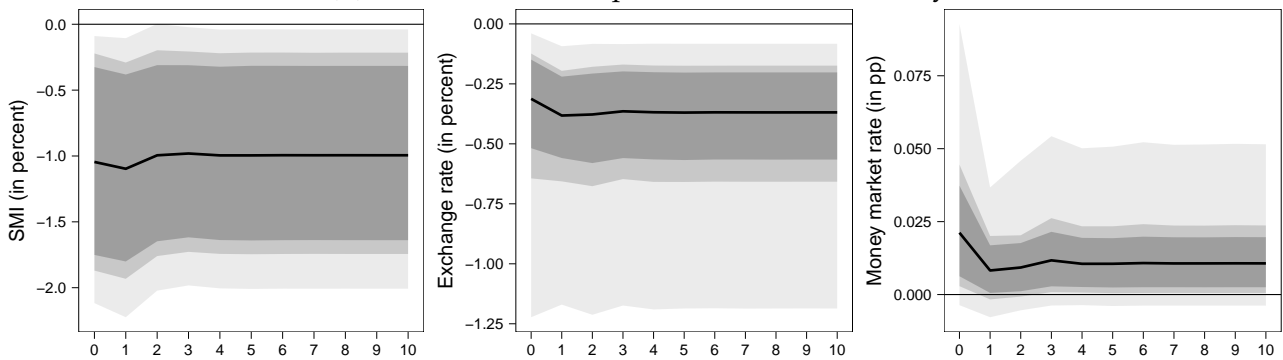
(a) VAR with exogenous variables

(b) VAR with four lags

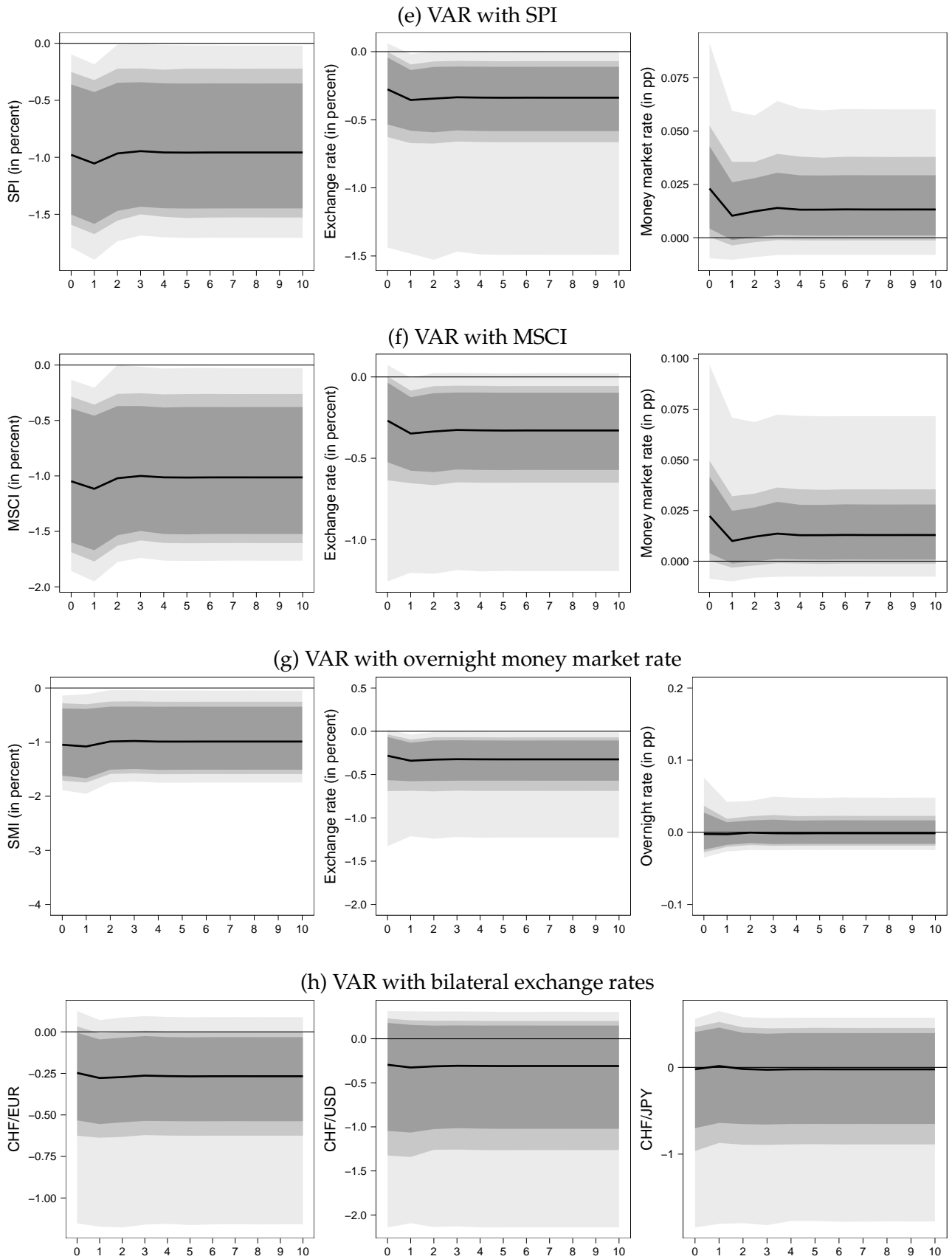### (c) Local projection with four lags

(d) VAR with bootstrap block of 40 business days

Notes: Estimated response for up to 10 business days to a 1 std. restrictive SNB Bill shock in black. The response of stock market prices and the exchange rate are measured in percent, the response of interest rates in percentage points. The shaded areas give 99%, 95%, and 90% confidence intervals based on a block bootstrap algorithm.
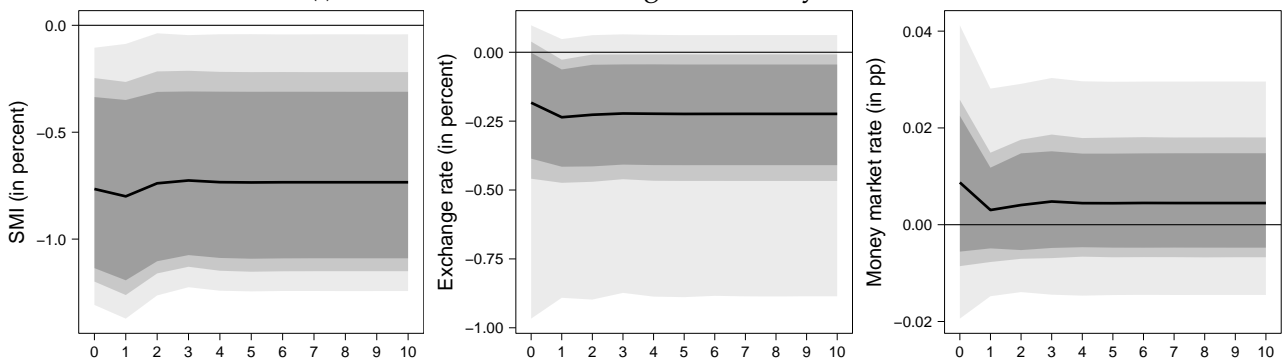
Figure E.2 — Robustness tests (contd.)

(e) VAR with SPI

(f) VAR with MSCI

(g) VAR with overnight money market rate

(h) VAR with bilateral exchange rates

Notes: Estimated response for up to 10 business days to a 1 std. restrictive SNB Bill shock in black. The response of stock market prices and the exchange rate are measured in percent, the response of interest rates in percentage points. The shaded areas give 99%, 95%, and 90% confidence intervals based on a block bootstrap algorithm.

Figure E.2 — Robustness tests (contd.)

(i) VAR without controlling for treasury bill auctions

Notes: Estimated response for up to 10 business days to a 1 std. restrictive SNB Bill shock in black. The response of stock market prices and the exchange rate are measured in percent, the response of interest rates in percentage points. The shaded areas give 99%, 95%, and 90% confidence intervals based on a block bootstrap algorithm.